Relative Performance and R&D Competition

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Abstract

We formulate a model where firms care about relative profits as well as their own profits. We investigate the relationship between the weight of relative performance and R&D expenditure. We find a non-monotone relationship between the weight of relative performance in their objectives and their R&D levels. Both highly reciprocal (altruism) and negative reciprocal attitudes yield high levels of R&D, while the intermediate situations yield low levels of R&D. Our result sheds light on the relationship between product market competition and innovation. Our result suggests that innovation is active in both competitive and collusive markets, while less active in the intermediate situations.

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1 Introduction

We investigate the relationship between the payoff functions of firms incorporating both positive and negative reciprocal preference and R&D expenditures. We also discuss the relationship between product market competition and innovation, which economists have long been interested in.1 Traditionally, many researchers believe that monopoly yields intensive R&D (monopoly view), and many other believe that competition yields intensive R&D (competition view). Both have presented many theoretical foundations and empirical (or anecdotal) evidences supporting their views.2 We find U-shaped relationship between competitiveness and the equilibrium level of R&D, i.e., both competitive and monopolistic situations yield higher levels of R&D than oligopolistic situations. Thus, our result explains both views consistently.

We consider a duopoly model where firms maximize relative profits rather than their own profits. Firm $i$'s payoff is $\pi_i - \alpha \pi_j$, where $\pi_i$ is its own profits, $\pi_j$ is the rival’s profits, and $\alpha \in (-1, 1)$. We investigate the relationship between $\alpha$ and the equilibrium R&D investment level. In the product market, firms are assumed to face Cournot competition. At the product market competition stage, the equilibrium outcome converges to the monopoly one when $\alpha$ converges to $-1$, and it converges to the perfectly competitive one (Walrasian) when $\alpha$ is close to $1$. Thus, we can interpret $\alpha$ as a parameter indicating the severity of competition. $\alpha = 0$ indicates the standard Cournot case, $\alpha = 1$ indicates the perfectly competitive case (Bertrand case), and $\alpha = -1$ indicates the monopoly case. Thus, a larger $\alpha$ indicates a more competitive market.3 The relative performance approach enables us to treat competitiveness as a continuous variable, and this approach contains three models, Cournot, Bertrand, and monopoly, as special cases.4

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1 See Aghion et al. (2005).

2 See, e.g., Schumpeter (1950) and Arrow (1962). See also Mateus and Moreira (2007), Cabral (2000), and works cited in these books.

3 Under the standard conditions in Cournot, the ratio between profit margin (price minus marginal cost) and the price, called as Lerner index, is decreasing in $\alpha$. This index is intensively used in the empirical literature as a measure of competitiveness in product markets.

4 The conjectural variation approach is another approach which contains three models as special cases. How-
In this paper, we investigate the relationship between $\alpha$ and the equilibrium R&D investment level. Although the literature dealing with strategic R&D competition is fairly abundant, most studies assume Cournot competition where firms maximize their own profits.\(^5\) Considering relative profits, we treat competitiveness as a continuous variable and analyze how competitiveness affects the innovative activities of firms. We find that a non-monotone (U-shaped) relationship. Given a nonpositive $\alpha$, an increase in $\alpha$ (an increase in competitiveness) reduces R&D. When $\alpha$ reaches a critical value (which is strictly larger than $\alpha = 0$ (the Cournot case)), the relationship is inverted. From the critical value of $\alpha$, an increase in $\alpha$ increases the levels of R&D. This result indicates that R&D are quite active in both collusive and competitive markets, while they are less active in the intermediate situations.

Our result sheds some insight on how reciprocal and negative reciprocal attitudes affect R&D. Given the pure selfish preferences of firms, an introduction of slight spiteful preferences reduces their innovative activities but an introduction of significant spiteful preferences stimulates their innovation.\(^6\)

We now present some rationales for discussing objective functions based on relative performance. First, evaluations of managers performances are often based on their relative performance as well as their absolute performance. Outperforming managers often obtain good positions in the management job markets. This rationalizes considering positive $\alpha$ in our model. Second, many laboratory (experimental) works pointed out the spiteful behavior as well as reciprocal variation model assumes that firm 1’s output affects that of firm 2 and vice versa. Needless to say, this assumption is inconsistent if we respect conjunctural variation model as a static model. The relative performance approach does not have this defect, and this is an important advantage of this approach.


