Relation Specific Investment with Cooperativity and Efficient Breach Remedy *

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Abstract

In this paper, we investigate efficient breach remedies for the buyer-seller relationship with one-sided relation specific investment which affects both the seller’s cost and the buyer’s value. We focus on the situation where renegotiation is possible.

To begin with, we analyze two standard rules that are the expectation damages (ED) and the reliance damages (RD). The efficiency of ED depends on purely selfishness of the investment, which means that the investment is beneficial only for the investor. On the contrary RD can achieve the first-best investment not only if the investment is purely cooperative which is Che and Chung (1999)’s result, but also even if the investment is partially selfish. Moreover, we show that if the court has ex post information sufficiently, there exists a breach remedy that can implement the first best investment no matter how the nature of the investment is.

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From these analyses, we conclude that breach remedies can be a useful solution for holdup. And we claim that the court who has ideal power and information can establish a foundation of incomplete contract as long as there are minute contract writing costs.

Keyword: Cooperative Investment, Damage Rules, Breach Remedy

JEL classification: D86, K12, K41, C7

1 Introduction

Relation specific investments are what create the trade value in a bilateral relationship. It is known that if the parties in a relationship cannot write an ex ante contract, they fail into the holdup problem where the parties invest less than the socially efficient levels. Once the investment was sunk, the parties are faced with ex post negotiation and share the trade surplus. Anticipating that what invested ex ante can only gain a partial trade surplus, the parties will invest under the efficient levels. An investment-contingent contract (hereafter complete contract) is one of the well-worked solution of this problem. Nevertheless there are many situations where complete contract cannot be written since investments are often nonmonetary and intangible and then cannot be objectively measured.

The holdup problem is still studied in many papers and the features and possibilities to solve holdups are cleared up. In some cases, holdups can be solved by simple contracts (fixed price contract, option contract) and/or the appropriate bargaining process\(^1\). On the other hand, however, there are some negative results as well. One of the meaningful results is demonstrated by Che and Hausch (1999). The efficiency results mentioned above are demonstrated in the case where the investment is purely selfish, which means that the

\(^{1}\)If you want to get the details, see the follows: Chung (1991) and Aghion et al. (1994) (endogenous bargaining and renegotiation process) and Nöldeke and Schmidt (1995, 1998) (buying option contract). See also Hart (1995), which is an excellent survey.
investment is beneficial only for the investor. However, they stress the cooperativity of the investment, which means that the investment directly affects the partner’s benefit, and show that if the investment is sufficiently cooperative and renegotiation cannot be prohibited, the parties cannot achieve the efficient investment by contracts indicating the trade quantity and the trade price.

As Che and Hausch (1999) state, cooperative investment is an important issue. For instance, standard moral hazard models include this cooperativity because the agent’s investment (or effort) affects the principal’s benefit. Moreover, many situations hold the cooperative feature in practice. One of the examples is that in supplier-maker relationships, where the supplier decides to build their plants. The distance between each factory affects the shipping cost and reliability, which is the supplier’s concern. Also the environmental factors such as temperature and water near the plant affect the quality of their goods, which is the maker’s concern.

Che and Hausch (1999) state that cooperative investments seriously induce holdups. We will challenge on this statement, i.e. how we can solve the inefficiency of cooperative investments. One of the instruments to solve this problem is a damage rule which is executed when either party breach their contract.

Most studies in contract theory assume that the court, which is the third party to enforce the contracts, will execute sufficient large punishment on the party defaulting their contract and then the parties will not break their contract as a consequence. However, these situations are somewhat strange because we can observe breaches of contracts in practice. Actually, in many lawsuits, we have observed two typical damage rules; one is the expectation damage, the other is the reliance damage. The expectation damage, which is often applied in the U.S., obliges the breaching party to make up the victim of the breach for the benefit the victim would have received if the contract had enforced. The reliance damage, on the other
hand, compensates the expenses in anticipation of the contract performance.

Some researchers show that the efficient investment can be achieved by using a simple contract under these damage rules. Edlin (1996) and Edlin and Reichelstein (1996) show that if the investment is purely selfish, the expectation damage can be a instrument to induce the efficient investment. On the other hand, Che and Chung (1999) show that if the investment is purely cooperative, the reliance damage can induce the efficient investment.

However, these investigation are not comprehensive because they study only situations of purely selfish or purely cooperative. As we mentioned the example of plant location above, we should also consider hybrid investments, which mean that the investment directly affects both parties’ benefit. One of our two main questions is that whether these two damage rules can work as a instrument to attain the efficient investment when the nature of the investment is hybrid.

We construct a model of a simple buyer-seller relationship with one-sided investment and analyze these damage rules. The answer is that the expectation damage cannot work in hybrid investment situation although the reliance damage can work as long as the investment is not only purely cooperative but also sufficiently cooperative. The difference between two damage rules is whether the fixed price contract can become a threat of inefficient investment or not. Because under the expectation damage the parties’ decision whether or not to breach depends on now desirable the trade is, the level of the price of the contract cannot affect the parties’ breach decision. Then the expectation damage, in general, cannot induce the efficient investment. On the other hand, under the reliance damage the parties’ decision whether or not to breach depends on the level of the price of the contract. Then if the

\[\text{\textsuperscript{2}}\]

The statement about the expectation damages in Edlin and Reichelstein (1996) itself may be negative result; they show that the expectation damages cannot induce the efficient investment under two-sided investments although the efficiency can be achieved under one-sided investment.

\[\text{\textsuperscript{3}}\]

Korakayama et al. (1986) and Ishiguro (1999) also show the efficiency of the expectation damages. The former includes hidden knowledge problem between the parties, and the latter does hidden action problem.
investment is sufficiently cooperative, the parties can induce the efficient investment through the appropriate price setting.

However, these results are somewhat pessimistic because even if the court can select the expectation damage or the reliance damage corresponding to the nature of the investment, there remain some possibilities that the parties cannot achieve the efficient investment. Then we permit the court to execute more general damage rule than the expectation damages and the reliance damages and investigate whether or not there is a damage rule under which the parties can induce the efficient investment. This is the other of our two main questions. The answer is that if the court has sufficient ex post information, we can implement the efficient investment for any nature of the investment. As we will call this procedure "always breaching implementation" below, in this procedure, the parties write a sufficiently high price contract, and the buyer always breaches and needs to pay the damage to the seller. At this time, the court sentences the buyer to pay so that the seller (the investor, here) were faced with the investment problem maximizing the trade surplus. Then the seller invests at the efficient investment level. The situation where the party always breach may seem to be peculiar. However, because we assume that renegotiation is always available through the analyses, the outcome of always breaching implementation can be interpreted as a result through renegotiation, which is a realistic interpretation.

These efficiency results offer us two realistic concerns. One is a judicial policy of breach remedies. We show that judicial intervention affects the reliance expenditure of the economic environments and that the economic parties always can achieve the efficiency through the judicial direct or indirect effects. Then it is important for the ex ante efficiency to ensure the appropriate rule. The other concern is a foundation of incomplete contract. Our result means that the parties can attain the first best even if they do not use complete contracts. Then they need not write complete contracts to attain the efficiency. As Tirole (1999) claims,
this phenomenon offers the reason why we often write only a simple contract in practice.

The rest of this paper is organized as follows. Section 2 describes our basic setup, and in section 3 we analyze the player’s breaching decisions. Section 4 presents the result under the standard breach remedies, and section 5 presents the efficiency of always breaching implementation we offer there. Section 6 discusses three concerns, the mechanism of each breach remedy, policy implications and foundations of incomplete contracts. In section 7 we extend our model and also show the efficiency of always breaching implementation. Section 8 concludes. Almost all the proofs are in the appendix.

