Endogenous Leader-Follower Structure of Privatization and Corporate Social Responsibility

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Abstract

This study investigates strategic relationships of corporate social responsibility acts and privatization policies and an endogenous timing structure of them. The result is that they can be either strategic complement or strategic substitute, depending on production efficiency of the firms and forms of CSR. Accompanying it, privatization-leader-CSR-follower structure is supported in the equilibrium as well as simultaneous structure. The results depends on parameters and specification, so policy makers have to check the market situation carefully for desired goals, especially in markets of low marginal costs.

JEL classification L33, M14, L21.

Keywords Endogenous timing, partial privatization, corporate social responsibility.

1 Introduction

Corporate Social Responsibility, henceforce CSR, prevails nowadays. According to KPMG Survey of Corporate Responsibility Reporting 2017, 93% of the world's largest 250 companies and 75% of the whole sample of 4900 companies issue Corporate Responsibility reports. The numbers were 39% and 12% in 1999 respectively. As the numbers show, private firms, which are conventionally modeled as pure profit maximizers, pay attention to social factors which directly has no positive effects on profits. This fact suggests CSR has indirect but positive effects on profits. José Luis Blasco, Global Head of KPMG Sustainability Service, points out in the report,"There was a time when corporate responsibility information was considered strictly "non-financial" and not relevant to include in annual financial reports.$\cdots$But times are changing".

Many papers try to figure out the channels of profit increase through CSR. Those papers include reputation, risk hedging, collusion, incomplete contract and so on[1].

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[1]See, for examples, Crifo and Forget (2015), Lambertini and Tampieri (2012), and papers cited therein.

Among them, strategic commitment usage of CSR has recently attracted researchers' attention in the field of oligopoly theory[2]. The basic rationale to adopt CSR in the literature is by doing so firms can commit to be aggressive and it helps rule out competitors from markets[3]. With natural language, when firms employ CSR, they try to serve consumers with lower prices by producing more than they do without CSR, and this extra amounts of production reduce competitors' residual demands. Hence one of the proposed channels is with CSR or by taking social factors into account firms can obtain larger market shares.

In contrast to private firms becoming concerned with social circumstances, state-owned enterprises have experienced so called privatization waves since 1980's, slightly before the CSR wave's arrival. The industries involved include but are not limited to, transportations, postal services, automobile manufactures, airports, financial institutions, oil companies telecommunications. To that emergence of privatized public enterprises, a number of economics studies respond like in cases of CSR. The one pioneering work is Matsumura (1998), which demonstrates, under fairly general quantity competition setting, full-nationalization is always suboptimal and hence privatization is required from a welfare perspective. The mechanism uncovered therein is that owing to privatization, the public firm reduces its production and it causes private production to increase through strategic substitute relationships. It is the private increment that improves welfare through lower prices and higher production efficiency. Following Matsumura, various papers apply and test this logic in situations such as spatial competition, free entry markets, endogenous timing and competition structure, and shadow cost of public funds and multi-market or multi-product environments, etc.

While economists piled up studies on CSR and privatization, they treat them independently until recently. However, since both CSR and privatization are now widespread, the overlaps of the two in single markets are also observed in reality. In French automobile market, for instance, partially privatized Renault compets against private PSA. Deutsche Telekom against Vodafone in German telecom market, and Japan Post against Yamato in Japanese postal service market are other examples. All of these real markets contain partially privatized state-owned enterprises and private CSR conscious firms. Besides, from theoretical perspectives, private CSR and privatization clearly share mirror images with each other. That is, profit maximizers take social matters into consideration by CSR and social welfare maximizers put substantive weights on own profits by privatization. These real and theoretical perspectives suggest the necessity of unified models to investigate interactive effects of CSR and privatization.

In this study, I present the unified model, which consists of two stages where in the first stage of two-stage game, private firms choose its degree of CSR commitment and a public firm choose its degree of privatization, then in the second stage they com-

[2] Baron (2001), Kopel and Brand (2012), Matsumura and Ogawa (2014) and so forth.

[3] Of course this is valid in submodular games e.g. Cournot competition games. In supermodular games, variants of standard CSR, such as Environmental CSR play the role of commitment device being less aggressive, in contrast. See, for example, Hirose et al. (2017).

pete in quantities. Using this model, I characterize strategic relationships of CSR and privatization. The result is that they can be either strategic complement or strategic substitute, depending on production efficiency of the firms and forms of CSR. Thereafter, the study introduces an endogenous timing choice *à la* Hamilton and Slutsky (1990) into the first stage. As a result, privatization-leader-CSR-follower structure is supported in the equilibrium as well as simultaneous structure. While I abstract CSR by consumer surplus up to this part, I test a social welfare version of CSR whether it can support the same results. It turns out that the social welfare formulation derives the simultaneous choice structure only.

