On patent licensing in spatial competition with endogenous location choice*

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Abstract

Using a standard linear city model with two firms, we consider how licensing activities affect the locations of the firms (i.e., the degree of product differentiation) and the incentive for R&D investment. In contrast to recent studies showing that R&D investment results in a large cost differential between firms, thereby leading to firm agglomeration, we find that licensing activities following R&D investment always lead to the maximum differentiation between firms and the mitigation of price competition. We also show that licensing activities induce the socially optimal effort level of R&D activity.

JEL classification: L13, O32, R32

Key words: licensing, oligopoly, R&D, location

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1 Introduction

For several decades, many researchers have emphasized that product differentiation is one of the more important strategies conducted by firms (see, for example, Porter (1980), Kotler (1999)). In order to differentiate their products, many firms attempt to create particular values based on various elements. These include, for example, brand image, product positioning, and customer service.

The spatial competition model a la Hotelling (1929) as one of the most important oligopoly models, has been viewed by many economics and marketing researchers as an attractive framework for the analysis of product differentiation (see, for example, d’Aspremont et al. (1979), Neven (1985), and Anderson et al. (1992), amongst others). The major advantage of this approach is that it allows for the explicit analysis of product selection. Of particular interest are the equilibrium pattern of product locations and the degree of product differentiation.

Two major papers in the literature (Hotelling (1929) and d’Aspremont et al. (1979)) provide quite different results concerning the degree of product differentiation. Hotelling (1929) finds that firms produce similar products (minimum differentiation). However, by considering two-stage location–price games, d’Aspremont et al. (1979) show that products are maximally differentiated when transport costs are quadratic. In their study, firms attempt to avoid cutthroat competition in the subsequent stage by never choosing minimal differentiation.

Along the lines of the spatial competition model a la Hotelling (1929), many researchers incorporate various factors into spatial models to investigate the degree of product differentiation. These include, for example: consumer distribution (Neven (1986) and Tabuchi and Thisse (1995)), transportation costs (Economides (1986)), demand uncertainty (Casado-Izaga (2000) and Meagher and Zauner (2004)), incomplete information (Boyer et al. (1994) and Boyer et al. (2003)), price regulation (Bhaskar (1997) and Brekke et al. (2006)), multiple buying (Guo (2006) and Kim and Serfes (2006)), and tournaments (Ganuza and Hauk (2006)).
Recently, several researchers have introduced quality uncertainty stemming from R&D activity into the duopoly location model. Gerlach et al. (2005) and Christou and Vettas (2005) provide interesting results concerning the relation between technological fluctuation (the uncertainty of the outcomes of R&D activity) and product positioning. They consider the following three-stage game. In the first stage, each firm chooses its location in a linear city; in the second stage, each firm’s product quality is probabilistically determined; and in the third stage, each firm sets its price.\(^1\) These show that spatial agglomeration (minimum differentiation) can appear in equilibrium.\(^2\)

We often observe that advanced firms which succeed in R&D transfer their technology to rivals, subject to a fee. That is, firms having advanced technologies often earn additional profits from licensing. Neither Gerlach et al. (2005) nor Christou and Vettas (2005) discuss these licensing activities, which are potentially important from the viewpoint of product positioning.

One purpose of our paper is to consider how licensing activities affect the location of firms. We consider how licensing activities following R&D affect the product choices of firms. Several modes of licensing currently exist: a royalty per unit of output produced with the transferred technology, a fixed fee that is independent of the quantity supplied with transferred technology, and a hybrid form such as a two-part tariff. In our paper, we consider a royalty per unit of output produced with the transferred technology.

\(^1\) The basic structures of these models do, however, differ. In Gerlach et al. (2005), if the R&D is unsuccessful, the firm becomes inactive in the market. In Christou and Vettas (2005) each firm has the ability to produce its product, even when its R&D project is unsuccessful.

\(^2\) There are other papers showing that minimum differentiation appears in equilibrium in other contexts. Price collusion after firms have made location choices is considered in Friedman and Thisse (1993). Mai and Peng (1999) consider cooperation between firms in the form of information exchange through communication. de Palma et al. (1985) introduce unobservable attributes in brand choice and consider situations where central agglomeration does not induce standard Bertrand competition with homogeneous products. They show that sufficient heterogeneity between firms induces central agglomeration. Bester (1998) introduces vertical quality characteristics, asymmetric information about the quality between sellers and consumers, and a limited number of repeat purchases by consumers. Bester (1998) finds that, in equilibrium, central agglomeration appears if the number of repeat purchases is more than one and not too large. Gerlach et al. (2005) and Christou and Vettas (2005) also deal with models of endogenous production costs (quality) with spatial competition. See, for instance, Brekke and Straume (2004) and Matsushima (2004, 2006).
Contrary to Gerlach et al. (2005) and Christou and Vettas (2005), we show that maximum differentiation always appears in equilibrium when each firm is able to transfer its own advanced technology to its rival via licensing. 3

We also consider whether the level of R&D activity under such a licensing scheme is excessive or insufficient from the viewpoint of social welfare. We show that the private and social gains from R&D investment are equal, so efficient R&D investment is achieved by firms without government intervention.

