Oligopoly Theory (7)
Product Differentiation and Spatial Competition

Aim of this lecture

(1) To understand the relationship between product differentiation and locations of the firms.
(2) To understand the difference between mill pricing and delivered pricing.

Outline of the 7th Lecture

7-1 Shopping Model and Shipping Model
7-2 Hotelling Model
7-3 Price-Setting Shopping Model
7-4 Circular-City Model
7-5 Agglomeration
7-6 Price-Setting Shipping Model
7-7 Quantity-Setting Shipping Model
7-8 Non-Spatial Interpretation of Shipping Model
7-9 Non-Spatial Product Differentiation Models
7-10 Mixed Strategy Equilibria
7-11 Linear and Circular City Models Revisited

Two Models of Spatial Competition

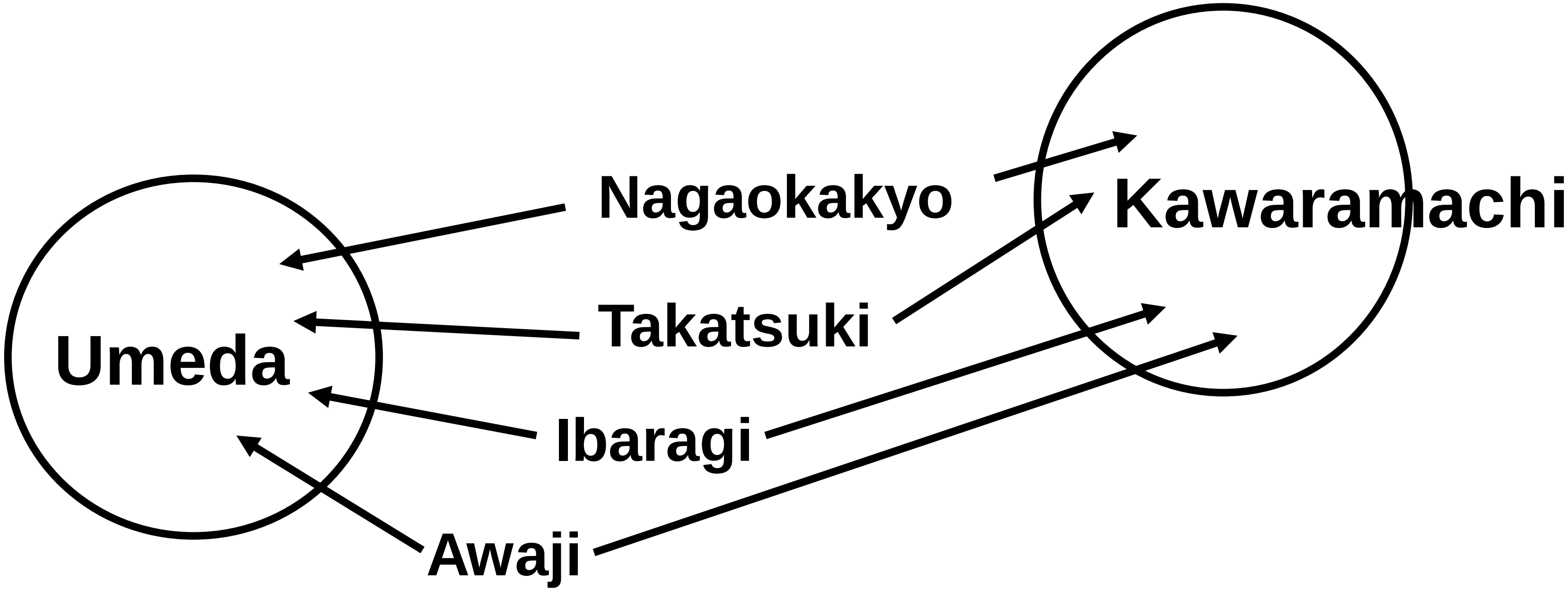
(1) Mill Pricing Model (Shopping Model)

Consumers pay the transport costs. Consumers go to the firm's shop.

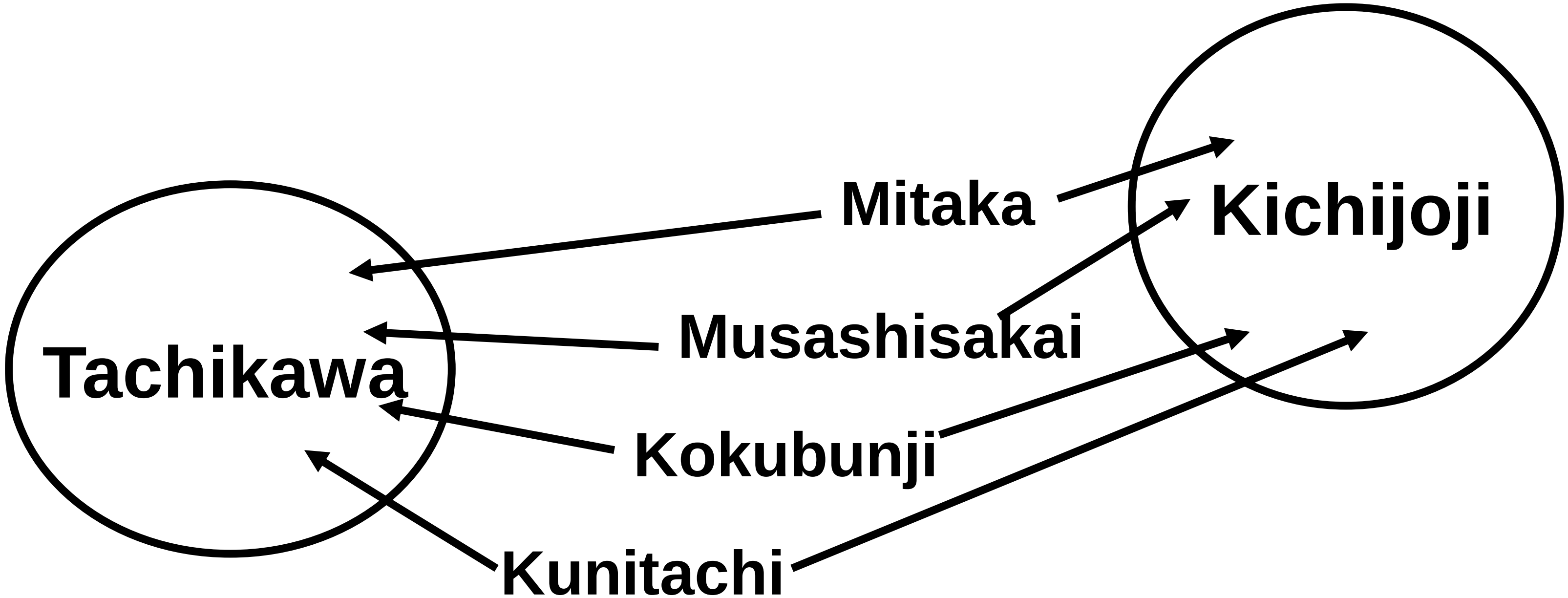
(2) Delivered Pricing Model (Shipping Model, Spatial Price Discrimination Model)

Firms pay the transport costs. Firms bring the goods to the markets.

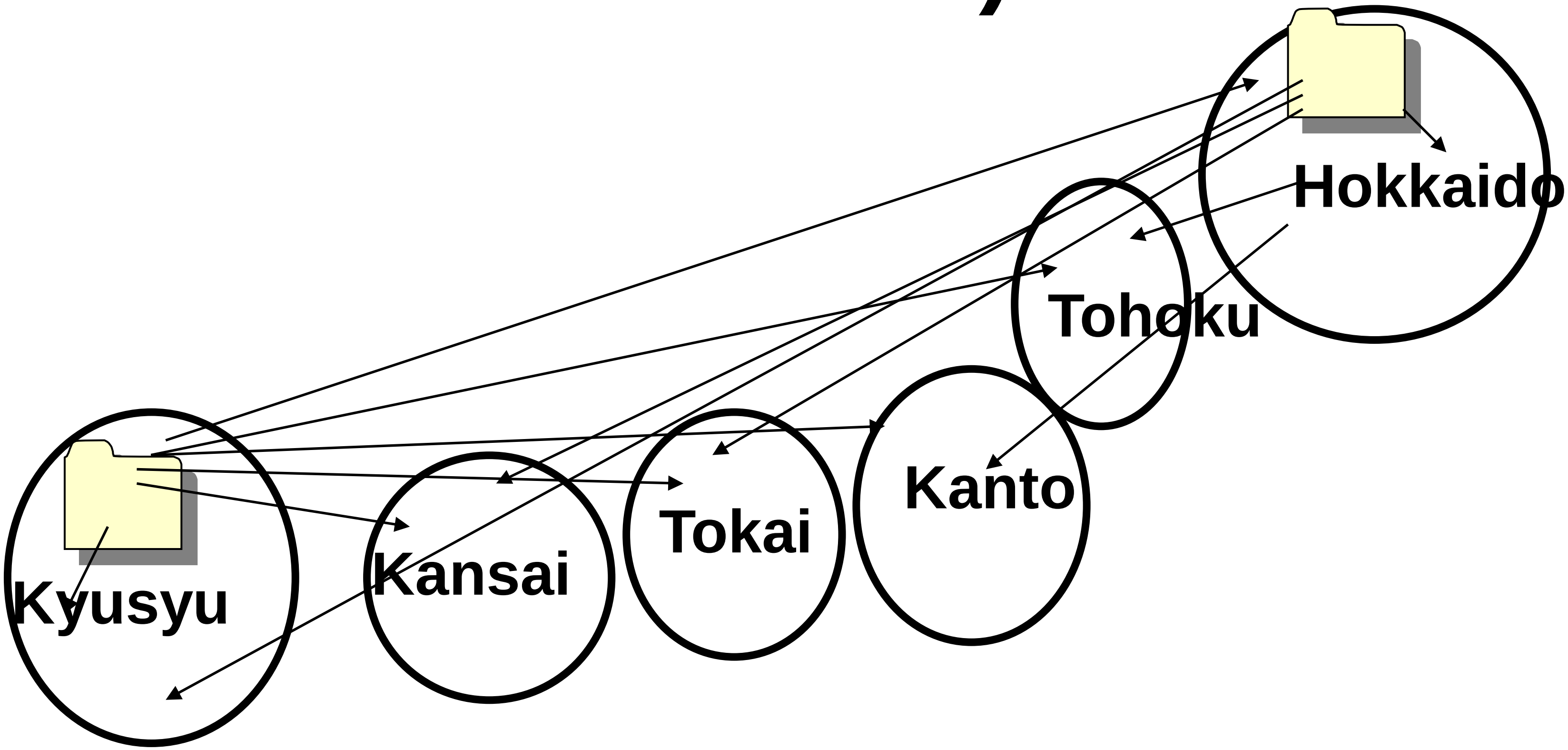
Mill Pricing Model (Shopping Model)



Mill Pricing Model (Shopping Model)



Delivered Pricing Model (Shipping Model, Spatial Price Discrimination Model)



Mill Pricing (Shopping) Models

Hotelling

Duopoly Model, Fixed Price Model, Shopping Model.

Consider a linear city along the unit interval [0,1], where firm 1 is located at x_1 and firm 2 is located at x_2.

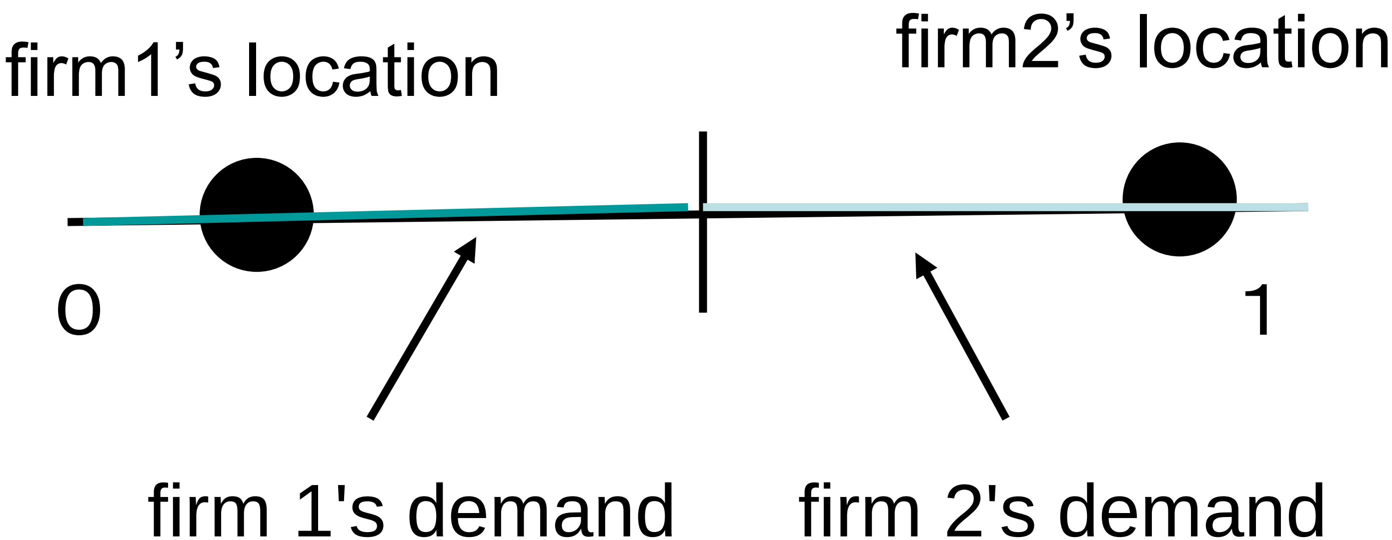
Consumers are uniformly distributed along the interval.

Each consumer buys exactly one unit of the good, which can be produced by either firm 1 or firm 2.

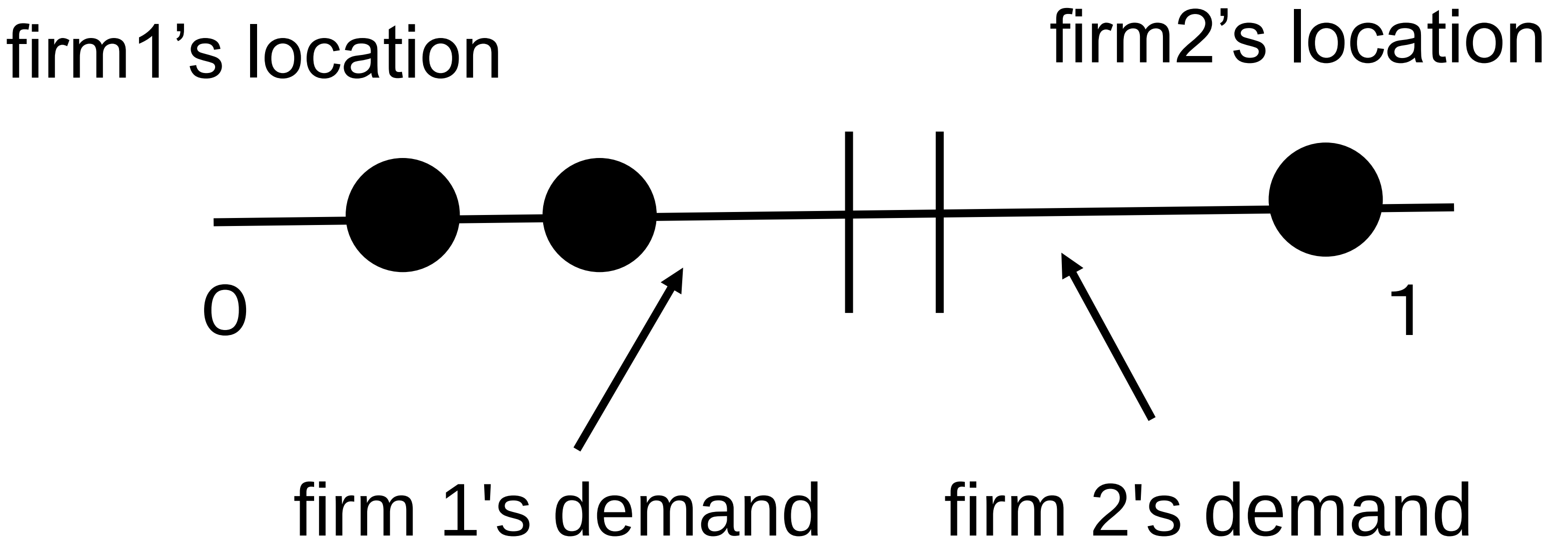
Each consumer buys the product from the firm that is closer to her.

Each firm chooses its location independently.

Hotelling


firm1’s location
firm2’s location
0
1
firm 1's demand
firm 2's demand

Relocation of Firm 1



This relocation increases the demand of firm 1, resulting in a larger profit of firm 1

Equilibrium

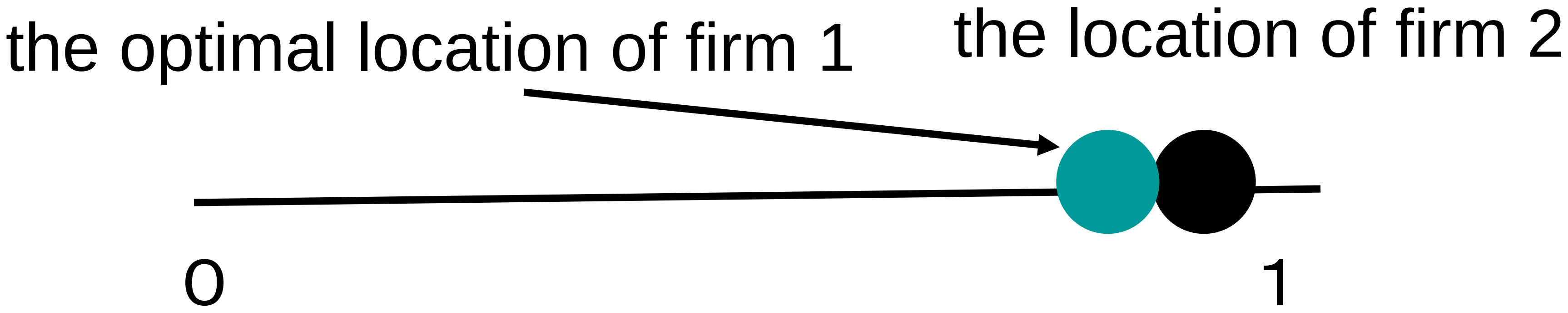
Firm 1's Best Response

If firm 2's location is larger than 1/2, then the location just left to it is firm 1's best reply.

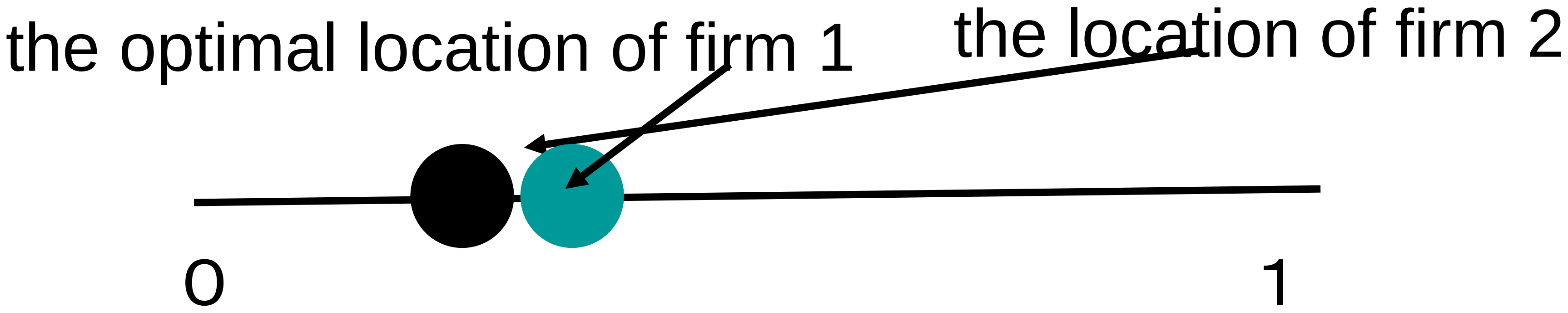
If firm 2's location is smaller than 1/2, then the location just right to it is firm 1's best reply.

→Two firms agglomerate at the central point.

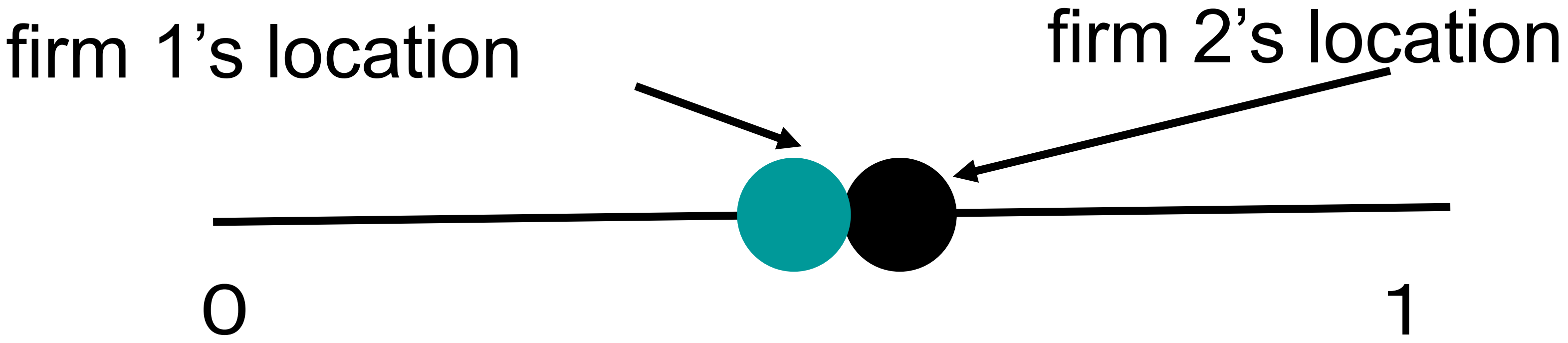
Best reply for firm 1



Best reply for firm 1



Equilibrium



Interpretation of the linear city

(1) city ~ spatial interpretation
(2) product differentiation ～ horizontal product differentiation
(3) political preference

(3)→interpretation of minimal differentiation
～The policies of two major parties become similar.