2 The Model

Consider a simple buyer-seller relationship a la Hart and Moore (1988). A buyer (female, hereafter B) and a seller (male, hereafter S) will trade one indivisible good. S has a investment decision prior to the trade. If S invests with cost \( a \in [0, \pi] \), the value of the good for B becomes \( v(a) \) and the cost of the good for S becomes \( c(a) \). We assume that \( v \) and \( c \) are twice differential and \( v - c \) is strictly increasing and strictly concave in \( a \). Moreover we also assume that \( v(0) < c(0) \) and \( c(\pi) < v(\pi) < +\infty \). We summarizing these setups, the trade is inefficient under no investment but the trade is efficient at a sufficient high investment level. And by the mean value theorem and strictly increasingness of \( v - c \), there uniquely exists \( \hat{a} \) such that \( v(\hat{a}) = c(\hat{a}) \), and

\[
\begin{align*}
v(a) &< c(a) \quad \text{for all } a \in [0, \hat{a}) \\
v(a) &\geq c(a) \quad \text{for all } a \in [\hat{a}, \pi].
\end{align*}
\]
We define $S(a)$ as ex post efficient surplus, that is to say

$$S(a) := \max\{v(a) - c(a), 0\}$$

$$= \begin{cases} 
  v(a) - c(a) & \text{if } a \in [\hat{a}, \bar{a}] \\
  0 & \text{if } a \in [0, \hat{a}). 
\end{cases}$$

In section 4, to ensure the interior solution\(^4\), we will focus on the following technology:

**Purely Selfish (PS)**: $v' = 0$ and $c' > 0$ for all $a$. In this case, $c$ must be strictly convex.

**Purely Cooperative (PC)**: $v' > 0$ and $c' = 0$ for all $a$. In this case, $v$ must be and strictly concave.

**Hybrid Investment (HI)**: $v' > 0$ and $c' > 0$ for all $a$. In this case, to assure an interior solution, we also assume that

- $\lim_{a \to +0} v'(a) = \infty$
- $\lim_{a \to +0} [-c'(a)] = \infty$
- Both $v$ and $c$ are strictly concave.

In addition, we also assume that $\lim_{a \to \pi-0}[v'(a) - c'(a)] = 0$ for each case.

Before the investment, the parties can write a contract $w := (t, p)$, that is, $t$ is up front fixed transfer from B to S (which may be negative) and $p$ is the price of the good. Because of $t$, we can apply Coase theorem by which we need not consider the participation constraint. However since the amount of $t$ is not an intrinsic factor in this paper, we ignore $t$ from now on.

\(^4\)In section 5 and 7, our analyses do not depend on the existence of the interior solution. Then we do not have to assume these conditions.
If either party (party $i$, here) decides to breach the contract, the other party (party $j$, here) must decide whether or not to go to the court to sue. When $j$ decides not to sue, trade is not formed and no damage occur. If $j$ decides to sue, the court sentences party $i$ to pay $j$ \( g_i(a \mid w) \). \{g_i(a \mid w)\}_{i=S,B} \) is the damage rule depending on $a$ and the parties’ contract $w$.\footnote{We will assume that the court has full information about this game structure. Then $g_i$ may implicitly depend on $v(a)$, $c(a)$ and $\alpha$ which is $S$’s bargaining power we will denote afterward.}

We assume that if the damages realize, the contract is voided.

Moreover, we assume that $S$ and $B$ can always renegotiate, and if they can Pareto-improve their payoff, the renegotiation is concluded. We postulate that when they renegotiate and try to share the surplus they produced by their renegotiation, $B$ and $S$ will achieve Nash bargaining solution and share it in the ratio of $(1 - \alpha)$ to $\alpha$ where $\alpha \in (0, 1)$.

We follow the timing of the model;

Stage -1: The court offers the damage rule $\{g_i(a \mid w)\}_{i=S,B}$.

Stage 0: If they want, the parties write a contract $w$.

Stage 1: $S$ invests $a$.

Stage 2: Each $B$ and $S$ (noncooperatively and simultaneously) decides whether or not to breach.

Stage 3: If either party breaches, the other party decides whether to sue or not. If (s)he sues, the damage $g_i$ is invoked.

Stage 4: The parties renegotiate.

Notice that, even if we permit renegotiation for all the stages, we can restrict our attention about effects of renegotiation to the stage after suing. The reason why we can do so is that in this setting, without loss of generality, there is no effect of renegotiation except for after suing. Renegotiation before the investment means only rewriting of the contract. And,
after the investment but before the litigation, the surplus becomes fixed value and the rule of sharing it is stipulated by the contract $p$. Then Pareto improvement can never realize through renegotiation.

We will derive subgame perfect equilibria of this game and, in the following analyses, we will mainly focus on the equilibrium investment. Before the derivation of the equilibrium, we now consider two benchmark investments.

First, we define the first best (efficient) investment $a^{FB}$ as the investment which maximize the surplus given the process of efficient breach decision. Then the first best investment maximizes the ex post surplus subtracted by the investment cost. Hence, $a^{FB} \in \arg \max_a [S(a) - a]$. We presume that $a^{FB} > 0$ which implies that $a^{FB}$ is the maximizer of $v(a) - c(a) - a^6$. Then we can use the first order condition to characterize $a^{FB}$. Thus,

$$v'(a^{FB}) - c'(a^{FB}) = 1.$$

Another benchmark is $a^{NO}$ which is the investment when the parties do not write any contracts. After the investment, the parties share the trade surplus under Nash Bargaining. Then S’s gains $\alpha S(a)$ by the trade. Foreseeing this outcome, S maximizes $[\alpha S(a) - a]$. Notice that $a^{NO}$ might be 0. Even if $a^{NO} > 0$, by taking the first order condition,

$$v'(a^{NO}) - c'(a^{NO}) = \frac{1}{\alpha},$$

which implies $a^{FB} > a^{NO}$. Then we can confirm that if the parties cannot write ex ante contracts, they must be confronted with a holdup problem.

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6) Notice that this assumption implies $a^{FB} \geq \hat{a}$ because if $a^{FB} < \hat{a}$, $S(a^{FB}) - a^{FB} = -a^{FB}$ which is less than $S(0)$. 

9
3 Breach Decision

We first consider stage 2 where B and S decide whether breach or not given \( w \) and \( a \).

The Case Where Both Decide to Not Breach   Suppose that both honor the contract. Then because B pays S \( p \) and they trade, B gains \( v(a) - p \) and S gains \( p - c(a) \). However if trade is inefficient, they can stop the trade and increase the surplus by \( c(a) - v(a) \) by renegotiation. Then the renegotiation surplus becomes \( \max\{c(a) - v(a), 0\} \) and since the parties split it by Nash bargaining, their total surpluses are, respectively, \( v(a) - p + (1 - \alpha) \max\{c(a) - v(a), 0\} \) and \( p - c(a) + \alpha \max\{c(a) - v(a), 0\} \).

The Case Where Either S or B Decides to Breach   If S decides to breach, B decides whether or not to sue. If B takes action, B gains \( g_S(a \mid w) \) from S, and if B does not, B has no gain from damage. Thus B sues if and only if \( g_S(a \mid w) \geq 0 \). Moreover, since contract is no longer enforced whether or not to sue, the renegotiation surplus is \( S(a) \). Then we can write the parties surplus as respectively, \( \max\{g_S(a \mid w), 0\} + (1 - \alpha)S(a) \) and \(-\max\{g_S(a \mid w), 0\} + \alpha S(a) \).

The case where B decides to breach can be analyzed similarly. The parties surplus are respectively, \(-\max\{g_B(a \mid w), 0\} + (1 - \alpha)S(a) \) and \( \max\{g_B(a \mid w), 0\} + \alpha S(a) \).

The Case Where Both Decide to Breach   We will postulate that if both parties breach, the contract is voided. Then they share the renegotiation surplus \( S(a) \). Hence the parties surplus are respectively, \((1 - \alpha)S(a) \) and \( \alpha S(a) \).

Table 1 is the game matrix at stage 2.