\(^6\) The payoff functions which are based on relative wage or relative wealth status have been intensively discussed in the macroeconomics context, too. See, among others, Akerlof and Yellen (1988), Corneo and Jeanne (1997, 1999) and Futagami and Shibata (1998). The relative performance approach is important in the political science. Obviously, a party cares about the number of votes obtained not in absolute term but in relative term. In addition in the context of international policies, the possibility that governments care about their relative performance as well as absolute performance are pointed out. See, among others, Grieco et al. (1993) and Mastanduno (1991).
ciprocal behavior or altruistic behavior, which is closely related to the objective functions based on relative performance (both positive and negative $\alpha$).\(^7\) Third, relative performance, especially the positive $\alpha$ case, is quite important from the viewpoint of evolutionary stability.\(^8\) Fourth, if we adopt the approach taken by Fershtman and Judd (1987) and replace sales with (minus) rival’s profit, the firms in fact adopt a positive $\alpha$.\(^9\) We believe that our formulation has sufficient importance for investigation on innovations.

2 The Model

We formulate a two-stage symmetric duopoly model. In the first stage firm $i$ ($i = 1, 2$) chooses its R&D level $I_i$. At the beginning of the second stage each firm observes the rival’s R&D. In the second stage firms produce perfectly substitutable commodities for which the market demand function is given by $p = a - Y$ (price as a function of quantity), where $Y$ is the total output of duopolists. Let $y_i$ denote the output of firm $i$. Firm $i$’s marginal production cost $c_i$ depends on $I_i$. Each firm $i$ chooses $y_i$ independently.

The payoff of firm $i$ ($i = 1, 2$) is given by $U_i = \pi_i - \alpha \pi_j$ ($i \neq j$), where $\pi_i$ is the profit of firm $i$ and $\alpha \in (0, 1)$ is a positive constant. $\alpha$ indicates the important of relative performance for firm $i$’s management. $\pi_i$ is given by $\pi_i = (a - Y)y_i - c_i(I_i)y_i - I_i$. We assume that $c_i'(I_i) \leq 0$ and $c_i''(I_i)$ is positive and sufficiently large so as to satisfy the second order condition. We also assume that $\lim_{I \to 0} c_i'(I) = -\infty$ and $\lim_{I \to \infty} c_i'(I) = 0$ so as to ensures the interior solution at the first stage.

\(^7\) See, among others, Brandts et al. (2004), Cason et al. (2002), and Coats and Neilson (2005).

\(^8\) See Alchian (1950) and Vega-Redondo (1997). Vega-Redondo (1997) also shows that Cournot competition with relative performance objectives yield the Bertrand outcome even in duopoly and this outcome is evolutionary stable.

\(^9\) See Kockesen et al. (2000).
3 Equilibrium

We solve the game by backward induction. First, we consider the second stage competition. Firm $i$ maximizes its payoff $U_i$. The first order condition is:

$$a - 2y_i - (1 - \alpha)y_j = c_i \quad (i = 1, 2, i \neq j).$$

We obtain the following reaction function:

$$y_i = R_i(y_j : \alpha) = \frac{a - c_i - (1 - \alpha)y_j}{2}.$$  

The second stage equilibrium outputs are:

$$y_{E1}^i = \frac{(1 + \alpha)a - 2c_1 + (1 - \alpha)c_2}{(1 + \alpha)(3 - \alpha)}, \quad y_{E2}^i = \frac{(1 + \alpha)a - 2c_2 + (1 - \alpha)c_1}{(1 + \alpha)(3 - \alpha)}.$$  

The resulting profit of firm $i$ is:

$$\Pi_i(c_i, c_j) = \frac{((1 - \alpha)a - (2 - \alpha)c_i + c_j)((1 + \alpha)a - 2c_i + (1 - \alpha)c_j)}{(3 - \alpha)^2(1 + \alpha)} - I_i.$$  