However, following the interpretation of (1) and (2), the model lacks the reality since consumers care about prices as well as the locations of the firms.

Vertical Product Differentiation

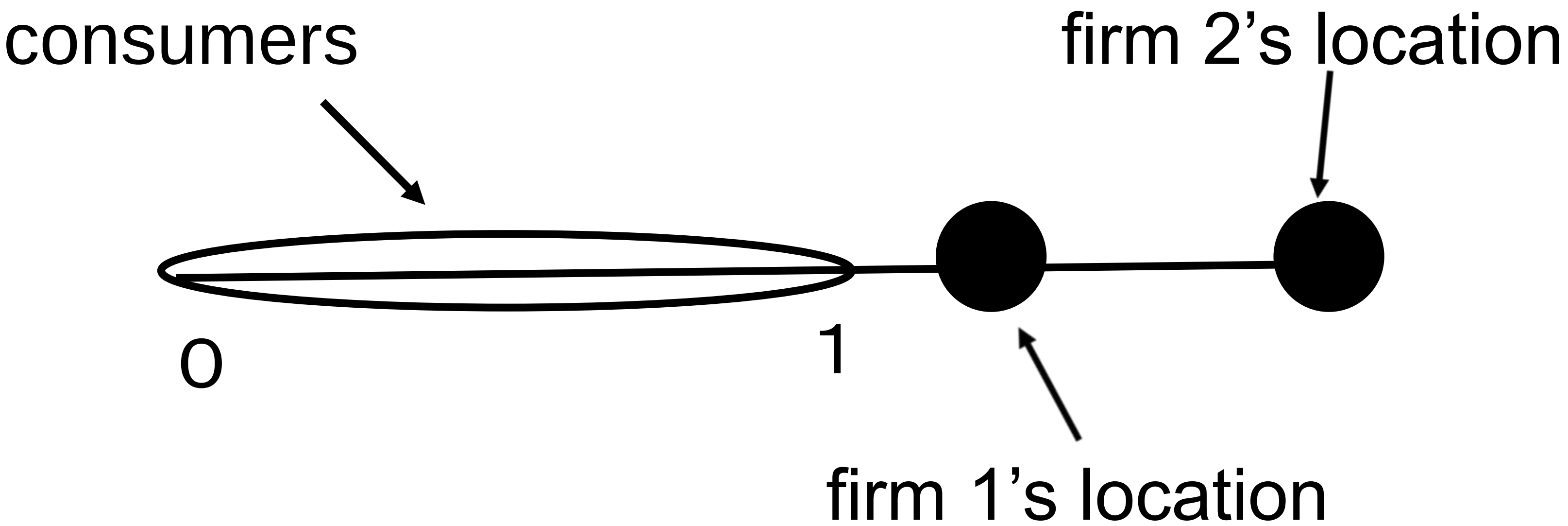
Vertical differentiation～higher quality product, lower quality product

If the prices of two products are the same and all consumers choose product A, not product B, then two products are vertically differentiated and product A is a higher product market.

We can formulate a vertically differentiated product model by the Hotelling line.

Vertical Product Differentiation

All consumers choose firm 1 if the prices of two firms are the same.



Endogenous Price

Duopoly Model, Shopping Model. Consider a linear city along the unit interval [0,1], where firm 1 is located at x_1 and firm 2 is located at x_2.

Consumers are uniformly distributed along the interval. Each consumer buys exactly one unit of the good, which can be produced by either firm 1 or firm 2. Each consumer buys the product from the firm whose real price (price +transport cost) is lower.

One-Stage Location-Price Model

Duopoly Model, Shopping Model. Consider a linear city along the unit interval [0,1], where firm 1 is located at x_1 and firm 2 is located at x_2.

Consumers are uniformly distributed along the interval. Each consumer buys exactly one unit of the good, which can be produced by either firm 1 or firm 2. Each consumer buys the product from the firm whose real price (price + transport cost) is lower.

Each firm chooses its location and price independently.

One-Stage Location-Price Model

No pure strategy equilibrium exists.

Given the price of the rival, each firm has an incentive to take a position closer to the rival's (the principle of the Hotelling).

Given the minimal differentiation, each firm names the price equal to its marginal cost, resulting in a zero profit. →Each firm has an incentive for locating far away each other. →Given the price of the rival, each firm again has an incentive to take a position closer to the rival's (the principle of the Hotelling).

Two-Stage Location then Price Model

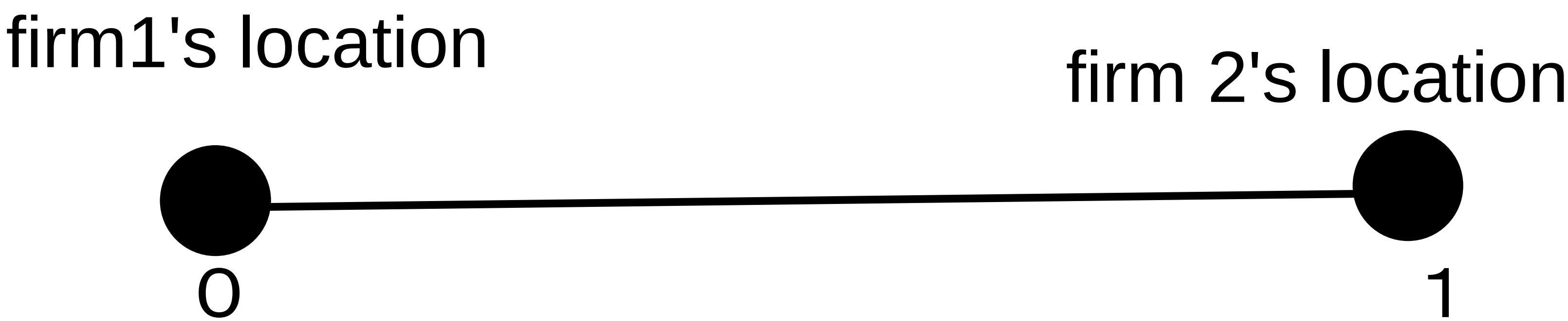
The same structure as the previous model except for the time structure. Each consumer buys the product from the firm whose real price (price +transport cost) is lower. Transport cost is proportional to (the distance)^2.~quadratic transport cost.

In the first stage, each firm chooses its location independently.

In the second stage they face Bertrand competition.

d'Aspremont, Gabszewics, and Thisse (1979, Econometrica)

Maximal Differentiation



Equilibrium

Maximal Differentiation

Each firm has an incentive to locate far away from the rival so as to mitigate price competition.

A decrease in $|x_2 - x_1|$ increases the demand elasticity ~ price becomes more important

An increase in the demand elasticity increases the rival's incentive for naming a lower price.

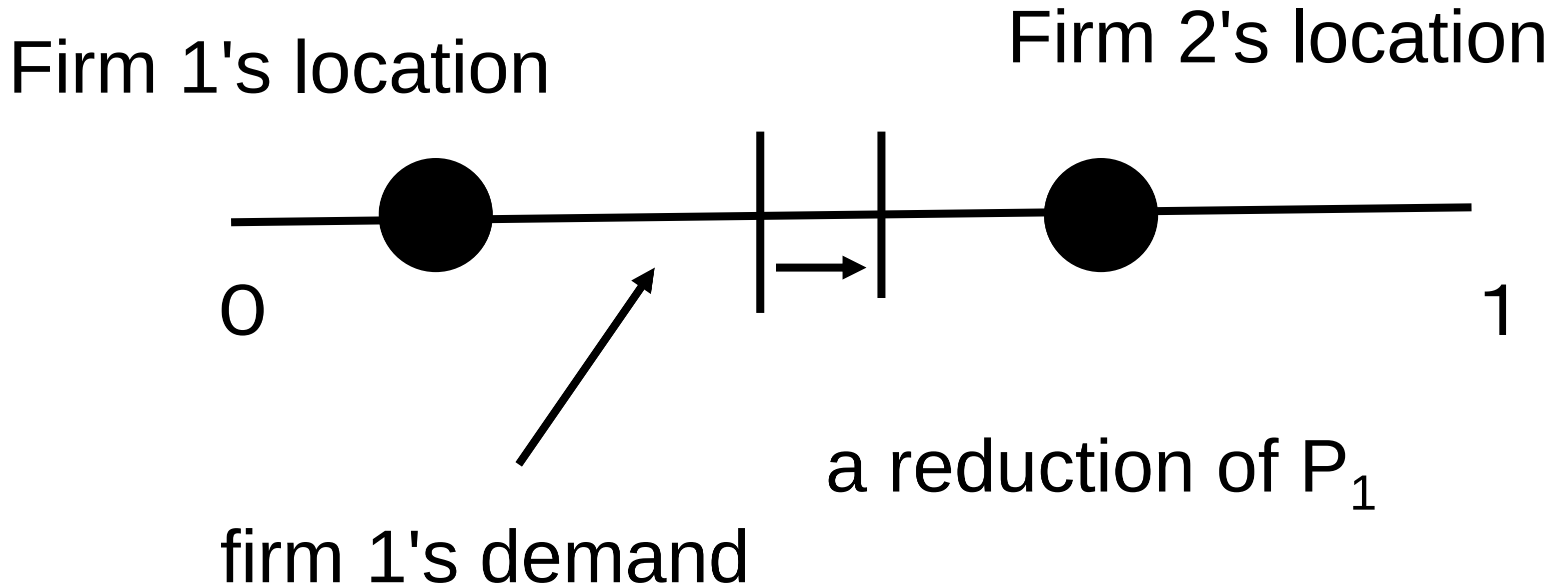
Through the strategic interaction (strategic complements), the rival's lower price increases the incentive for naming a lower price. → further reduction of the rival's price

Why Quadratic?

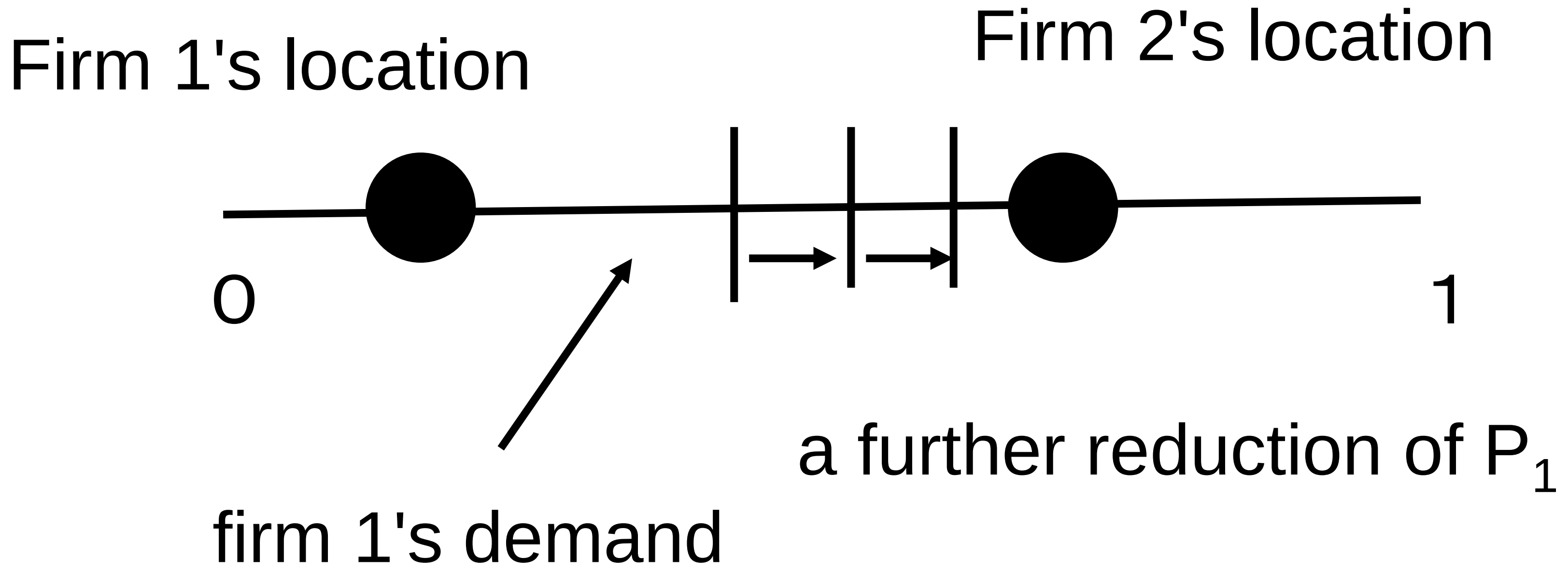
Why do we use quadratic transport cost function?
Hotelling himself use linear (proportional to the distance)

If we use linear transport cost, the payoff function becomes non-concave
→no pure strategy equilibrium exists

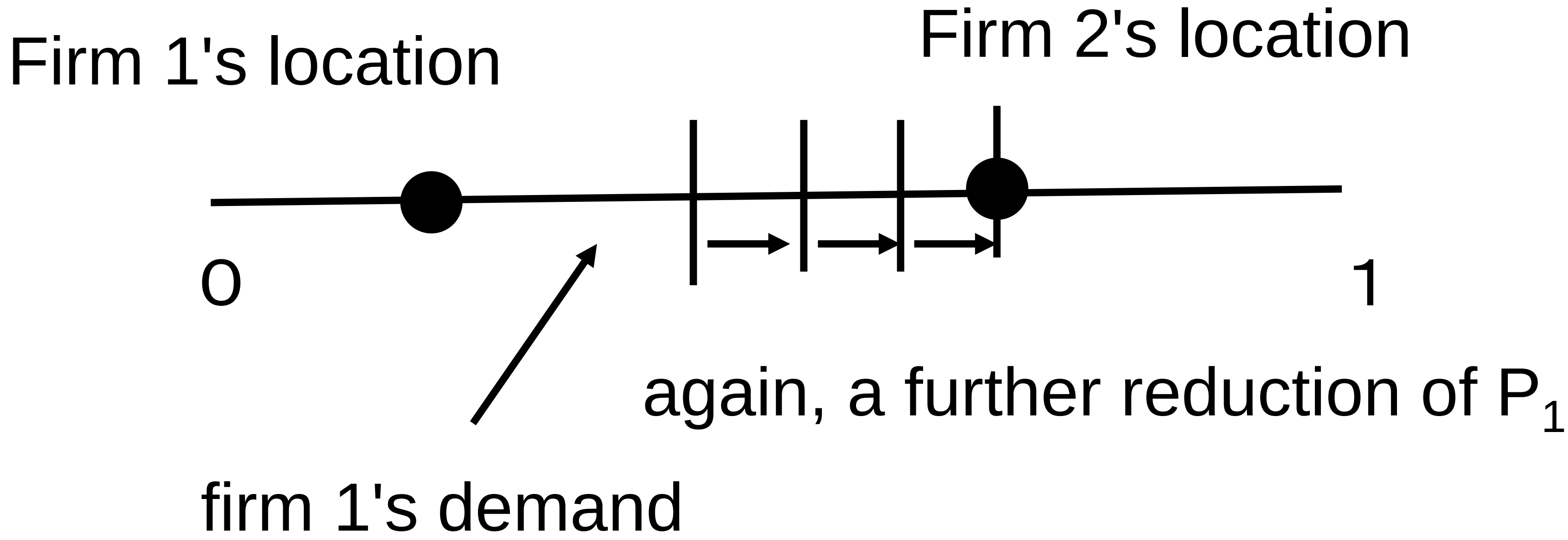
second stage subgame



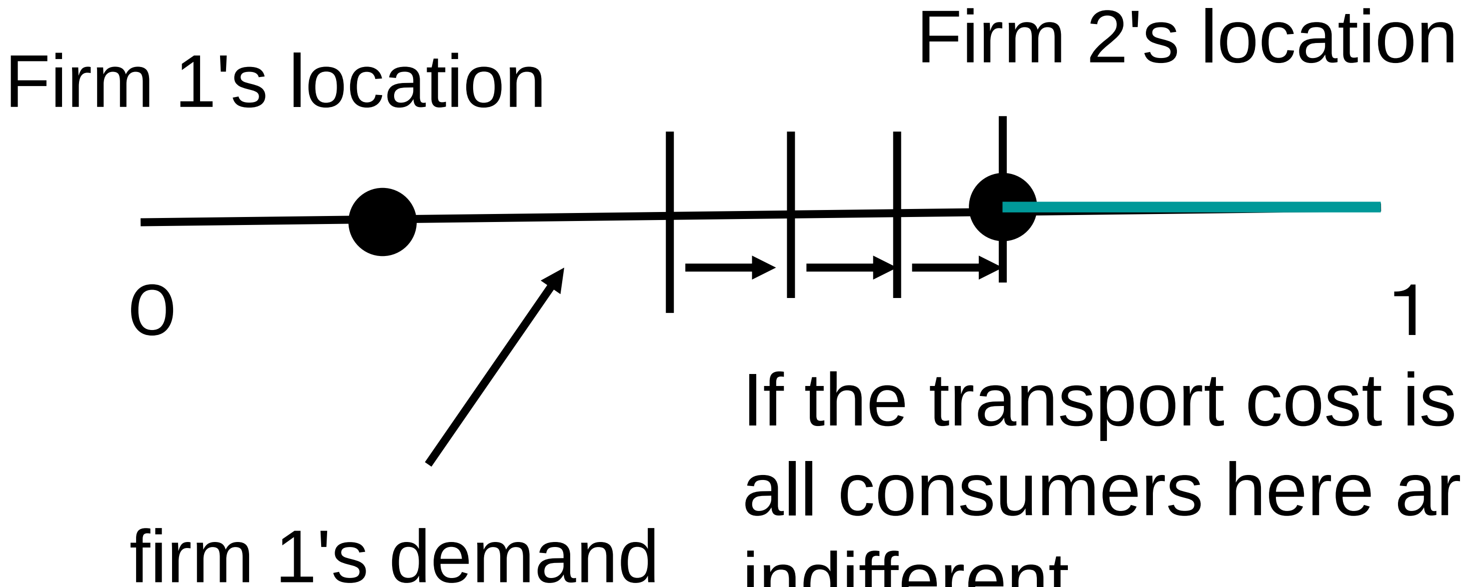
second stage subgame



second stage subgame

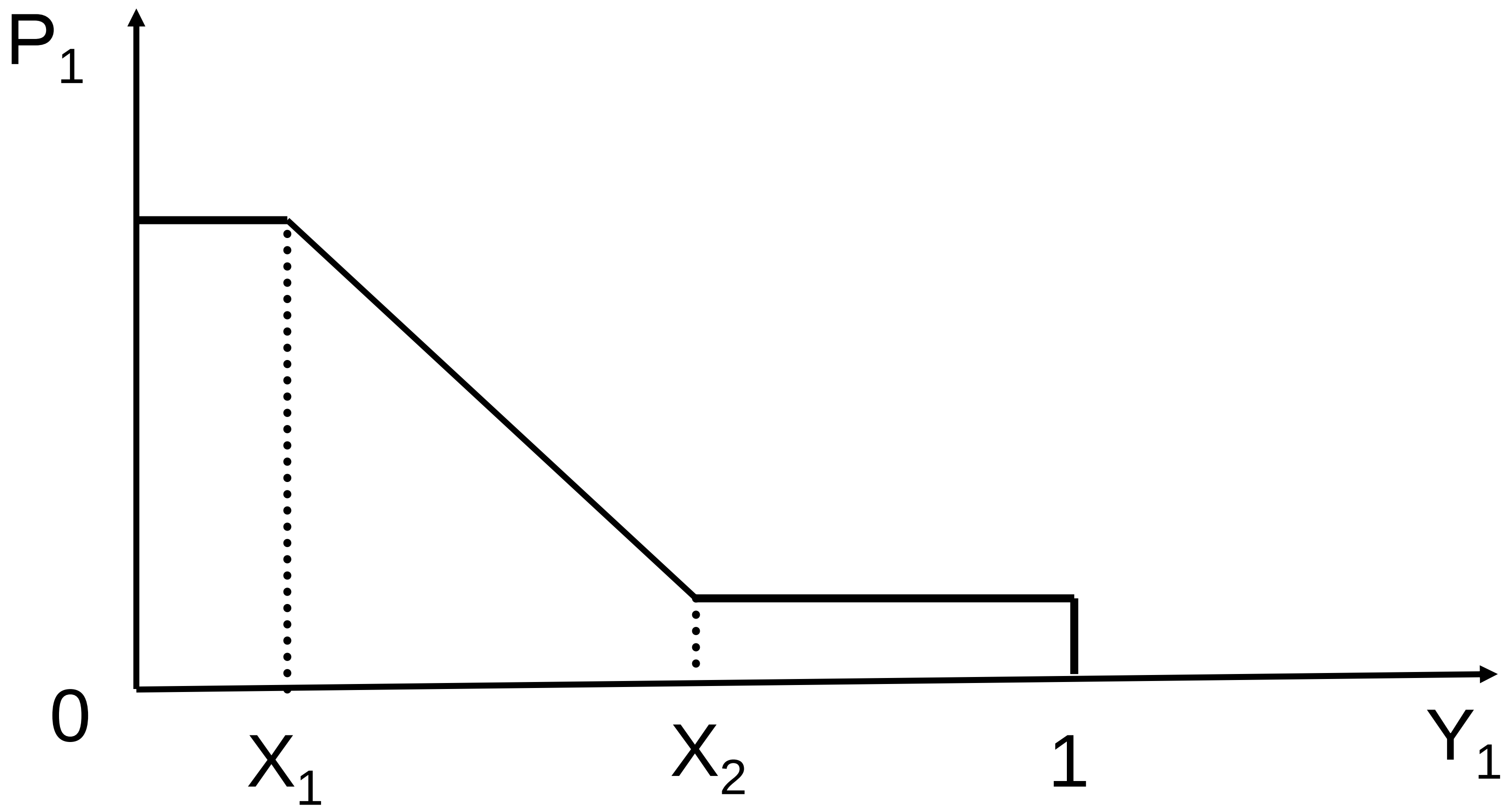


second stage subgame



If the transport cost is linear, all consumers here are indifferent.