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7) We assume that the victim go to the court if \( g_S(a \mid w) = 0 \).
8) How the court should do in the both breaching case is still controversial. See Edlin (1996) for details.
9) “b” means “breach”, and “nb” means “not breach”. And for each cell, the upper is B’s payoff and the lower is S’s payoff (not including his investment cost). We will adopt this expression in the following matrixes.
Table 1: Game matrix at stage 2

<table>
<thead>
<tr>
<th></th>
<th>b</th>
<th>nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>$(1 - \alpha)S(a)$</td>
<td>$-\max{g_B(a</td>
</tr>
<tr>
<td>nb</td>
<td>$\max{g_S(a</td>
<td>w), 0} + (1 - \alpha)S(a)$</td>
</tr>
<tr>
<td></td>
<td>$\alpha S(a)$</td>
<td>$p - c(a) + \alpha \max{c(a) - v(a), 0}$</td>
</tr>
</tbody>
</table>

In the breach stage, we will lay two assumptions. One is that the action profile that both parties breach is never realized even though it might weakly be Nash equilibrium theoretically. Second, if breaching is indifferent to no-breaching for the party, (s)he will choose no-breaching.

4 Under the Standard Breach Remedies

We now compare the investment level when the court takes standard breach remedies, which are the expectation damages (hereafter ED) and the reliance damages (hereafter RD).

Before considering the damage rules, we define the following concept for the convenience of our explanation.

**Definition 1** We call $\hat{p}(a; \alpha) := \alpha v(a) + (1 - \alpha)c(a)$ the renegotiation price.

This renegotiation price is what B pays S if renegotiation is profitable. As we argue in the following, it is important that the renegotiation price may be a criterion of the parties breach decision.

**Specific Performance** Before ED and RD, we will show the investment under the specific performance rule (hereafter SP) which always forces the parties to carry out the contract.

In this case, we can interpret that the damages $g_i(a | w) (i = B, S)$ are sufficiently high, or extremely $+\infty$. Thus the equilibrium breach decision must become (nb , nb) in table 1.
Then S invests to maximize \( p - c(a) + \alpha \max\{c(a) - v(a), 0\} - a \), which is

\[
\begin{cases} 
  p - c(a) - a & \text{if } a \geq \hat{a} \\
  p - \hat{p}(a; \alpha) - a & \text{if } a < \hat{a} 
\end{cases}
\]

By the way, if the nature of the investment is (PS) or (HI), there uniquely exists \( a \in A \) such that \(-c'(a) = 1\). We define \( \bar{a}^{SP} \) as

\[
\begin{cases} 
  -c'(\bar{a}^{SP}) = 1 & \text{for (PS) or (HI)} \\
  \bar{a}^{SP} = 0 & \text{for (PC)}.
\end{cases}
\]

Notice that \( \bar{a}^{SP} \) is the maximizer of \( p - c(a) - a \). Using \( \bar{a}^{SP} \), we can show the equilibrium investment under SP.

**Lemma 1** The equilibrium investment under SP, which is \( a^{SP} \), satisfies

\[
\begin{cases} 
  a^{SP} \leq \bar{a}^{SP} & \text{if } \bar{a}^{SP} \geq \hat{a} \\
  a^{SP} \leq \hat{a} & \text{if } \bar{a}^{SP} < \hat{a}.
\end{cases}
\]

**Proof** See the appendix. \( \square \)

Notice that \( a^{FB} \geq \bar{a}^{SP} \geq 0 \) with first equality only if the nature of investment is (PS)\(^{10}\). Then this lemma implies that the parties can implement the first best investment only when the investment is purely selfish\(^{11}\).

\(^{10}\) In (PS) or (HI) case, \(-c'(\bar{a}^{SP}) = 1 = \nu'(a^{FB}) - c'(a^{FB}) \geq -c'(a^{FB}) \) with equality only if the nature of investment is (PS).

\(^{11}\) Che and Hausch (1999) shows that even if the investment is not purely selfish, the first best investment can be achieved under SP as long as the investment is not only purely selfish, but also sufficiently selfish. The difference between our result and theirs is that, in Che and Hausch’s model, the parties are faced with decision making of the quantity traded between them. If we permit the parties to commit stochastic trading by the contract (i.e. they can write a probability of trade in the contract), our result is consistent to that of Che and Hausch (1999). However, here we ignore the stochastic trading of indivisible goods because it is unrealistic.
**Expectation damages**  ED forces the breaching party to compensate the expected profit the breach victim would have gain if the breach party had not breached. This expected profit is $p - c(a)$ for $S$ and $v(a) - p$ for $B$. Then we can write $g_B(a \mid w) = p - c(a)$ and $g_S(a \mid w) = v(a) - p$. Table 2 is the game matrix at stage 2 under ED.

<table>
<thead>
<tr>
<th>B \ S</th>
<th>b</th>
<th>nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>$(1 - \alpha)S(a)$</td>
<td>$- \max{p - c(a), 0} + (1 - \alpha)S(a)$</td>
</tr>
<tr>
<td></td>
<td>$\alpha S(a)$</td>
<td>$\max{p - c(a), 0} + \alpha S(a)$</td>
</tr>
<tr>
<td>nb</td>
<td>$\max{v(a) - p, 0} + (1 - \alpha)S(a)$</td>
<td>$v(a) - p + (1 - \alpha)\max{c(a) - v(a), 0}$</td>
</tr>
<tr>
<td></td>
<td>$- \max{v(a) - p, 0} + \alpha S(a)$</td>
<td>$p - c(a) + \alpha \max{c(a) - v(a), 0}$</td>
</tr>
</tbody>
</table>

Table 2: Game matrix at stage 2 under ED

Analyzing whether or not to breach, we can prove following lemma.

**Lemma 2** *Suppose that the court applies ED. Then if trade is efficient (i.e. $a \geq \hat{a}$), breaches never occur. And if trade is inefficient (i.e. $a < \hat{a}$), breaches always occur except for the case where $p = \hat{p}(a; \alpha)$. Given $a < \hat{a}$, if $p > (\geq)\hat{p}(a; \alpha)$, $B$ ($S$) breaches.*

**Proof**  *See the appendix.*  □

This lemma implies that when $a \geq \hat{a}$, $S$’s payoff is $p - c(a) - a$. And when $a < \hat{a}$, $S$’s payoff depends on the difference between $p$ and $\hat{p}(a; \alpha)$, which is summarized as follows;

\[
\begin{align*}
\begin{cases}
    p - c(a) - a & \text{if } a \geq \hat{a} \\
    p - c(a) - \alpha[c(a) - v(a)] - a & \text{if } a < \hat{a} \text{ and } p = \hat{p}(a; \alpha) \\
    - \max\{v(a) - p, 0\} + \alpha S(a) - a & \text{if } a < \hat{a} \text{ and } p < \hat{p}(a; \alpha) \\
    \max\{p - c(a), 0\} + \alpha S(a) - a & \text{if } a < \hat{a} \text{ and } p > \hat{p}(a; \alpha).
\end{cases}
\end{align*}
\]

However, the second line of the equations above is $-a$ because of $\hat{p}(a; \alpha)$, and the third equation(respectively, fourth one) is equal to $- \max\{v(a) - p, 0\} - a$ (resp. $\max\{p - c(a), 0\} - a$).
a) since \( S(a) = 0 \). Then we can replace above equations as

\[
\begin{cases}
p - c(a) - a & \text{if } a \geq \hat{a} \\
-a & \text{if } a < \hat{a} \text{ and } p = \hat{p}(a; \alpha) \\
-\max\{v(a) - p, 0\} - a & \text{if } a < \hat{a} \text{ and } p < \hat{p}(a; \alpha) \\
\max\{p - c(a), 0\} - a & \text{if } a < \hat{a} \text{ and } p > \hat{p}(a; \alpha).
\end{cases}
\]

Here, we can prove that S’s investment under ED, \( a^{ED} \), has a property similar to that of SP.

**Proposition 1** Suppose that the court applies ED. The equilibrium investment \( a^{ED} \), satisfies

\[
\begin{cases}
a^{ED} \leq \pi^{SP} & \text{if } \pi^{SP} \geq \hat{a} \\
\pi^{SP} \leq \hat{a} & \text{if } \pi^{SP} < \hat{a}.
\end{cases}
\]

**Proof** See the appendix. □

By the same logic of SP situation, the first best investment can be achieved only if the investment is (PS) which implies \( a^{FB} = \pi^{SP} \). The efficiency result of ED by Edlin (1996) and Edlin and Reichelstein (1996) depends on purely selfishness of the investment\(^{12}\).

