Oligopoly Theory (6)
Endogenous Timing in Oligopoly

The aim of this lecture

(1) To understand the basic idea of endogenous timing games.

(2) To understand the relationship between the first mover and the second mover advantage and timing games

(3) To understand the difference among four representative timing games

Outline of the 6th Lecture

6-1 Cournot or Stackelberg
6-2 Timing Games
6-3 Stackelberg's Discussion on the Market Instability
6-4 Observable Delay Game
6-5 Action Commitment Game
6-6 Infinitely Earlier Period Model
6-7 Seal or Disclose
6-8 Two Production Period Model

Stackelberg or Cournot

Cournot (Bertrand) model and Stackelberg model yield different results.

Simultaneous move model and sequential move model yield different results.

Which model should we use ? Which model is more realistic?

An incumbent and a new entrant compete →sequential-move model

There is no such asymmetry between firms →simultaneous-move model

However, in reality, firms can choose both how much they produce and when they produce.

Timing Games

Firms can choose when to produce.

Formulating a model where both Cournot (simultaneous-move game) and Stackelberg (sequential-move game) outcomes can appear, and investigating whether Cournot or Stackelberg appears in equilibrium.

Stackelberg Duopoly

Firm 1 and firm 2 compete in a homogeneous product market.

Firm 1 chooses its output $Y_1 \in [0, \infty)$. After observing Y_1, firm 2 chooses its output $Y_2 \in [0, \infty)$.

Each firm maximizes its own profit Π_i.

$\Pi_i = P(Y)Y_i - C_i(Y_i)$, P: Inverse demand function,

Y: Total output, Y_i: Firm i's output, C_i: Firm i's cost function

I assume that $P' + P''Y_1 < 0$ (strategic substitutes)

⇒First-Mover Advantage

Stackelberg's discussion on the market instability

In the real world, it is not predetermined which firm becomes the leader.

Because of the first-mover advantage, both firms want to be the leaders.

Straggle for becoming the leader make the market instable.

～This is just an idea of endogenous timing game.

However, he did not present a model formally.

Some papers discussing this problem appeared since the end of 70s.

Four representative timing games

(1) Observable delay game
(2) Action commitment game
(3) Infinitely earlier period model
(4) Seal or disclose
(5) Two production period model

Observable Delay Game

Hamilton and Slutsky (1990, GEB)

Duopoly

First stage: Two firms choose period 1 or period 2.

Second Stage: After observing the timing, the firm choosing period 1 chooses its action.

Third Stage: After observing the actions taking at the second stage, the firm choosing period 2 chooses its action.

Payoff depends only on its action (not period).

Possible Outcomes

Both firms choose period 1 ⇒Cournot
Both firms choose period 2 ⇒Cournot
Only firm 1 chooses period 1 ⇒Stackelberg
Only firm 2 chooses period 1 ⇒Stackelberg

Equilibrium in Observable Delay Game

Strategic Substitutes

⇒Both firms choose period 1 (Cournot)
since Leader ≫ Cournot ≫ Follower

Strategic Complements

⇒Only firm1 chooses period 1 (Stackelberg) or
Only firm2 chooses period 1 (Stackelberg)
since Leader ≫ Cournot
and Follower ≫ Cournot.

Equilibrium in Observable Delay Game

Strategic Substitutes

Question: Suppose that Firm 1 chooses period 1. Given this strategy, firm 2's best reply is choosing (period 1, period 2)

Equilibrium in Observable Delay Game

Strategic Substitutes

Question: Suppose that Firm 1 chooses period 2. Given this strategy, firm 2's best reply is choosing (period 1, period 2)

Equilibrium in Observable Delay Game

Strategic Complements

Question: Suppose that Firm 1 chooses period 1. Given this strategy, firm 2's best reply is choosing (period 1, period 2)

Equilibrium in Observable Delay Game

Strategic Complements

Question: Suppose that Firm 1 chooses period 2. Given this strategy, firm 2's best reply is choosing (period 1, period 2)

Asymmetric Cases

It is possible that two firms have different payoff ranking.
e.g., Price Leadership (5th Lecture)
Suppose that firm 1 has a Cost Advantage.
Firm 1 Leader≫Follower≫Bertrand
Firm 2 Follower≫Leader≫Bertrand~Ono (1978,1982, Economica)

Firm 2 Leader≫Follower≫Bertrand
Firm 1 Follower≫Leader≫Bertrand~Hirata and Matsumura (2011, JoE)
It is quite natural to think that firm 1 becomes a leader (follower) in the former (latter) setting in equilibrium.
Is it true?

Matsumura and Ogawa (2009, JoE)

Assumption $U_i^L \geqq U_i^C$

Result If $U_1^L > U_1^F$ and $U_2^F > U_2^L$,

(i) firm 1's leadership is the unique equilibrium outcome,

(ii) equilibrium outcomes other than firm 1's leadership is supported by weakly dominated strategies,

or (iii) firm 1's leadership is risk dominant

⇒Pareto dominance implies risk dominance in the observable delay game. ～foundation for Ono's discussion.

Pareto efficient outcome can fail to be an equilibrium in general contexts

2

		C	D
1	C	(3,3)	(0,4)
	D	(4,0)	(1,1)

Pareto dominant equilibrium can fail to be the risk dominant equilibrium in general contexts

2

		C	D
	C	(3,3)	(-100,-1)
1	D	(-1,-100)	(1,1)

Pareto Dominance →(C,C)
Risk Dominance →(D,D)

Risk Dominance

2

1

	C	D
C	(3,3)	(-100,-1)
D	(-1,-100)	(1,1)

Consider a mixed strategy equilibrium. Suppose that in the mixed strategy equilibrium each firm independently chooses C with probability q. Then (C,C) is risk dominant if and only if $q < 1/2$.

Observable Delay

2

		1	2
1	1	(A,a)	(C,b)
	2	(B,c)	(A,a)

$C \geqq A$, $c \geqq a$.

Observable Delay, Matsumura (2003, BER) mixed duopoly, foreign private firm

2

1		1	2
	1	(A,a)	(C,b)
	2	(B,c)	(A,a)

C > A > B, c > a, b > a

Question: Derive the equilibrium outcome.

Observable Delay, Pal (1998, Economics Letters) mixed duopoly, domestic private firm

2

		1	2
1	1	(A,a)	(C,b)
	2	(B,c)	(A,a)

B > C > A, c > a, b > a

Question: Derive the equilibrium outcome.

Action Commitment Game (1)

Hamilton and Slutsky (1990, GEB)

Duopoly

First stage: Two firms choose period 1 or period 2.

Second Stage: Without observing the timing, the firm choosing period 1 chooses its action.

Third Stage: After observing the actions taking at the second stage, the firm choosing period 2 chooses its action.

Payoff depends only on its and the rival's actions (not period).

Action Commitment Game (2)

Duopoly

First stage: Each firm chooses whether it takes actions in period 1 or not. Firms choosing period 1 take their actions.

Second Stage: After observing the actions taking in period 1, the firm choosing period 2 takes its action.

Payoff depends only on its and the rival's actions (not period).

Two Action Commitment Games

There is no difference if we consider a two-period model.

However, there is an important difference between two models if we consider a three or more period model.

Model 1~The firm that does not take its action period 1 have already decided whether it takes its action in period 2 or in period 3.

Model 2~The firm that does not take its action in period 1 again chooses whether it takes its action in period 2 or waits until period 3.

Equilibrium in the Action Commitment Game-Two Period Model

(1) Both firms choose period 1 (Cournot)
(2) Only firm 1 chooses period 1 (Stackelberg)
(3) Only firm 2 chooses period 1 (Stackelberg)

Except for one outcome where both firms choose period 2 can be equilibrium outcomes.
This result does not depend on R' (whether strategic substitute or complement)

Equilibrium(1)

(1) Both firms choose period 1 (Cournot)

Suppose that firm 1 deviates from the equilibrium strategy and chooses period 2.

Firm 2 has already chosen its output before observing this deviation and it is Cournot output.

Firm 1 chooses the same output before the deviation in period 2.