Firm 1's demand



Linear Transport Costs

Difficulties

(1) Demand function (and so profit function) is not differentiable. ~ Analysis becomes complex substantially.

(2) Non-concavity of the profit function

Problem (1) disappears as long as the transport cost function is strictly convex, while (2) takes place if t'' (distance) is small.

→It is possible that no pure strategy equilibrium exists even when t'' >0.

Strong Convexity

Difficulty when t'' is too large.

If t'' is too large, given the moderate price p_2, firm 1 can monopolize the market near to its location. Thus, it has an incentive to name a high price and obtains the market near to its location only.

→Given this high price, firm 2 raises the price

→Given firm 2's high price, firm 1 reduces the price substantially and obtains a larger market.

→Given firm 1's low price, firm 2 has an incentive to raise the price and obtain the market near to its location only. →firm 1 raises the price.

～similar to Edgeworth Cycle.

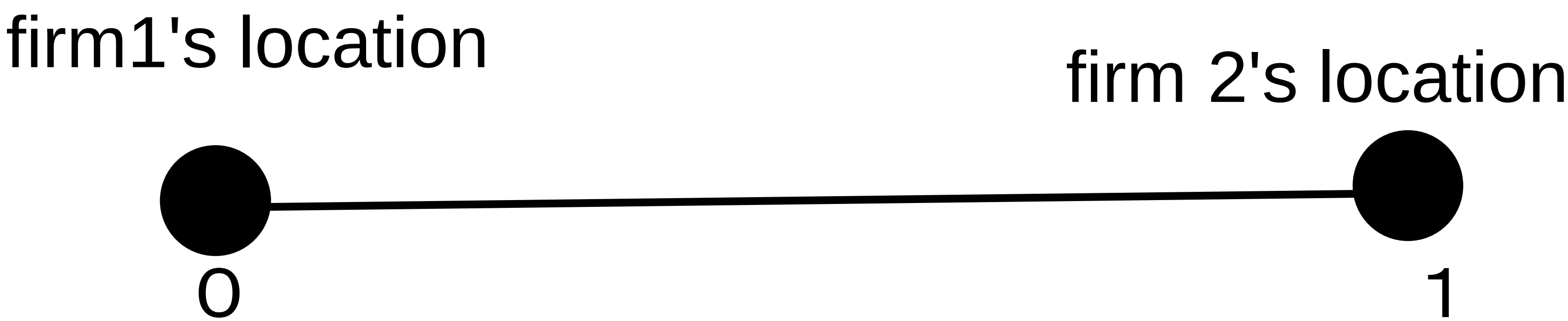
Linear-Quadratic Transport Costs

Linear-quadratic transport cost
$= \alpha(\text{distance}) + \beta(\text{distance})^2$

(1) Pure strategy equilibrium exits unless β is too small (α is too large).
(2) Non-maximal differentiation can appear in equilibrium.

Anderson (1988,Economica)

Possible Location of the Outside of the Linear City, Tabuchi and Thisse (1995), Lambertini (1997)



This is a corner solution. Firms may be able to locate outside the city. Then a further product differentiation appears in equilibrium.

Strategic Behavior in Two Spatial Models

Two models (a model allowing firms to locate outside the city (model 2) and a model not allowing (model 1) yield the contrasting implications.)

In the first stage, firms make strategic commitments. In the second stage, firms chooses their locations. In the third stage, firms chooses their prices.

Matsumura and Matsushima (2013a,b)

In Model 1, commitment inducing aggressive behavior (lower production cost, a positive weight for sales in management reward, and so on) induces aggressive pricing, resulting in a lower profits. ⇒Firms avoid such a commitment.

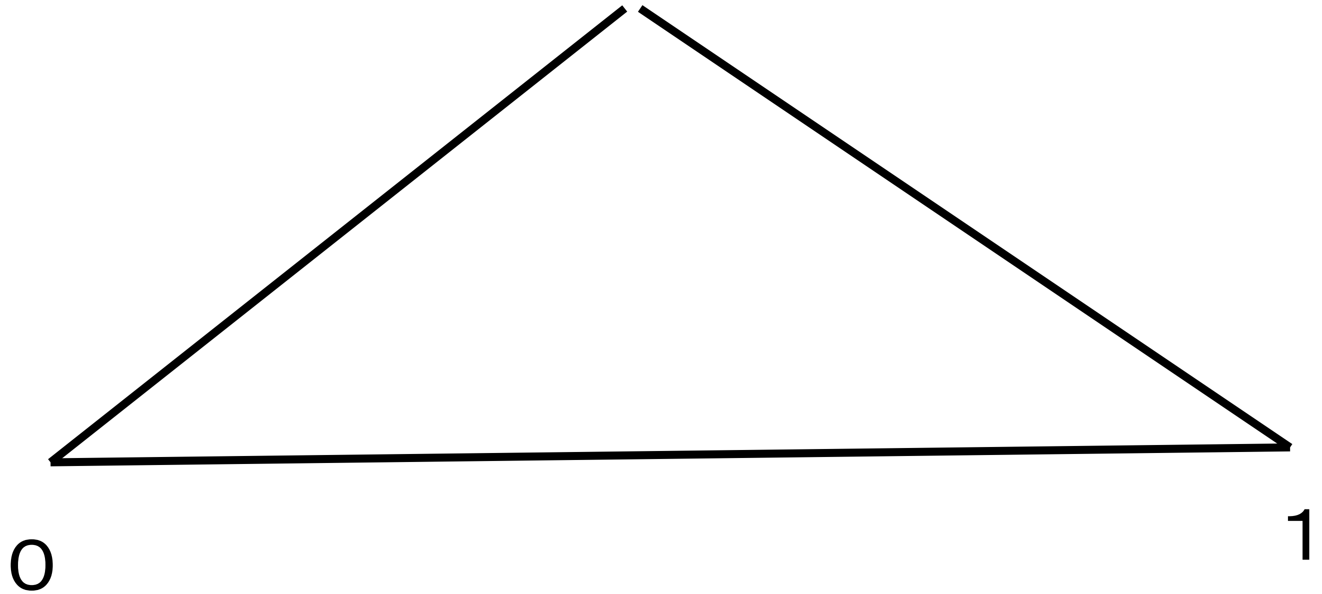
In Model 2, commitment inducing aggressive behavior induces aggressive location choice as well as aggressive pricing. The former induces less aggressive location of the rival and increases its profit. The former effect dominates the latter and firms make such commitment. ~ Model 2 may result in larger CS.

Non-Uniform Distribution of Consumers

Suppose that consumers agglomerate at the center of the city.

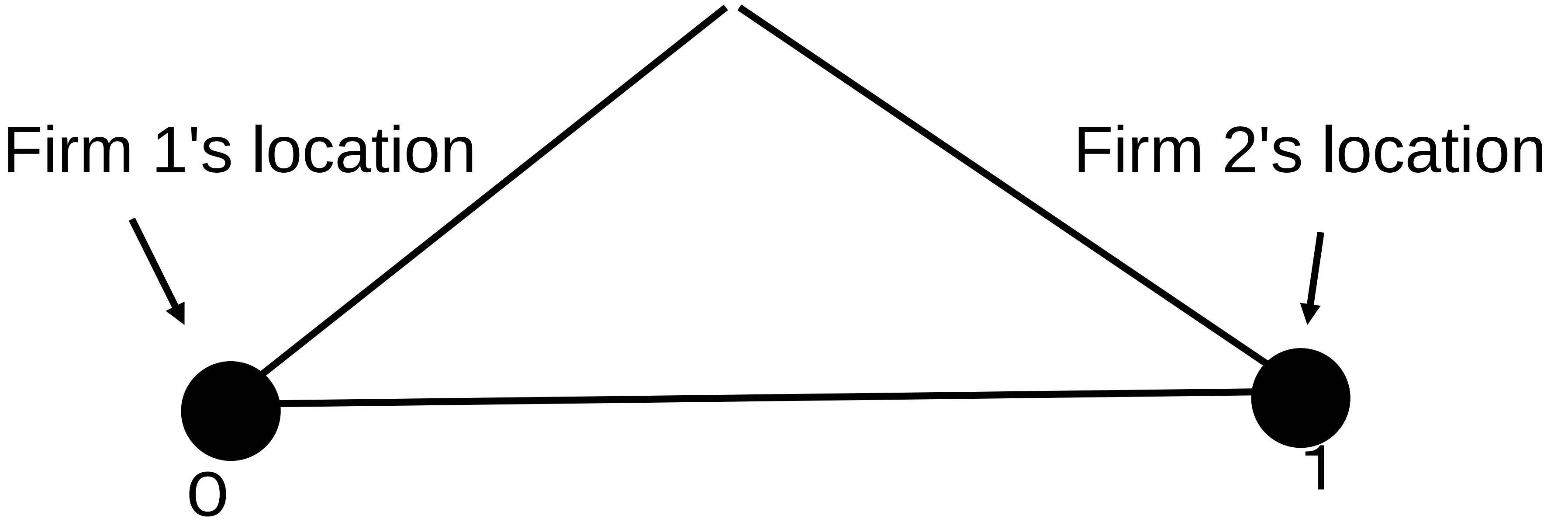
Non-Uniform Distribution of Consumers

Tabuchi and Thisse (1995, IJIO)



Non-Uniform Distribution of Consumers

Tabuchi and Thisse (1995)



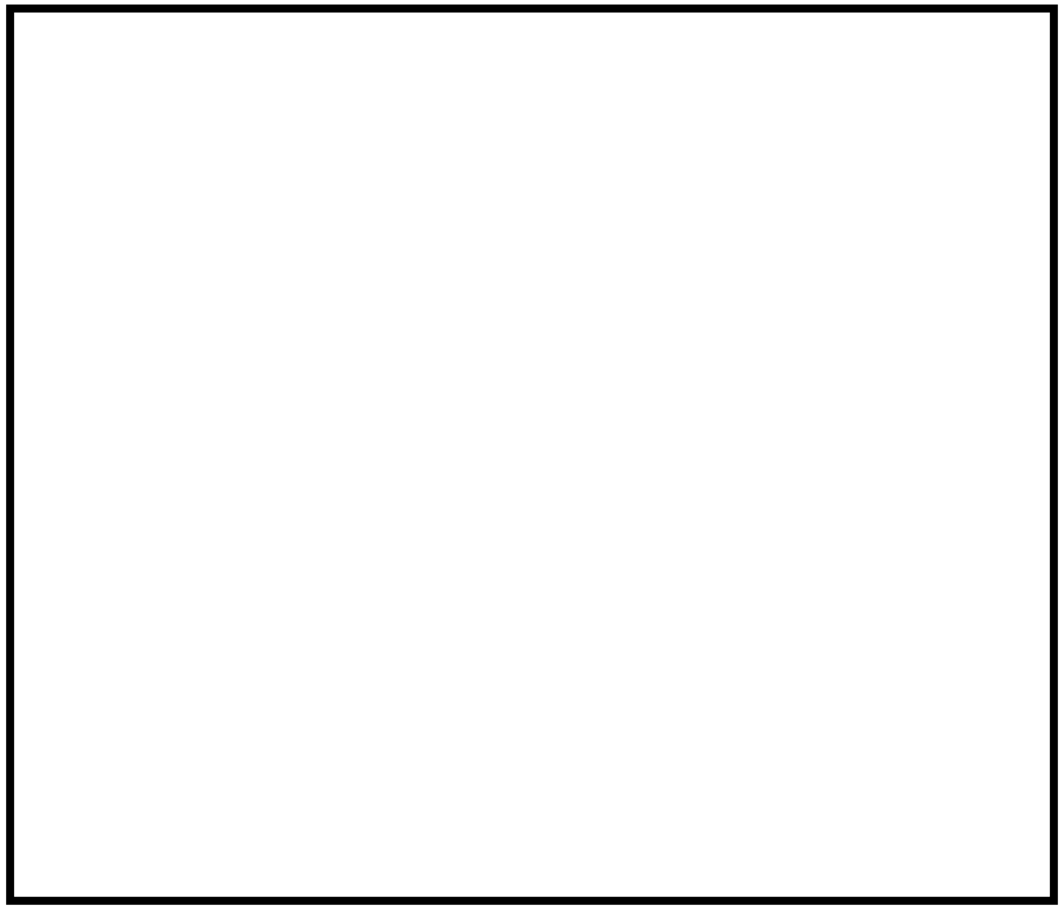
Question: The competition is (more, less) severe under this distribution than under the uniform distribution.

Non-Uniform Distribution and Competition

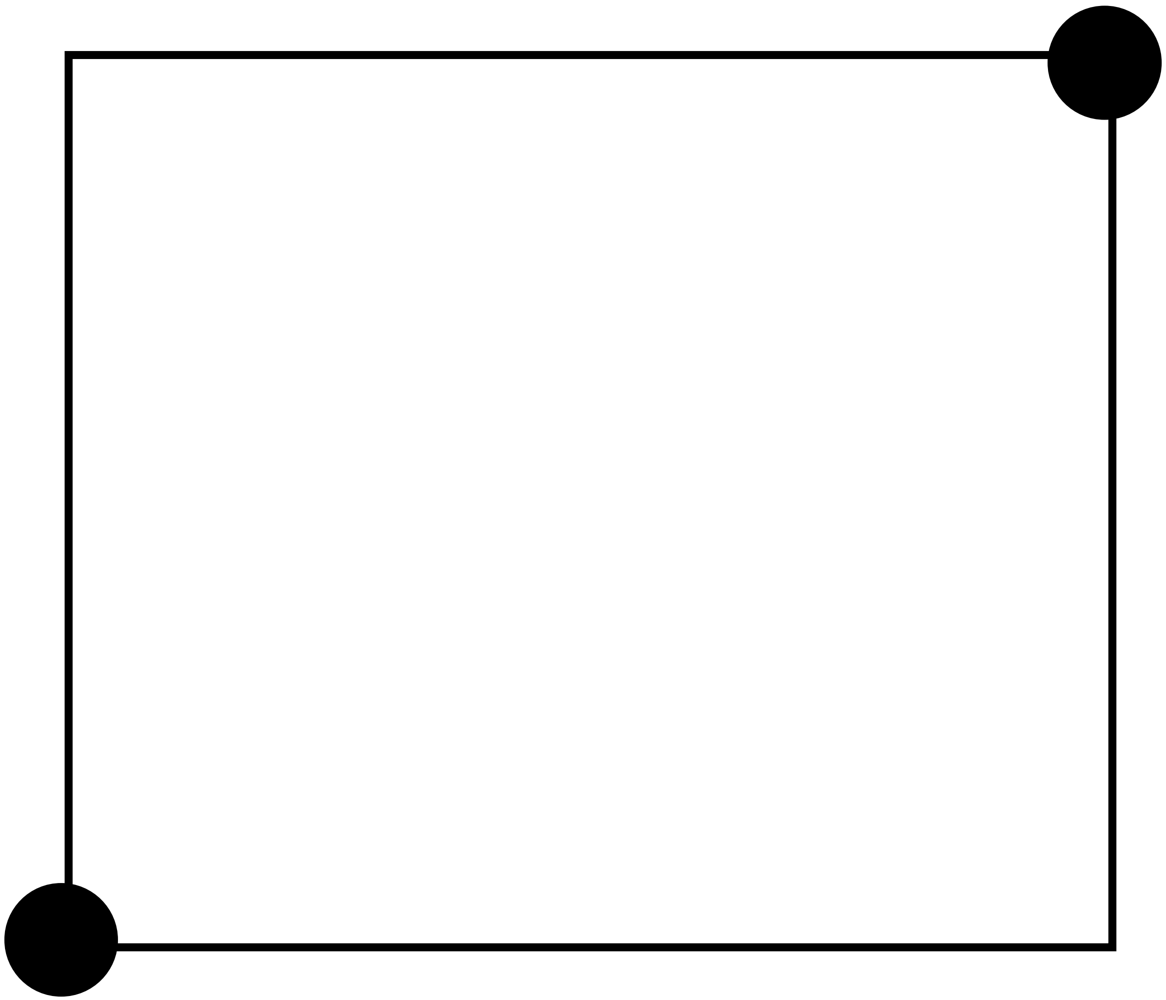
Suppose that $p_1 = p_2 = p^E$ in equilibrium under uniform distribution.

Given $p_2 = p^E$, firm 1's optimal price (best response) is (higher, lower) than p^E under non-uniform distribution (triangle distribution) in the previous sheet.

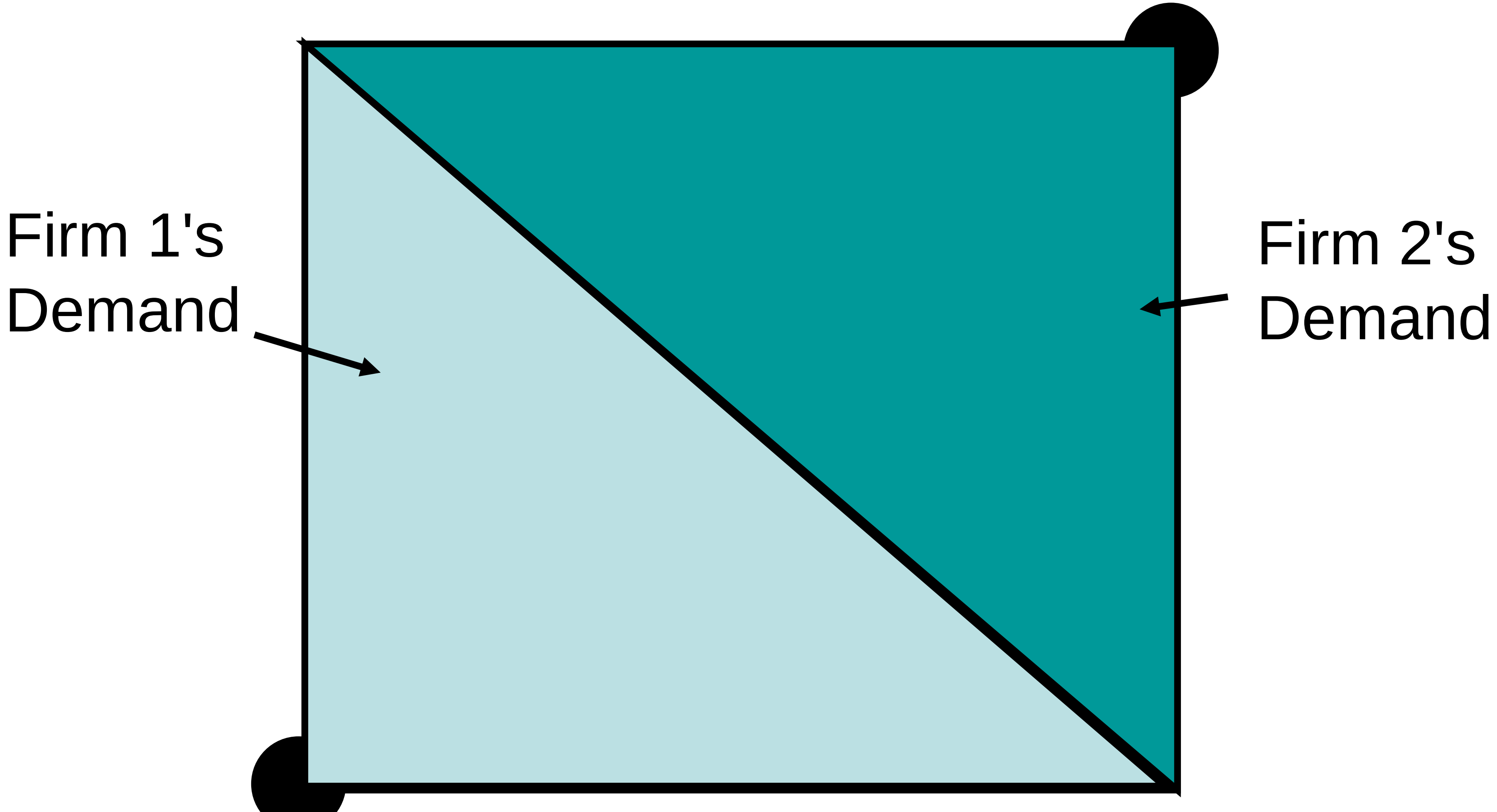
Two-Dimension Space
Tabuchi (1994, RSUE)