Since the seminal work of Arrow (1962), a vast literature (see Kamien (1992)) has arisen that focuses on the optimal licensing arrangement for the patentee in a wide variety of situations. There are papers that analyze various aspects of patent licensing (for instance, insider and outsider patentees, incentives for innovation, and market structures (monopoly, oligopoly, and competitive)). 4 However, few of these studies consider endogenous product positioning.

To the best of our knowledge, only Poddar and Sinha (2004) employ the Hotelling model with cost asymmetry in the analysis of licensing. 5 Although these analyses consider the optimal licensing strategy of both outsider and insider patentees, the locations of firms are exogenously given. We therefore believe that the setting discussed herein provides additional insights for the licensing literature.

In addition, there is another important difference between the current paper and Poddar

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3 If a fixed fee independent of the quantity supplied with transferred technology is adopted, the result of maximal differentiation is absolutely obvious because there is no cost difference between the two firms after licensing. Our result indicates that even when a royalty per unit of output or hybrid (two-part tariff licensing) is adopted, the maximal differentiation result still holds. In addition, in our model, firms choose a per unit of output royalty contract if they can choose the type of contract.

4 In the literature on licensing, the interactions between patentees and licensees are intensively discussed: whether the licensees are insiders or outsiders is also an important consideration (see, for instance, Hernández-Murillo and Llobet (2006), Kamien et al. (1992), Kamien and Tauman (1984, 2002), Katz and Shapiro (1986), Muto (1993), Sen and Tauman (2007), Wang (1998, 2002), and Wang and Yang (1999), among others). We briefly discuss the latter topic.

5 Some papers on licensing include the role of product differentiation. See, for instance, Caballero-Sanz et al. (2002), Fauli-Oller and Sandonis (2002), Mukherjee and Balasubramanian (2001), Muto (1993) and Wang and Yang (1999). Most of these studies do not use location models, with the exception of Caballero-Sanz et al. (2002) who consider an exogenous location model.
and Sinha (2004). In our analysis, the firm that holds the patent chooses its price to maximize total profits (profits from its own products plus revenue from the license fee). In contrast, Poddar and Sinha (2004) assume that the firm chooses its price so as to maximize its profit from its own products and neglects the license fee revenue (although the firm still maximizes total profit when it chooses the level of the license fee).\footnote{Poddar and Sinha (2004) do not explicitly state this assumption, but their result does not hold without it. See also footnote 9.} We show that one of their results for nonneutrality does not hold in a more plausible setting.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Sections 3 and 4 detail the results under the exogenous and endogenous location models, respectively. Section 5 provides the main result of the paper. Sections 6 and 7 discuss the levels of R&D investment, and Section 8 concludes.

2 The model

Consider a linear city along the unit interval $[0, 1]$, where firm 1 is located at $x_1$ and firm 2 is located at $1 - x_2$. Without loss of generality, we assume that $x_1 \leq 1 - x_2$. Consumers are uniformly distributed along the interval. Each consumer buys exactly one unit of the good, which can be produced by either firms 1 or 2. The utility of the consumer located at $x$ is given by:

$$u_x = \begin{cases} -T(|x_1 - x|) - p_1 & \text{if bought from firm 1,} \\ -T(|1 - x_2 - x|) - p_2 & \text{if bought from firm 2,} \end{cases}$$

(1)

where $T(\cdot)$ represents the transport cost incurred by the consumer. $T(\cdot)$ is an increasing function. For a consumer living at $x(p_1, p_2, x_1, x_2)$, where

$$-T(|x_1 - x(p_1, p_2, x_1, x_2)|) - p_1 = -T(|1 - x_2 - x(p_1, p_2, x_1, x_2)|) - p_2,$$

(2)

the utility is same whichever of the two firms is chosen. Thus, the demand facing firm 1, $D_1$, and that facing firm 2, $D_2$, are given by:

$$D_1(p_1, p_2, x_1, x_2) = \min\{\max(x(p_1, p_2, x_1, x_2), 0), 1\},$$
$$D_2(p_1, p_2, x_1, x_2) = 1 - D_1(p_1, p_2, x_1, x_2).$$

(3)
We assume that one of the firms (denoted as firm 1) has a cost-reducing innovation. Assume the pre-innovation marginal costs of both firms are \(c_1 = c_2 = c\). Firm 1’s cost-reducing innovation lowers its marginal cost by \(d > 0\), so that post-innovation, \(c_1 = c - d\) and \(c_2 = c\).