As described, the results are mixed and depend on parameter values and specifications. This is because changing each degree of commitment causes two effects, namely, first and second order effects. The first order effect denotes an effect on the firm which changes the intensity of commitment, whereas the second order effect an effect on the competing firm. For example, an increase of CSR makes private firms more aggressive in the production stage as repeatedly pointed out in the literature. This is the first order effect of incremental CSR. At the same time, it induces competing public firms to be less aggressive through strategic interaction. This is, I call, the second order effect. In the literature, the mutual strategic substitute relationship of CSR and privatization is already shown under some specifications. That is, the best response of public firms to the incremental CSR is nationalizing rather than privatizing, and vice versa. In other words, the public firms try to be more aggressive if the private firms become more aggressive too. This is because the second order effects of incremental CSR, decreasing public production, relatively dominate the first order effects, increasing private one. However, the strategic complement relationships can also hold in this paper, as mentioned above, this is because of the more generalized functional form than in the literature. Indeed, the results of strategic relationships of CSR and privatization depend on relative sizes of the two effects. The relative sizes in turn depend on marginal cost functions, or specific forms of CSR of private firms and so forth. Hence, policy makers have to check carefully the market situation they face in order to, say, foresee the effect of privatization.

The new findings of the strategic relationships yield a new consequence of endogenous timing game, which is a privatization-leader-CSR-follower structure. Considering frequencies of adjusting government shareholding rates in partially privatized corporations and revising CSR policies in financial reports, there seem to be many cases for this structure to hold in reality. Regarding the companies in the above example, Renault and Deutsche Telekom have retained their government shareholding rates almost the same these 10 years[4], and Japan Post sold its government stock twice in the meantime[5], whereas all competing private firms issue CSR reports every year. From theoretical points of view also, the persistency of privatization policies compared to

[4]Precisely, the rates fluctuate under the .5 per cent range. For more information, visit their web cites (https://www.telekom.com/en/ for Deutsche Telekom and https://group.renault.com/en/ for Renault).

[5]https://www.japanpost.jp/en/

flexible CSR is natural because the former involves administrative procedures which are typically more complicated and the latter has an even wider variety of ways of implementation.

Simultaneously with this paper, several papers study the interaction of privatization and CSR. Ouattara (2017) and Kim et al. (2017) analyze strategic incentives of privatization to CSR. They show the result that privatization is a strategic substitute to CSR and if CSR is fixed at some high degrees then full-nationalization can be optimal in contrast to Matsumura (1998). Kim et al. (2017) additionally points out, yet, that the optimal degree of privatization exhibits non-monotonic relationship with the degree of CSR if private firms have different magnitudes of CSR. Itano (2017) also studies strategic relationship of privatization to CSR but also the one of CSR to privatization as well. He concludes that CSR is also a strategic substitute to privatization. To this literature, my study contributes in three ways; showing cost parameters are crucial for the strategic relationships of CSR and privatization by generalizing cost function, analyzing endogenous timing game of the two, and comparing consumer surplus type CSR and social welfare type CSR with respect to strategic commitment devices.

After Introduction part, the game model of strategic commitment and its equilibrium are described. The the game is extended to the endogenous timing version in Section 4. Then, Section 5 investigates welfare type of CSR, instead of consumer surplus type, whether the type matters to the results. Section 6 concludes.

2 Model

2.1 Game flow

The game consists of two stages. In the first stage, both the public and the private firms choose either a degree of privatization to maximize welfare if public or CSR to do its own profit if private. The choice is done simultaneously except an endogenous timing part. In the second stage, both firms simultaneously choose production quantities to maximize own objective functions, which are possibly different from pure welfare or profit ones in accordance with the result of the first stage. An equilibrium concept is a subgame perfect equilibrium.

2.2 Specification

Following the literature, this study employs simple linear demand and quadratic cost formulations. Precisely, both firms have its profit function of the following form,

$$\pi_i = (1 - q_i - q_j)q_i - \frac{\gamma q_i^2}{2}, \quad i = 0, 1. \tag{1}$$

Here, $i = 0$ stands for a state-owned public enterprise and $i = 1$ a competing private firm and q_i denotes each amount of production. On top of a canonical formulation, it introduces γ in the cost functions. This signifies an efficiency of firms' production technology. In the literature, $\gamma = 1$ is a standard assumption because of its parsimony. However as shown later, generalizing γ provides new findings which are in a stark contrast to some results in the literature. Thus γ is allowed to take any strictly positive value, i.e., $\gamma > 0$.

As an abstraction of corporate social responsibility act, the private firm takes care of consumer surplus too in its objective function, let alone own profit. Thus, the objective function to be maximized in the production stage is

$$U_1 = \alpha_1 \pi_1 + (1 - \alpha_1) CS, \tag{2}$$

$$where \;\; CS = \frac{(q_0 + q_1)^2}{2}. \tag{3}$$

α_1 indicates an inverse degree of CSR of the private firm. The lower is α_1, the more CSR is the private concerned with. To guarantee positive productions for both firms with any γ, the study restricts an attention to $\alpha_1 \in [1/2, 1]$.

For the public sector, following Matsumura (1998), its objective function has a possibility of partial privatization in the following form,

$$U_0 = \alpha_0 \pi_0 + (1 - \alpha_0) SW, \tag{4}$$

$$where \;\; SW = \pi_0 + \pi_1 + CS. \tag{5}$$

In turn, $\alpha_0 \in [0, 1]$ is a degree of privatization, so higher α_0 means the public firm sells larger amount of its stock to private investors and thus takes care its own profit more. The public firm maximizes U_0 in the production stage.