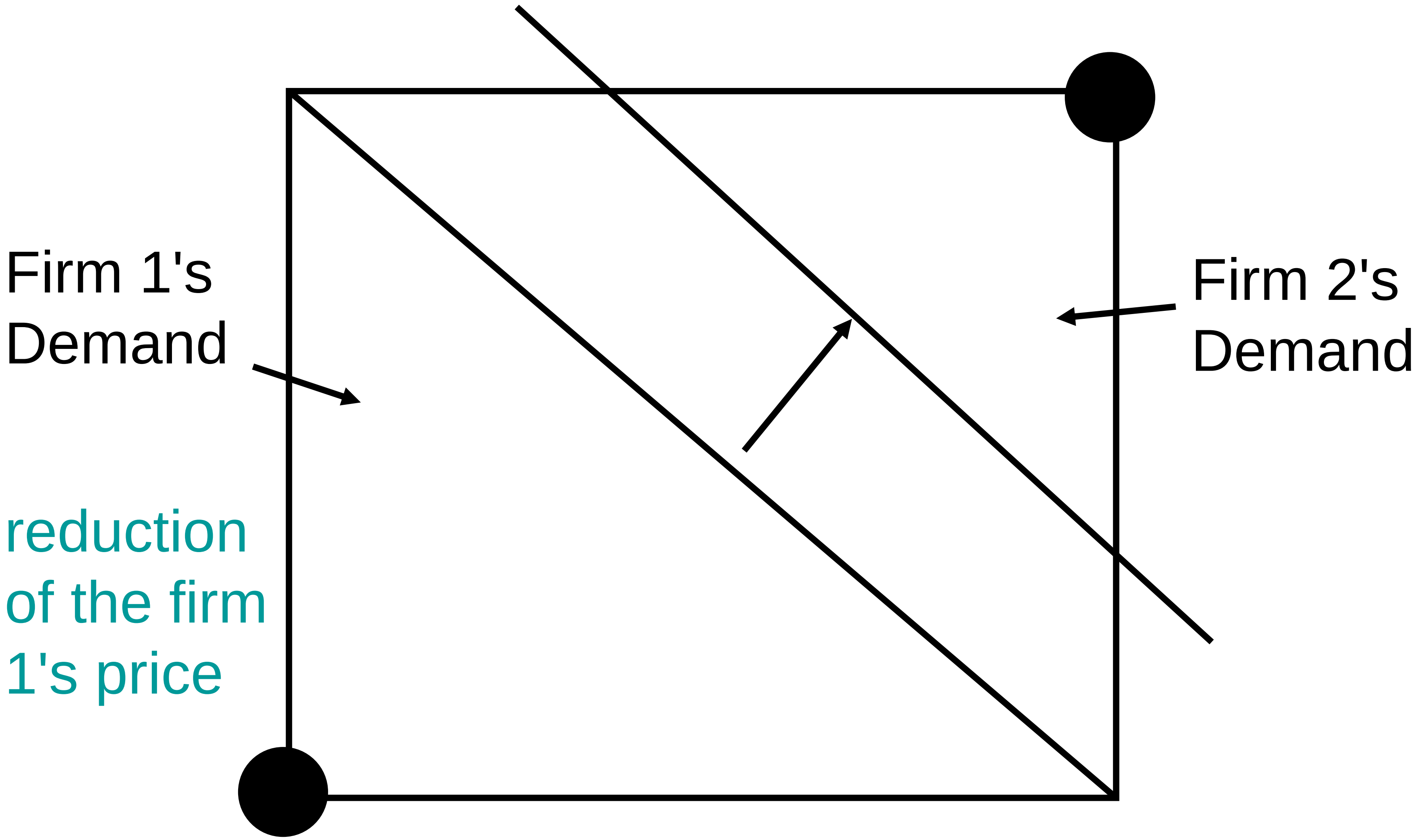
Maximal Differentiation



Maximal Differentiation

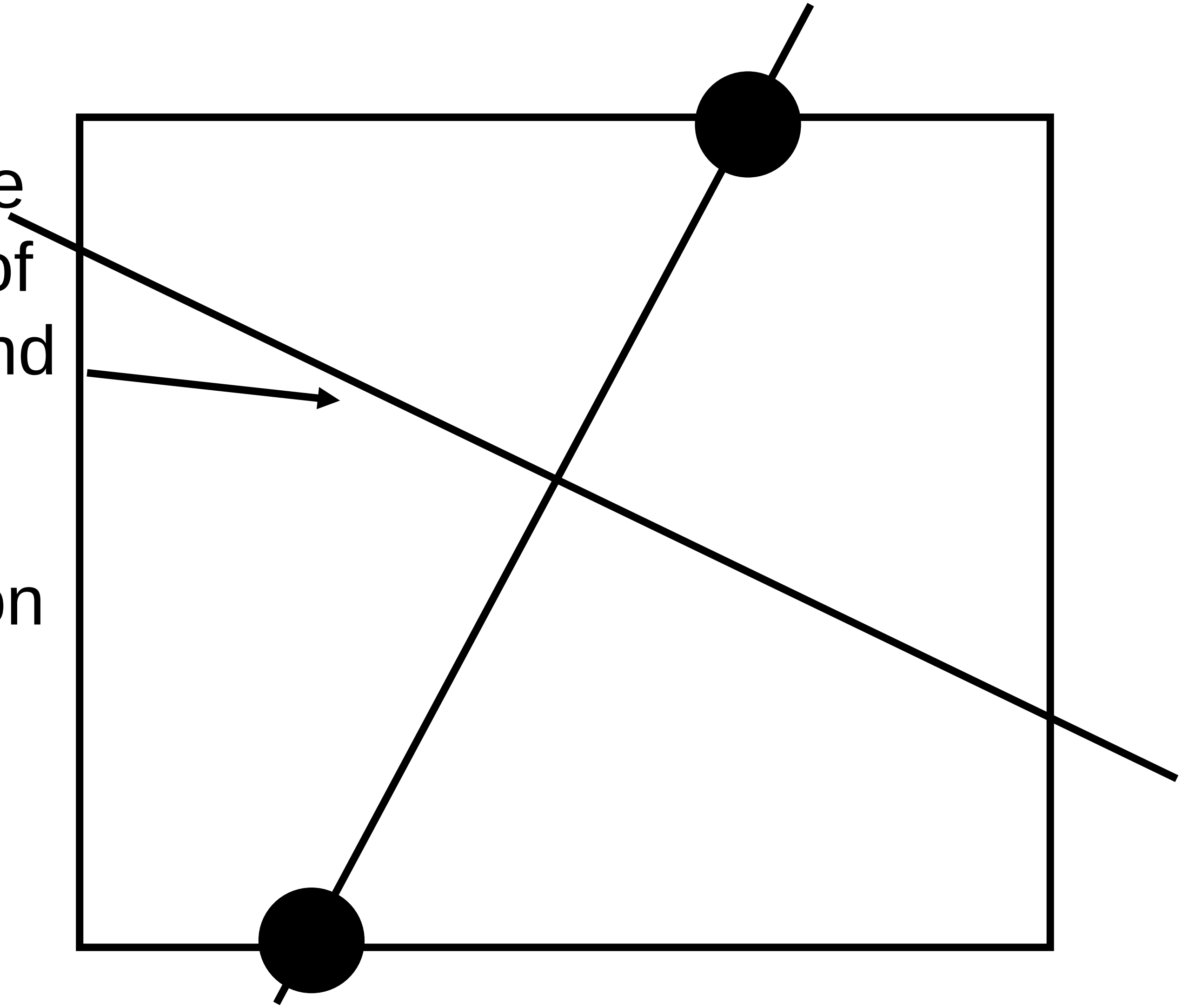


Maximal Differentiation

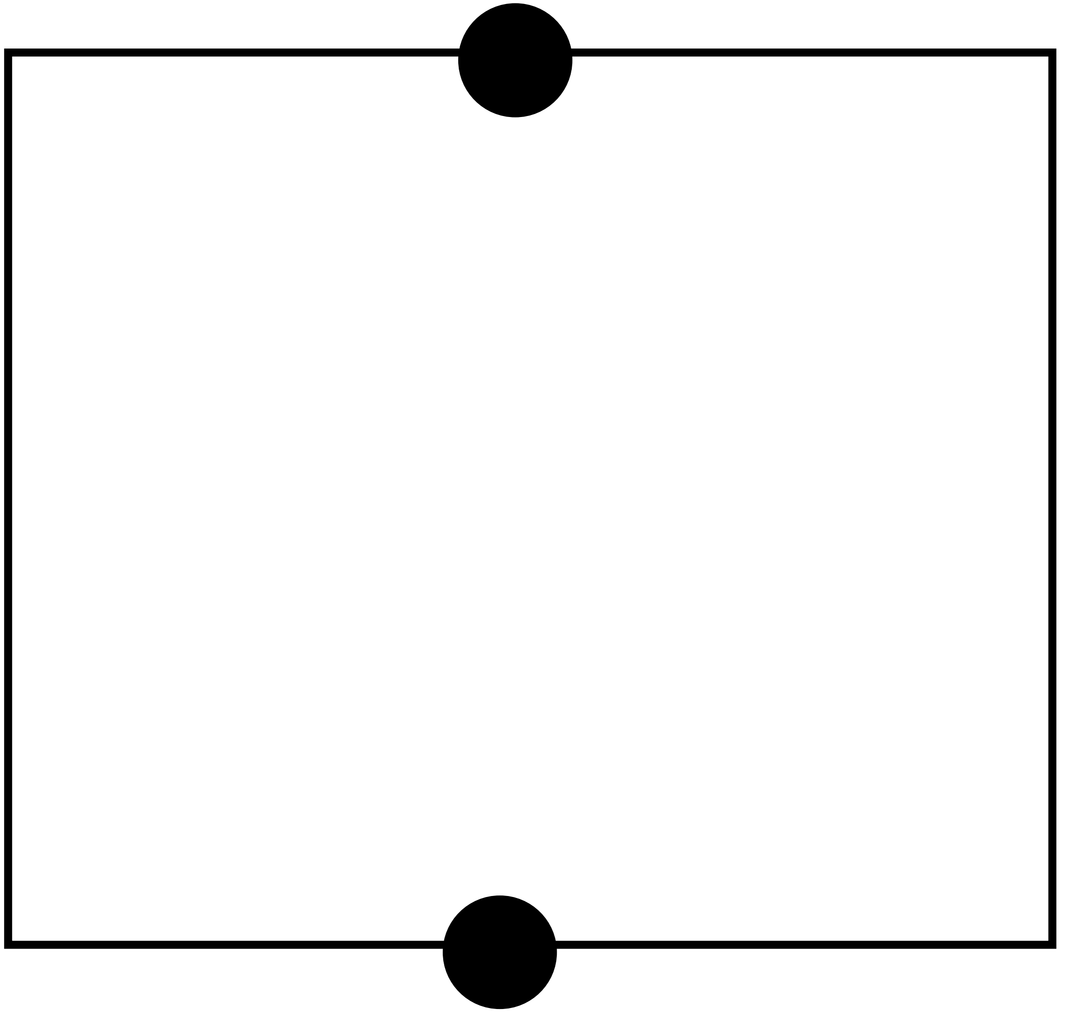


Non-Maximal Differentiation

lower price elasticity of the demand →it mitigates competition

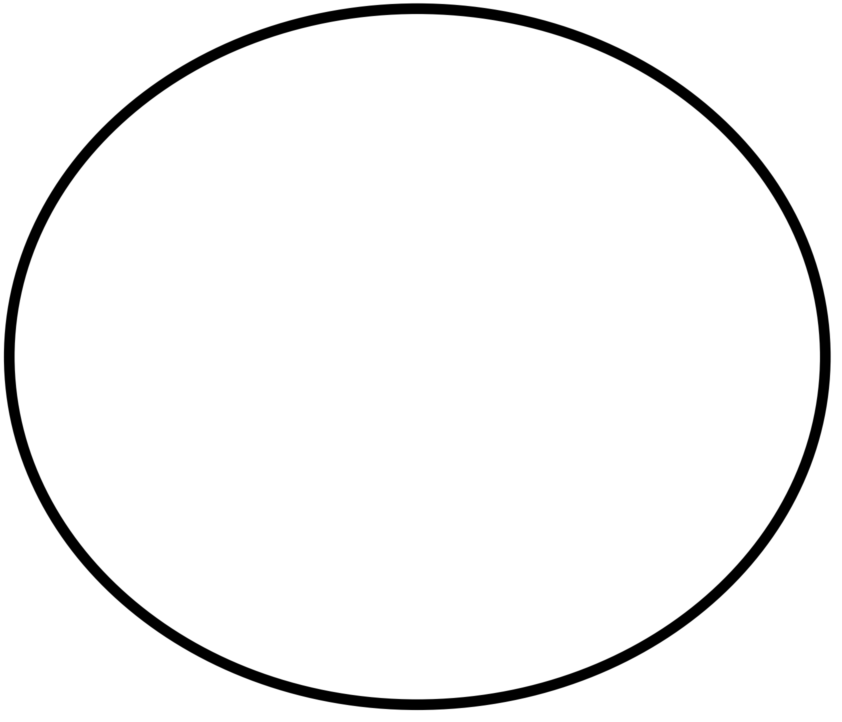


Equilibrium



Circular-City Model

Vickrey (1964), Salop (1979)



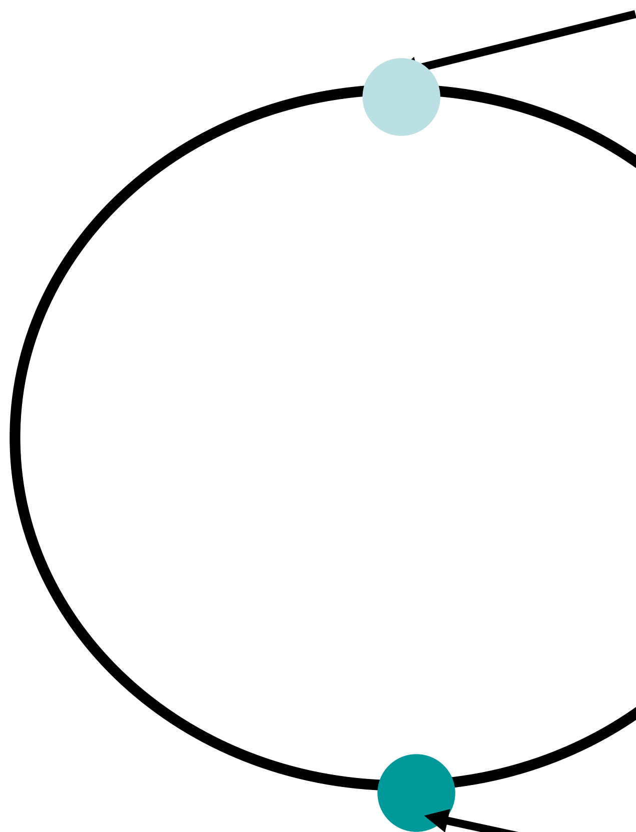
Properties of Circular-City Model

(1) Symmetry ~ no central- periphery structure
→Advantage for analyzing n-firm oligopoly modes.
(2) Pure strategy equilibrium can exist when transport cost function is linear or even concave.

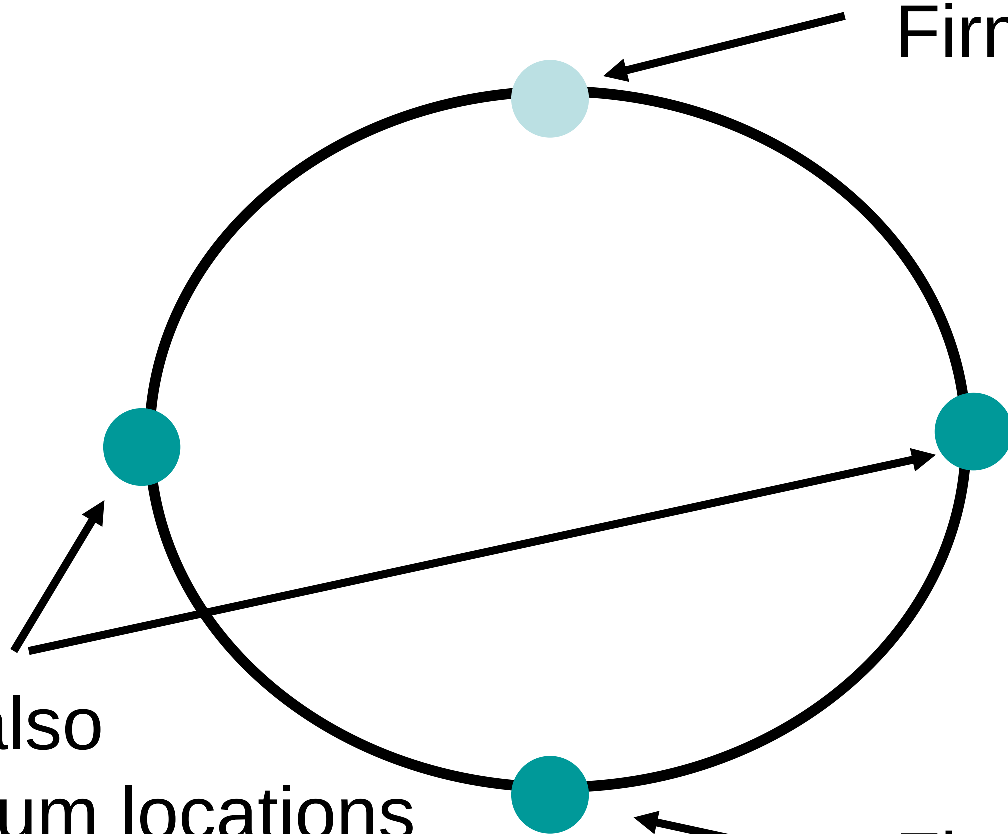
Equilibrium locations under linear-quadratic transport cost

Both strictly convex and concave transport cost usually yield this type of equilibrium

De Frutos et al (1999,2002, RSUE)



Equilibrium locations under linear transport cost



Agglomeration

In reality firms often agglomerate (firms often produce homogeneous products).

- There are other factors of product differentiation, which are not represented by the linear city.
  →Products are differentiated even if firms agglomerate at the center.～de Palma et al. (1985)
- Externality ～ Mai and Peng (1999)
- Delivered Pricing, Cournot～Hamilton et al. (1989)
- Uncertainty
- Location then Collusion
- Cost Asymmetry

Matsumura and Matsushima (2009)

The same structure except for asymmetric costs between duopolists.

Firm 1's unit cost is 0, Firm 2's is c >0

- Small cost difference→Maximal Differentiation
- Large cost difference→No Pure Strategy

Under large cost difference, the major firm (lower cost firm) prefers agglomeration, whereas the minor firm still prefers maximal differentiation→conflict of interests→No pure strategy equilibrium

mixed strategy equilibrium: Firms randomly choose both edges of the city→agglomeration with probability ½.

Friedman and Thisse (1993, Rand)

Duopoly Model, Location then Price Model, Symmetric Firms

Firms choose locations

Firms collude. They divide their collusive profits according to the relative profits at status quo.

→agglomeration

Many (Japanese) legal scholars think that non-product differentiation and collusion are closely related.

This model supports this view.

Intuition behind agglomeration

Firm 1 moves from the edge to the center →Its profit decreases and the rival's profit also decreases

Its own profit～Hotelling effect (positive)+ competition accelerate effect (negative)

Rival's profit～Hotelling effect (negative)+ competition accelerate effect (negative)

→improves bargaining position of firm 1.

This is why agglomeration appears in location-collusion model.

Subsequent works

Jehiel (1992, IJIO)

Nash Bargaining →central agglomeration without side payment

Rath and Zhao (2003, GEB)

egalitarian solution and Kalai-Smorodinsky solution →multiple equilibria including central agglomeration exist.

These result does not hold under even slight cost difference between two firms (Matsumura and Matsushima, 2011, GEB)

Delivered Pricing (Shipping) Models

delivered-pricing model

Consider a symmetric duopoly.

Transport cost is proportional to both distance and output quantity (linear transport cost).

In the first stage, each firm chooses its location independently.

In the second stage, each firm chooses its price independently.

Each point has an independent market, and the demand function is linear demand function, P=A-Y. No consumer's arbitrage. Production cost is normalized as zero. A is sufficiently large.

Interpretation of pricing strategy delivered-pricing vs uniform pricing

Delivered Pricing~ Personalized Pricing
⇒Personal Discounting based on Personal Data.

Then, firms can choose which strategies they adopt (I will explain later).