Lemma 2 implies that ex post deciding whether or not to breach is almost always efficient\(^{13}\). For selfish investment, ED can sufficiently incentivize the investor to invest because she can be compensated for her cost reduction\(^{14}\). For cooperative investment, however, she

\(^{12}\)Stochastic trading according to the contract can lead the first best investment even if the investment is not purely selfish. This is same as SP case.

\(^{13}\)This ex post efficiency are consistent to previous literatures such as Edlin (1996) and Che and Chung (1999).

\(^{14}\)Shavell (1980) and Rogerson (1984) show that when the investment is purely selfish, overinvestment (which is larger investment than the first best) can occur under ED which cannot occur in our model. This difference is caused by uncertainty after investment which determines whether or not trade is efficient. The parties make an efficient decision about breaching even in their model. Then the investor is assured his payoff under the contract enforced, regardless breach occurs or not, and then the return from the investment becomes too large. Thus if investment is sufficiently selfish, the investor has an incentive to overinvest above the first best.
cannot be compensated for value-increasing investment even if investment is efficient or not. Then ED cannot work to motivate cooperative investment.

Notice also that if $\bar{a}^{SP} \leq \hat{a}$, S never invests at the level more than $\hat{a}$. Then the equilibrium investment $a^{ED}$ satisfies

$$S(a^{ED}) - a^{ED} = -a^{ED} \leq 0$$

which means social surplus with $a^{ED}$ is worse than that of no investment. This futile investment can occur not only when $\bar{a}^{SP} \leq \hat{a}$, but also when $v(\bar{a}^{SP}) - c(\bar{a}^{SP}) - \bar{a}^{SP} < 0$ \(^{15}\). If so, the damage rule and their ex ante contract have only the bad effect to on the investment.

**Reliance Damage**  RD forces the breach party to compensate the victim the loss so as to return to the situation before the contract. This loss is $a$ for S and 0 for B. Then we can write $g_B(a \mid w) = a$ and $g_S(a \mid w) = 0$. Table 3 is the game matrix at stage 2 under RD.

<table>
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<tbody>
<tr>
<td>b</td>
<td>$(1 - \alpha)S(a)$</td>
<td>$-a + (1 - \alpha)S(a)$</td>
</tr>
<tr>
<td></td>
<td>$\alpha S(a)$</td>
<td>$a + \alpha S(a)$</td>
</tr>
<tr>
<td>nb</td>
<td>$(1 - \alpha)S(a)$</td>
<td>$v(a) - p + (1 - \alpha)\max{c(a) - v(a), 0}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha S(a)$</td>
<td>$p - c(a) + \alpha \max{c(a) - v(a), 0}$</td>
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Table 3: Game matrix at stage 2 under RD

Analyzing whether or not to breach, we can prove following lemma.

**Lemma 3** Suppose that the court applies RD. Then breaches do not occur when $\hat{p}(a; \alpha) + a \geq p \geq \hat{p}(a; \alpha)$. B breaches if and only if $p > \hat{p}(a; \alpha) + a$ and S breaches if and only if $p < \hat{p}(a; \alpha)$.

**Proof** See the appendix. □
Lemma 3 means that $\hat{p}(a;\alpha)$ is the central criterion of breaching under RD, although whether or not breaches occur seldom depends on $\hat{p}(a;\alpha)$ under ED.

S’s payoff is then

$$
\begin{cases}
\alpha S(a) & \text{if } p > \hat{p}(a;\alpha) + a \\
 p - c(a) - a + \alpha \max\{c(a) - v(a), 0\} & \text{if } \hat{p}(a;\alpha) + a \geq p \geq \hat{p}(a;\alpha) \\
\alpha S(a) - a & \text{if } p < \hat{p}(a;\alpha).
\end{cases}
$$

Under RD, can the parties achieve the efficient investment? Che and Chung (1999) show that if the investment is (PC), the equilibrium investment becomes the first best. Now we will show that this result is robust even if the investment is not (PC)

**Proposition 2** Suppose that the court applies RD. If $\partial \hat{p}(a;\alpha)/\partial a > 0$ for all $a$, then there exists $w$ such that the first-best investment can be achieved.

**Proof** See the appendix. □

Then the sufficient condition of the efficiency of RD is $\partial \hat{p}(a;\alpha)/\partial a > 0$ for all $a$. Notice that because (PC) implies $\hat{p}(a;\alpha) > 0$, our result includes that of Che and Chung (1999).

How do the parties achieve the efficient investment under RD? As we indicate in the proof, the contract which can induce the first best is $p = \hat{p}(a^{FB};\alpha) + a^{FB}$. Then as lemma 3 shows, whether or not B honors the contract is dependent on whether or not the investment is above the first best level. Then under RD, the parties can use the contract as a threat of the punishment to the inefficient investment.

What does $\partial \hat{p}(a;\alpha)/\partial a$ mean? $\partial \hat{p}(a;\alpha)/\partial a > 0$ implies $\alpha v'(a) > |(1-\alpha)c'(a)|$. It means that the effect of the investment to B’s value weighted on S’s bargaining power (i.e. $\alpha v'(a)$) is larger than that of S’s cost weighted on B’s bargaining power (i.e. $|(1-\alpha)c'(a)|$). We can interpret $v'(a)$ as sensitivity of the investment to B’s value (or ”cooperativeness”) and $c'(a)$...
as sensitivity of the investment to S’s cost (or ”selfishness”). Then to satisfy \( \partial \hat{p}(a; \alpha)/\partial a > 0 \), we need that the degree of cooperativeness \( (v'(a)) \) and S’s bargaining power \( \alpha \) are sufficient high relative to the degree of selfishness \( (c'(a)) \). As long as this condition holds, Che and Chung’s efficiency result is robust even if the investment is not purely cooperative.

If the investment is sufficiently cooperative and S has large bargaining power, the renegotiation price is increasing in \( a \). Then S’s benefit through renegotiation is also increasing in \( a \). Then, if \( p \) is high, S has no incentive to underinvestment because B breaches. However \( a \) reaching some level, because the payment of RD and the renegotiation price is relatively high to the contract price \( p \), B will honor the contract. If B will not breach, S does not have an incentive to investment more because the investment affects only B’s value. Then overinvestment does not occur. Setting such \( a \) mentioned above, the parties can attain the first best.

5 Efficiency of Always Breaching Implementation

In the previous section, we will focus on some specific damage rules that are ED and RD. Under these damage rules, the situation where the first best investment can be attained is limited. Now, we will find more general damage rule which can implement the first-best investment for any nature of the investment.

Is there such a damage rule? In fact if the court can know the parties situation ex post, there is a damage rule which can leads the first best. We call the implementation through such a damage rule ”always breaching implementation”. As the name indicating, in this implementation, either party always breaches on the equilibrium.

Jumping to a conclusion, this implementation consists of a damage rule \( g_B(a \mid w) = (1 - \alpha)S(a) + constant \) and sufficiently high \( p \) which leads B to breach always. To confirm
that it induces the first best investment, now come back to the breach decision at stage 2.

If \( g_B(a \mid w) = (1 - \alpha)S(a) + \beta \), we can rewrite table 1 as table 4.