Through the second stage, an equilibrium pair of quantities $(q_0^*(\alpha_0, \alpha_1), q_1^*(\alpha_0, \alpha_1))$ and resultant values materialize. Anticipating them, both firms choose each α_i to maximize each objective function. Namely, the public firm's problem is,

$$\max_{\alpha_0} \; SW \left(q_0^*(\alpha_0, \alpha_1), q_1^*(\alpha_0, \alpha_1) \right) \tag{6}$$

and the private one is,

$$\max_{\alpha_1} \; \pi_1 \left(q_0^*(\alpha_0, \alpha_1), q_1^*(\alpha_0, \alpha_1) \right). \tag{7}$$

3 Equilibrium

Since an equilibrium concept is SPE, the game is solved backwards.

3.1 Production stage

Given each α_i, each firm maximizes each objective function with its own quantities, namely, the public does (4) with q_0 and the private (2) with q_1. The FOCs are,

$$\begin{aligned}\frac{\partial U_0}{\partial q_0} &= \alpha_0\frac{\partial \pi_0}{\partial q_0} + (1-\alpha_0)\frac{\partial SW}{\partial q_0}\\ &= \alpha_0(1-q_1-(2+\gamma)q_0) + (1-\alpha_0)(1-q_1-(1+\gamma)q_0) = 0, \qquad (8)\\ \frac{\partial U_1}{\partial q_1} &= \alpha_1\frac{\partial \pi_1}{\partial q_1} + (1-\alpha_1)\frac{\partial CS}{\partial q_1}\\ &= \alpha_1(1-q_0-(2+\gamma)q_1) + (1-\alpha_1)(q_0+q_1) = 0. \qquad (9)\end{aligned}$$

SOCs are satisfied. The reaction functions and the equilibrium pair of quantities are

$$q_0^R = \frac{1-q_1}{1+\gamma+\alpha_0}, \quad q_1^R = \frac{\alpha_1-(2\alpha_1-1)q_0}{3\alpha_1-1+\alpha_1\gamma}, \tag{10}$$

$$\begin{aligned}q_0^*(\alpha_0,\alpha_1) &= \frac{\alpha_1(2+\gamma)-1}{\alpha_0(3\alpha_1-1+\alpha_1\gamma)+\alpha_1(1+4\gamma+\gamma^2)-\gamma},\\ q_1^*(\alpha_0,\alpha_1) &= \frac{\alpha_1(-1+\alpha_0+\gamma)+1}{\alpha_0(3\alpha_1-1+\alpha_1\gamma)+\alpha_1(1+4\gamma+\gamma^2)-\gamma}.\end{aligned} \tag{11}$$

The above equations leads to the following lemma, which confirms an aggressive role of CSR and less-aggressive role of privatization.

Lemma 1.

$$\frac{\partial q_i^*(\alpha_0,\alpha_1)}{\partial \alpha_i} < 0, \quad \frac{\partial q_i^*(\alpha_0,\alpha_1)}{\partial \alpha_j} > 0 \quad for\ i = 0,1,\ i \neq j. \tag{12}$$

Proof. See Appendix. □

3.2 Commitment stage

Firms solve each maximizing problem, either (6) or (7), which is subject to the constraint (11). The reaction functions in this stage are,

$$\begin{aligned}\alpha_0^R(\alpha_1) &= \max\left\{\frac{(6\alpha_1^2-7\alpha_1+2)\,\gamma}{\alpha_1\,(\alpha_1\,(\gamma^2+5\gamma+2)-2\gamma-1)}, 0\right\},\\ \alpha_1^R(\alpha_0) &= \frac{\alpha_0^2+\alpha_0(3\gamma+2)+\gamma(2\gamma+3)}{\alpha_0^2+3\alpha_0(\gamma+1)+2\gamma(\gamma+2)}.\end{aligned} \tag{13}$$

Differentiating both reaction functions with respect to each argument provides the next lemma, which claim a strategic relationship of privatization and CSR and, that is why, is a key to a main proposition.

Lemma 2.

1. *For $\gamma > 0$, α_0 is a strategic complement to α_1. In other words, a degree of privatization is a strategic substitute to a degree of CSR.*

2. *For $\gamma \geq 1$, α_1 is a strategic complement to α_0. In other words, a degree of CSR is a strategic substitute to a degree of privatization.*

3. *For $0 < \gamma < 1$, α_1 is a strategic substitute in a range of $0 \leq \alpha_0 < \sqrt{\gamma} - \gamma$ and complement otherwise.*

Proof. See Appendix. □

The first part of Lemma 2 can be understood in the following way. The public firm fundamentally faces a dilemma that it tries to improve CS by increasing production while at the same time to improve production cost efficiency by sharing total production with the private firm as equally as possible. Since when $\alpha_1 = 1$, that is, the private is purely own profit oriented, q_1 is the smallest from Lemma 1, the public also needs to be moderate in production not to worsen cost inefficiency. Thus, again from Lemma 1, it privatizes the most. Yet, as the private starts CSR, it increases production, so the public has less anxiety for cost inefficiency and can increase production. Thus in this case, as CSR strengthened, the partial privatized firm become more public.