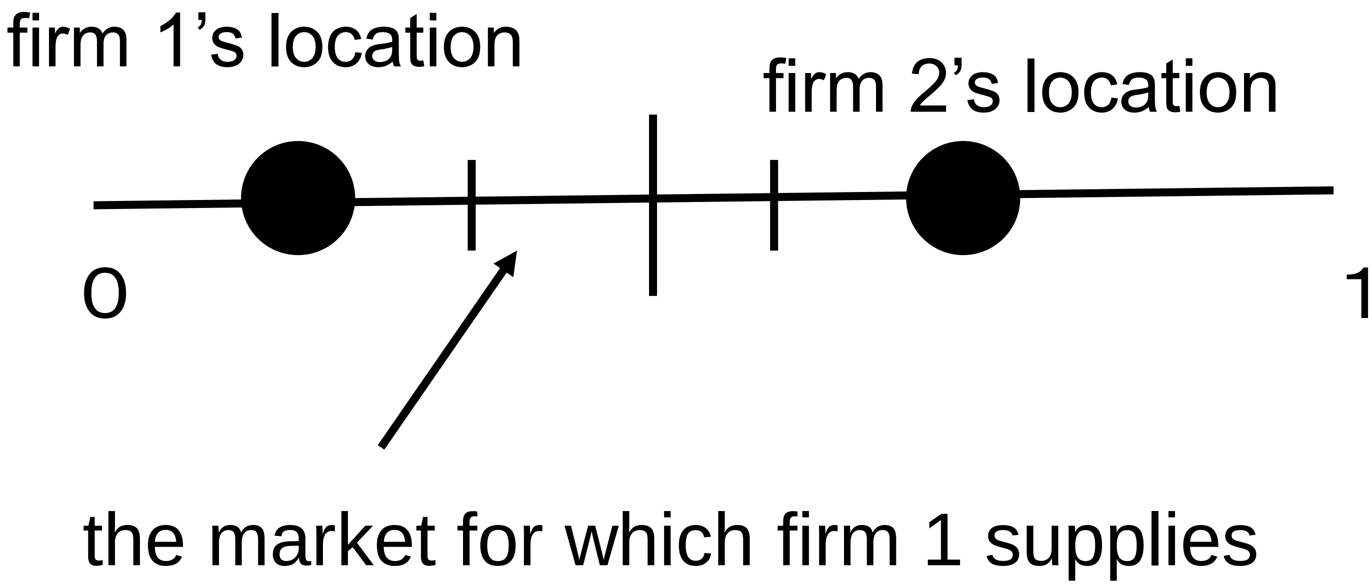
second stage subgames

The structure is the same as the Bertrand Model in a homogeneous product market.

The firm closer to the market (the firm with lower transport cost to the market) obtains the whole market and the price is equal to the rival's cost.

～The price depends on the rival's location only (does not depends on its location) as long as it supplies for the market.

second stage subgame



Equilibrium Prices

Suppose that the unit transport cost is T=td where d is the distance between the market and the location of the firm.

Suppose that $x_1=1/4$ and $x_2=3/4$.

Question: Derive the equilibrium price at the market x ($0 \leqq x \leqq 1/2$).

Equilibrium Prices

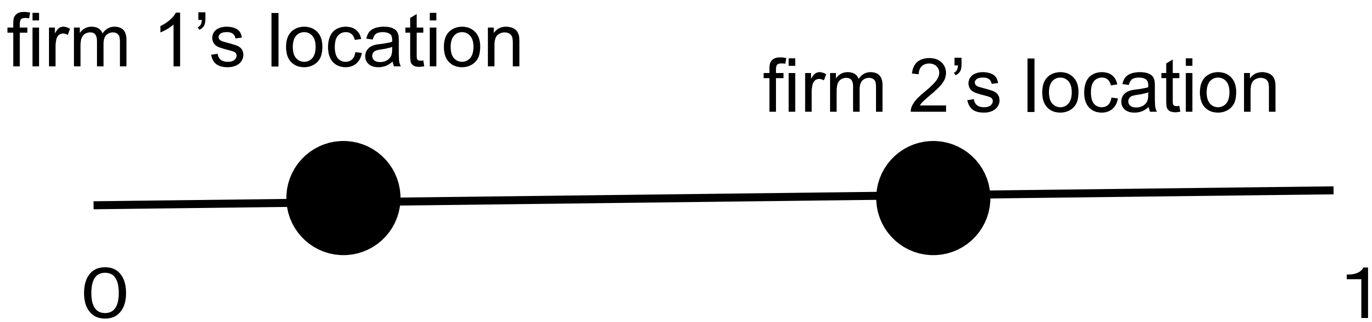
Suppose that the unit transport cost is T=td where d is the distance between the market and the location of the firm.

Suppose that $x_1=1/4$ and $x_2=3/4$.

Question: Derive the equilibrium price at the market x ($0 \leqq x \leqq 1/2$).

Question: ~the price at the central point is (lower, higher) than the price at the edge.

Equilibrium Location



Equilibrium location of firm 1 is larger than 1/4
Hamilton et al (1989).

Equilibrium Location

Firm 1 chooses its location so as to minimize the transport cost given the prices of the rival.

If the demand is inelastic, firm 1 chooses 1/4 (central point of its supply area).

If the demand is elastic, firm 1 put a larger weight on the market for which it supplies larger output.

→Firm 1 chooses a location closer to the central point 1/2.

Equilibrium Location

The relocation affects the supply area.

Should firm 1 consider this effect when it chooses its location rather than considering transport cost only.

→The profit from the marginal market is zero, so the marginal expansion of the supply area does not affect the profits.

→Firms care about its transport costs only.

Spatial Cournot Model

Consider a symmetric duopoly. Transport cost is proportional to both distance and output quantity (linear transport cost).

In the first stage, each firm chooses its location independently. In the second stage, each firm chooses its output independently.

Each point has an independent market, and the demand function is linear demand function, P=A-Y. No consumer's arbitrage. Production cost is normalized as zero. A is sufficiently large.

Hamilton et al. (1989, RSUE), Anderson and Neven (1991, IER)

Properties of Spatial Cournot Model

Market overlap ~ Two firms supply for all markets
Market share depends on the locations of the two firms.

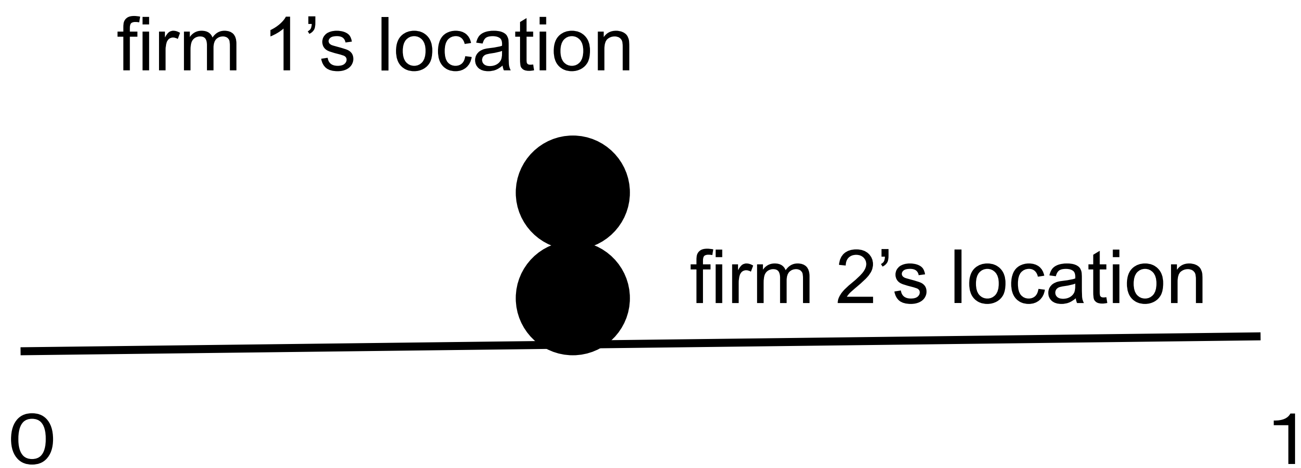
Second-Stage Competition

Suppose that the unit transport cost is T = td where d is the distance between the market and the location of the firm.

Suppose that $x_1 = 1/4$ and $x_2 = 3/4$.

Question: The market share of firm 1 at point 0 market is (larger than, smaller than, equal to) that at point 1.

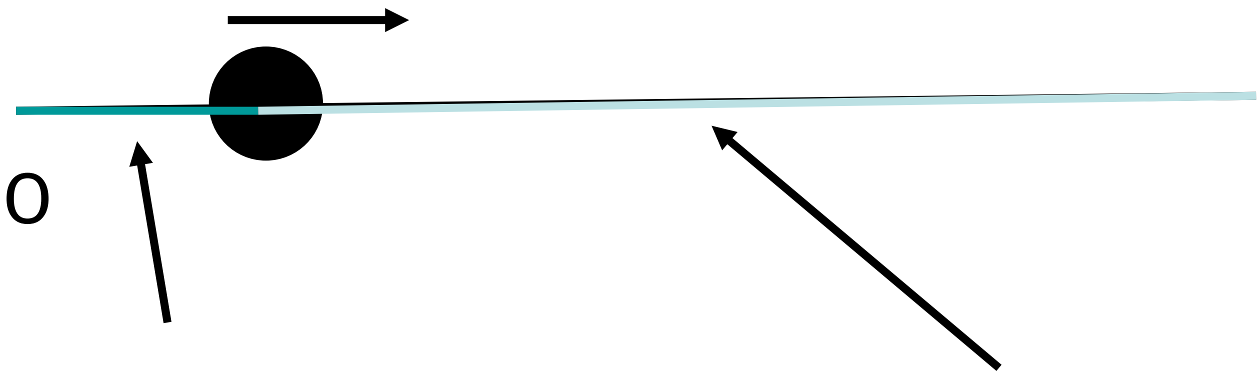
Equilibrium Location



Two firms agglomerate at the central points.
similar result in oligopoly. Anderson and Neven (1991).

Location and Transport Costs

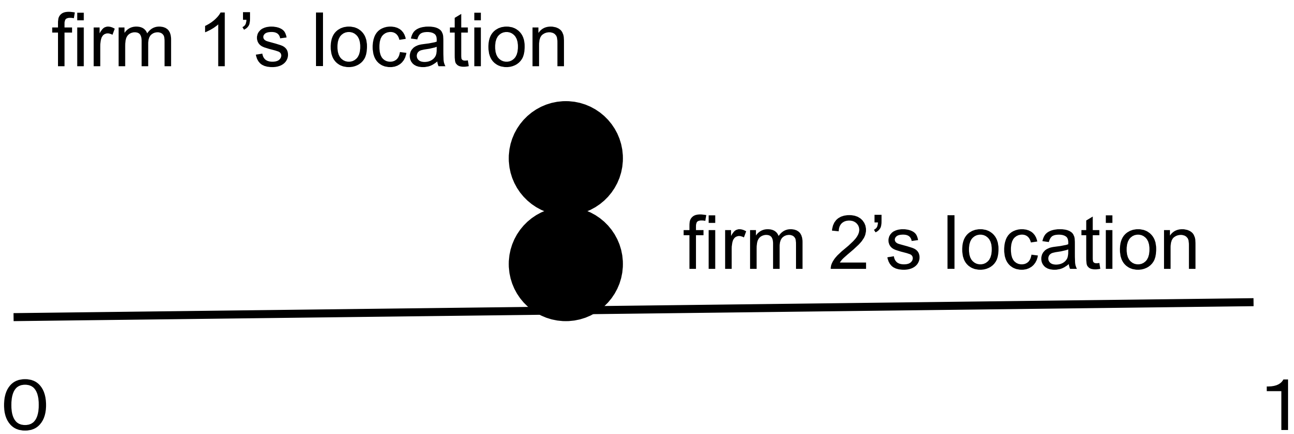
A slight increase of x_1



The area for which the relocation increases the transport cost of firm 1

The area for which the relocation decreases the transport cost of firm 1

Non-Uniform Distribution of Population



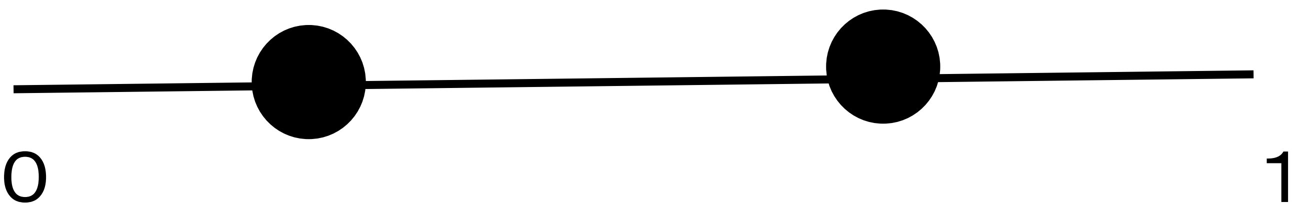
Suppose that population density is higher at central, like Tabuchi and Thisse (1995).

→ more incentive for central agglomeration

Non-Uniform Distribution of Population

firm 1’s equilibrium location

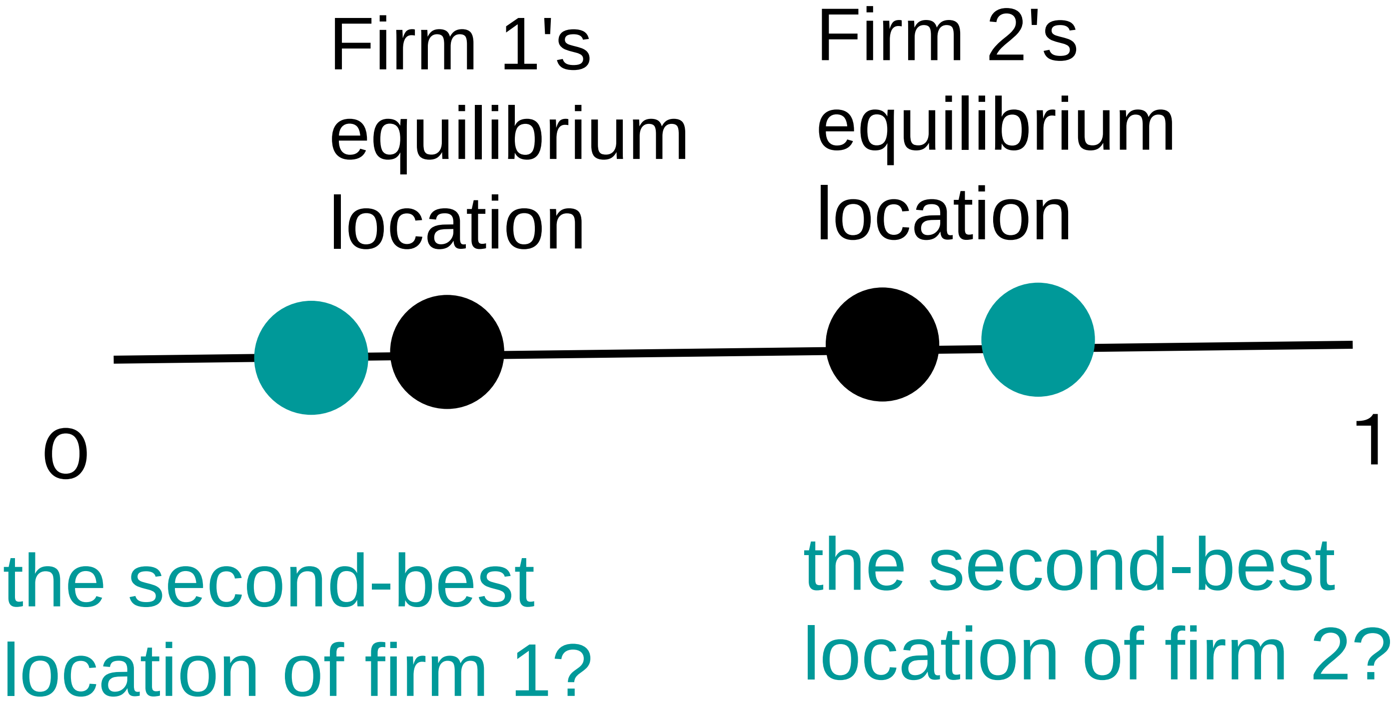
firm 2’s equilibrium location



Suppose that population density is higher at the end points, barbell model.

→ Firms may far away from the central point.

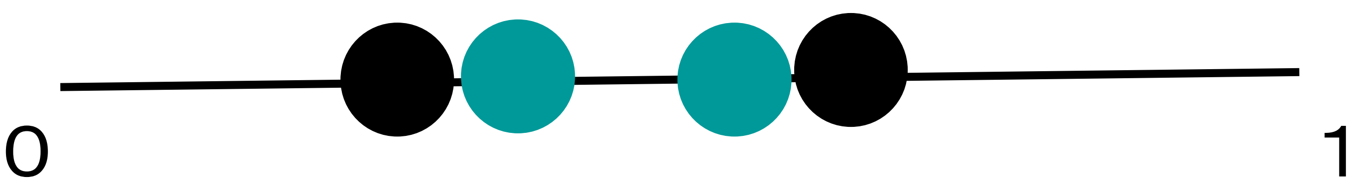
Welfare Implications in Cournot Matsumura and Shimizu (2005, Economics Letters)



Welfare Implications in Cournot
Matsumura and Shimizu (2005)

Firm 1's equilibrium location

Firm 2's equilibrium location



the second-best location of firm 1?

the second-best location of firm 2?

Welfare Implications in Bertrand Matsumura and Shimizu (2005, Economics Letters)

Firm 1's equilibrium location

Firm 2's equilibrium location

the second-best location of firm 1?

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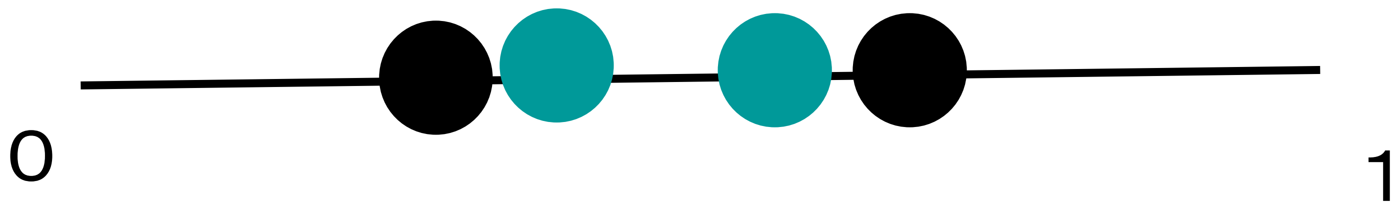
Welfare Implications in Bertrand Matsumura and Shimizu (2005)

Firm 1's equilibrium location

Firm 2's equilibrium location



the second-best location of firm 1?

the second-best location of firm 2?

Spatial Cournot with Circular-City

Consider a symmetric duopoly.

Transport cost is proportional to both distance and output quantity (linear transport cost).

In the first stage, each firm chooses its location independently on the circle.

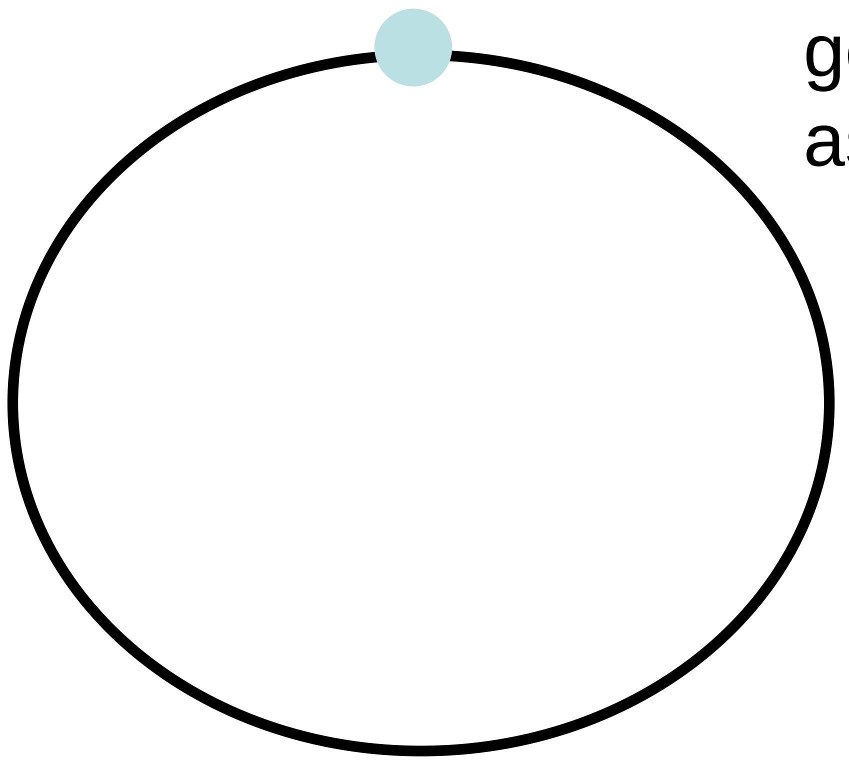
In the second stage, each firm chooses its output independently.

Each point has an independent market, and the demand function is linear demand function, P=A-Y. No consumer's arbitrage. Production cost is normalized as zero. A is sufficiently large.