<table>
<thead>
<tr>
<th>B \ S</th>
<th>b</th>
<th>nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>( (1 - \alpha)S(a) )</td>
<td>(- \max{(1 - \alpha)S(a) + \beta, 0} + (1 - \alpha)S(a) )</td>
</tr>
<tr>
<td>nb</td>
<td>( \max{g_S(a \mid w), 0} + (1 - \alpha)S(a) )</td>
<td>( v(a) - p + (1 - \alpha)\max{c(a) - v(a), 0} )</td>
</tr>
<tr>
<td></td>
<td>(- \max{g_S(a \mid w), 0} + \alpha S(a) )</td>
<td>( p - c(a) + \alpha \max{c(a) - v(a), 0} )</td>
</tr>
</tbody>
</table>

Table 4: Game matrix at stage 2 under always breaching implementation

Let \((b, nb)\) be a Nash equilibrium of the game at stage 2. If \((b, nb)\) is an equilibrium, B must have an incentive to breach, and then

\[
v(a) - p + (1 - \alpha)\max\{c(a) - v(a), 0\} < -\max\{(1 - \alpha)S(a) + \beta, 0\} + (1 - \alpha)S(a)
\]

\[\iff p > \hat{p}(a; \alpha) + \max\{(1 - \alpha)S(a) + \beta, 0\}. \quad (1)\]

When B breaches, S’s payoff (including his investment) becomes \( \max\{(1 - \alpha)S(a) + \beta, 0\} + \alpha S(a) - a \). The first term \( \max\{(1 - \alpha)S(a) + \beta, 0\} \) means the compensation from B to S, and he has an incentive to sue if and only if\(^{17}\)

\[(1 - \alpha)S(a) + \beta \geq 0. \quad (2)\]

Thus, S’s payoff is in detail,

\[
\begin{cases}
\alpha S(a) - a & \text{if (2) does not hold} \\
S(a) - a + \beta & \text{if (2) holds}.
\end{cases}
\]

Given a, if (1) and (2) hold, S’s ex post payoff becomes \( S(a) - a + \beta \). Since \( \beta \) is constant

\(^{16}\) \( \beta \) might be negative.

\(^{17}\) Notice that we assume that if this decision is indifferent, S goes to sue.
in a, the maximizer of this payoff coincides \( a^{FB} \). Then if \( a^{FB} \) satisfies (1) and (2), S invests \( a^{FB} \) rather than a which satisfies (1) and (2)

What a satisfies (1) and (2)? If (2) holds, (1) is

\[
\begin{align*}
  p > v(a) + \beta & \quad \text{if } v(a) \geq c(a) \\
  p > \hat{p}(a; \alpha) + \beta & \quad \text{if } v(a) < c(a)
\end{align*}
\]

Then (1) and (2) are equivalent to

\[
\begin{align*}
  p - v(a) > \beta \geq -(1 - \alpha)S(a) & \quad \text{if } v(a) \geq c(a) \\
  p - \hat{p}(a; \alpha) > \beta \geq -(1 - \alpha)S(a) & \quad \text{if } v(a) < c(a).
\end{align*}
\]

(3) shows that whether a satisfies (1) and (2) depends on p and \( \beta \). However, we can show that if p is sufficient high, there exists \( \beta \) such that (3) holds for all \( a \in A \). In short, given appropriate \( \beta \), if the parties set p sufficiently high, (1) and (2) hold for all a and then S’s payoff becomes \( S(a) - a + \beta \) for all a. Expecting this payoff, S invests \( a^{FB} \) which maximizes \( S(a) - a + \beta \).

The claim we expand above can be summarized as following proposition.

**Proposition 3** There exist a contract \( w \) and damage rules \((g_B(a \mid w), g_S(a \mid w))\) such that the first best investment realizes.

**Proof** The proof of almost part was discussed above. Mathematically precise proof is in the appendix. □

In this always breaching procedure, B always breaches. If B always breaches, the renegotiation surplus is born and S’s share of it is \( \alpha S(a) \). At this point, if the damage rule is \( g_B(a \mid w) = (1 - \alpha)S(a) + \beta \), S can be compensated with it and S’s payoff becomes ex
post surplus $S(a)$ together with some constant gain (or loss) $\beta$. Then S’s object coincides with maximization of trading surplus. To impose the damage can internalize S’s return of the investment. It is necessary for this internalization to force B to breach. However, if $p$ is sufficient high, B always has an incentive to breach. Then he never pays $p$. Instead, he ”pays the damage” for trading.

This implementation is a powerful solution for holdup problems because any technology such that the first best investment level exists can be applied for this method and attained the first best investment. Then this implementation can lead the first best regardless of concavity or convexity of $v$ and $c$. Moreover this implementation can lead efficiency even if the investment $a$ is multi-dimensional and some ex post uncertainty is introduced\(^{18}\). Unfortunately, this implementation needs at least two assumptions which are renegotitable, and ex post full information of the court. However the former is not serious because if renegotiation is impossible, the parties can achieve the first best by message contingent contract which is called ”shoot the liar” mechanism Che and Hausch (1999) insist. On the other hand, the latter assumption seems to be somewhat severe. And yet this assumption is almost equivalent to one with which the court can execute ED. In order to enable to execute ED, the court must know $v(a)$ and $c(a)$ ex post. However, if it is possible the court can also execute the damage of always breaching implementation.

6 Discussion

Before extending our model to see the robustness of always breaching implementation, we will discuss three concerns: the mechanism of each breach remedy, optimal damage rules and foundations of incomplete contracts.

\(^{18}\)We will discuss it in section 7.
The Difference of The Mechanism between Each Breach Remedy

Here, we will mention about the difference of the mechanism of ED, RD, and always breaching implementation. To begin with, focus on the investment level below which B breaches. Under ED, because the parties decide whether or not to breach efficiently by lemma 2\(^{19}\)), this investment level is \(\hat{a}\). Under RD, if \(\hat{p}(a; \alpha) > 0\) and they write a contract to attain the first best, this investment level is \(a^{FB}\). In always breaching implementation, there is no such a investment level because B always breaches.

If there is a threshold of the investment below which B, who is not an investor, has an incentive to breach, S’s payoff as well as B’s payoff can change as such at this threshold. Then this threshold means a threat of S’s underinvestment. However, under ED, the parties cannot control this threshold because efficient breach realizes in any case. Then ED cannot work as the device which incentivizes the investment thorough the breach procedure. Efficiency results of ED in Edlin (1996) are, so to speak, accidental and depend on pure selfishness of the investment. On the other hand, under RD, the parties can control this threshold somewhat by controlling the price of contract. This breaching device can work to incentivize to investment, if the investment is sufficiently cooperative, (even if not purely cooperative).

Then how is always breaching implementation? Because B always breaches in this implementation, we cannot state that there is a device which threatens of S’s underinvestment. Instead, in this implementation, the court can directly incentivize the investor to invest efficiently through internalizing the return of his investment. This direct internalization by the court can perform whether the investment is selfish or cooperative.

From more contract theoretical view point, always breaching implementation can be interpreted as complete contract situation through breaching and litigation procedure. In

\(^{19}\)Strictly speaking, even if trade is inefficient, the contract is enforced when the renegotiation price is equal to the price of the contract. Here, however, we ignore this situation.
other words, the court offers the damage which looks like a complete contract to the party and one party always breaches to receive this damages. This interpretation may be somewhat away from reality. However, this damage is indeed not a contract but a judgment of the court\textsuperscript{20}. Then if writing a complete contract is costly (and the court has only an ability to know only the parties’ ex post information), it is strictly more beneficial for them to write only a simple contract and then to go in the litigation process than to write a complete contract ex ante. Hence this interpretation is not so ridiculous\textsuperscript{21}.

**Optimal Damage Rules** In previous sections, we showed that the efficient relation specific investment can be achieved by breach remedies under some conditions. However although it is important to offer a suggestion for the court policy, we did not discuss what damage rule we should apply for breach remedy. We can state two conclusion concerning this issue.

One is that, in the economic party to be faced with traded quantity decision, we can achieve the efficiency only by the reliance damage in many situations. This statement is considerably related to the result of Che and Hausch (1999).

We usually interpret the results of Che and Hausch (1999) as negative results, that is, when the investment is sufficient cooperative and the investor have sufficient bargaining power, any contract which is not contingent on investment can not induce the first best investment. However, in other word, if this condition does not hold, the parties can achieve the first best investment by a contract indicating the trade quantity and price under SP.

If the inefficiency in the sense of Che and Hausch (1999) occurs, can we solve this by

\textsuperscript{20}If the litigation process is costly, the parties will renegotiate before either goes to the court. However, this renegotiation is concluded so as to be the result of the court’s judgement because the court plays a role as a threat point of renegotiation. Then the outcome through renegotiation is same as that of costless litigation.

\textsuperscript{21}We will discuss this interpretation connected with a foundation of incomplete contract in section 6.
another method? Although we did not analyze formally, the condition of the inefficiency of Che and Hausch (1999) is similar to the efficiency of RD in our paper. Then in this situation, the RD can solve this inefficiency. Then we can conclude that the parties can achieve the efficiency by using quantity contract (a la Che and Hausch (1999)) and fixed price contract or judgement of enforcing RD.