⇒Firm 1 obtains the same profit before the deviation. = No improvement of the payoff.

Equilibria(2)(3)

(2) Only firm1 chooses period 1 (Stackelberg)

(a) Suppose that firm 2 deviates from the above strategy and chooses period 1. Firm 1 has already chosen its output before observing this deviation. Firm 2 chooses the same output before the deviation in period 1. ⇒Firm 2 obtains the same profit before the deviation.=No improvement of the payoff.

(b) Suppose that firm 1 deviates from the above strategy and chooses period 2. Firm face Cournot competition. Firm 1 obtains the smaller profit before the deviation.=No improvement of the payoff.

Instability of Cournot Outcome in the Action Commitment Game

(1) Both firms choose period 1 (Cournot)

Suppose that firm 1 deviates from the equilibrium strategy and chooses period 2.

Firm 2 has already produces Cournot output in period 1→Firm 1 chooses Cournot output in period 2⇒Firm 1 obtains the same payoff as before.

What happens off the equilibrium path？

Instability of Cournot Outcome in the Action Commitment Game

off path:

Suppose that firm 2 chooses period 2.

⇒After and before deviation the outcome is Cournot.
~The deviation does not change the payoff.

Suppose that firm 2 chooses period 1 and chooses the output that is not equal to the Cournot output.
⇒the deviation improves payoff.

Choosing period 1 and producing Cournot output is weakly dominated by choosing period 2.

Cournot is not robust.

Introducing Small Interest Costs

Suppose that the firm pays additional cost e>0 if it produces in period 1, may be inventory cost or interest cost.

→Waiting until period 2 strictly dominates producing Cournot output in period 1.

⇒(1) fails to be an equilibrium.

~Cournot is not robust.

Introducing Small Incomplete Information

Suppose that each firm obtains additional information on the cost of rival. In period 1, each firm knows its own cost. It also knows that the rival's cost is cN with probability 1-e and is cA with probability e∈(0,1). In period 2 each firm knows its rival's cost.

→Waiting until period 2 strictly dominates producing Cournot output in period 1.

⇒(1) fails to be an equilibrium.

~Cournot is not robust

Instability of Cournot Outcome in the Action Commitment Game Revisited,

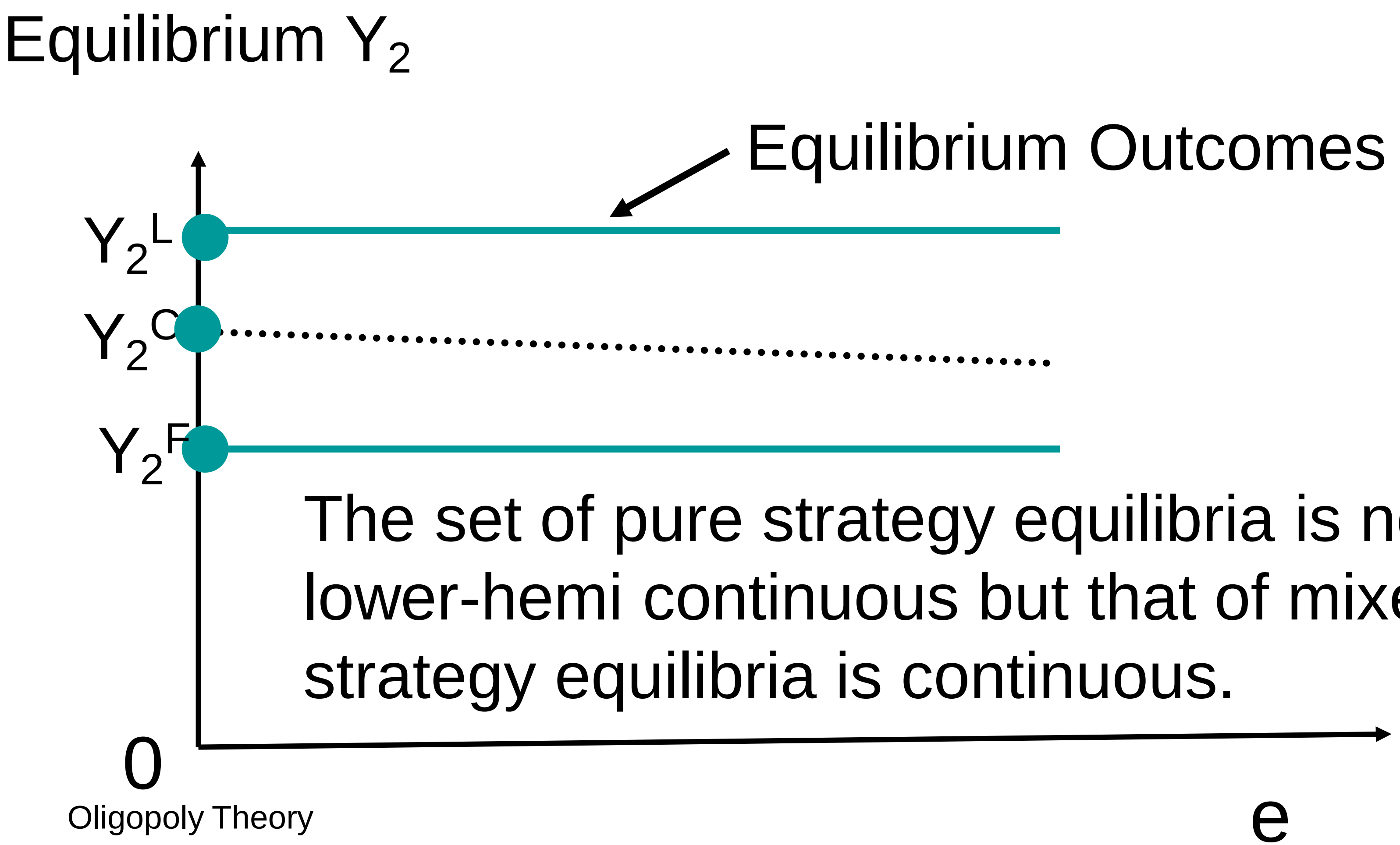
Matsumura et al. (2011, ORL)

There are two pure strategy equilibria with positive waiting gain. →There must be a mixed strategy equilibria.

If waiting gain e converges to zero, the mixed strategy equilibrium converges to the Cournot.