Firm 1 licenses its new technology to firm 2 at a royalty rate \(r\). The total royalty firm 2 pays will depend on the quantity supplied by firm 2 using the new technology. In this case, firm 1’s marginal cost of production is \(c - d\) and firm 2’s marginal cost of production becomes \(c - d + r\). The royalty \(r \leq d\), otherwise firm 2 will not accept the contract.

The game runs as follows. In the first stage, firm \(i\) \((i = 1, 2)\) chooses its location \(x_i \in [0,1]\) simultaneously. In the second stage, \(r\) is determined (Firm 1 offers the royalty \(r\) to firm 2. Firm 2 chooses whether or not to accept).\(^7\) In the third stage, firm \(i\) chooses its price \(p_i \in [c_i, \infty)\) simultaneously.\(^8\)

3 Exogenous locations

In this section, we first discuss the case where the firms’ locations are exogenously given by \(x_1 = x_2 = 0\) so as to clarify the effect of royalty licensing on the quantities supplied by the firms. We introduce location choices explicitly in Section 4. We label \(x(p_1,p_2,x_1,x_2)\) \(x(p_1,p_2)\). The profit functions of the firms are:

\[
\pi_1 = (p_1 - (c - d))x(p_1,p_2) + r(1 - x(p_1,p_2)) \\
= (p_1 - (c - d) - r)x(p_1,p_2) + r, \\
\pi_2 = (p_2 - (c - d) - r)(1 - x(p_1,p_2)).
\]

The first-order conditions are:

\[
\frac{\partial \pi_1}{\partial p_1} = x(p_1,p_2) + (p_1 - (c - d) - r)\frac{\partial x(p_1,p_2)}{\partial p_1} \\
= x(p_1,p_2) - (p_1 - (c - d) - r) \times \frac{1}{T'(1 - x(p_1,p_2)) + T'(x(p_1,p_2))} = 0,
\]

\(^7\) The order of the first and second stages is interchangeable. If firm 1 chooses \(r\) and then the firms choose their locations, all our results hold true.

\(^8\) Choosing \(p_i < c_i\) is weakly dominated by choosing \(p_i = c_i\), so we assume that the lower bound of the price is its cost.
\[ \frac{\partial \pi_2}{\partial p_2} = 1 - x(p_1, p_2) - (p_2 - (c - d) - r) \times \frac{1}{T'(1 - x(p_1, p_2)) + T'(x(p_1, p_2))} = 0. \]

The first-order conditions lead to the following proposition.

**Proposition 1** If the locations of the firms and the royalty rate is \( r \) are exogenously given, the equilibrium prices and profits are:

\[ p_1 = p_2 = c - d + r + T' \left( \frac{1}{2} \right), \quad x(p_1, p_2) = \frac{1}{2}, \quad \pi_1 = \frac{T'(1/2)}{2} + r, \quad \pi_2 = \frac{T'(1/2)}{2}. \]  

Proposition 1 indicates that the quantities supplied by the firms do not depend on the level of the royalty fee, \( r \). Proposition 1 also indicates that the profit of firm 2 does not depend on \( r \). We now explain the intuition behind this result. When the licensing firm sets its price, it takes into account the per unit licensing fee, \( r \). \( r(1 - x) \) is the profit from selling its own technology to the rival firm. When the licensing firm decreases its price and increases its output level \( (x) \), the rival’s supply \( (1 - x) \) decreases. As a result, the profit from licensing \( r(1 - x) \) decreases. The licensing fee can be interpreted as the opportunity cost of the licensing firm.\(^9\) Since firm 1’s unit cost (the real cost plus the opportunity cost) is \( c - d + r \) and firm 2’s unit cost is \( c - d + r \), both firms face the same unit cost, leading to equal market share. An increase in \( r \) increases the per unit cost of each firm and raises equilibrium prices. This is why the profit of firm 2 does not depend on \( r \).

This property is similar to that found in Sappington (2005). Using a simple duopoly model, Sappington (2005) considers the problem of access charge and the entrant firm’s make-or-buy decision. The vertically integrated incumbent firm charges the input price \( w \) to the entrant firm. Sappington (2005) shows that the level of \( w \) does not affect the quantities supplied by the firms.