While the first part of Lemma 2 is from the public side's dilemma, the second and third part is owing to the one of the private side, a revenue and a cost. Consider infinite small increment of privatization from the point $(0, \alpha_1^R(0))$, where the public is fully nationalized and the private optimally respond to it with CSR. Due to the privatization, q_0 decreases in its first order while q_1 increases in its second order. With the first order effect only, the price would rise with other values fixed. This motivates the private firm to acquire more revenue by increasing production, so it does to promote CSR more. On the other hand, with the second order effect only, the price would drop and the cost inflate, so the private would like to withhold CSR. The total effect is a sum of both, thus ambiguous. However, as the marginal cost become heavier because of higher γ or q_1 (because of higher α_0), the second order tends to dominate the first. Therefore, beyond the threshold, CSR comes to a strategic substitute to partial privatization from a strategic complement.

Lemma 2 is about global properties of the reaction functions, and next Lemma 3 states the local property at the equilibrium.

Lemma 3.

$$\exists \gamma^* \in (0,1) \quad s.t. \quad 0 < \gamma < \gamma^* \Leftrightarrow \left.\frac{d\alpha_1}{d\alpha_0}\right|_{\alpha=\alpha^*} < 0, \quad \gamma^* \leq \gamma \Leftrightarrow \left.\frac{d\alpha_1}{d\alpha_0}\right|_{\alpha=\alpha^*} \geq 0. \tag{14}$$

Proof. See Appendix. □

The approximate value of γ^* is .8. Lemma 3 claims the reversibility of strategic relationship of CSR to privatization matters on the equilibrium, as well as off-path. As is well known, the strategic relationships of strategic values are crucial in an equilibrium result of an endogenous timing game, which follows in the next section.

4 Endogenous timing game

Now the study proceeds to an investigation of an endogenous timing game of CSR and privatization *à la* Hamilton and Slutsky (1990). The previous first stage of the game is split into two stages, thus hereafter the game consists of three stages as a whole. In the first stage, both firms have choices with respect to timing of commitment of either privatization or CSR, whether early of later. The second, ex-first, stage is played simultaneously or sequentially depending of the result of the first stage. For example, if in the first stage, say, the public firm chooses early and the private chooses later, then in the second stage, the public privatizes first and the private decides the amount of CSR following it, and vice versa. Alternatively, if both of them choose the same timing, early or later, in the first stage, the second stage is a plain simultaneous game as illustrated above. Following the two commitment stages, the third, production stage are played simultaneously.

Proposition 1. *In the endogenous timing game, where the public firm and the private choose timing of privatization of CSR commitment, the equilibrium is that,*

- *For* $0 < \gamma < \gamma^*$*, the public firm commits as a leader and the private follower.*
- *For* $\gamma^* \leq \gamma$*, both try to be a leader thus the simultaneous structure occurs.*

Proof. See Appendix. □

The resultant timing structure of Proposition 1 fundamentally owes to Lemma 3, which is new in the literature. Therefore the leader-follower structure of privatization and CSR in the range of high cost efficiency has not ever been supported until this study, at least with this type of game. However, as examples in Introduction shows, it seems the real structure with plausibility. The behind intuition is following. When the public firm knows that the private adjusts its degree of CSR in accordance with the one of privatization, the public predicts incremental CSR follows the privatization (in a relevant range) if the costs are sufficiently efficient. This is because of the private firm's rationale illustrated in Lemma 2. The incremental CSR compensates the welfare loss due to privatization. From the view point of the private also, the leader-follower structure is welcome, since the market share of the private in the production stage increases, inducing higher profits. Therefore the leader-follower structure is supported by both firms, so it is the equilibrium. The resultant amounts of CSR and privatization are summarized in the following corollary.

Corollary 1. *Comparing to the simultaneous choice version, the public leader and private follower structure induces,*

- *more CSR and privatization if costs are efficient enough,*
- *less CSR and more privatization otherwise.*

Proof. See Appendix. □

5 Welfare type of CSR

While up to this point the consumer surplus represents CSR in this study, there also are papers in which social welfare plays that role. Which specifications match the reality better seems under discussion yet. Therefore this section is devoted to compare the welfare one to the consumer surplus one by checking if the welfare one supports the public leader and private follower structure as the examples and the CS specification does. It turns out the result is negative, so the CS has more plausibility than SW in this context. The reasons of the difference is addressed after showing the result.

5.1 Equilibrium with SW type CSR

The game structure is exactly the same as before. The formulation is also the same with the one exception. It is SW instead of CS the private takes into account in its objective function of the production stage. Accordingly, the domain of α_1 becomes $[0,1]$ as α_0. In order to highlight valus are calculated under SW type CSR setting, overlines are added appropriately.

The FOCs, reaction functions, and equilibrium quantities in the third stage are,

$$\begin{aligned}\frac{\partial U_i}{\partial q_i} &= \overline{\alpha}_i \frac{\partial \pi_i}{\partial q_i} + (1-\overline{\alpha}_i)\frac{\partial SW}{\partial q_i} \\ &= \overline{\alpha}_i(1-\overline{q}_j-(2+\gamma)\overline{q}_i) + (1-\overline{\alpha}_i)(1-\overline{q}_j-(1+\gamma)\overline{q}_i) = 0,\end{aligned} \tag{15}$$

$$\overline{q}_i^R = \frac{1-\overline{q}_j}{1+\gamma+\overline{\alpha}_i}, \tag{16}$$

$$\overline{q}_i^*(\overline{\alpha}_i,\overline{\alpha}_j) = \frac{\overline{\alpha}_i+\gamma}{(\overline{\alpha}_i+\overline{\alpha}_j)(1+\gamma)+\overline{\alpha}_i\overline{\alpha}_j+\gamma(2+\gamma)}. \tag{17}$$

Then, the simultaneously played second stage generates the following reaction functions of CSR and privatization.

$$\overline{\alpha}_0^R(\overline{\alpha}_1) = \frac{\overline{\alpha}_1\gamma}{\gamma+(\overline{\alpha}_1+\gamma)^2}, \quad \overline{\alpha}_1^R(\overline{\alpha}_0) = \frac{\overline{\alpha}_0+\gamma}{1+\overline{\alpha}_0+\gamma}. \tag{18}$$

Given the reaction functions, the next lemma, which is a CSR-by-SW counterpart to Lemma 2 in the CSR-by-CS case, holds.