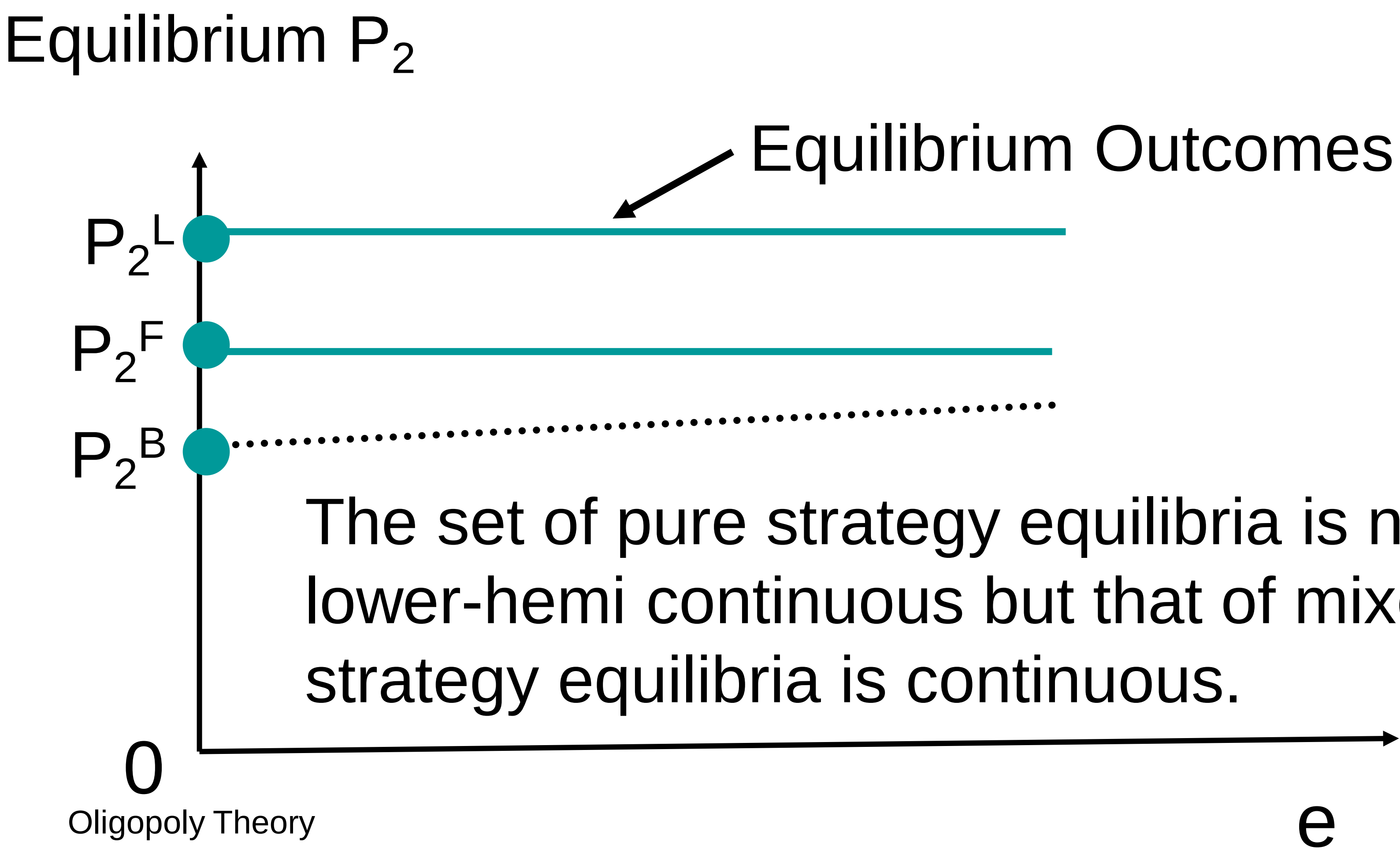
In the action commitment game, (1) is a degenerated mixed strategy equilibrium.

The Set of Equilibria in Quantity-Setting Game



The set of pure strategy equilibria is not lower-hemi continuous but that of mixed strategy equilibria is continuous.

The Set of Equilibria in Price-Setting Game



The set of pure strategy equilibria is not lower-hemi continuous but that of mixed strategy equilibria is continuous.

Why do observable delay and action commitment yield such different equilibrium outcome in mixed strategy equilibria

Observable Delay Game

Consider a mixed strategy equilibria. When firm 1 chooses period 1, firm 1 chooses its price after observing whether firm 2 chooses period 1 or period 2. → firm 1's action is either Stackelberg leader's or Bertrand.

Two actions are indifferent only when the probability that the rival chooses period 1 is $q \in (0,1)$ for small ε.

Why do two games yield such different equilibrium outcome in mixed strategy equilibria

Action Commitment Game

Consider a mixed strategy equilibria. When firm 1 chooses period 1, firm 1 chooses its quantity or price before observing whether firm 2 chooses period 1 or period 2. → firm 1's action is between Stackelberg leader's and Bertrand (Cournot). It converges to Bertrand (Cournot) when the probability that the rival chooses period 1 →1. Thus, the loss of not choosing 2 converges to zero when ε →0.

Action Commitment Game in Oligopolies

First stage: n firms choose period 1 or period 2.

Second Stage: Without observing the timing, the firm choosing period 1 chooses its action.

Third Stage: After observing the actions taking at the second stage, the firm choosing period 2 chooses its action.

Payoff depends only on its and the rivals' actions (not period).

Action Commitment Game in Oligopolies - two period model

Oligopoly
Strategic Complements or Substitutes
Question：How many firms become leaders in equilibrium?

Question 1：Does the outcome where all firms choose period 2 become an equilibrium?

Action Commitment Game in Oligopolies

Oligopoly
Strategic Complements or Substitutes
Question: How many firms become leaders in equilibrium?

Question 2: Does the outcome where only firm 1 chooses period 1 become an equilibrium?

Action Commitment Game in Oligopolies

Strategic Complements or Substitutes

Question：How many firms become leaders in equilibrium?

Question 3：Suppose that n=3. Does the outcome where only firm 3 chooses period 2 become an equilibrium?

Action Commitment Game in Oligopoly

Oligopoly

Strategic Complements or Substitutes

Question：How many firms become leaders in equilibrium?

Question 3：Suppose that n=3. Does the outcome where all firms choose period 1 become an equilibrium?

Action Commitment Game in Oligopolies

Strategic Complements or Substitutes

Question：Consider an n-firm oligopoly. How many firms become leaders in equilibrium?

Action Commitment Game with more than two periods

Consider an m-period version of the Action Commitment Game (1). Strategic Substitutes, m period, duopoly, sufficiently small but positive interest cost (later production has advantage)

Suppose that $m>2$. Then no pure strategy equilibrium exits→Stackelberg Instability

Matsumura (2002, JoE)

Action Commitment Game with more than two periods

Suppose that firm 1 chooses period t and firm 2 chooses period t'>t.

Then t = t'-1. Otherwise, firm 1 can economize the inventory cost by delaying the production without affecting firm 2's behavior.

t'=m since otherwise firm 2 can economize the inventory cost by delaying the production without affecting firm 2's behavior.

Action Commitment Game with more than two periods

Given that firm 1 chooses period m-1, firm 2 can increase its payoff by choosing period m-2 and being the leader (first-mover advantage).

→non-existence of pure strategy equilibrium.

Infinitely Earlier Period Model

Robson(1990,IER)

There is no first period. Firm 1 can choose any period t, t-1,t-2,t-3,...

Interest cost e(s), where e is decreasing in s and e(t)=0 and $\lim_{s\to -\infty} e(s)=\infty$. (advantage of later production）

The same structure of the Action Commit Game (2).

Symmetric Duopoly

Infinitely Earlier Period Model

Equilibrium（Second-Mover Advantage)

Firm 2 chooses period t. Firm 1 chooses period t-1.

Equilibrium (First-Mover Advantage)

Firm 2 chooses period t. Firm 1 chooses period t' such that the difference of the profit of first-mover and the second mover is larger than the inventory cost e(t') and smaller than e(t-1).

~Resulting payoff of the first mover is close to that of the second mover.

Seal or Disclose

Anderson and Engers (1992, IJIO)

Firm 1 chooses its output. Then firm 1 chooses whether or not to reveal its output to the rival. Then firm 2 chooses its output.

→ If firm 1 seals, two firms face Cournot competition. If it discloses, they face Stackelberg competition.

Question：Does firm 1 seal or disclose its output in equilibrium?（Does the answer depend on whether strategic substitutes or complements?)

Seal or Disclose

Firm 1 chooses its output. Then firm 1 chooses whether to reveal its output to the rival. Then firm 2 chooses its output.
→ If firm 1 seals, two firms face Cournot competition. If it discloses, they face Stackelberg competition.

Question：Does firm 1 seal or disclose its output in equilibrium?（Does the answer depend on whether strategic substitutes or complements?)

Answer：firm 1 always discloses.

Two-Production Period Model

Other models～Each firm produces in one period only.

This model, formulated by Saloner (1987, JET) ～Each firm can produce both in periods 1 and 2.

First Stage: Firm i chooses its first period production $Y_i(1) \in [0,\infty)$.