Note that the result that the quantities supplied by the firms do not depend on the level of the licensing fee \( r \) differs from that in Poddar and Sinha (2004). Using the Hotelling model with cost asymmetry, Poddar and Sinha (2004) consider the optimal licensing strategy of both outsider and insider patentees. Although Poddar and Sinha (2004) present several interesting insights on optimal licensing strategies, they do not consider the effect of the licensing fee on pricing strategy. Therefore, in their paper the quantity supplied by the licensed firm is \( (3 - r)/6 \), where \( r \) is the royalty rate (Poddar and Sinha 2004, p. 215). In our paper, we assume the patentee is an insider (firm 1 holds the patent). If we consider an outsider patentee, we can show that the patentee’s profit is \( d \): this is exactly the same as the additional profit of an insider patentee. This implies that neutrality (both insider and outsider patentees have the same incentive for R&D) holds in our spatial model.
We now discuss the equilibrium level of $r$. Firm 1 must set $r \leq d$, otherwise firm 2 never uses firm 1’s advanced technology and firm 1 cannot obtain the license fee. Since $r \leq d$, firm 1 naturally sets $r = d$.\footnote{In Section 5 (Lemma 1), we show that firm 1’s profit is larger under licensing than without licensing.} Since an increase in $r$ does not reduce the profit of firm 2, and increases that of firm 1, firm 1 chooses the maximal royalty fee and firm 2 has no reason to reject it. Note that this result does not depend on the distribution of bargaining power between firms 1 and 2. We summarize the above properties in the following proposition.

**Proposition 2** If the locations of the firms are exogenously given, firm 1 transfers its technology to firm 2 and sets the royalty rate at $r = d$. The profits of the firms are:

$$
\pi_1 = \frac{T'(1/2)}{2} + d, \quad \pi_2 = \frac{T'(1/2)}{2}.
$$

(5)

4 Endogenous locations

In this section, we endogenize the firms’ locations. To derive the location equilibrium explicitly, we assume that the transport costs of consumers are quadratic in distance.

Before solving this problem, we explain a closely related result provided by Ziss (1993). Using the Hotelling model with cost asymmetry, Ziss (1993) shows that pure strategy equilibria do not exist if and only if $d \geq (6 - 3\sqrt{3})t$ ($t$ is the standard parameter concerning transportation cost and later assumed in equation (6)). That is, no pure strategy exists if the cost difference between the firms $d$ is sufficiently large.\footnote{For the mixed strategy equilibrium under cost differences, see Matsumura and Matsushima (2007).} We show that when the efficient firm is able to license its efficient technology to the inefficient rival, maximum differentiation appears in equilibrium, regardless of $d$.

The utility of the consumer living at $x$ is given by:

$$
u_x = \begin{cases} 
-t(x_1 - x)^2 - p_1 & \text{if bought from firm 1,} \\
-t(1-x_2 - x)^2 - p_2 & \text{if bought from firm 2.}
\end{cases}
$$

(6)

For a consumer living at:

$$
x = \frac{1 + x_1 - x_2}{2} + \frac{p_2 - p_1}{2t(1-x_1 - x_2)},
$$

(7)
total utility is the same regardless of which of the two firms is chosen. Given the values of \(x_1, x_2, r\), the profits of the firms are:

\[
\pi_1 = (p_1 - (c - d))x + r(1 - x)
\]
\[
= (p_1 - (c - d) - r)\left(\frac{1 + x_1 - x_2}{2} + \frac{p_2 - p_1}{2t(1 - x_1 - x_2)}\right) + r,
\]
(8)

\[
\pi_2 = (p_2 - (c - d) - r)(1 - x)
\]
\[
= (p_2 - (c - d) - r)\left(\frac{1 - x_1 + x_2}{2} + \frac{p_1 - p_2}{2t(1 - x_1 - x_2)}\right).
\]
(9)

In the third stage, each firm sets its price simultaneously. The first-order conditions lead to:

\[
p_1 = (c + r - d) + \frac{t(1 - x_1 - x_2)(3 + x_1 - x_2)}{3},
\]
\[
p_2 = (c + r - d) + \frac{t(1 - x_1 - x_2)(3 - x_1 + x_2)}{3}.
\]

Substituting the prices into the profit functions in (8) and (9), we have:

\[
\pi_1 = \frac{t(1 - x_1 - x_2)(3 + x_1 - x_2)^2}{18} + r,
\]
\[
\pi_2 = \frac{t(1 - x_1 - x_2)(3 - x_1 + x_2)^2}{18}.
\]

In the second stage, firm 1 sets the level of \(r\). For the same reason discussed in the previous section, firm 1 transfers its advanced technology to firm 2 and sets the level of \(r\) at \(r = d\). (Firm 1 has an incentive to license. This is explicitly discussed in the following section in Lemma 1.)