Lemma 4.

1. *For* $0 < \gamma < \left(\sqrt{5}-1\right)/2$, $\overline{\alpha}_0$ *is a strategic complement to* $\overline{\alpha}_1$ *in a range of* $0 < \sqrt{\gamma^2+\gamma}$ *and complement otherwise.*

2. *For* $\left(\sqrt{5}-1\right)/2 \leq \gamma$, $\overline{\alpha}_0$ *is a strategic complement to* $\overline{\alpha}_1$*. In other words, a degree of privatization is a strategic substitute to a degree of CSR.*

3. *For* $\gamma > 0$, $\overline{\alpha}_1$ *is a strategic complement to* $\overline{\alpha}_0$*. In other words, a degree of CSR is a strategic substitute to a degree of privatization.*

In comparison with Lemma 2, there are two main differences. The first one is that the strategic relation of privatization to CSR depends on the cost efficiency. The other one is that of CSR to privatization does not. Since Lemma 4 is about the global properties of both reaction functions as Lemma 2 in the CS case, the differences also change subsequent results of local properties at the equilibrium and endogenous timing game. Therefore to understand roots of the differences at this point is necessary. Though the differences are two, they share one and the same root. It is expressed in the following lemma and corollary.

Lemma 5.

$$\forall \alpha_1 \in (1/2, 1) \;\; \forall \overline{\alpha}_1 \in [0,1) \;\; \frac{\partial^2 q_1^R(q_0)}{\partial \alpha_1 \partial q_0} < 0 < \frac{\partial^2 \overline{q}_1^R(q_0)}{\partial \overline{\alpha}_1 \partial q_0}. \tag{19}$$

Proof. Though proving on the specification herein is straightforward, we provide a proof on more general assumptions. Suppose for $i, j = 0, 1$ $i \neq j$, $(\partial^2 \pi_i / \partial {q_i}^2)$, $(\partial^2 \pi_i / \partial q_i \partial q_j)$, $(\partial^2 SW / \partial {q_i}^2)$, and $(\partial^2 SW / \partial q_i \partial q_j)$ all exist and are negative and $(\partial^2 CS / \partial {q_i}^2)$ and $(\partial^2 CS / \partial q_i \partial q_j)$ exist and positive. Suppose additionally, α_i is defined in the intersectional range of $(\partial^2 U_i / \partial {q_i}^2) < 0$ and $[0, 1]$.

Under these assumptions, the two cross derivatives are,

$$\frac{\partial^2 q_1^R(q_0)}{\partial \alpha_1 \partial q_0} = \frac{-\left(p' - c_i''\right)\left(p' + p''(q_i + q_j)\right)}{\left(\partial^2 U_i / \partial q_i^2\right)^2} = \frac{(\partial^2 SW / \partial q_i^2)(\partial^2 CS / \partial q_i^2)}{\left(\partial^2 U_i / \partial q_i^2\right)^2} < 0, \tag{20}$$

$$\frac{\partial^2 \overline{q}_1^R(q_0)}{\partial \overline{\alpha}_1 \partial q_0} = \frac{{p'}^2 + p'' q_1 c_1''}{\left(\partial^2 U_i / \partial q_i^2\right)^2} > \frac{(p' + p'' q_1)^2}{\left(\partial^2 U_i / \partial q_i^2\right)^2} > 0. \tag{21}$$

The first inequality in (21) is from strict concavity of the profit function. Q.E.D. □

Corollary 2.

$$\forall \alpha_1 \in (1/2, 1) \;\; \forall \overline{\alpha}_1 \in [0,1) \;\; 0 > \frac{\partial \overline{q}_1^R(q_0)}{\partial q_0} > \frac{\partial q_1^R(q_0)}{\partial q_0}. \tag{22}$$

If firms try to maximize consumer surplus only, all they have to do is just producing infinite amounts regardless of competitors' actions. In contrast, if firms take care welfare only like as the fully nationalized firm, they have to more flexibly adjust own amounts of production in response to competitors' ones than the pure profit maximizer. Therefore, as the private firm doing CSR with CS, it pays less attention to the competitor and with SW vice versa. It is this difference of sensivity to competitors' actions that generates a number of differet results depending on the CS type CSR or SW type.