Next, we consider the first stage competition. Each firm $i$ chooses $I_i$ so as to maximize $U_i = \Pi_i - \alpha \Pi_j$. We restrict our attention to the symmetric equilibrium. The sufficiently large $c''_i$ guarantees that the unique equilibrium is symmetric. The first order condition is:

$$-\frac{((1 + \alpha)a - 2c_i + (1 - \alpha)c_j)(4 - 3\alpha + \alpha^2)}{(3 - \alpha)^2(1 + \alpha)}c'_i = 1.$$  

Define $H(\alpha, c_i, c_j) \equiv ((1 + \alpha)a - 2c_i + (1 - \alpha)c_j)(4 - 3\alpha + \alpha^2)/((3 - \alpha)^2(1 + \alpha))$. A larger $H$ and a larger $|c'|$ implies a higher marginal benefit of R&D. Since the payoff function is assumed to be concave with respect to $I_i$ ($c''$ is large enough), a larger $H$ (as well as larger $|c'|$) yields a higher level of R&D investment.

We then consider how $\alpha$ affects the equilibrium R&D level. We find a non-monotone relationship between the equilibrium level of R&D and the weight of relative performance $\alpha$.

**Proposition 1** The equilibrium R&D investment level is decreasing in $\alpha$ for $\alpha < 1/3$ and is increasing in $\alpha$ for $\alpha > 1/3$. 


**Proof:** Substituting \( c_j = c_i = c \) into \( H(\alpha, c_i, c_j) \) yields:

\[
H(\alpha, c) = \frac{(1 + \alpha)a - (1 + \alpha)c(4 - 3\alpha + \alpha^2)}{(3 - \alpha)^2(1 + \alpha)}.
\]

Differentiating \( H(\alpha, c) \) with respect to \( \alpha \), we have:

\[
\frac{\partial H(\alpha, c)}{\partial \alpha} = \frac{(1 - c)(3\alpha - 1)}{(3 - \alpha)^3}.
\]  \( (6) \)

(6) is negative for \( \alpha < 1/3 \) and positive for \( \alpha > 1/3 \). This implies that, given \( c_1 = c_2 = c \), marginal benefit of R&D is decreasing (increasing) in \( \alpha \) for \( \alpha < (>) 1/3 \). This yields Proposition 1.

Q.E.D.

We explain the intuition behind Proposition 1. Since \( y_i^E \) is increasing in \( \alpha \), the cost minimizing level of R&D, which is derived by minimizing \( c_i y_i^E + I_i \), is increasing in \( \alpha \). Thus for the purpose of cost minimization (minimization of production cost plus R&D cost), firm \( i \) has a stronger incentive for R&D when \( \alpha \) is larger. On the other hand, firm \( i \) has a weaker incentive of innovation for strategic purpose.\(^{10}\) From (2), we have that \( |R'_i| \) is decreasing in \( \alpha \). This implies that the strategic value of R&D is decreasing in \( \alpha \). The former dominates when \( \alpha \) is large, while the latter dominates when \( \alpha \) is small. This yields the U-shaped relationship between \( \alpha \) and the equilibrium R&D level.

Proposition 1 has a quite rich implication. Both aggressive competition and collusive situation yield high levels of R&D. There are two influential views for the relationship between competitiveness and R&D. One view is monopoly view. The monopoly yields intensive R&D investments. The other is competition view. Severe competition accelerates innovation. Both have presented many theoretical foundations and empirical (or anecdotal) evidences supporting their views. We consistently explain that both views can be right by the single simple model.

In our setting, R&D is minimized when \( \alpha = 1/3 \). Starting Cournot competition \( (\alpha = 0) \) a slight increase in \( \alpha \) decreases R&D investments, while a large increase in \( \alpha \) increases. Thus, an

\(^{10}\) From (3) we see that an increase in \( I_i \) (a decrease in \( c_i \)) decreases \( y_i^E \), and it results in an increase in \( \pi_i \), \((i, j \in \{1, 2\}, i \neq j)\). This is the strategic value of R&D. For this strategic effect of R&D, see Brander and Spencer (1983).
envy society (positive $\alpha$) where people care about their relative performances as well as their absolute performances can yield either more or less aggressive R&D activities.