The other of our conclusions is simply to use always breaching implementation. To be sure, the court needs sufficient information about the parties’ environment to use this implementation. However, if the court knows it, the inefficiency derived by incomplete contract can be eliminated easily. Furthermore, if renegotiation is costless, always breaching implementation does not have to pass the litigation procedure which may be troublesome in actuality. This damage rule needs only to play a role of threat point of their renegotiation.

Foundations of Incomplete Contracts Some papers try to construct foundations of incomplete contracts. This endeavor have mainly two approach.

One is that because writing a simple contract is not second best, then we can claim that the parties do not write any contracts. For example Hart and Moore (1999) and Segal (1999) explain the reason from the complexity of trading environment. Or Che and Hausch (1999) states that the cooperativity of the investment induces incomplete contracts.

On the other hand, Tirole (1999) claims another approach. It is that if the parties can achieve the first best by writing not a complete contract but a less sophisticated contract, they need not write a complicated contract to achieve the first best, and then we can claim that incomplete contracts occur.

We can apply the latter approach to our model and explain the reason of incomplete contracts. Our analyses show that always breaching implementation can lead the first best investment even if the parties can write only a fixed price contract. This implies that even
if the parties can write a complete contract, they need not write a complete contract. Then we insist that incomplete contracts can occur.

This statement is, however, justly dependent on an assumption that always breaching implementation can be utilized by the parties. To enable always breaching implementation, the court must have sufficient information. It is certain that this informed court assumption may be disputable.

Then if constructing foundations of incomplete contract more plausibly, we have to expand more elaborated argument. Then let us introduce writing cost into our model.

Suppose that there exist costs of specifying and writing the contingency in ex ante contracts. For example, suppose that you want to build a new office and write a sales contract with a construction company. You may be able to add a clause that "if there is a defect in the building you have to repair it for free". It is, however, often difficult to specify and to write the degree of the defect to be repaired. We can interpret this specifying cost as writing cost. Then if these transaction parties attempt to achieve the efficient investment of the company through a state-contingent contract, they must incur the writing cost.

However comparing to ex ante, it may be easy to acquire the information ex post. If the court has ex post information about the parties, they can use always breaching implementation. Then they can achieve the efficient investment without writing costs through our mechanism. Then the parties will not write a state-contingent contract.

Will the company necessarily breach in fact? Actually we may not be able to say so. But we expect that they will renegotiate instead they breach. This renegotiation results in as if a breach occurs because the breaching party is confronted with the threat to have to compensate the other party by the court’s damage rule.

In short, if there exist costs to specify the contingency and the court acquires ex post information easily, foundations of incomplete contracts can be justified by our model.
In this section, to confirm the versatility of always breaching implementation we will introduce multidimensional investment and ex post uncertainty into our model.

First we set the investment space as $A = \prod_{i=1}^{M} [0, \bar{a}_m]$ which is $M$-dimensional compact set. For all $a := (a_1, \ldots, a_M) \in A$, we define $S$’s investment cost as $d(a)$ instead of $a$.

Second we introduce the uncertainty into $B$’s value which is then $v(a, \theta)$ where $\theta \in [0, 1]$ is uncertain state. After the investment stage we call stage 1.5, this state realized according to the cumulative distribution function $F(\theta)$. We assume $v$ is continuous and strictly increasing in $\theta$.

Given $a$ and $\theta$, ex post trading value is $v(a, \theta) - c(a)$. We assume that $v(a, \theta) - c(a)$ is increasing in $a_i$ for all $i = 1, \ldots, M$ and $v(a, 0) - c(a) < 0 < v(a, 1) - c(a)$. Then given $a$, by the mean value theorem and strictly increasingness of $v - c$, there uniquely exists $\hat{\theta}(a)$ such that $v(a, \hat{\theta}(a)) - c(a) = 0$ and

$$
\begin{cases}
  v(a, \theta) < c(a) & \text{for all } \theta \in [0, \hat{\theta}(a)) \\
  v(a, \theta) \geq c(a) & \text{for all } \theta \in [\hat{\theta}(a), 1].
\end{cases}
$$

We define $S(a, \theta)$ as ex post efficient surplus which is

$$
S(a, \theta) := \max\{v(a, \theta) - c(a), 0\}
$$

$$
= \begin{cases}
  v(a, \theta) - c(a) & \text{if } \theta \in [\hat{\theta}(a), 1] \\
  0 & \text{if } \theta \in [0, \hat{\theta}(a)).
\end{cases}
$$
The first best investment maximizes ex ante surplus, and then this satisfies

\[
a^{FB} \in \arg \max_a \left[ E_\theta \left[ \max \{ v(a, \theta) - c(a), 0 \} \right] - d(a) \right]
\]

\[
= \arg \max_a \left[ \int_{\hat{\theta}(a)}^1 (v(a, \theta) - c(a))dF(\theta) - d(a) \right]
\]

\[
= \arg \max_a \left[ \int_{\hat{\theta}(a)}^1 v(a, \theta)dF(\theta) - \left(1 - F(\hat{\theta}(a))\right)c(a) - d(a) \right].
\]

We assume that there exists such \(a^{FB} \in A\).

In the analysis of the basic model, we virtually only have to consider whether or not to breach after stage 2. Here we must also consider the breach decision before stage 1.5. However here we can also state that always breaching implementation can lead the efficient investment.

**Proposition 4** In the extension model, there exist a contract \(w\) and damage rules \((g_B(a \mid w), g_S(a \mid w))\) such that the first best investment realizes.

**Proof** See the appendix. □

This proposition shows that always breaching implementation can be a solution of holdup even if the environment is more complicated. Especially it is important that even if ex post uncertainty exists, the efficiency can be achieved. Ex post uncertainty brings the parties accidental breaching which means unwillingly breaching because of, for example, some accidents, as well as strategic breaching which means deliberately breaching to exploit the surplus although to enforce the contract is efficient. The latter has been already analyzed in the deterministic model of the previous section. We can also argue that always breaching implementation can lead the efficient investment even if accidental breaching problem may occur.
Notice that the court damage rule \( \{ (g_i(a \mid w))_{i=S,B} \} \) do not have to depend on \( \theta \). There are some cases where not all parties can know \( \theta \) in the standard hidden action model. However, this implementation can be utilized in such situations.

8 Conclusion and Future Research

This paper analyzes the possibility of the efficient cooperative investment through damage rules. Challenging the inefficiency of the cooperative investment claimed by Che and Hausch (1999), we shows two theoretical results.

First, we show the robustness of the result of Che and Chung (1999). The efficiency of the reliance damage, Che and Chung (1999) show, can be achieved when the renegotiation price, which is the price at the parties’ Nash bargaining outcome, is increasing in the investment. This condition is also satisfied when the investment is purely cooperative. An incentive to underinvestment can be turned out by the reliance damage. Besides, when the renegotiation price is increasing in the investment, the appropriate price contract can prevent the investor overinvesting. The combination of this effect can induce the efficient investment.

Second, we can always implement the efficient investment by always breaching implementation. If the price of the contract is sufficiently high, the buyer will always breach the contract. At this point, by appropriate sentence of the damage for breaching, the return of the investment for the investor results in as if not the bargaining surplus but the social surplus. Thus the damage rule can internalize the investor’s return well. This internalization can work regardless of natures of the investment, dimensions of the investment, and ex post uncertainty, and then this implementation can be expected as a powerful solution of holdup problems.

Although we extend discussions about realistic concerns in section 6, we must realize
that our analyses are too lack to conclude the realistic concerns precisely. To elaborate the
discussions, there are some future research issues. One is that we must introduce quantity
decision and uncertainty into our basic model. This extended model is consistent to Edlin
and Reichelstein (1996) and Che and Hausch (1999), who use a quantity decision as a
solution of holdups. Also, relationship between the reliance damage and quantity decision
is an interesting issue. The other is about the court’s information. In order to implement
always breaching implementation, the court must know almost all the structure of the game.
If we can implement the efficient investment with less informed court, the argument about
foundations of incomplete contracts will become more solid.