Pal (1998, Economics Letters)

Equilibrium Location

Without loss of generality. we assume x_1=0

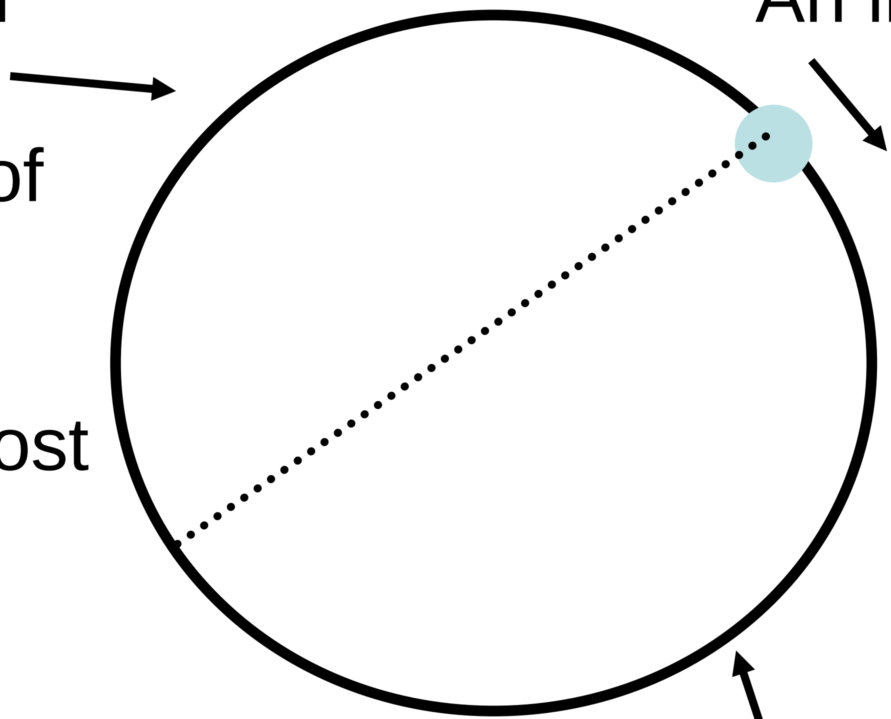


Consider the best reply for firm 2.

Location and Transport Costs

the area for which the relocation of firm 2 increases transport cost

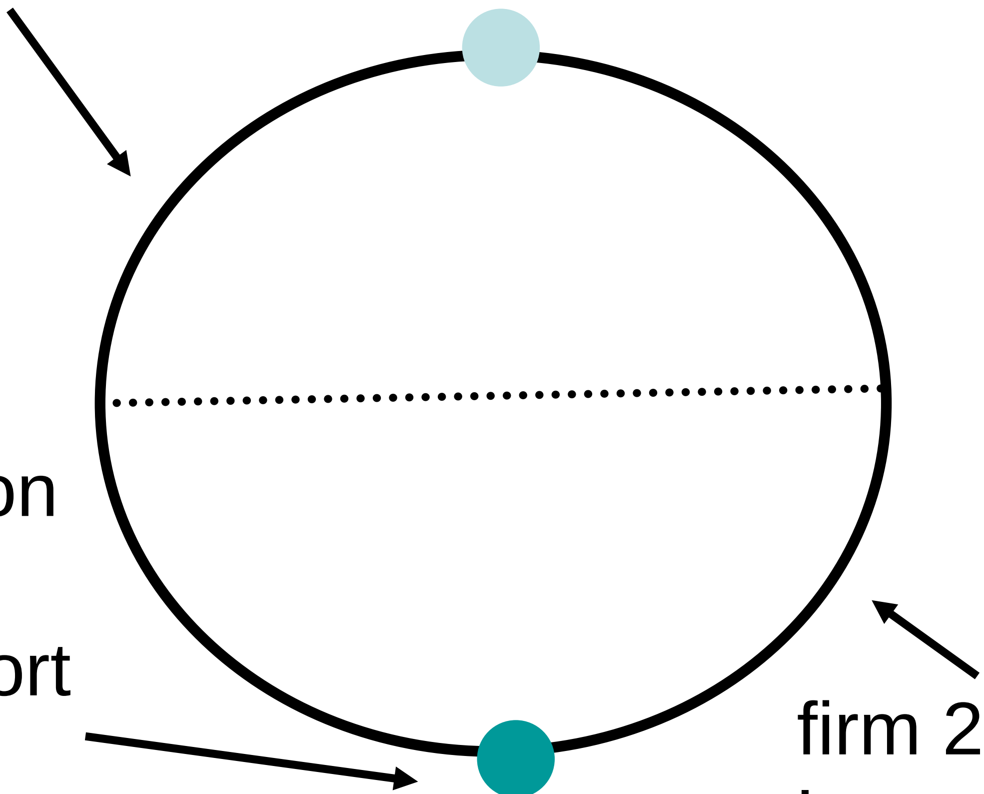
An increase of x_2



the area for which the relocation of firm 2 decreases transport cost

Equilibrium Location

firm 2's output is small

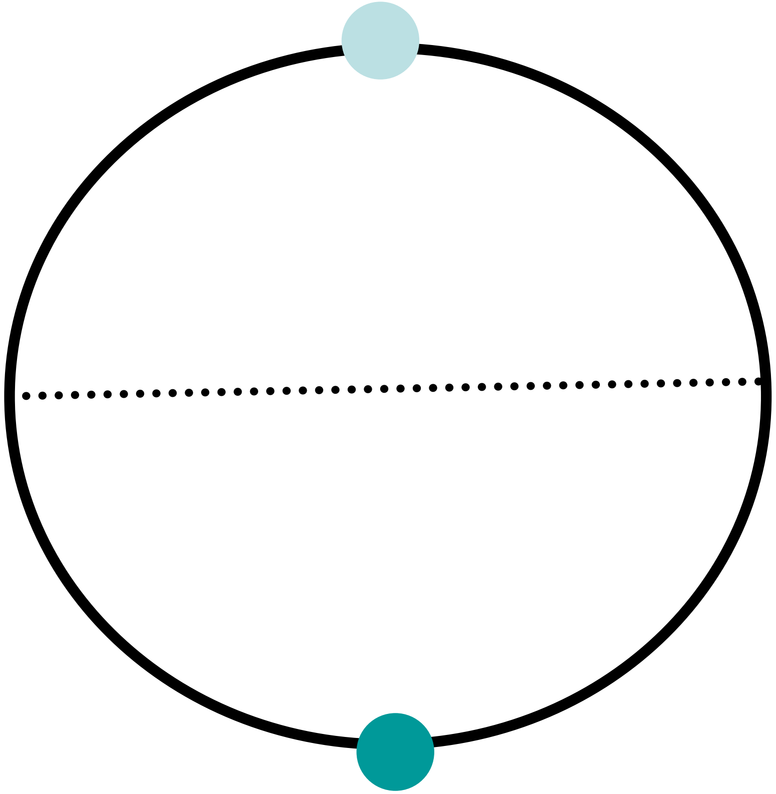


The location minimizing the transport cost of firm 2.

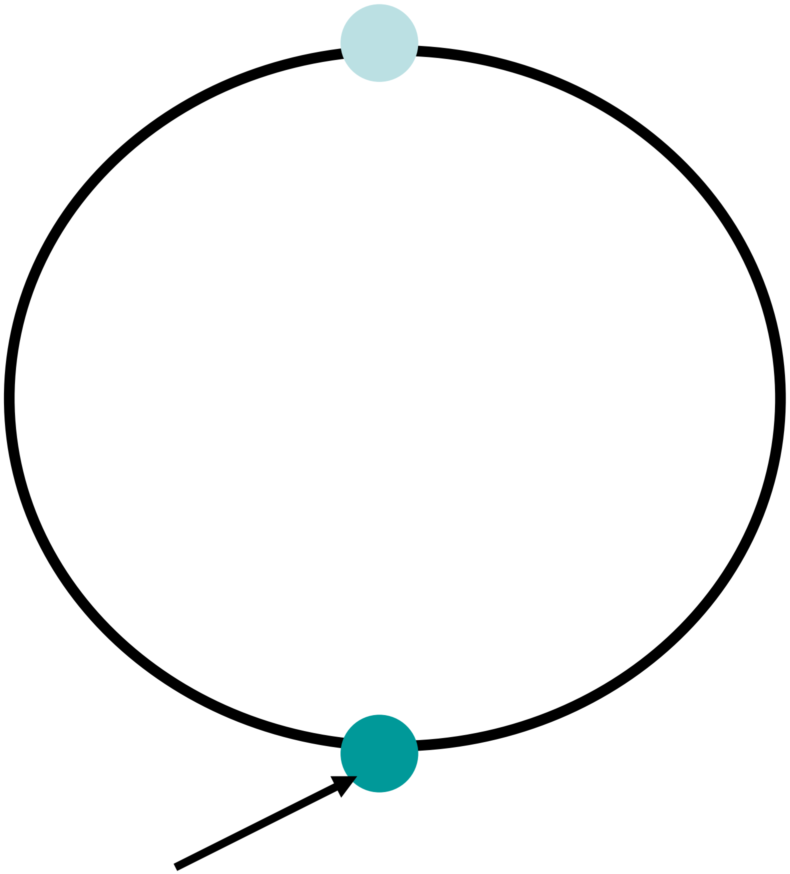
firm 2's output is large

Equilibrium Location

Question:
The resulting market price at market 0 is (lower than, higher than, equal to) that at market 1/4.



Equilibrium Location

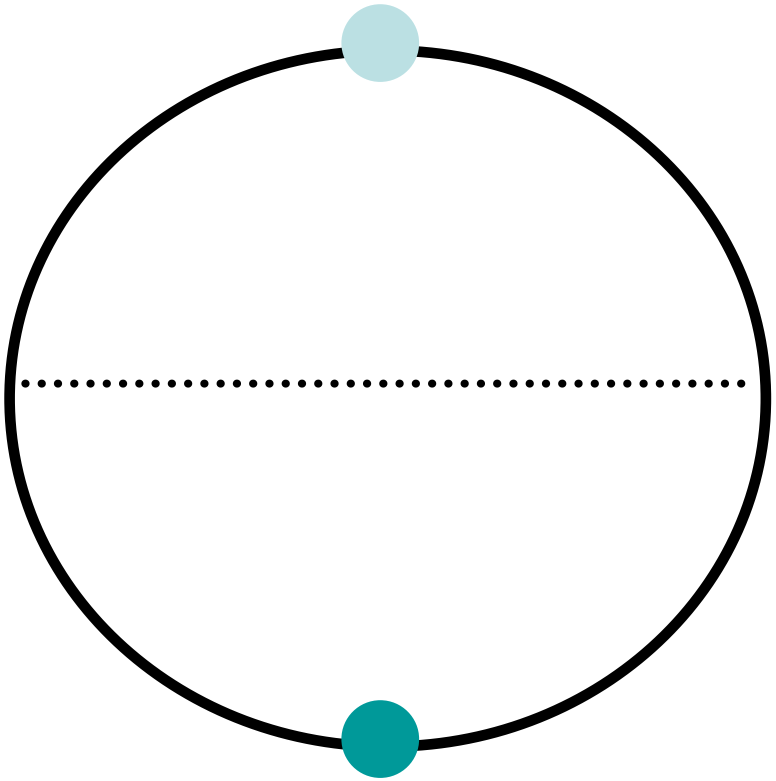


Maximal distance is the unique pure strategy equilibrium location pattern as long as the transport cost is strictly increasing.

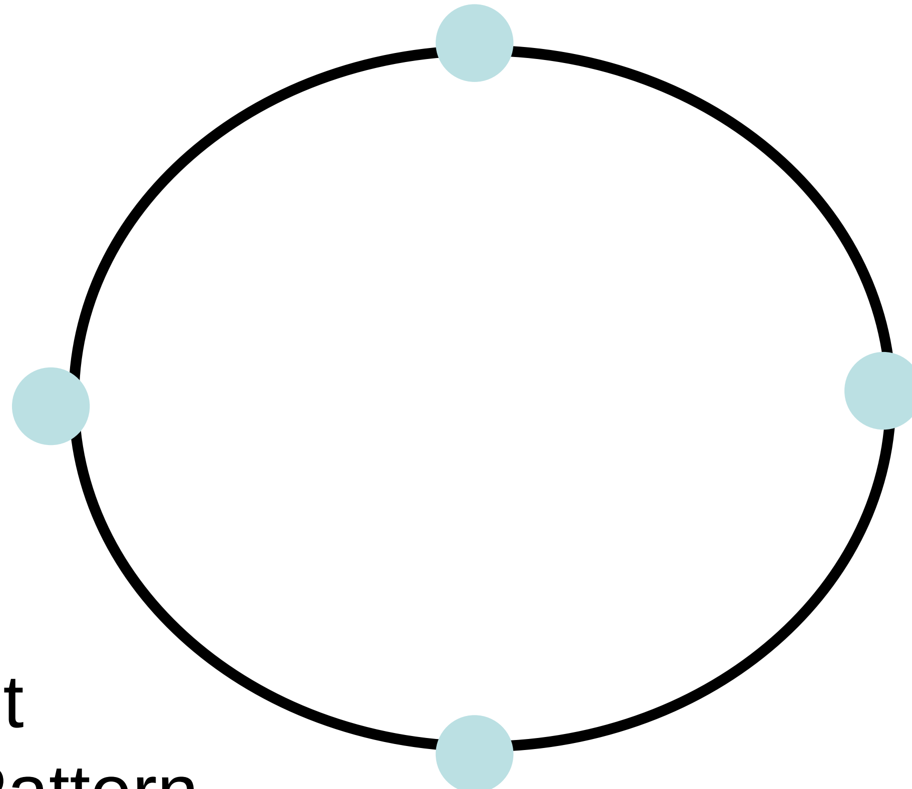
firm 2's equilibrium location

Equilibrium Location

Question:
Suppose that the unit transport cost is concave with respect to the distance. The resulting market price at market 0 is (lower than, higher than, equal to) that at market 1/4.

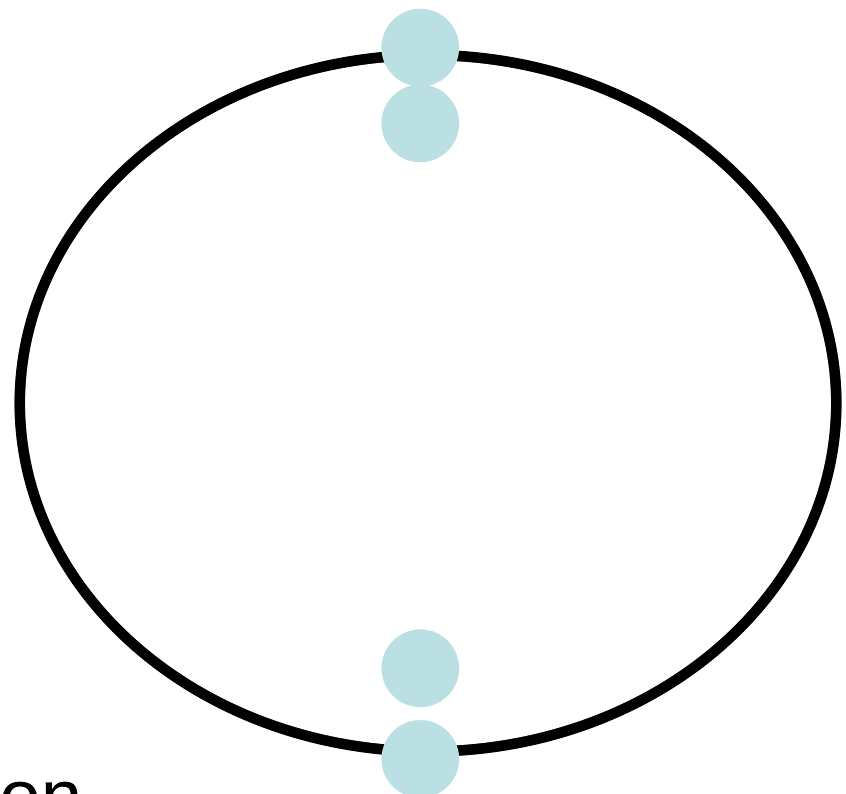


Equilibrium Location in Oligopoly



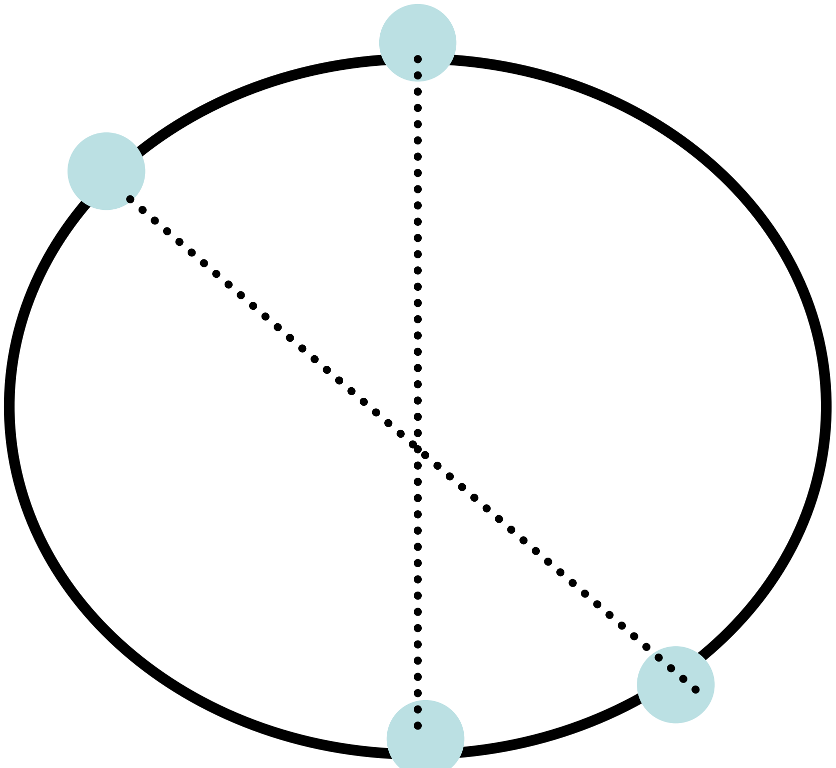
Equidistant
Location Pattern

Equilibrium Location in Oligopoly



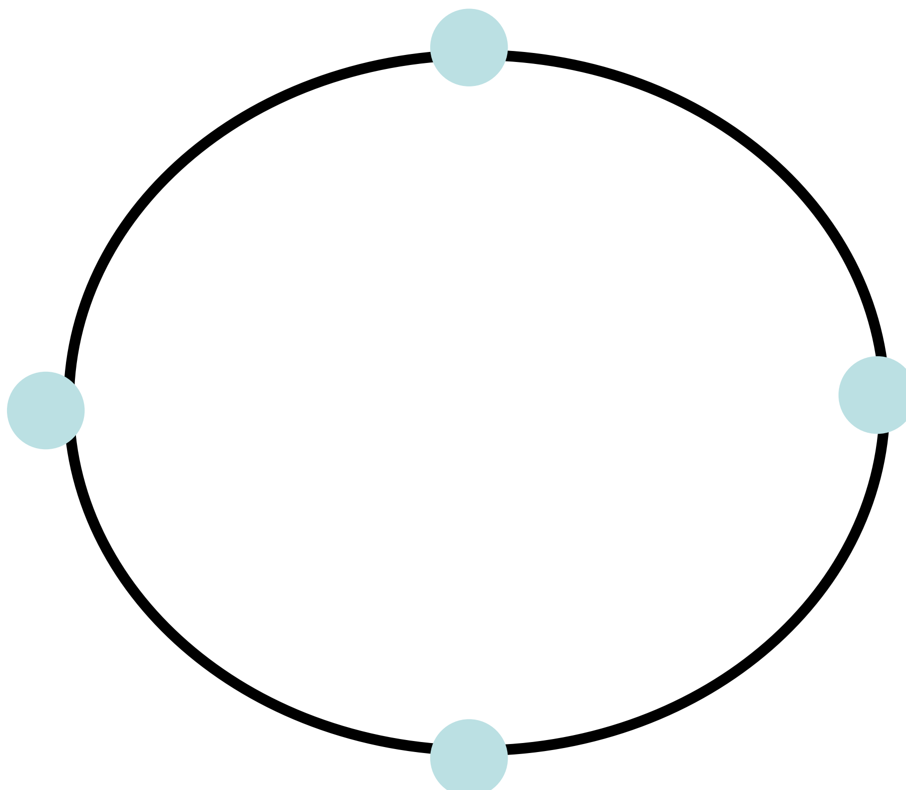
Partial
Agglomeration
～Matsushima (2001, Economics Letters)