To see how the sensivity matters in the first difference in Lemma 4 and Lemma 2, the strategic relation of privatization to CSR depends if SW type and not if CS type, the point of $(\alpha_0^R(1), 1)$ is important. Since, at that point, the private firm behaves as a pure private and the public responds with its optimally adjusted objective, the difference of SW and CS has no effects on reaction functions, resultant productions and other values in the production stage. Once the private starts CSR, the difference emerges. The initiation of CSR only of course improve welfare through both total production and cost efficiency increases. However, more importantly, the public become able to improve welfare more by adjusting privatization in response to the CSR. On its choice, there are potentially two opposite directions, either increasing or reducing its production, i.e., nationalizing or privatizing more. Since at the status quo, the public produces more than the private, increasing production improve consumer surplus but worsen the efficiency and vice versa. Hence, the public directions to pursue depends on the relative sizes of two improvement effects.

The determinant relative sizes in turn depends on the slope of the private reaction function, or sensitivity to the public production. The steeper slope, which is the case with the SW type CSR, makes the efficiency improvement larger, because tiny reduction of the public production induces substantial increase of the private production. At the same time, it also makes the total production costly to increase, due to the harsh crowding out effect. On the other hand, with the flatter slope, as with the CS type, the opposite holds. Therefore, with the CS type CSR entailing the lower sensitivity of the private, the total production effect ends up always dominating the efficiency one as depicted in Lemma 2. With SW type and the higher sensitivity, however, it reverses in some range if γ is enough small. The reversion is limited in a range with small CSR and small γ. The former is because as CSR increases and privatization follows the production gap shrinks, so does the efficiency improvement. The latter is because with large γ, the public cannot produce large amounts in the first place, so the production gap is again small.

Secondly, the other difference in Lemma 4 and 2, the strategic relation of CSR to privatization in turn does not depend with the SW type CSR, is also due to the sensitivity difference. As noted in the exposition after Lemma 2, the strategic relation depends on the relative sizes of the first and second order effects of privatization on the private profit. If the first order one is large, that is, the q_0 decrease is large, then the private would like to promote CSR and be aggressive. If, in contrast, the second order, the q_1 increase, is so, the private withhold it and be less aggressive. Therefore, with SW type CSR, i.e., with the steeper reaction curve, the latter case is more likely to happen. This sensitive reaction is so dominant a factor in the specification for the CSR to be a strategic substitute to privatization regardless of γ.

As in the CS type CSR, the next lemma characterizes the equilibrium.

Lemma 6.

$$\forall\gamma > 0 \quad \left.\frac{d\overline{\alpha}_i}{d\overline{\alpha}_j}\right|_{\overline{\alpha}=\overline{\alpha}^*} > 0 \quad i,j = 0,1,\ i \neq j. \tag{23}$$

Proof. See Appendix. □

Lemma 6 claims that though the strategic relationship of privatization for CSR depends, at the equilibrium it is always a strategic substitute, as in Lemma 3. That of CSR for privatization also does not depend on the marginal cost, but this is in contrast in Lemma 3 owing to the difference explained above.

5.2 Endogenous Timing Game with SW type CSR

Finally, we check if the SW type CSR can endogenously support the privatization leader and CSR follower structure like as the CS type one.

Proposition 2. *In the endogenous timing game in the commitment stage with privatization and the welfare type CSR, the equilibrium is a simultaneous structure in consequence of a common insentive for the leader.*

Proof. See Appendix. □

Because of the mutual strategic substitutability induced by SW type CSR, both try to be a leader, otherwise their objectives being impaired. Therefore, if the privatization leader CSR follower structure is imposed for some reason, it has a possibility of crowding out private CSR acts, though improving overall welfare.

6 Conclusion

Appendix

Proof of Lemma 1

Differentiating two equations in (11) by each degree yields the jaccobian matrix,

$$\begin{pmatrix} \frac{\partial q_0^*}{\partial \alpha_0} & \frac{\partial q_0^*}{\partial \alpha_1} \\ \frac{\partial q_1^*}{\partial \alpha_0} & \frac{\partial q_1^*}{\partial \alpha_1} \end{pmatrix} = \frac{1}{A}\begin{pmatrix} -(\alpha_1(\gamma+2)-1)(\alpha_1(\gamma+3)-1) & \alpha_0+2\gamma+1 \\ (2\alpha_1-1)(\alpha_1(\gamma+2)-1) & -(\alpha_0^2+\alpha_0(3\gamma+2)+2\gamma^2+3\gamma+1) \end{pmatrix} \tag{24}$$

where $A = \left(\alpha_0(\alpha_1(\gamma+3)-1)+\alpha_1\left(\gamma^2+4\gamma+1\right)-\gamma\right)^2 > 0.$

Therefore for $\alpha_0 \in [0,1]$, $\alpha_1 \in (1/2,1]$, and $\gamma > 0$ the statement holds. Q.E.D.