In the first stage, each firm chooses its location. Differentiating \(\pi_i\) with respect to \(x_i\), we have:

\[
\frac{\partial \pi_1}{\partial x_1} = -\frac{t(1 + 3x_1 + x_2)(3 + x_1 - x_2)}{18} < 0,
\]
\[
\frac{\partial \pi_2}{\partial x_2} = -\frac{t(1 + x_1 + 3x_2)(3 - x_1 + x_2)}{18} < 0.
\]

The optimal locations of the firms are \(x_1 = x_2 = 0\). We summarize the result as follows.

**Proposition 3** If the locations of the firms are endogenously determined, the equilibrium locations of the firms are \(x_1 = x_2 = 0\); that is, the maximum differentiation appears in
equilibrium. The profits of the firms are:

\[ \pi_1 = \frac{t}{2} + d, \quad \pi_2 = \frac{t}{2}. \]

When we take into account licensing activity, the well-known maximum differentiation result appears, even though the cost differential between the firms is significant. This result is quite different from that found in Ziss (1993).

5 R&D investments and location choice

In this section, we show that the result of maximal differentiation still holds when R&D activities are endogenized. We now consider the following four-stage game: first, each firm chooses its location in a linear city; second, each firm chooses whether it engages in R&D investment that probabilistically affects its production cost; third, one of the firms that has a cost advantage over its rival chooses whether it licenses its technology to the rival, and chooses the royalty fee when it licenses; and fourth, each firm sets its price.\(^{12}\)

In the third stage, one of the firms that has a cost advantage over its rival chooses whether it licenses its technology to the rival. Suppose that firm 1 has a cost advantage following its investment in R&D and that the difference between the two firms’ marginal costs is \(d\). We have already shown that, given locations and marginal costs, if the advantaged firm licenses its technology, the profits of the firms are (see the second and the third stages in the previous section):

\[
\pi_1^L = \frac{t(1-x_1-x_2)(3+x_1-x_2)^2}{18} + d, \\
\pi_2^L = \frac{t(1-x_1-x_2)(3-x_1+x_2)^2}{18}.
\]

If the advantaged firm does not license its technology, the profits of the firms are:\(^{13}\)

\[
\pi_1^N = \begin{cases} 
    d - t(1-x_2-x_1)(1-x_1+x_2), & \text{if } d \geq t(1-x_2-x_1)(3-x_1+x_2), \\
    \frac{(d + t(1-x_1-x_2)(3+x_1-x_2))^2}{18t(1-x_1-x_2)}, & \text{if } d < t(1-x_1-x_2)(3-x_1+x_2),
\end{cases}
\]

\(^{12}\) If firms engage in R&D before choosing their location, the analysis of the previous sections directly applies to the problem of location choice and the maximal differentiation result is straightforwardly derived.

\(^{13}\) For the derivations, see Matsumura and Matsushima (2007).
\[ \pi_2^N = \begin{cases} 
0, & \text{if } d \geq t(1 - x_1 - x_2)(3 - x_1 + x_2), \\
\frac{(t(1 - x_1 - x_2)(3 - x_1 + x_2) - d)^2}{18t(1 - x_1 - x_2)}, & \text{if } d < t(1 - x_1 - x_2)(3 - x_1 + x_2). 
\end{cases} \]

From the equations, we have the following lemma.

**Lemma 1** For any \( x_1 \) and \( x_2 \), \( \pi_1^L \geq \pi_1^N \). The equality holds only if \( x_1 + x_2 = 1 \) (i.e., two firms produce homogeneous goods).

**Proof:** A simple calculation leads to

\[ \pi_1^F - \pi_1^N = \begin{cases} 
\frac{t(9 - x_1 + x_2)(3 - x_1 + x_2)(1 - x_1 - x_2)}{18} \geq 0, & \text{if } d \geq t(1 - x_2 - x_1)(3 - x_1 + x_2), \\
\frac{d(2t(6 - x_1 + x_2)(1 - x_1 - x_2) - d)}{18t(1 - x_1 - x_2)} > \frac{d(9 - x_1 + x_2)}{18} > 0, & \text{if } d < t(1 - x_1 - x_2)(3 - x_1 + x_2). 
\end{cases} \]

Therefore, for any \( x_1 \) and \( x_2 \) except \( x_1 + x_2 = 1 \), \( \pi_1^F - \pi_1^N \) is positive. If \( x_1 + x_2 = 1 \), \( \pi_1^F - \pi_1^N = 0 \). Q.E.D.