Equilibrium Location in Oligopoly



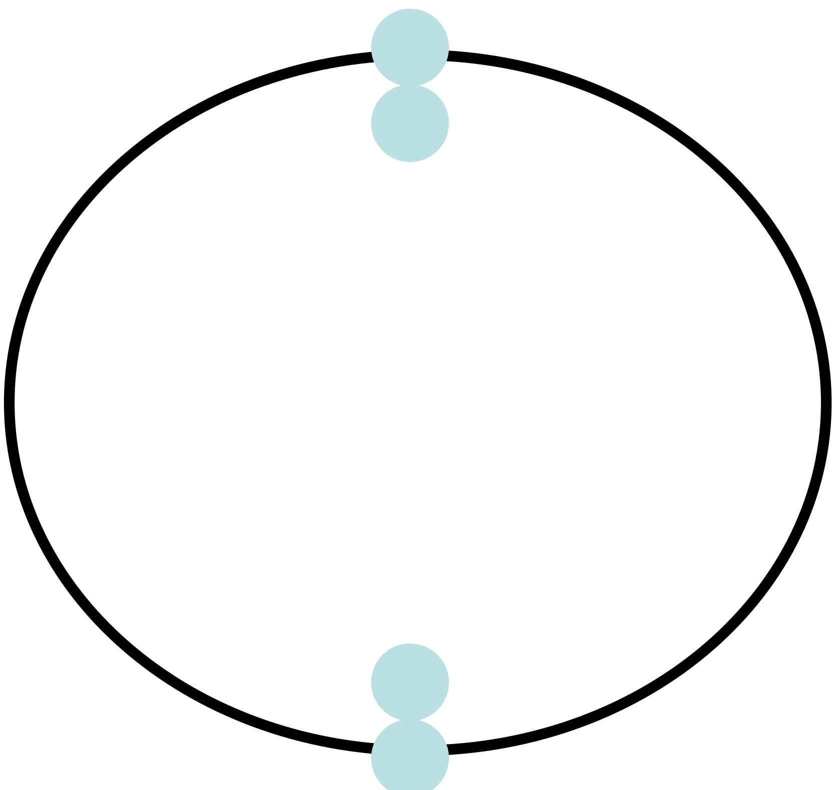
a continuum of equilibria exists
～Shimizu and Matsumura (2003), Gupta et al. (2004)

Equilibrium Location in Oligopoly



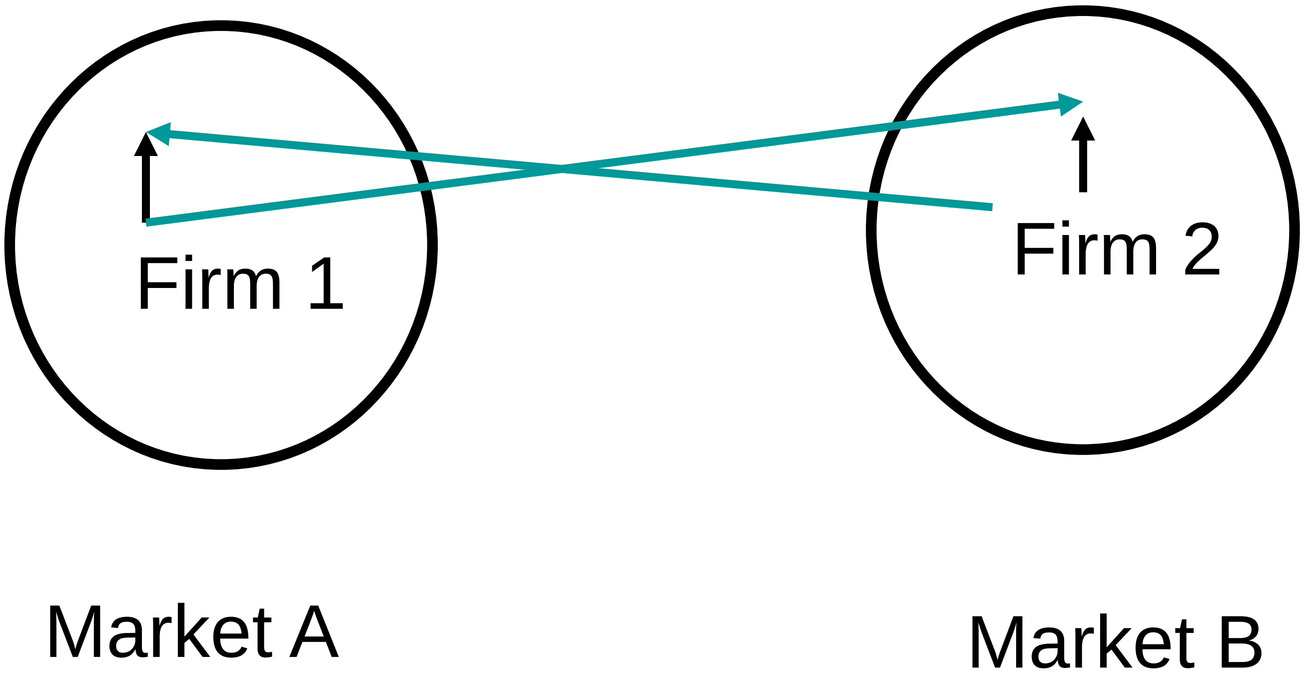
Under non-liner transport cost

Equilibrium Location in Oligopoly

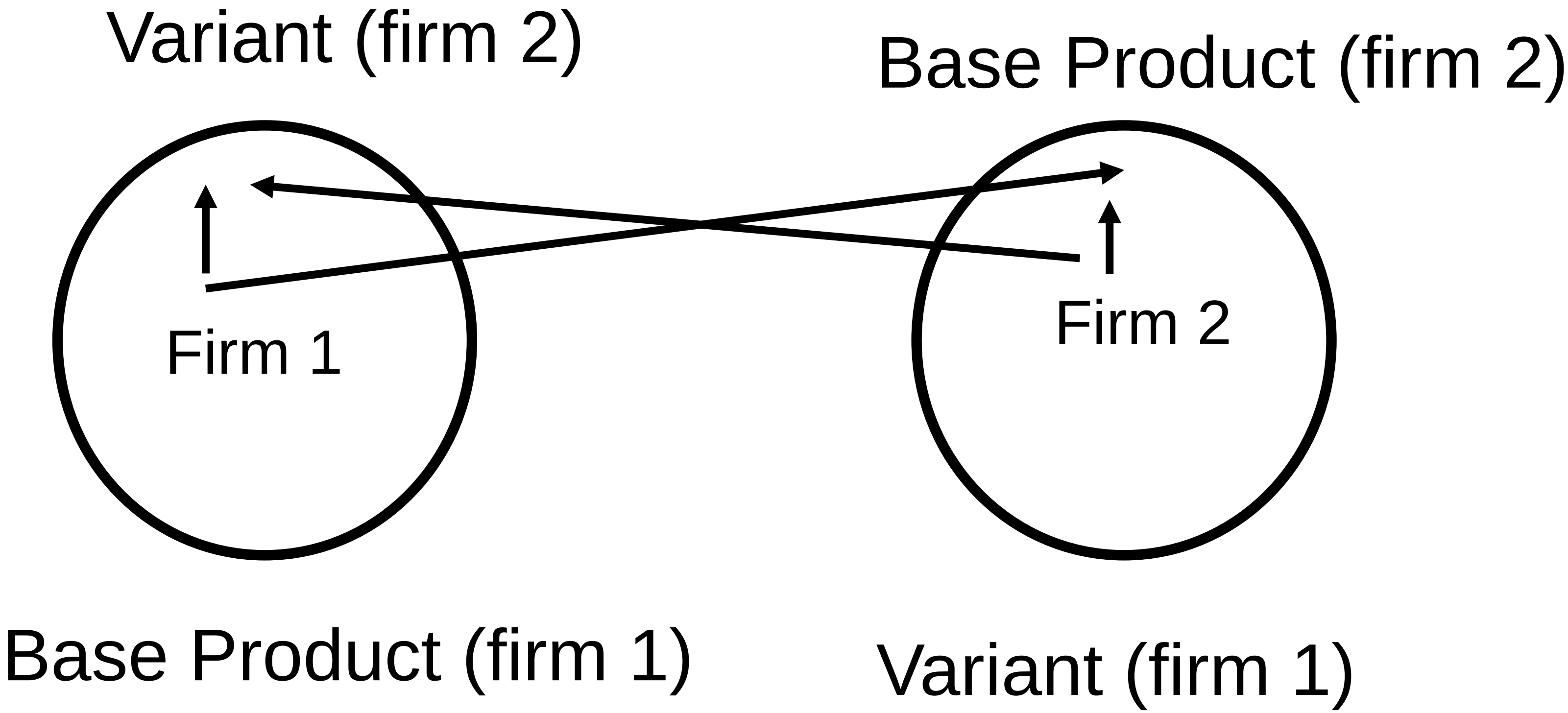


Under non-linear transport cost

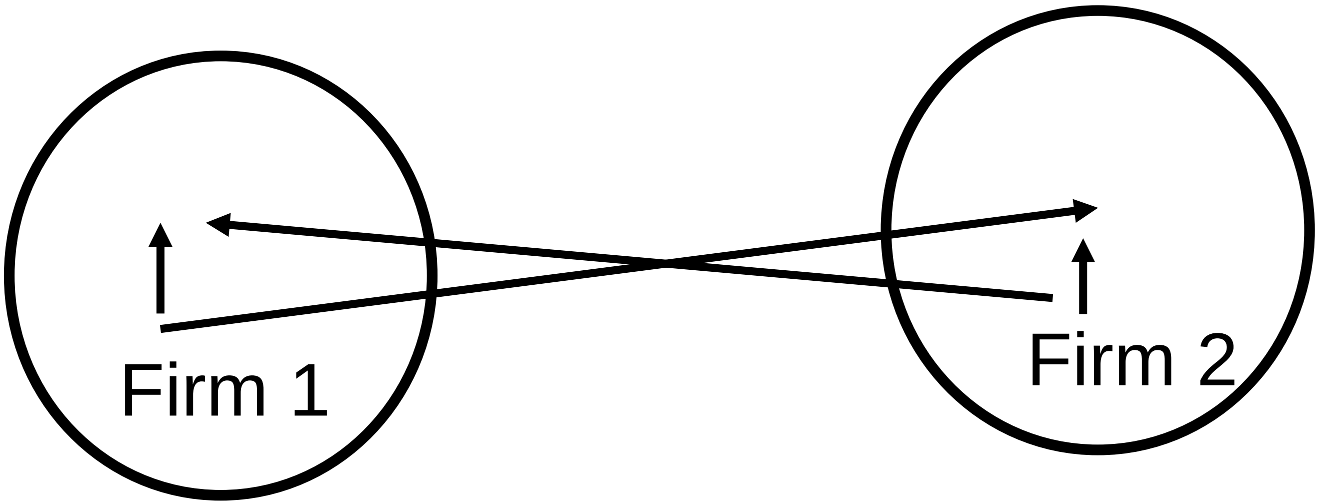
Spatial Interpretation of Shipping Model



Non-Spatial Interpretation of Shipping Model: FMS Eaton and Schmitt (1994)



Non-Spatial Interpretation of Shipping Model: Technological Choice (Matsumura (2004))



Market A:

Small Car

Market B:

Large Car

Mixed Strategy Equilibria

Uniqueness of the Equilibrium

Shopping, Hotelling, quadratic transport cost, uniform distribution （standard Location-Price Model)

The unique pure strategy equilibrium location pattern is maximal differentiation.

However, there are two pure strategy equilibria.

$(x_1, x_2)=(0,1)$, $(x_1, x_2)=(1,0)$

→Mixed strategy equilibria may exist.

In fact, many (infinite) mixed strategy equilibria exist Bester et al (1996).

Cost Differential between Firms

Consider a production cost difference between two firms.

When the cost difference between two firms is small, the maximal differentiation is the unique pure strategy equilibrium location pattern.

When the cost difference between two firms is large, no pure strategy equilibrium exists.

Suppose that firm 1 is a lower cost firm and the cost difference is large. The best location of firm 1 is $x_1=x_2$ (minimal differentiation), while that of firm 2 is either $x_2=1$ or $x_2=0$ (maximal differentiation).

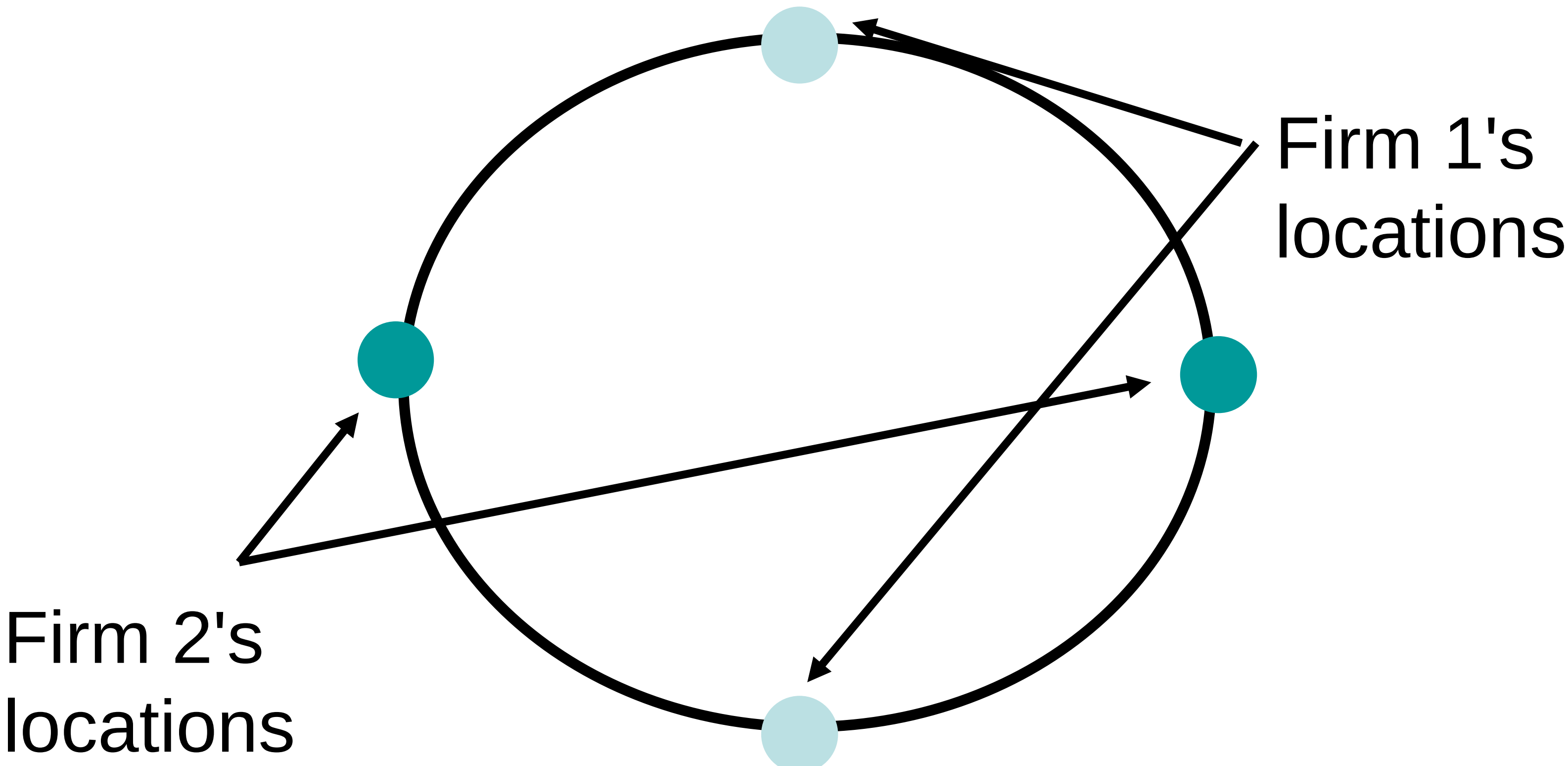
Cost Differential between Firms

Consider a production cost difference between two firms.

When the cost difference between two firms is large, no pure strategy equilibrium exists.

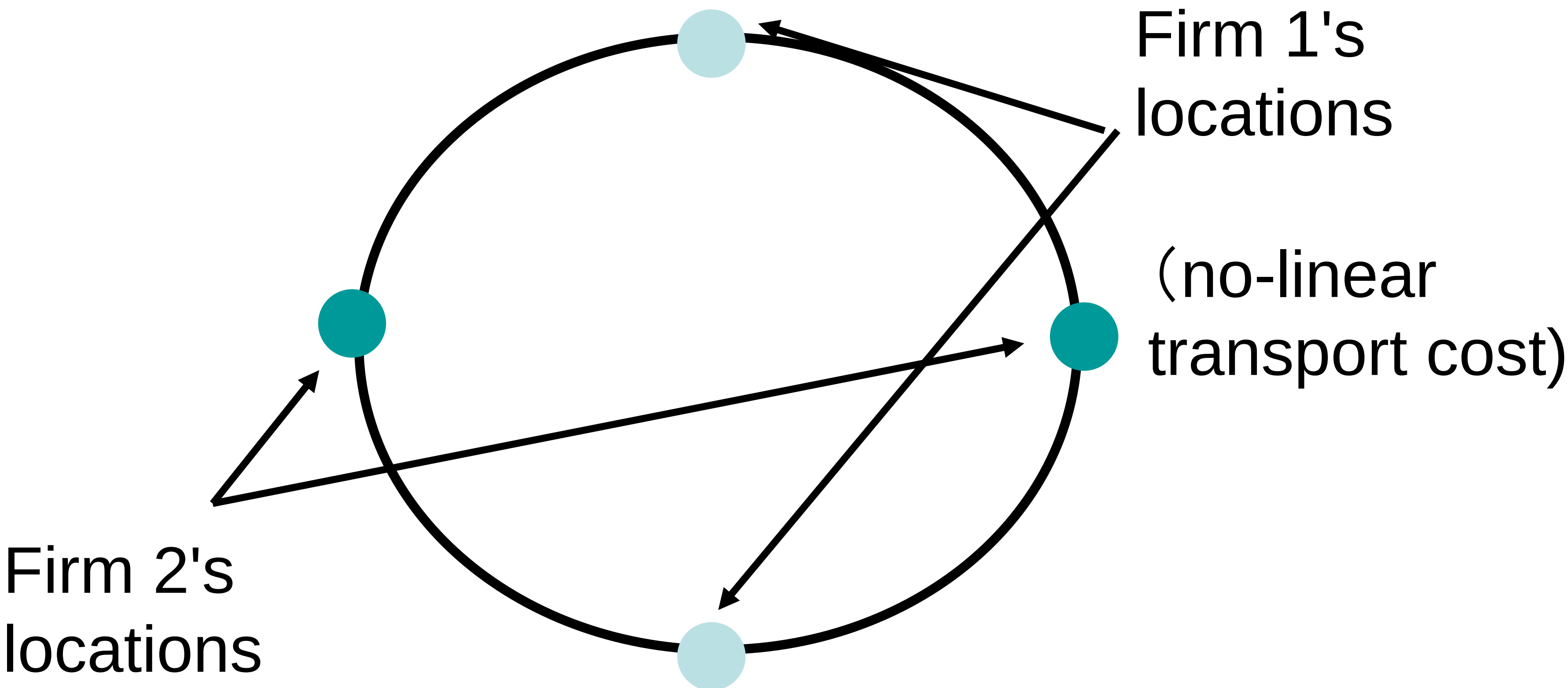
In this case, the following constitutes a mixed strategy equilibrium.
Both firms choose two edges with probability 1/2.
This does not constitute a mixed strategy equilibria without cost difference.

mixed strategy equilibria under quadratic transport cost (Shopping, Bertrand)

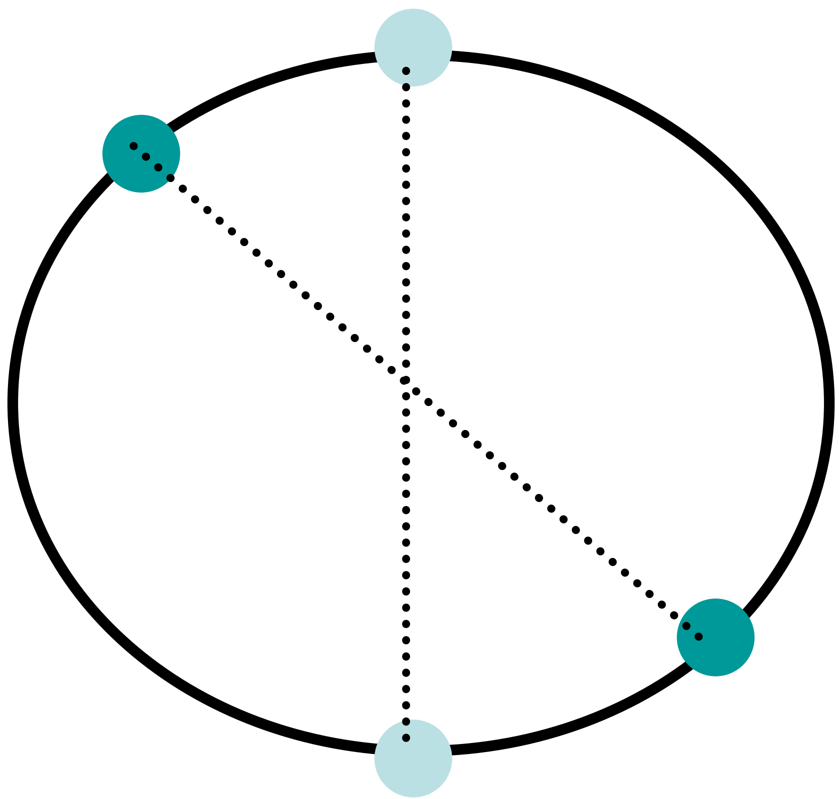


non-maximal differentiation, Ishida and Matsushima (2004,RSUE).

mixed strategy equilibria (Shopping, Cournot)



mixed strategy equilibria (linear transport cost)



a continuum of equilibria exists ～Matsumura and Shimizu (2008,JER)

Two Standard Models of Space

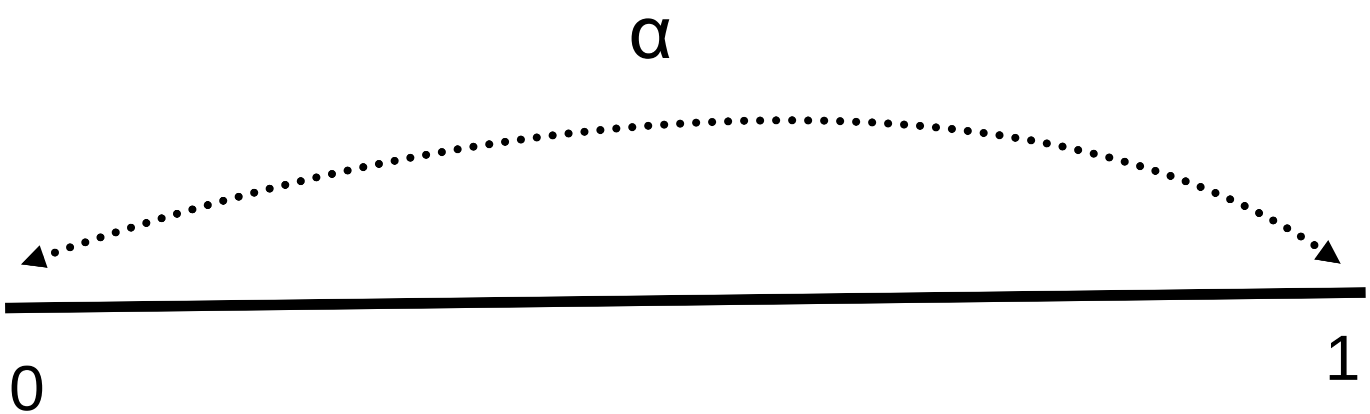
(1) Hotelling type Linear-City Model

(2) Salop type (or Vickery type) Circular-City Model

Linear-City has a center-periphery structure, while every point in the Circular-City is identical.