Proof of Lemma 2

Differentiating the reaction function of α_0 in (13), we have,

$$\frac{d\alpha_0}{d\alpha_1} = \begin{cases} \dfrac{\gamma\left(\alpha_1^2\left(7\gamma^2+23\gamma+8\right)-4\alpha_1\left(\gamma^2+5\gamma+2\right)+4\gamma+2\right)}{\alpha_1^2\left(-\alpha_1\left(\gamma^2+5\gamma+2\right)+2\gamma+1\right)^2}, & \alpha_1 \in [2/3,1], \\ 0 & \textit{otherwise}. \end{cases} \tag{25}$$

The sign in the range of $\alpha_1 \in [2/3,1]$ is determined by the numerator. Note that the numerator takes U-shape in terms of α_1, and its value at $\alpha_1 = 2/3$ is $2\gamma\left(2\gamma^2+4\gamma+1\right)/9 > 0$. Thus the first part of Lemma 2 is proven. Then, for the other reaction function, we have,

$$\frac{d\alpha_1}{d\alpha_0} = \frac{\alpha_0^2+2\alpha_0\gamma+(\gamma-1)\gamma}{\left(\alpha_0^2+3\alpha_0(\gamma+1)+2\gamma(\gamma+2)\right)^2} \lesseqgtr 0 \quad \Leftrightarrow \quad \alpha_0 \lesseqgtr \sqrt{\gamma}-\gamma. \tag{26}$$

Therefore when $\gamma \in (0,1)$ α_1 has a range of strategic complement to α_0 near the full nationalization, and otherwise substitute. Q.E.D.

Proof of Lemma 3

If $\gamma > 1$, the result is trivial, so we restrict our attention to the case $\gamma \in (0,1]$. In the following we compare the point $(\sqrt{\gamma}-\gamma, \alpha_1(\sqrt{\gamma}-\gamma))$ and the point $(\alpha_0(\alpha_1(\sqrt{\gamma}-\gamma)), \alpha_1(\sqrt{\gamma}-\gamma))$ along with α_0 axis. If $\sqrt{\gamma}-\gamma < \alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))$, then the reaction functions intersect at the point of mutual strategic complement, and otherwise α_1 is strategic substitute to α_0.

To begin with, from the strategic complementarity of α_0 to α_1, we have,

$$0 < \alpha_0(\alpha_1(\sqrt{\gamma}-\gamma)) < \alpha_0(1) = \frac{\gamma}{1+3\gamma+\gamma^2} < \gamma. \tag{27}$$

So the public reaction point under consideration is bounded from above by γ. In turn, the private point has following property.

$$\frac{d(\sqrt{\gamma}-\gamma)}{d\gamma} = \frac{1}{2\sqrt{\gamma}} - 1 \gtreqless 0 \Leftrightarrow \gamma \lesseqgtr \frac{1}{4}. \tag{28}$$

Note that $\sqrt{\gamma}-\gamma = 1/4$ at $\gamma = 1/4$. Therefore in the range of $(0,1/4]$, $\sqrt{\gamma}-\gamma > \alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))$. For $1/4 < \gamma \leq 1$, $\sqrt{\gamma}-\gamma$ decreases to 0, so if $\alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))$ is increasing in this range, the existence of the unique threshold is guaranteed.

$$\begin{aligned} &\frac{d\alpha_0(\alpha_1(\sqrt{\gamma}-\gamma))}{d\gamma} \\ &= \frac{80\gamma^{\frac{3}{2}}+138\gamma^{\frac{5}{2}}+7\gamma^{\frac{7}{2}}-47\gamma^{\frac{9}{2}}-8\gamma^{\frac{11}{2}}-\gamma^6-26\gamma^5-44\gamma^4+86\gamma^3+129\gamma^2+32\gamma+6\sqrt{\gamma}}{\left(\gamma+2\sqrt{\gamma}+2\right)^2\left(6\gamma^{\frac{3}{2}}+2\gamma^{\frac{5}{2}}+\gamma^3+5\gamma^2+5\gamma+2\sqrt{\gamma}+1\right)^2} \\ &> \frac{80\gamma^{\frac{3}{2}}+138\gamma^{\frac{5}{2}}+7\gamma^{\frac{7}{2}}+86\gamma^3+3\gamma^2+32\gamma+6\sqrt{\gamma}}{\left(\gamma+2\sqrt{\gamma}+2\right)^2\left(6\gamma^{\frac{3}{2}}+2\gamma^{\frac{5}{2}}+\gamma^3+5\gamma^2+5\gamma+2\sqrt{\gamma}+1\right)^2} > 0. \end{aligned}$$

Q.E.D.

Proof of Proposition 1 and Corollary 1

In preparation for the proof of the endogenous timing game, we put the next lemma.

Lemma 7.

$$\begin{aligned}
\frac{dSW(\alpha_0(\alpha_1),\alpha_1)}{d\alpha_1} &= \frac{\partial SW}{\partial \alpha_0}\frac{d\alpha_0}{d\alpha_1} + \frac{\partial SW}{\partial \alpha_1} = \left.\frac{\partial SW}{\partial \alpha_1}\right|_{\alpha_0=\alpha_0(\alpha_1)} \\
&= -\frac{(3\alpha_1-2)\gamma\left(2\alpha_1\gamma^2+7\alpha_1\gamma+\alpha_1-2\gamma\right)}{\left(\alpha_1^2\left(\gamma^3+7\gamma^2+15\gamma+1\right)-2\alpha_1\gamma(\gamma+5)+2\gamma\right)^2} < 0, \qquad (29)\\
\frac{d\pi_1(\alpha_0,\alpha_1(\alpha_0))}{d\alpha_0} &= \frac{\partial \pi_1}{\partial \alpha_0} + \frac{\partial \pi_1}{\partial \alpha_1}\frac{d\alpha_1}{d\alpha_0} = \left.\frac{\partial \pi_1}{\partial \alpha_0}\right|_{\alpha_1=\alpha_1(\alpha_0)} \\
&= \frac{(\alpha_0+\gamma)(\alpha_0(\gamma+1)+\gamma(\gamma+2))}{(\alpha_0+\gamma+1)^2(\alpha_0(\gamma+2)+\gamma(\gamma+3))^2} > 0. \qquad (30)
\end{aligned}$$