That is, the advantaged firm has the incentive to license its technology to its rival in the third stage. In the first and the second stages, each firm anticipates the profit functions \( \pi_1^L \) and \( \pi_2^L \).

In the second stage, the results concerning R&D investment affect the value of \( d \) only in \( \pi_1^L \).

In the first stage, the locations are independent from the results in the second stage that are related to the value of \( d \). Thus, the equilibrium locations must be exactly the same as those in Section 4. We summarize the result as follows.

**Proposition 4** Under any cost-reducing R&D firm technology, the optimal locations of the firms are \( x_1 = x_2 = 0 \); that is, maximum differentiation appears in equilibrium.

This result lies in sharp contrast to that of Gerlach et al. (2005) and Christou and Vettas (2005). The possibility of licensing completely changes the equilibrium location.

In this section, we show that for any cost-reducing activities, the maximal differentiation result holds under licensing and the advanced firm has, in fact, an incentive to license its technology. In the next section, we discuss the welfare implications of R&D activity.
6 Efficiency of R&D competition and licensing

In this section we specify how R&D activity affects the firm’s costs and discuss the welfare implications of R&D competition.

The unit cost of the product for each firm is $c_i$ ($i = 1, 2$): this is determined by its investment. Each firm chooses whether to invest to reduce its production costs. Through investment, firm $i$ has to incur the fixed cost of the investment $F$, and it stochastically reduces its marginal production cost from $c$ to $c_i = c - h$ ($h \in [0, \bar{c}]$). $h$ is stochastic and its density function is $f(h)$. After the firms invest, Nature determines $c_1$ and $c_2$.

We can summarize the situation with the following payoff matrix ($I$ and $N$ indicate ‘invest’ and ‘not invest’ respectively).

<table>
<thead>
<tr>
<th></th>
<th>firm 2</th>
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<tbody>
<tr>
<td></td>
<td>$I$</td>
</tr>
<tr>
<td>firm 1</td>
<td>$E[\pi_1(I, I)], E[\pi_2(I, I)]$</td>
</tr>
<tr>
<td>$I$</td>
<td>$E[\pi_1(N, I)], E[\pi_2(N, I)]$</td>
</tr>
</tbody>
</table>

Whether firm 1 invests depends on firm 2’s decision. Thus, we have to evaluate:

$$\Delta_1 \equiv E[\pi_1(I, N)] - E[\pi_1(N, N)], \quad \Delta_2 \equiv E[\pi_1(I, I)] - E[\pi_1(N, I)].$$

$\Delta_1$ is the expected additional profit from the investment given that the rival firm does not invest. $\Delta_2$ is the expected additional profit given that the rival firm does invest. If $\Delta_i \geq F$, the firm has an incentive to invest given that the $i - 1$ firms invest.

We consider two cases: (1) the private and social gains from R&D investment given that the rival firm does not invest; (2) the private and social gains from R&D investment given that the rival firm invests.

**Case (1): Profit** As discussed earlier, if a firm has a cost advantage, it has the incentive to license its own advanced technology for its rival. The firm earns additional profit $d$ from the license, where $d$ is the difference between the marginal costs of the firms. On the other hand, the profit of a firm using the licensed technology is constant, $t/2$. Under Case 1, the
expected additional profit of the investing firm ($\Delta a^1$) is:

$$\Delta a^1 = \int_0^\bar{c} \left( v + \frac{t}{2} \right) f(v)dv - \frac{t}{2} = \int_0^\bar{c} v f(v)dv.$$ 

**Case (1): Welfare**  We now suppose that the social planner anticipates the licensing regime. Regardless of the result of the R&D activities, the quantity supplied by each firm is 1/2 and the prices set by the firms are identical. Therefore, R&D only affects the production costs of the firms. In the licensing regime, the most efficient firm’s technology is available for all firms. For instance, when an innovating firm’s marginal cost becomes $c - h$, then the other firm’s marginal cost also becomes $c - h$ and the social benefit of the R&D is $h \times 1 = h$ (1 is the total quantity supplied by the firms). If one firm invests, the gross benefit of the R&D investment is:

$$\Delta W a^1 = \int_0^\bar{c} v f(v)dv.$$ 

**Comparison (1)**  The difference between $\Delta a^1$ and $\Delta W a^1$ is 0. Thus, we have the following proposition.

**Proposition 5**  Given that a firm does not invest, the private gain from R&D investment by the other firm is equal to the social gain from that R&D.