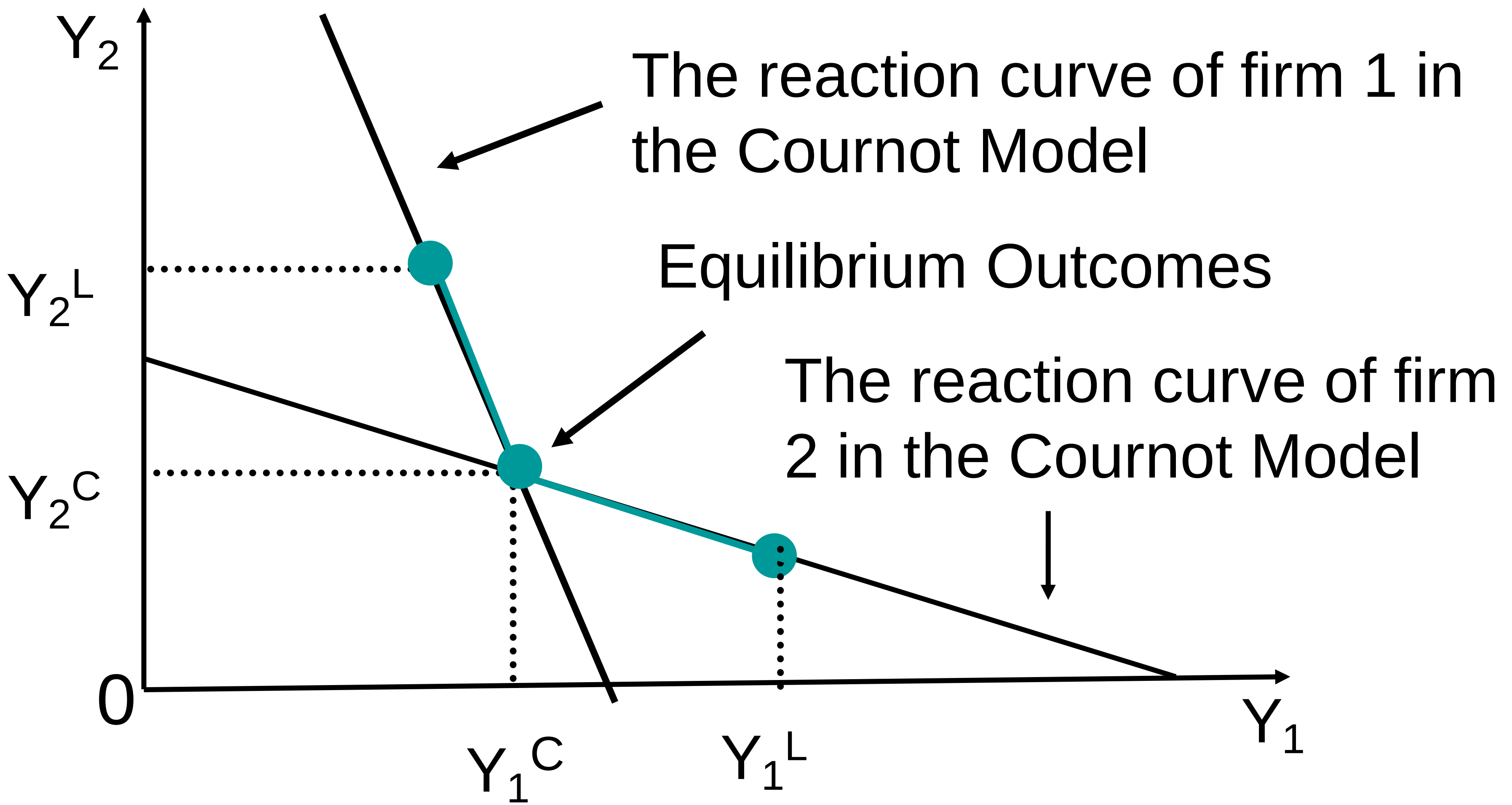
Second Stage: Firm i chooses its second period production $Y_i(2) \in [0,\infty)$.

At the end of the game, the market opens and each firm i sells $Y_i \equiv Y_i(1) + Y_i(2)$.

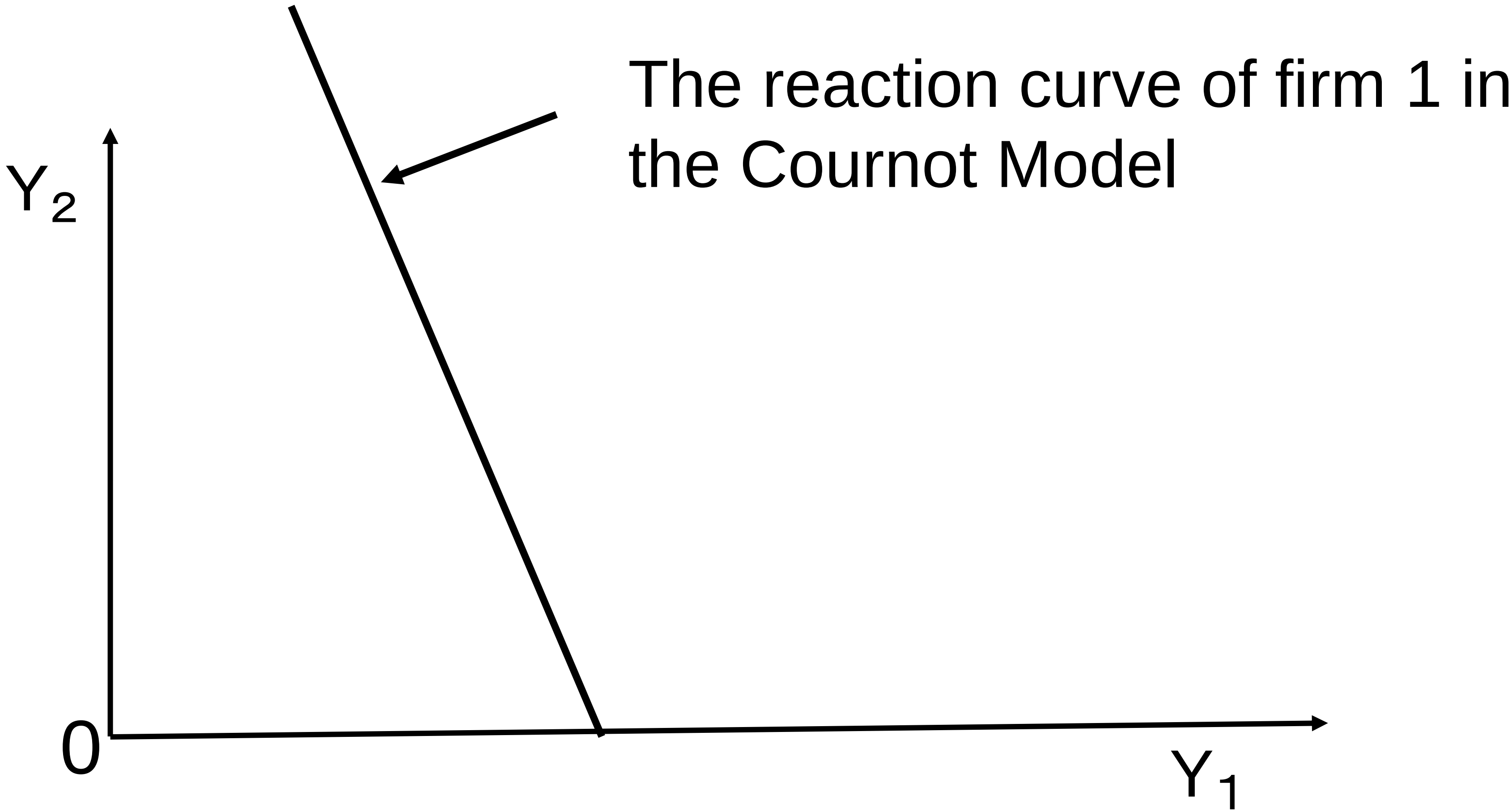
Each firm can increase but not decrease its total output.

We assume that the profit function of the Stackelberg leader is concave.