Second inequalities in each equation are from Envelope theorem. What the lemma means is that along each reaction function each ultimate objective increases or decreases with competitor's commitment. Given this, the hypothetical leader firm picks the best point on the competitor's reaction curve. Let (α_i^L, α_j^F) denote that best point when the firm i is a leader and firm j follower. Then it necessarily satisfies the property either $\alpha_1^* \geq \alpha_1^F$ or $\alpha_0^* \leq \alpha_0^F$. For expository simplicity, take a public leader case. Suppose the leader choose the point such as $\alpha_1^F > \alpha_1^*$, then $SW(\alpha_0^L, \alpha_1^F) < SW(\alpha_0(\alpha_1^F), \alpha_1^F) < SW(\alpha_0^*, \alpha_1^*)$. The first inequality comes from the definition of reaction functions and the second one from Lemma 7. Hence the point (α_0^L, α_1^F) is strictly dominated by the simultaneous one, this is contradiction. Similar arguments hold in the private leader case. As for α_i^L to α_i^*, the relative positions depend on whether the reaction curve of firm j has a positive or negative slope.

Lastly, owing to the following equations, the two weak inequalities above are actually strict ones, except $\gamma = \gamma^*$ case.

$$sgn\left(\left.\frac{dSW(\alpha_0,\alpha_1(\alpha_0))}{d\alpha_0}\right|_{(\alpha_0^*,\alpha_1^*)}\right) = sgn\left(\frac{\partial SW}{\partial \alpha_0} + \frac{\partial SW}{\partial \alpha_1}\frac{d\alpha_1}{d\alpha_0}\right) = -sgn\left(\frac{d\alpha_1}{d\alpha_0}\right), \qquad (31)$$

$$sgn\left(\left.\frac{d\pi_1(\alpha_0(\alpha_1),\alpha_1)}{d\alpha_1}\right|_{(\alpha_0^*,\alpha_1^*)}\right) = sgn\left(\frac{\partial \pi_1}{\partial \alpha_0}\frac{d\alpha_0}{d\alpha_1} + \frac{\partial \pi_1}{\partial \alpha_1}\right) = sgn\left(\frac{d\alpha_0}{d\alpha_1}\right). \qquad (32)$$

Therefore if, say, the private is a leader, the point chosen is upper right to the simultaneous point in the $\alpha_0 - \alpha_1$ coordinate plane. At that point the private profit of course increases compared to the simultaneous one, but the welfare on the other hand decreases as in Lemma 7. Hence, the public deviates from its follower role. The similar results hold for the public leader case if α_1 is strategic complementary to α_0 at the simultaneous equilibrium. However, if it is strategic substitute at the intersection, then the private profit also increases, so it has no deviation incentive from the follower position. Q.E.D.

Proof of Lemma 4

Differentiating two reaction functions in (18) yields,

$$\frac{d\alpha_0^R}{d\alpha_1} = \frac{\gamma\left(-\alpha_1^2+\gamma^2+\gamma\right)}{\left(\alpha_1^2+2\alpha_1\gamma+\gamma^2+\gamma\right)^2}, \quad \frac{d\alpha_1^R}{d\alpha_0} = \frac{1}{(\alpha_0+\gamma+1)^2}. \tag{33}$$

The left is straightforward as in the proof of Lemma 2. Q.E.D.

Proof of Lemma 6

The next equation demonstrates that the private reaction curve passes under, if any, the corner of the public one.

$$For\ \gamma > 0, \quad \sqrt{\gamma^2+\gamma} - \alpha_1^R(\alpha_0^R(\sqrt{\gamma^2+\gamma})) = \frac{(3\gamma+2)\sqrt{\gamma^2+\gamma}-\gamma}{3\gamma+4} > 0. \tag{34}$$

Q.E.D.

Proof of Proposition 2

The counterpart of Lemma 7 is following.

$$\frac{dSW(\alpha_0^R(\alpha_1),\alpha_1)}{d\alpha_1} = \left.\frac{\partial SW}{\partial \alpha_1}\right|_{\alpha_0=\alpha_0^R(\alpha_1)} = \frac{-\alpha_1\gamma^2(\alpha_1+\gamma+1)}{\left(\alpha_1^2(\gamma+1)+2\alpha_1\gamma(\gamma+2)+\gamma\left(\gamma^2+3\gamma+2\right)\right)^2} < 0, \tag{35}$$

$$\frac{d\pi_1(\alpha_0,\alpha_1^R(\alpha_0))}{d\alpha_0} = \left.\frac{\partial \pi_1}{\partial \alpha_0}\right|_{\alpha_1=\alpha_1^R(\alpha_0)} = \frac{(\alpha_0+\gamma)(\alpha_0(\gamma+1)+\gamma(\gamma+2))}{(\alpha_0+\gamma+1)^2(\alpha_0(\gamma+2)+\gamma(\gamma+3))^2} > 0. \tag{36}$$

Thus, the similar arguments as in Proposition 1 remain valid. Q.E.D.

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