As mentioned earlier, when an innovating firm’s marginal cost becomes $c - h$, then the other firm’s marginal production cost also becomes $c - h$ and the social benefit of the R&D is $h \times 1 = h$. By licensing, the innovating firm completely takes the social benefit of the R&D, $h$. Therefore, the private gain from R&D is equal to its social gain, thereby resulting in efficient R&D investment. The same principle applies to the following case.

**Case (2): Profit**  The more efficient firm earns additional profit $d$ (the cost difference between the two firms) from the license. In Case 2, the expected additional profit of the second investing firm ($\Delta a^2$) is:

$$\Delta a^2 = \int_0^\bar{c} \int_0^{\bar{c}} (v - \min\{v, t\}) f(t)df(v)dv$$

$$= \int_0^\bar{c} \int_0^v (v - t) f(t)df(v)dv.$$
\( v - \min\{v, t\} \) is the level of the \textit{ex post} cost advantage by the investment, where \( t \) is the level of the rival firm’s cost reduction and \( v \) is the level of its own cost reduction. If \( v > t \), the level is positive and the firm gains additional profit \( v - t \), otherwise the additional gain is zero.

**Case (2): Welfare** If the most efficient innovating firm’s marginal cost becomes \( c - h \), then the other firm’s marginal cost also becomes \( c - h \) and the social benefit of the R&D is \( h \times 1 = h \) (1 is the total quantity supplied by the firms). If both firms invest, the \textit{gross} benefit of the R&D investment is:

\[
\Delta W^a_2 = \int_0^c \int_0^c \max\{v, t\} f(t) df(v) dv
\]

\[
= \int_0^c \int_0^v v f(t) df(v) dv + \int_0^c \int_v^c t f(t) df(v) dv.
\]

\( \max\{v, t\} \) is the level of the efficient firm’s cost reduction. If \( v > t \), the decrease in the social marginal cost is \( v \), otherwise \( t \). To derive the \textit{additional} benefit of the second R&D investment, we change the expression \( \Delta W^a_1 \) as follows:

\[
\Delta W^a_1 = \int_0^c \left( \int_0^v v f(t) dt + \int_v^c v f(t) dt \right) f(v) dv.
\]

The additional social benefit of the second R&D investment is:

\[
\Delta W^a_2 - \Delta W^a_1 = \int_0^c \int_0^c (t - v) f(t) dt f(v) dv.
\]

**Comparison (2)** The difference between \( \Delta^2_2 \) and \( \Delta W^a_2 - \Delta W^a_1 \) is:

\[
\Delta^2_2 - (\Delta W^a_2 - \Delta W^a_1) = \int_0^c \int_0^c v f(t) df(v) dv + \int_0^c \int_0^c v f(t) df(v) dv
\]

\[
- \left( \int_0^c \int_0^v t f(t) df(v) dv + \int_0^c \int_v^c t f(t) df(v) dv \right)
\]

\[
= \int_0^c \int_0^v v f(t) df(v) dv - \int_0^c \int_0^c t f(t) df(v) dv
\]

\[
= \int_0^c v f(v) dv - \int_0^c t f(t) dt \times \int_0^c f(v) dv
\]

\[
= 0.
\]

From the discussion above, we have the following proposition.
**Proposition 6** Given that a firm invests, the private gain from R&D investment by the other firm is equal to the social gain from that R&D.

Propositions 5 and 6 indicate that socially efficient R&D investment is achieved by the firms without governmental intervention. Our result suggests that government intervention in the advanced firm’s licensing strategies (such as regulation of the license fee) reduces the incentive for innovation. This results in a loss of social surplus.\(^{14}\) This result is crucially dependent on the assumption that only the patentee has bargaining power. If the licensee, along with the patentee, has bargaining power, the incentive for R&D investment is insufficient. Even so, any government intervention that reduces the profits or bargaining power of the patentee in our setting reduces total social surplus.

### 7 R&D investments with and without licensing

Finally, we discuss whether the licensing stimulates R&D investment. We consider the following two cases: (a) each firm licenses its technology whenever it has a cost advantage; (b) neither firm licenses its advanced technology. We compare the R&D incentives in case (a) to case (b).

The basic setting is the same as the former section, except for the following simplification. By its investment, firm \(i\) reduces its marginal production cost at \(c_i = c - \bar{c}\) with probability \(p\); \(c_i = c - \bar{c}/2\) with probability \(q\); \(c_i = c\) with \(1-p-q\). As mentioned in the former section, to check the incentive to invest, we now evaluate:

\[
\begin{align*}
\Delta_1^b &= E[\pi_1(I, N)] - E[\pi_1(N, N)], \\
\Delta_2^b &= E[\pi_1(I, I)] - E[\pi_1(N, I)].
\end{align*}
\]

\(\Delta_1^b\) is the expected additional profit from the investment given that the rival firm does not invest. \(\Delta_2^b\) is the expected additional profit given that the rival firm invests. If \(\Delta_1^b \geq F\), the firm has the incentive to invest, given that \(i - 1\) firm invests.