A Appendix: Proofs

Proof of Lemma 1 Consider the case where $\pi^{SP} \geq \hat{a}$. Then obviously $\arg \max_{a \in [\hat{a}, \bar{a}]} [p - c(a) - a] = \{\pi^{SP}\}$ by the first order condition. Then S never invests over $\pi^{SP}$.

Next consider the case where $\pi^{SP} < \hat{a}$. Then the maximization problem of $p - c(a) - a$
subject to $a \in [\hat{a}, \bar{a}]$ has a unique solution which is $\hat{a}$ because $p - c(a) - a$ is single peaked.
Thus $\hat{a}$ dominates for all $a > \hat{a}$ that is never invested as a result.

Proof of Lemma 2 Given $p$ and $a$, $(nb, nb)$ is a unique equilibrium breach decision if and
only if

$$
\left\{ \begin{array}{l}
- \max\{p - c(a), 0\} + (1 - \alpha)S(a) \leq v(a) - p + (1 - \alpha)\max\{c(a) - v(a), 0\} \\
- \max\{v(a) - p, 0\} + \alpha S(a) \leq p - c(a) + \alpha \max\{c(a) - v(a), 0\}
\end{array} \right.
\left\{ \begin{array}{l}
(1 - \alpha)[v(a) - c(a)] \leq v(a) - p + \max\{p - c(a), 0\} \\
\alpha [v(a) - c(a)] \leq \max\{v(a) - p, 0\} + p - c(a).
\end{array} \right.
$$

(4)
Suppose that $a \geq \hat{a}$. Then $v(a) \geq c(a)$. Using this inequality,

$$v(a) - p + \max\{p - c(a), 0\} \geq v(a) - p + p - c(a)$$

$$= v(a) - c(a)$$

$$\geq (1 - \alpha)[v(a) - c(a)]$$

and

$$\max\{v(a) - p, 0\} + p - c(a) \geq v(a) - p + p - c(a)$$

$$= v(a) - c(a)$$

$$\geq \alpha[v(a) - c(a)].$$

Thus (4) is automatically satisfied when $a \geq \hat{a}$.

Next suppose that $a < \hat{a}$. Then $v(a) < c(a)$. When $p \geq c(a)$,

$$v(a) - p + \max\{p - c(a), 0\} = v(a) - p + p - c(a)$$

$$= v(a) - c(a)$$

$$< (1 - \alpha)[v(a) - c(a)]$$

which contradicts (4). When $p \leq v(a)$,

$$\max\{v(a) - p, 0\} + p - c(a) = v(a) - p + p - c(a)$$

$$= v(a) - c(a)$$

$$< \alpha[v(a) - c(a)].$$
which also contradicts (4). At last, when $c(a) > p > v(a)$, (4) becomes

\[
\begin{align*}
\begin{cases}
(1 - \alpha)[v(a) - c(a)] & \leq v(a) - p \\
\alpha[v(a) - c(a)] & \leq p - c(a).
\end{cases}
\end{align*}
\]

\[\iff\]

\[
\begin{align*}
\begin{cases}
p \geq \hat{p}(a; \alpha) \\
p \leq \hat{p}(a; \alpha)
\end{cases}
\end{align*}
\]

\[\iff\]

\[
p = \hat{p}(a; \alpha).
\]

Thus if $a < \hat{a}$, the parties do not breach only when $p = \hat{p}(a; \alpha)$.

Next we will investigate who breaches when $a < \hat{a}$. S breaches if and only if

\[
-\max\{v(a) - p, 0\} + \alpha S(a) > p - c(a) + \alpha \max\{c(a) - v(a), 0\}
\]

\[\iff\]

\[
\alpha[v(a) - c(a)] > \max\{v(a) - p, 0\} + p - c(a)
\] (5)

and B breaches if and only if

\[
-\max\{p - c(a), 0\} + (1 - \alpha)S(a) > v(a) - p + (1 - \alpha) \max\{c(a) - v(a), 0\}
\]

\[\iff\]

\[
(1 - \alpha)[v(a) - c(a)] > v(a) - p + \max\{p - c(a), 0\}.
\] (6)

When $p > \hat{p}(a; \alpha)$, $p > v(a)$ holds. Then (5) becomes

\[
\alpha[v(a) - c(a)] > \max\{v(a) - p, 0\} + p - c(a)
\]

\[=\]

\[
p - c(a)
\]

\[=\]

\[
\hat{p}(a; \alpha) - c(a)
\]

\[=\]

\[
\alpha[v(a) - c(a)]
\]
which implies a contradiction. And (6) becomes

\[(1 - \alpha)[v(a) - c(a)] > v(a) - p + \max\{p - c(a), 0\}\]

\[\geq v(a) - p + p - c(a)\]

\[= v(a) - c(a)\]

which is always satisfied because \(v(a) < c(a)\). Then, when \(p > \hat{p}(a; \alpha)\), \((nb, b)\) is an unique equilibrium.

By similar argument, we can confirm that \((b, nb)\) is an unique equilibrium if and only if \(p > \hat{p}(a; \alpha)\).

**Proof of Proposition 1**  
S’s payoff is

\[
\begin{cases}
    p - c(a) - a & \text{if } a \geq \hat{a} \\
    -a & \text{if } a < \hat{a} \text{ and } p = \hat{p}(a; \alpha) \\
    - \max\{v(a) - p, 0\} - a & \text{if } a < \hat{a} \text{ and } p < \hat{p}(a; \alpha) \\
    \max\{p - c(a), 0\} - a & \text{if } a < \hat{a} \text{ and } p > \hat{p}(a; \alpha).
\end{cases}
\]

If \(\bar{a}^{SP} \geq \hat{a}\), S prefers \(\bar{a}^{SP}\) to any \(a > \bar{a}^{SP}\) because \(\bar{a}^{SP}\) satisfies the first order condition.

On the other hand, If \(\bar{a}^{SP} < \hat{a}\), \(\hat{a}\) is the solution of the problem \(\max_{a\in[\hat{a}, \bar{a}]}[p - c(a) - a]\) because this objective function is single peaked. Then any \(a\) strictly larger than \(\hat{a}\) never becomes the equilibrium investment.

**Proof of Lemma 2**  
By confirming the incentive to breach as the same procedure of proof of Lemma 2, we can prove this lemma.
Proof of Proposition 2  Consider that the price in the contract is \( p = \hat{p}(a^{FB}; \alpha) + a^{FB} \).
By defining \( \pi^{RD} := \max\{a \in A \mid \hat{p}(a; \alpha) \leq \hat{p}(a^{FB}; \alpha) + a^{FB}\} \). We can rewrite S’s payoff as follows:

\[
\begin{cases}
\alpha S(a) & \text{if } a < a^{FB} \\
p - c(a) - a & \text{if } a^{FB} \leq a \leq \pi^{RD} \\
\alpha S(a) - a & \text{if } a > \pi^{RD}.
\end{cases}
\]

(Notice that because \( a^{FB} > \hat{a} \), \( \max\{c(a) - v(a), 0\} = 0 \).)

Since \( a^{FB} > \overline{a}^{SP} \) and \( p - c(a) - a \) is single peaked, for \( a \geq a^{FB} \), \( a^{FB} \) maximizes S’s payoff and the value \( p - c(a^{FB}) - a^{FB} = \alpha(v(a^{FB}) - c(a^{FB})) \). For all \( a < a^{FB} \),

\[
\alpha(v(a^{FB}) - c(a^{FB})) - \alpha S(a) > 0
\]

because \( v - c \) is increasing in \( a \), and for all \( a > \pi^{RD} \),

\[
\alpha(v(a^{FB}) - c(a^{FB})) - [\alpha S(a) - a] = \alpha(v(a^{FB}) - c(a^{FB})) - [\alpha(v(a) - c(a)) - a] \\
> \alpha(v(a^{FB}) - c(a^{FB})) - a^{FB} - [\alpha(v(a) - c(a)) - a] \\
> 0
\]

where the reason of the last inequality is that \( a^{NO} < a^{FB} < a \) and \( \alpha(v - c) - a \) is single peaked.