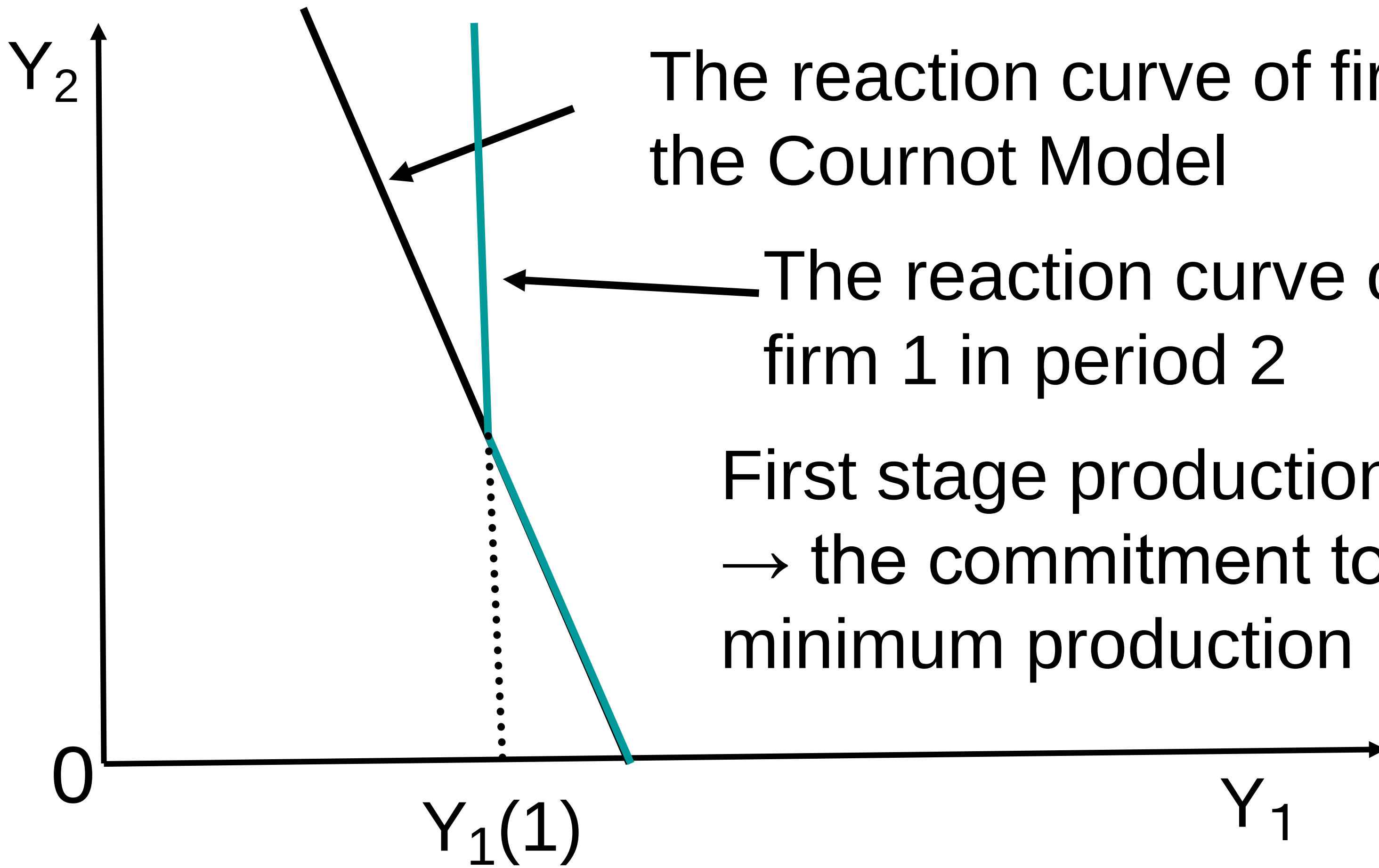
Equilibrium Outcomes



Firm 1's reaction curve in period 2



Firm 1's reaction curve in period 2



The reaction curve of firm 1 in the Cournot Model

The reaction curve of firm 1 in period 2

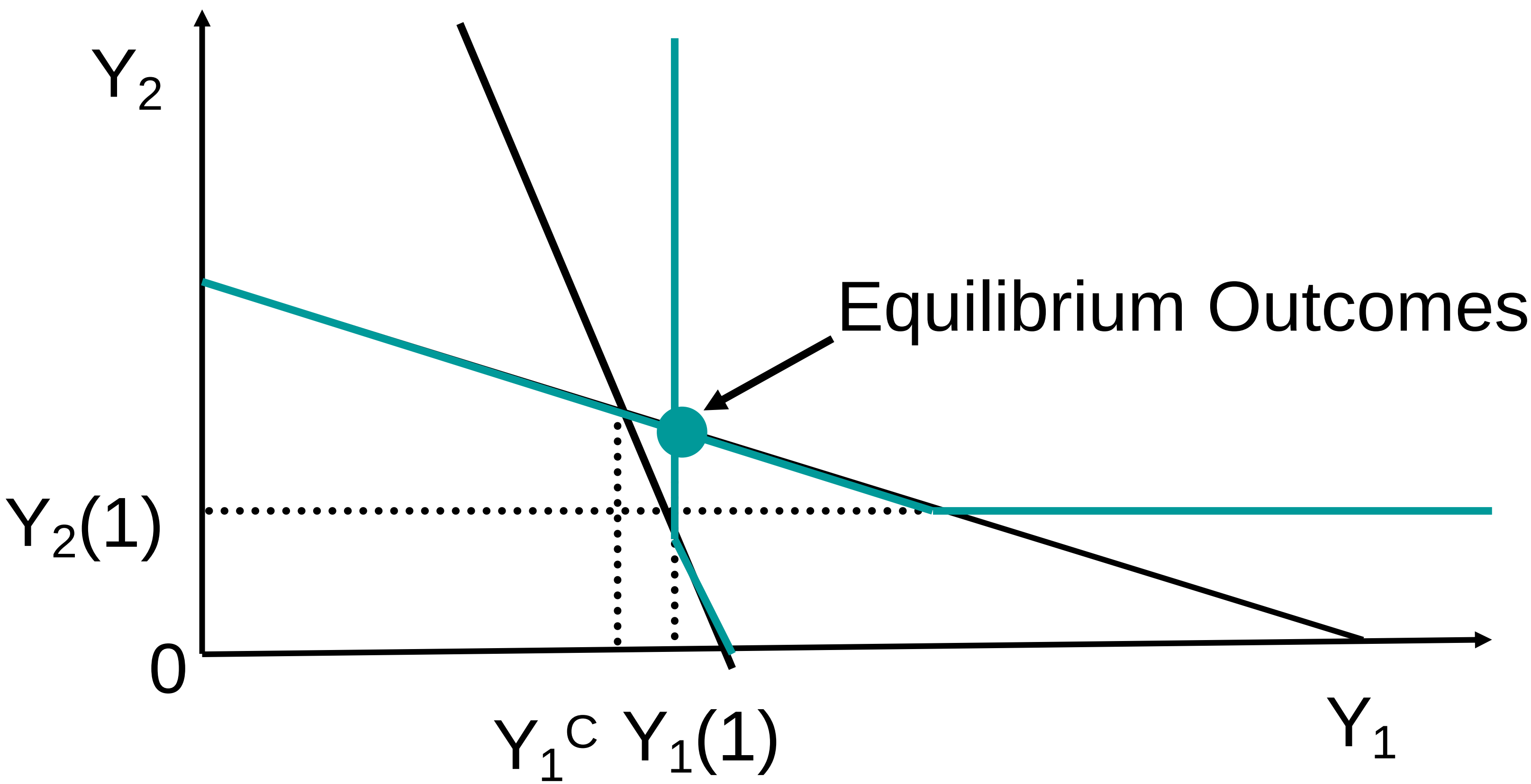
First stage production
→ the commitment to the minimum production level

Second Stage Subgame(1)