**Case (1)** Suppose that firm 2 does not invest. We compare firm 1’s (gross) marginal gain for R&D investment between cases (a) and (b). Given the difference between the (gross)

\(^{14}\) For example, in 1975 the US Federal Trade Commission ordered Xerox to license, at a nominal cost, its patents on electrostatic copying to all entrants.
marginal gains in the cases (a) and (b) is:

$$\Delta_1^a - \Delta_1^b = \bar{c}\left(4(12t - \bar{c})p + (24t - \bar{c})q\right) > 0,$$

where $\Delta_1^a$ is defined in Section 6. Regardless of $p$, $q$, $t$, and $h$, the R&D incentive under the licensing case is stronger than under the noncooperative case (the without licensing case). There are two effects behind this result. When the innovating firm licenses its technology, both firms’ costs are reduced and the licensor obtains all of the gain. On the contrary, without licensing, only one firm’s cost reduces, resulting in a smaller gain for the innovator. The second effect is that the innovation accelerates competition in the without licensing case, while no such effect exists in the licensing case. This yields another disincentive for innovation in the without licensing case.

**Case (2)** Suppose that firm 2 invests. We compare firm 1’s (gross) marginal gain for R&D investment between cases (a) and (b). The difference between the (gross) marginal gains in cases (a) and (b) is:

$$\Delta_2^a - \Delta_2^b = \bar{c}\left[12\left(2p + q\right) - 3\left(2p^2 + 2pq + q^2\right)\right]t - (4p + q - 2(2p + q)^2)\bar{c} > 0, \tag{11}$$

where $\Delta_2^a$ is defined in Section 6. The sign in $\Delta_2^a - \Delta_2^b$ depends on the values of $p$, $q$, $t$, and $h$. Figure 1 depicts the regions where the sign is negative and positive when $\bar{c} = t/10$.15

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15 The shape does not depend very much on the value of $\bar{c}/t$. When $p + q$ is not large enough, the R&D incentive under the licensing case is stronger than under the noncooperative case. We explain the underlying intuition as follows. Consider a case in which $p+q$ is large enough, that is, the probability of success in R&D is large enough. In the licensing case, given the rival firm invests, the probability that the other firm has a cost advantage over its rival is small, because the probabilities of success are high enough to allow both firms to succeed in their R&D investments. For instance, if the marginal cost
of an investing firm is \( c - d \) with probability 1, the additional benefit of the second R&D investment is zero. On the contrary, in the nonlicensing case, the additional gain of the second R&D investment is strictly positive, even when \( p + q = 1 \). As opposed to the licensing case catching up the advanced firm increase the profit of the firm with cost disadvantage.

8 Concluding remarks

We have investigated the relationship between licensing activities and equilibrium locations in a product differentiation model. We also discuss the welfare implications of R&D investment. We show that the level of royalty fee does not affect equilibrium locations or the equilibrium profit of the licensor, and that firms choose the maximal license fee. We show that licensing activities after R&D always lead to maximum differentiation between firms and mitigate price competition. Furthermore, we show that these licensing activities induce socially optimal R&D.

Our results have testable implications for the licensing literature. We have shown that an efficient firm has an incentive to transfer its advanced technology to its rival and that the transfer mitigates competition between these firms. In our setting, we have implicitly assumed that the technology is transferable. On the other hand, we believe that the results derived by Gerlach et al. (2005) and Christou and Vettas (2005) are suitable for industries where technological transfer is unavailable or difficult. That is, spatial agglomeration leading to tough competition appears if technological transfers are difficult.

Given our results, we regard the following as an important research question: are the price–cost margins of firms positively related to technological transferability which, in turn, is positively related to the propensity to license? If technologies are transferable, price competition is moderate and firms are able to earn greater than normal profits. On the other hand, if technologies are not transferable, tough competition appears and firms may earn normal or subnormal profits. Further research is required to consider this problem.

In this paper, we assume that the licensor can transfer advanced technology without any transaction costs. However, some technologies are difficult to transfer, and firms can choose whether they invest in nontransferable or transferable technology. We believe that
the competitive structure affects this choice and this is an important problem in the context of R&D investment. This remains a topic for future research.
Figure 1: R&D incentives

Horizontal: $p$, Vertical: $q$, The shaded area: $\Delta_2^g - \Delta_2^h < 0$, The white area: $\Delta_2^g - \Delta_2^h > 0$
References