Therefore S invests at \( a = a^{FB} \) on the equilibrium.

Proof of Proposition 3  Consider the \( w \) and \( (g_B(a \mid w), g_S(a \mid w)) \) such that

\[ p = \max\{v(\overline{a}), \overline{p}(a, \alpha)\} + \varepsilon \text{ where } \overline{p}(a, \alpha) = \sup_{a \in [0, \hat{a}]}(\alpha v(a) + (1 - \alpha)c(a)) \text{ and } \varepsilon > 0 \]
• \( g_B(a \mid w) = (1 - \alpha)S(a) \)

• \( g_S(a \mid w) = z \geq \max_a \hat{p}(a; \alpha) + p. \)

Consider stage 3. Given for all \( a \), if B breaches, when he does not breach, S’s surplus from the trade is

\[
(1 - \alpha)S(a) + \alpha S(a) = S(a)
\]

and when he breaches,

\[
\alpha S(a) \leq S(a).
\]

Then if B breaches, S choose not to breach.

On the other hand, suppose that S does not breach. When B does not breach, her surplus of trade is

\[
v(a) - p + (1 - \alpha) \max\{c(a) - v(a), 0\} = \max\{\hat{p}(a; \alpha), v(a)\} - p
\]

\[
= \begin{cases} 
\hat{p}(a; \alpha) - p & \text{if } v(a) < c(a) \\
v(a) - p & \text{if } v(a) \geq c(a)
\end{cases}
\]

and when B breaches,

\[-(1 - \alpha)S(a) + (1 - \alpha)S(a) = 0.\]

We compare these two B’s surplus. When \( v(a) < c(a), \ a < \hat{a}. \) Then

\[
\hat{p}(a; \alpha) - p = \hat{p}(a; \alpha) - \max\{v(\bar{\pi}_S), \bar{p}(a, \alpha)\} - \varepsilon
\]
Thus B will breach.

When $v(a) \geq c(a)$,

$$v(a) - p = v(a) - \max \{ v(\alpha_S), p(a, \alpha) \} - \varepsilon$$

$$\leq v(a) - v(\alpha_S) - \varepsilon$$

$$\leq -\varepsilon$$

$$< 0.$$

Then B will also breach.

Suppose that $(nb, b)$ (i.e. S breaches and B does not breach) is another equilibrium. However when B does not breach. S has incentive to breach if and only if

$$-z + \alpha_S(a) > p - c(a) + \alpha \max \{ c(a) - v(a), 0 \}$$

$$\iff z < \hat{p}(a; \alpha) - p.$$
\[ g_B(a, \theta \mid w) = (1 - \alpha) \left[ \int_{\hat{\theta}(a)}^{1} v(a, \theta)dF(\theta) - \left\{ 1 - F(\hat{\theta}(a)) \right\} c(a) \right] \]

\[ g_S(a, \theta \mid w) = z > \alpha v(a, 1) + (1 - \alpha)c(a) - p \quad \text{where} \quad z \text{ is constant for any } a \text{ and } w. \]

Notice that \( g_B(a, \theta \mid w) \geq 0. \)

To begin with, consider the breach decision after \( \theta \) is decided. At this stage, the game matrix is identical with table 1 except that \( v(a) \) becomes \( v(a, \theta) \) and \( S(a) \) becomes \( S(a, \theta) \).

Then we can rewrite breaching stage game as table 5.

<table>
<thead>
<tr>
<th>B \ S</th>
<th>b</th>
<th>nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>((1 - \alpha)S(a, \theta))</td>
<td>(-\max{g_B(a \mid w), 0} + (1 - \alpha)S(a, \theta))</td>
</tr>
<tr>
<td>nb</td>
<td>(\max{g_S(a \mid w), 0} + (1 - \alpha)S(a, \theta))</td>
<td>(v(a, \theta) - p + (1 - \alpha)\max{c(a) - v(a, \theta), 0})</td>
</tr>
</tbody>
</table>

Table 5: Game matrix at stage 2 under the extension model

B always breaches and S never breaches for all \( \theta \). Let us confirm it. When S decides not to breach, B’s payoff (not including the investment cost) is \( v(a, \theta) - p + (1 - \alpha)\max\{c(a) - v(a, \theta), 0\} \) when she does not breach and \(-\max\{g_B(a \mid w), 0\} + (1 - \alpha)S(a, \theta) \) when she breaches. Notice that

\[
- \max\{g_B(a \mid w), 0\} + (1 - \alpha)S(a, \theta) - [v(a, \theta) - p + (1 - \alpha)\max\{c(a) - v(a, \theta), 0\}]
\]

\[
= -(1 - \alpha) \left[ \int_{\hat{\theta}(a)}^{1} v(a, \theta)dF(\theta) - \left\{ 1 - F(\hat{\theta}(a)) \right\} c(a) \right] + (1 - \alpha)S(a, \theta)
\]

\[
- \left[ v(a, \theta) - \max_a \left\{ \alpha v(a, 1) + (1 - \alpha) \left[ \int_{\hat{\theta}(a)}^{1} v(a, \theta)dF(\theta) + F(\hat{\theta}(a))c(a) \right] \right\} - \varepsilon
\]

\[
+ (1 - \alpha)\max\{c(a) - v(a, \theta), 0\} \right]
\]

\[
= \max_a \left\{ \alpha v(a, 1) + (1 - \alpha) \left[ \int_{\hat{\theta}(a)}^{1} v(a, \theta)dF(\theta) + F(\hat{\theta}(a))c(a) \right] \right\}
\]

\[
- \left[ \alpha v(a, \theta) + (1 - \alpha) \left[ \int_{\hat{\theta}(a)}^{1} v(a, \theta)dF(\theta) + F(\hat{\theta}(a))c(a) \right] \right] + \varepsilon
\]

\[
\geq \max_a \left\{ \alpha v(a, 1) + (1 - \alpha) \left[ \int_{\hat{\theta}(a)}^{1} v(a, \theta)dF(\theta) + F(\hat{\theta}(a))c(a) \right] \right\}
\]
Then $B$ always breaches. Similarly, we can confirm that the action profile that $S$ breaches and $B$ does not breach cannot be equilibrium since $z$ is too high.

Next, consider the breach decision before stage 1.5. If a breach occurs, the parties will go in renegotiation after ward. Then their payoffs consist of the damage payment and expected renegotiation surplus. If no breach occur, $B$ will breaches at stage 2 as analyzed above. Then their payoffs are stage 2’s payoff integrated with respect to $F(\theta)$.

Then, we can represent this situation as following matrix.

<table>
<thead>
<tr>
<th>B \ S</th>
<th>b</th>
<th>nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>$(1 - \alpha) \int S(a, \theta) dF - \alpha \int S(a, \theta) dF$</td>
<td>$- \max{g_B(a</td>
</tr>
<tr>
<td>nb</td>
<td>$z + (1 - \alpha) \int S(a, \theta) dF$ - $z + \alpha \int S(a, \theta) dF$</td>
<td>$- \max{g_B(a</td>
</tr>
</tbody>
</table>

Table 6: Game matrix before stage 1.5

In this game, equilibrium are (b, nb) and (nb, nb) at both of which the parties attain same payoff.

Considering breach decision mentioned above, Then the S’s payoff (including the investment cost) is

$$\max\{g_B(a | w), 0\} + \alpha \int_0^1 S(a, \theta) dF(\theta) - d(a)$$

$$= (1 - \alpha) \left[ \int_{\hat{\theta}(a)}^1 v(a, \theta) dF(\theta) - \left\{ 1 - F(\hat{\theta}(a)) \right\} c(a) \right] + \alpha \int_0^1 S(a, \theta) dF(\theta) - d(a)$$

$$= \left[ \int_{\hat{\theta}(a)}^1 v(a, \theta) dF(\theta) - \left\{ 1 - F(\hat{\theta}(a)) \right\} c(a) \right] - d(a)$$
which is identical with the objective function of the first best. Then $S$ invests $a^{FB}$.

**Reference**