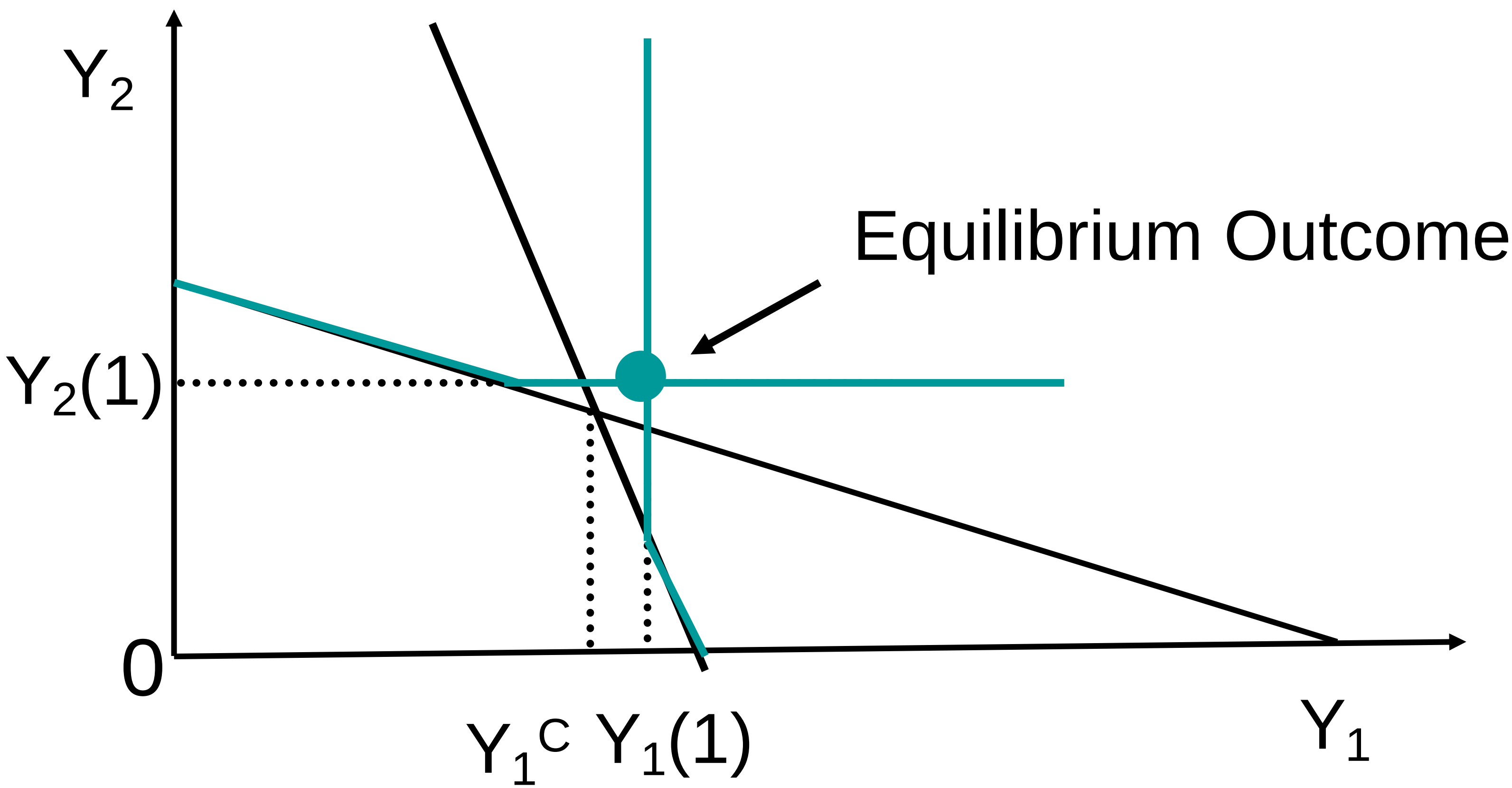
If $Y_i(1) \geqq Y_i^C$, then $Y_i(2)=0$.

If a firm chooses the output larger than the Cournot output in period 1, then it does not produce in period 2, regardless of the rival's production in period 1.

Equilibrium outcome at the second stage subgame



Equilibrium outcome at the second stage subgame



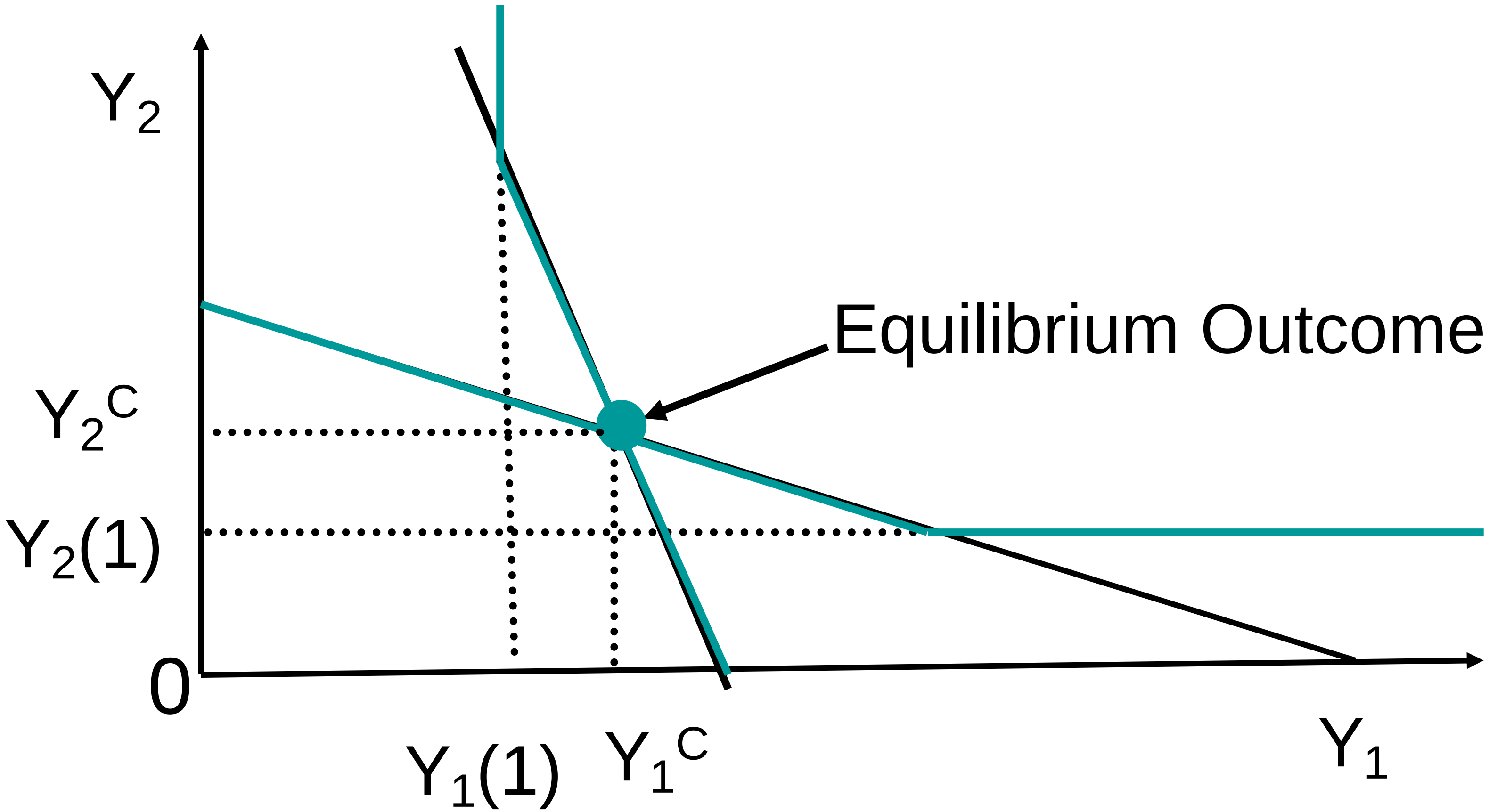
Second Stage Subgame(2)

If $Y_1(1) < Y_1^C$, and $Y_2(1) < Y_2^C$, then $Y_i = Y_i^C$.

If both firms choose the outputs smaller than the Cournot outputs, then the equilibrium is Cournot.

The constraint that total output is never smaller than the first period production is never binding.