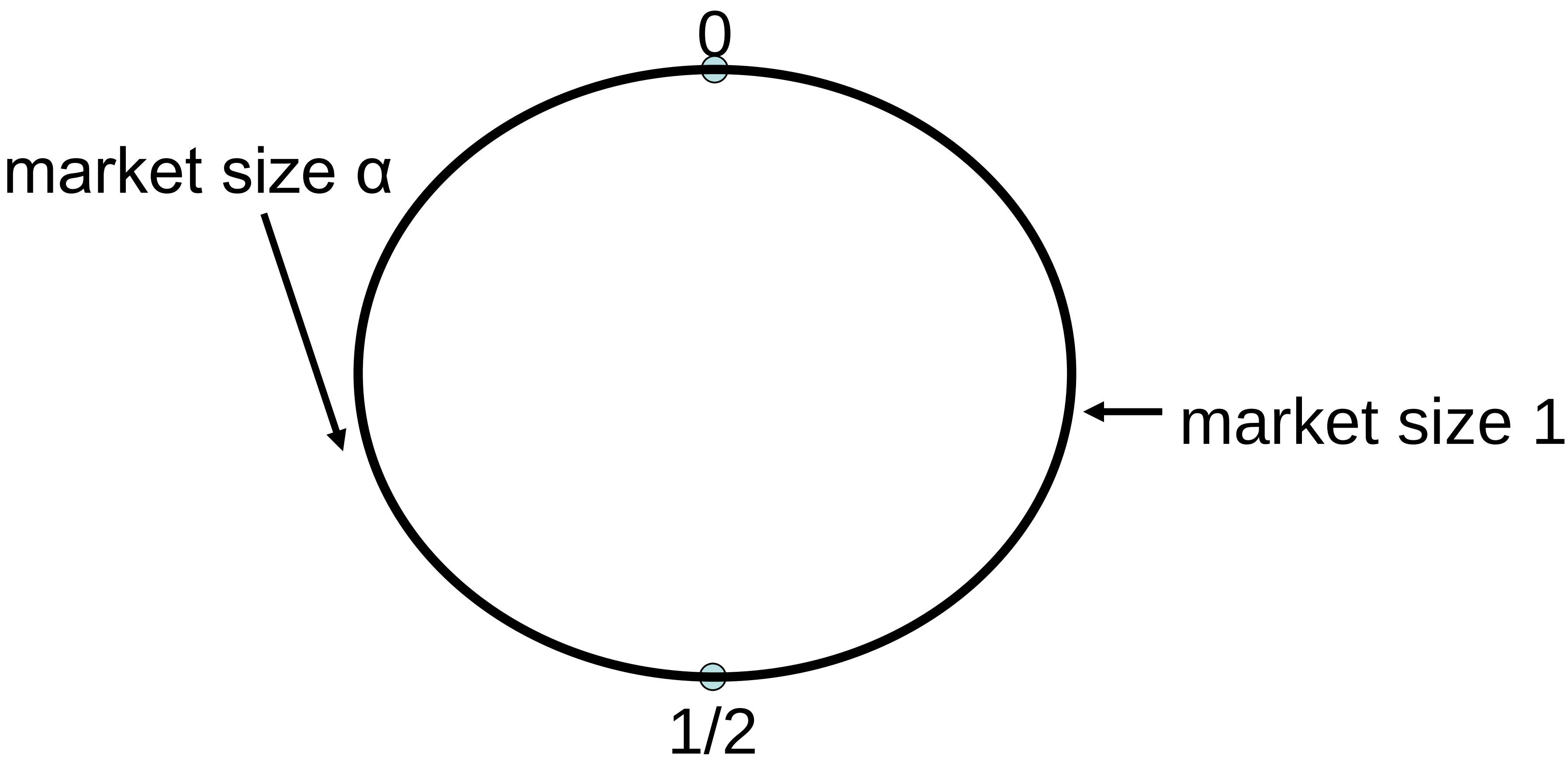
→Circular Model is more convenient than Linear Model for discussing symmetric oligopoly except for duopoly.

General Model (1)



It costs α to transport from 0 to 1.
The transport cost from 0 to 0.9 is min(0.9t, α + 0.1t).
If α = 0, this model is a circular-city model. If α > t, this model is a linear-city model.

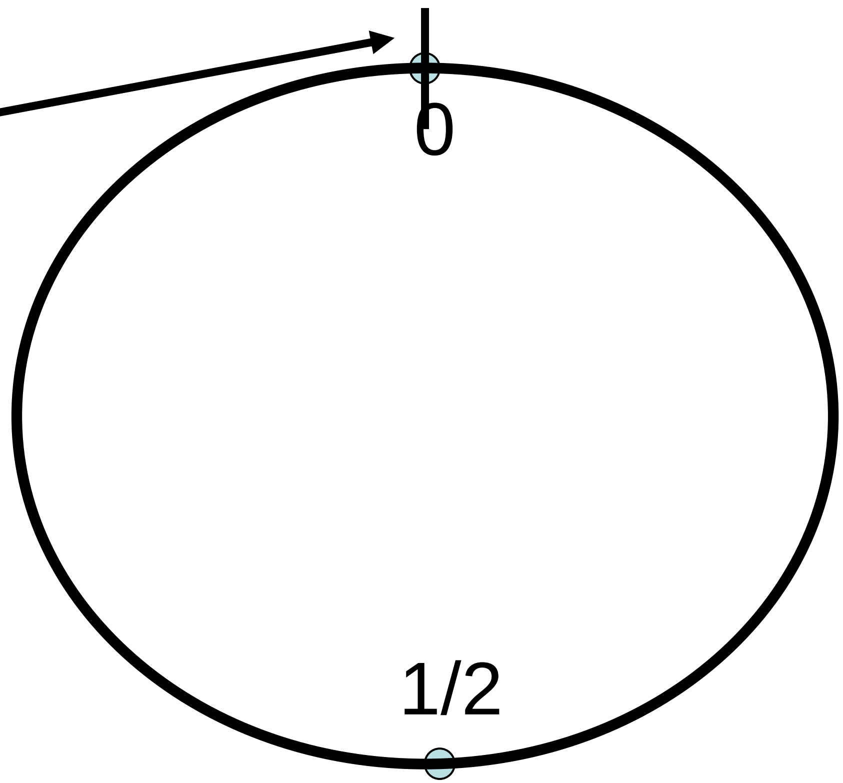
General Model (2)



If α =0, this model is a linear-city model. If α=1, this model is a circular-city model.

General Model (3)

It costs α to across this point



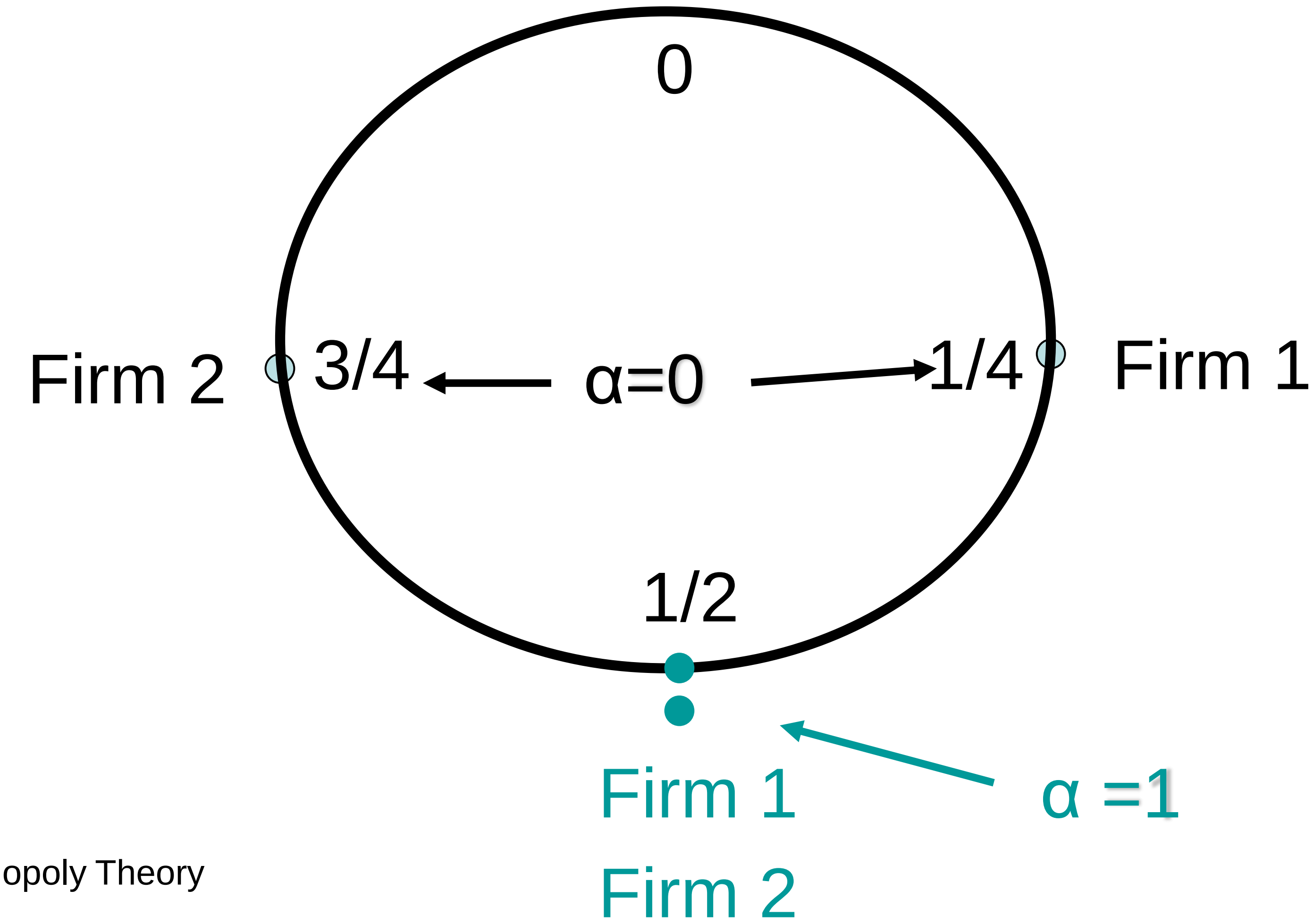
If α=0, this model is a circular-city model. If α>1, it is a linear-city model. (essentially the same model as (1)).

Application

In the mill pricing (shopping) location-price models, both linear-city and circular-city models yield maximal differentiation.

delivered pricing model (shipping model) →linear-city model and circular-city model yield different location patterns~ We discuss this shipping model.

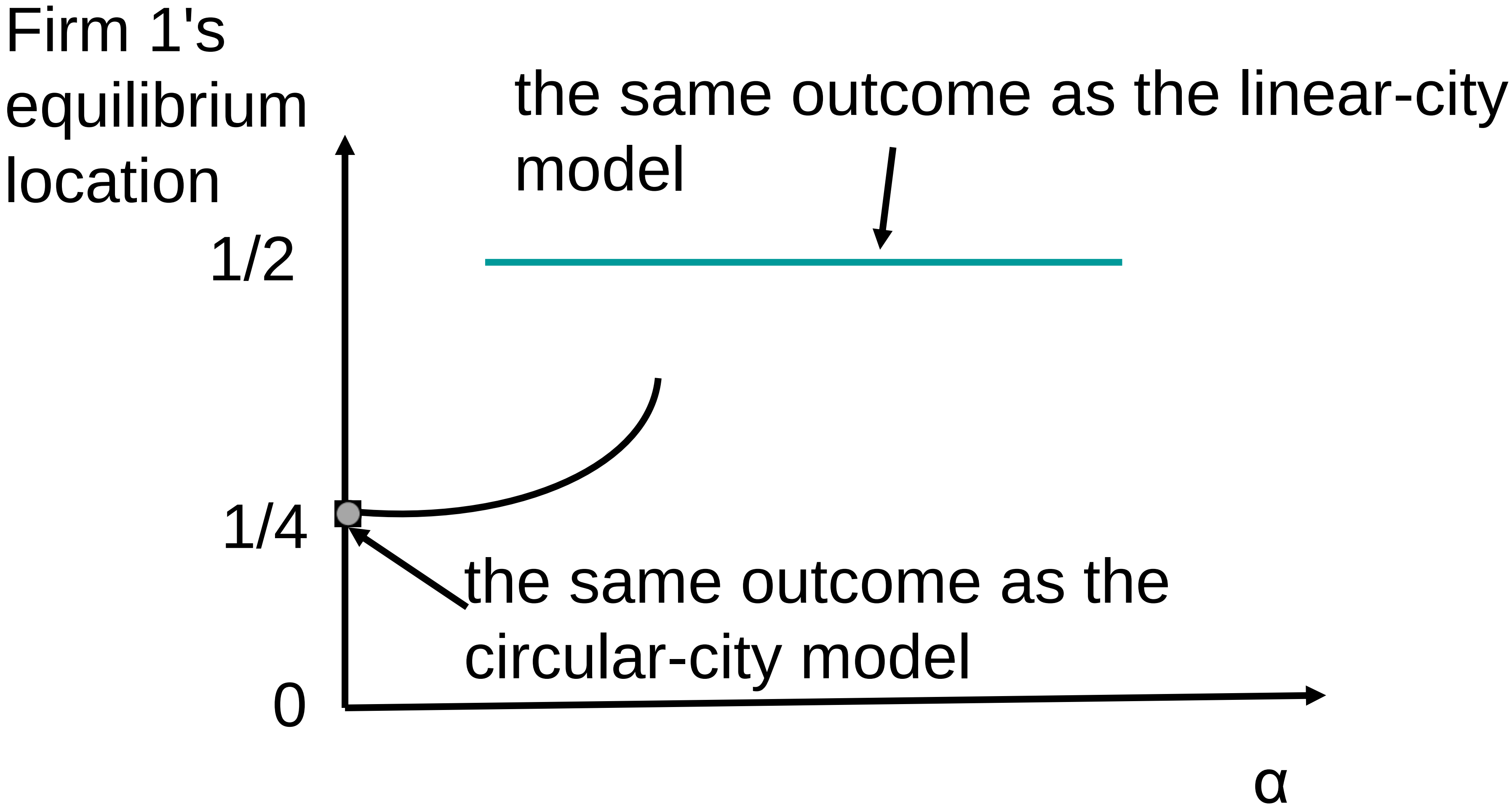
Location-Quantity Model



Results

・The equilibrium locations are symmetric.
・The equilibrium location pattern is discontinuous with respect to α (A jump takes place).
・Multiple equilibria exist. Ebina et al. (2011, PiRS)

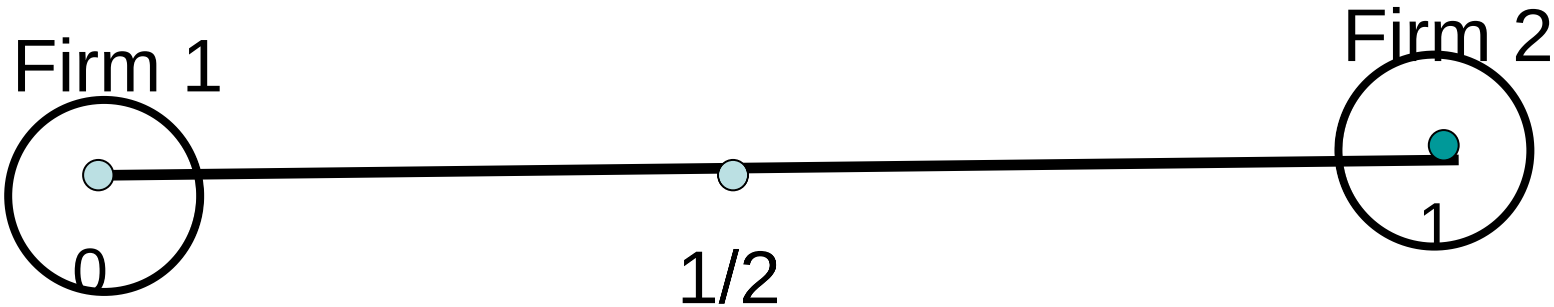
Results


Firm 1's equilibrium location
the same outcome as the linear-city model
1/2
1/4
the same outcome as the circular-city model
0
α

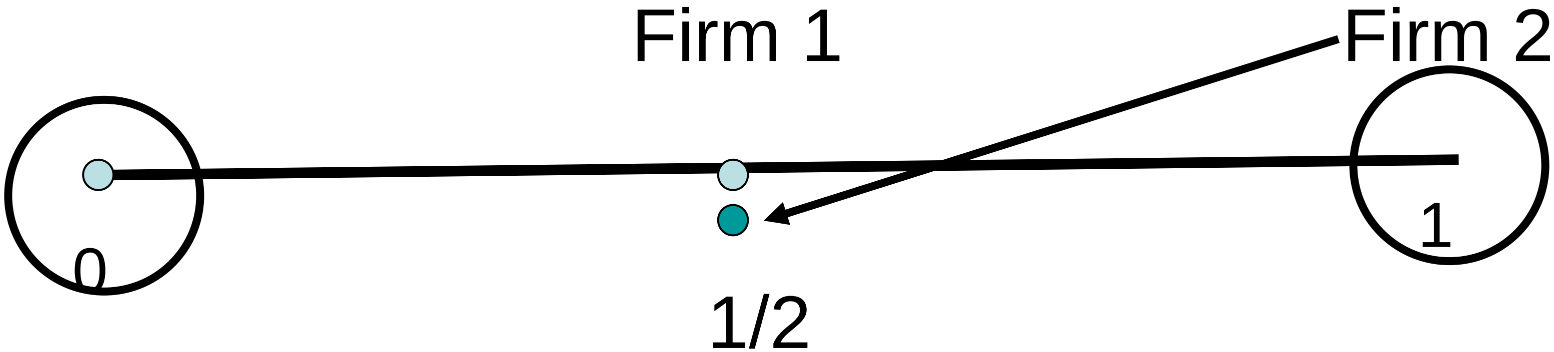
Intuition

Why discontinuous (jump)?
Why multiple equilibria？
←strategic complementarity
Suppose that firm 1 relocate from 0 to 1/2.
It increases the incentive for central location of firm 2.
～Matsumura (2004, RDE)

Complementarity
Matsumura (2004, RDE)



Complementarity
Matsumura (2004)



Central location by firm 1 increases the value of market 0 and decreases that of market 1 for firm 2→it increases the incentive for central location by firm 2.

Shopping or Shipping

Shopping → Uniform pricing, FOB pricing: the price does not depend on the location or personal properties.

Shipping→Spatial price discrimination, CIF pricing: the prices depend on the location or personal properties.

Firms may be able to choose their pricing strategies.

Thisse and Vives (1988, AER) endogenize this choice.

Endogenous Pricing Strategies

Thisse and Vives (1988) endogenize this choice.

(1) Both firms choose delivered pricing (personal pricing)

(2) Uniform pricing is mutually beneficial for firms (prisoners' dilemma)

These may not hold under asymmetry of the firms （Matsumura and Matsushima, 2015, JEMS)