4 Joint R&D Implementation

In this section, we investigate whether or not our result depends on the assumption about the formation of R&D activities. We consider the case in which the firms cooperatively determine their investment level $I$ to maximize their joint profits, while they non-cooperatively compete in the product market.\footnote{Explicit collusion in product markets is usually illegal, while cooperation at R&D stage is often permitted by anti-monopoly legislations.} Firm $i$’s profit is given by $\Pi_i = (a - Y - d(I))y_i - I$, where $d(I)$ is the common marginal cost of the firms. We again assume that $d'(I) < 0$ and $d''(I)$ is positive and large enough. We also assume that $\lim_{I \to 0} I'(I) = -\infty$ and $\lim_{I \to \infty} d'(I) = 0$ so as to ensure the interior solution at the first stage.

We consider a two-stage game. In the first stage, the firms choose their R&D level cooperatively. In the second stage, the firms non-cooperatively produce perfectly substitutable commodities.

The second stage competition has already discussed in the previous section. In the first stage, the firms chooses their investment level $I$. The first order condition is:

$$-\frac{4(1-c)(1-\alpha)^2}{(3-\alpha)^2}d'(I) = 1. \quad (7)$$

The left-hand side in (7) is the marginal benefit of their joint R&D investment and the right side in (7) is the marginal cost of the investment. Define $G(\alpha, d) = 4(1-c)(1-\alpha)^2/(3-\alpha)^2$. Because of the concavity of the payoff function, which is guaranteed by the assumption of sufficiently large $d''$, a larger $G$ implies a higher level of investment. We investigate how $\alpha$ affects the equilibrium R&D level. We find a monotone relationship between the R&D level and the weight of relative performance $\alpha$.

**Proposition 2** If the firms make their R&D investments cooperatively, the equilibrium R&D investment level is decreasing in $\alpha$ for $\alpha \in (-1, 1)$.
Proof: Differentiating $G(\alpha, d)$ with respect to $\alpha$, we have:

$$\frac{\partial G(\alpha, d)}{\partial \alpha} = -\frac{16(1 - c)(1 - \alpha)}{(3 - \alpha)^2}.$$  \hfill (8)

(8) is negative for any $\alpha \in (-1, 1)$.

Q.E.D.

We explain the intuition behind Proposition 2. A decrease in $c$ (the common cost of two firms) lowers the price. Since $dp/dc = (a(1 - \alpha) + 2c)/(3 - \alpha)$, $dp/dc$ is increasing in $\alpha$. In other words, when $\alpha$ is large, a cost reduction by R&D reduces the equilibrium price substantially, thus firms lose incentives for R&D. This yields a smaller R&D level as $\alpha$ is larger.

Preposition 2 suggests that under joint implementation of R&D, perfect competition in product markets can be obstacles for innovations.

5 Concluding Remarks

In this paper, we investigate R&D competition in oligopoly markets where firms’ payoffs depend on both absolute and relative profits. Firm $i$’s payoff is $\pi_i - \alpha \pi_j$, where $\pi_i$ is its own profit, $\pi_j$ is the rival’s profit, and $\alpha \in (-1, 1)$. We find that U-shaped relationship between $\alpha$ and the levels of R&D investments when the firms choose the levels of R&D investments independently.

We can interpret $\alpha$ as a parameter indicating how severe the competition in the product market is. $\alpha = 0$ indicates the Standard Cournot case, $\alpha = 1$ indicates perfectly competitive case (Bertrand case), and a negative $\alpha$ indicates collusive (less competitive) situation.\footnote{For the discussions on the relationship between competitiveness and non-profit maximizing payoffs, see Bolton and Ockenfels (2000) and Fehr and Schmidt (1999).} Thus, a larger $\alpha$ indicates more competitive market. The relative performance approach enables us to treat competitiveness as a continuous variable, and this approach contains three models, Cournot, Bertrand, and monopoly, as special cases. Following this interpretation, our result shows that R&D levels are high in both competitive and collusive markets. This result, however, does not hold when firms jointly implement the R&D. In this situation, collusion in product markets promotes innovation.
In this paper we adopt relative performance approach. We think that this approach is important in more general contexts. We can also interpret that $\alpha$ indicates the degree of reciprocal attitude or altruism. An increase in $\alpha$ indicates less reciprocal attitude. We believe that this approach is applicable to broad areas of social sciences.
References