Equilibrium outcome at the second stage subgame

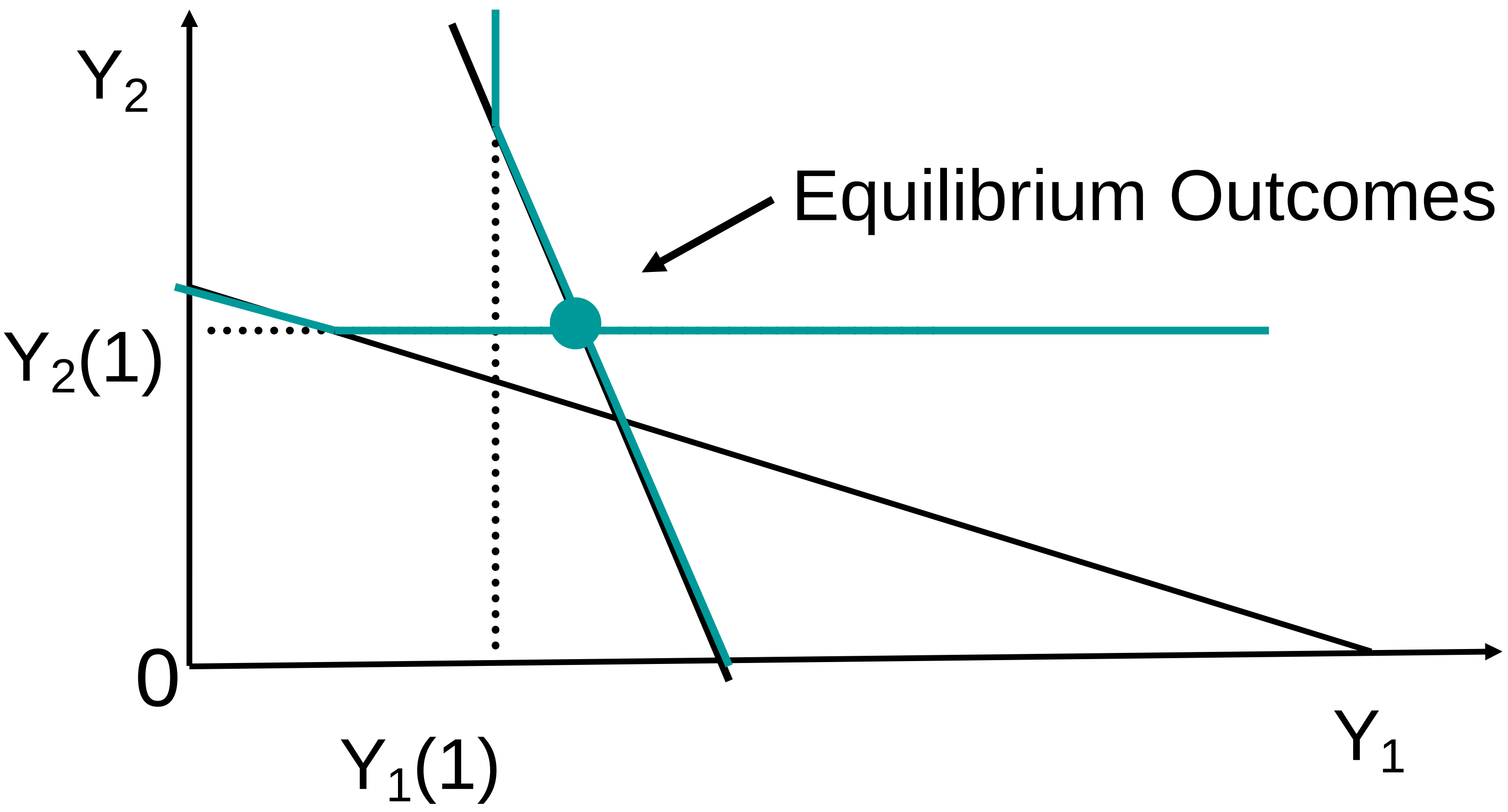


Second Stage Subgame(3)

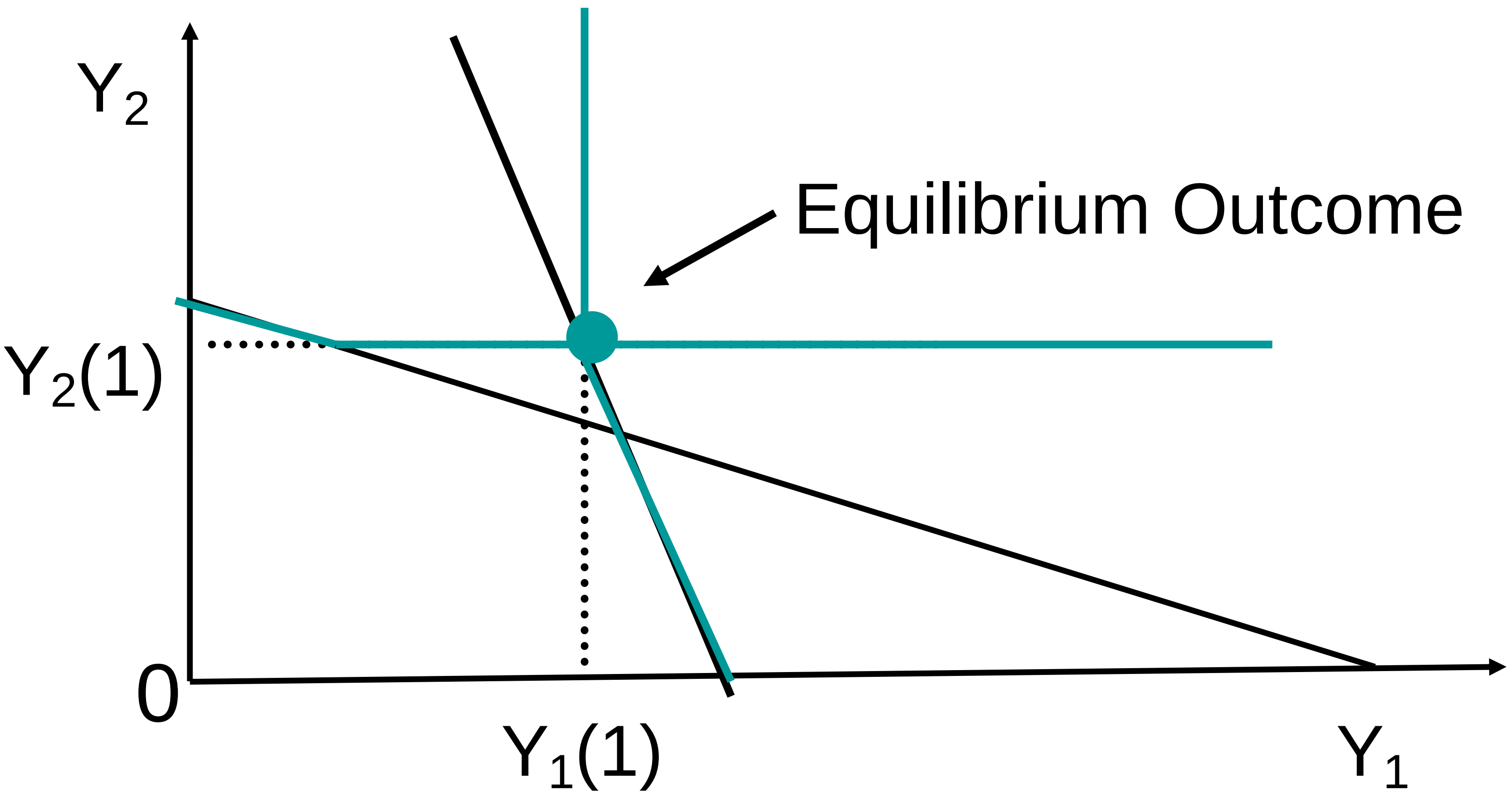
If $Y_1(1) < Y_1^C$, and $Y_2(1) \geqq Y_2^C$, then
$Y_1 = \max(R_1(Y_2(1)), Y_1(1))$.

If a firm chooses the output smaller than the Cournot output in period 1 and the rival chooses the output smaller than the Cournot output in period 1, then the firm chooses the output that is best reply to the rival's first stage production, or does not produce in period 2.

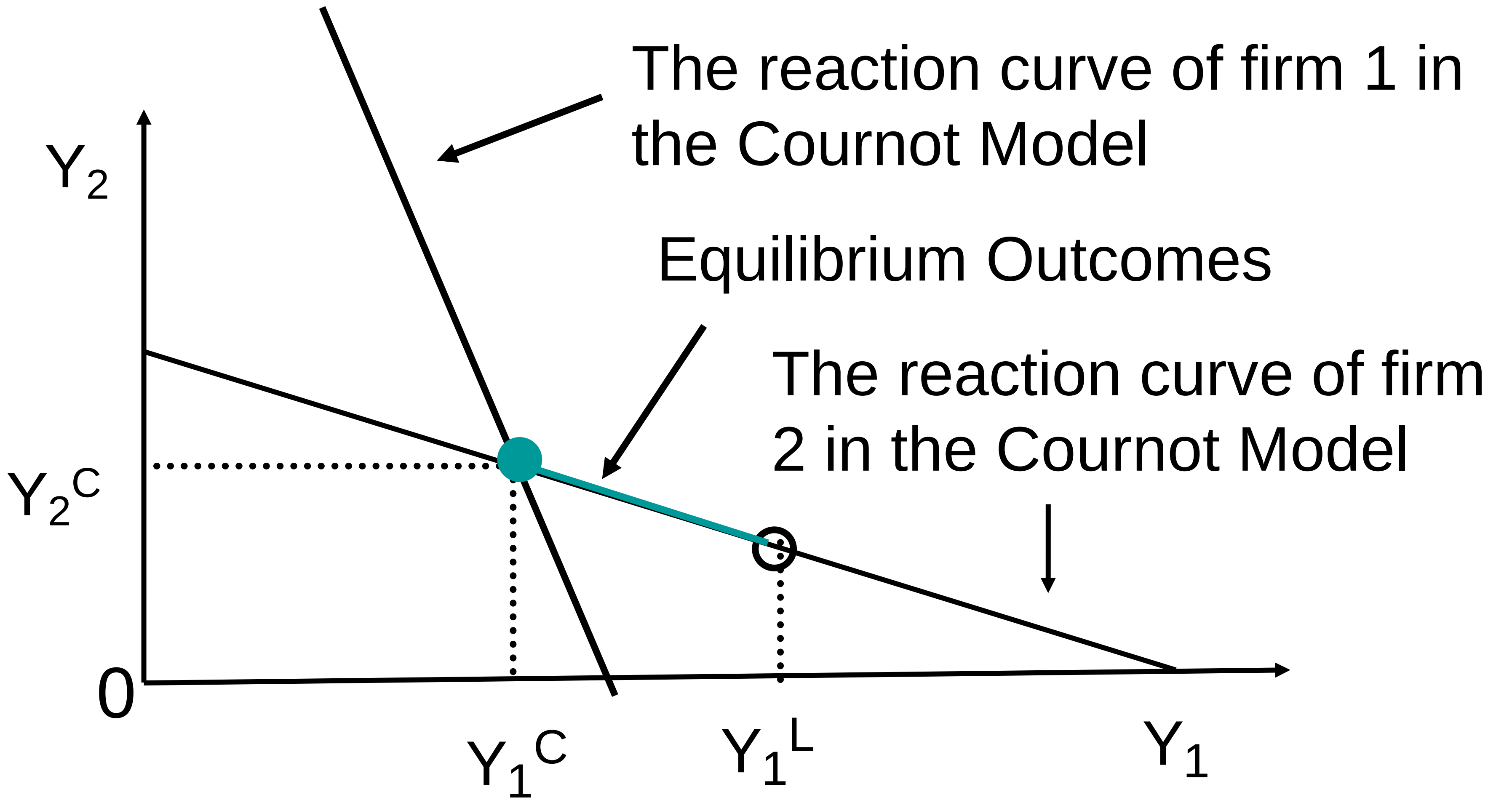
Equilibrium outcome at the second stage subgame



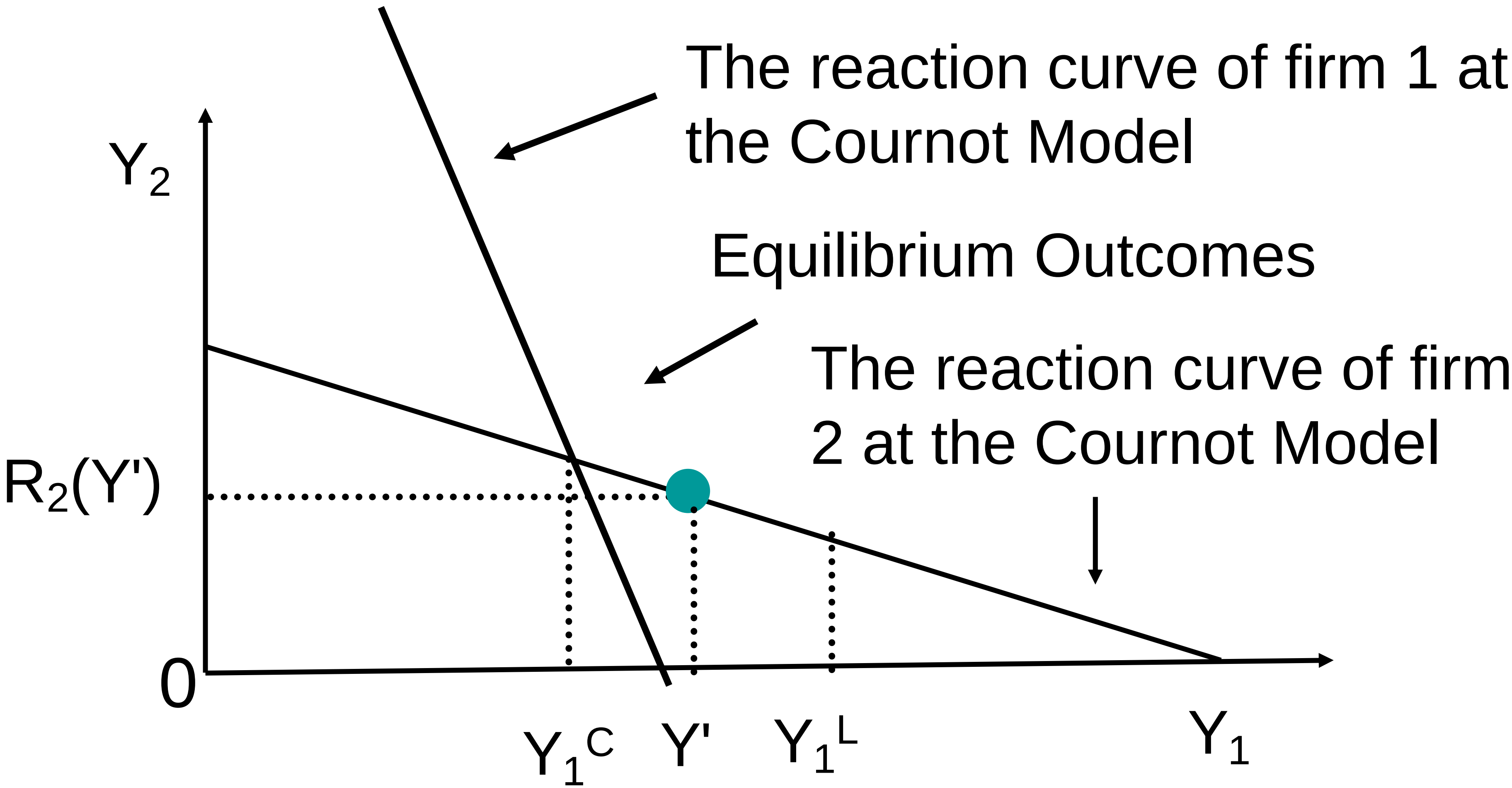
Equilibrium outcome at the second stage subgame



Equilibrium Outcomes



Equilibrium Outcomes



Equilibrium

$Y_1(1) = Y' \geqq Y_1^C$, and $Y_2(1) = R_2(Y')$.

First, we show that firm 2 does not improve its payoff by deviating the above strategy.
Since firm 1's total output does not depend on $Y_2(1)$, the deviation never improves its payoff.
Remember the following result:
If $Y_i(1) \geqq Y_i^C$, then $Y_i(2) = 0$.

Equilibrium

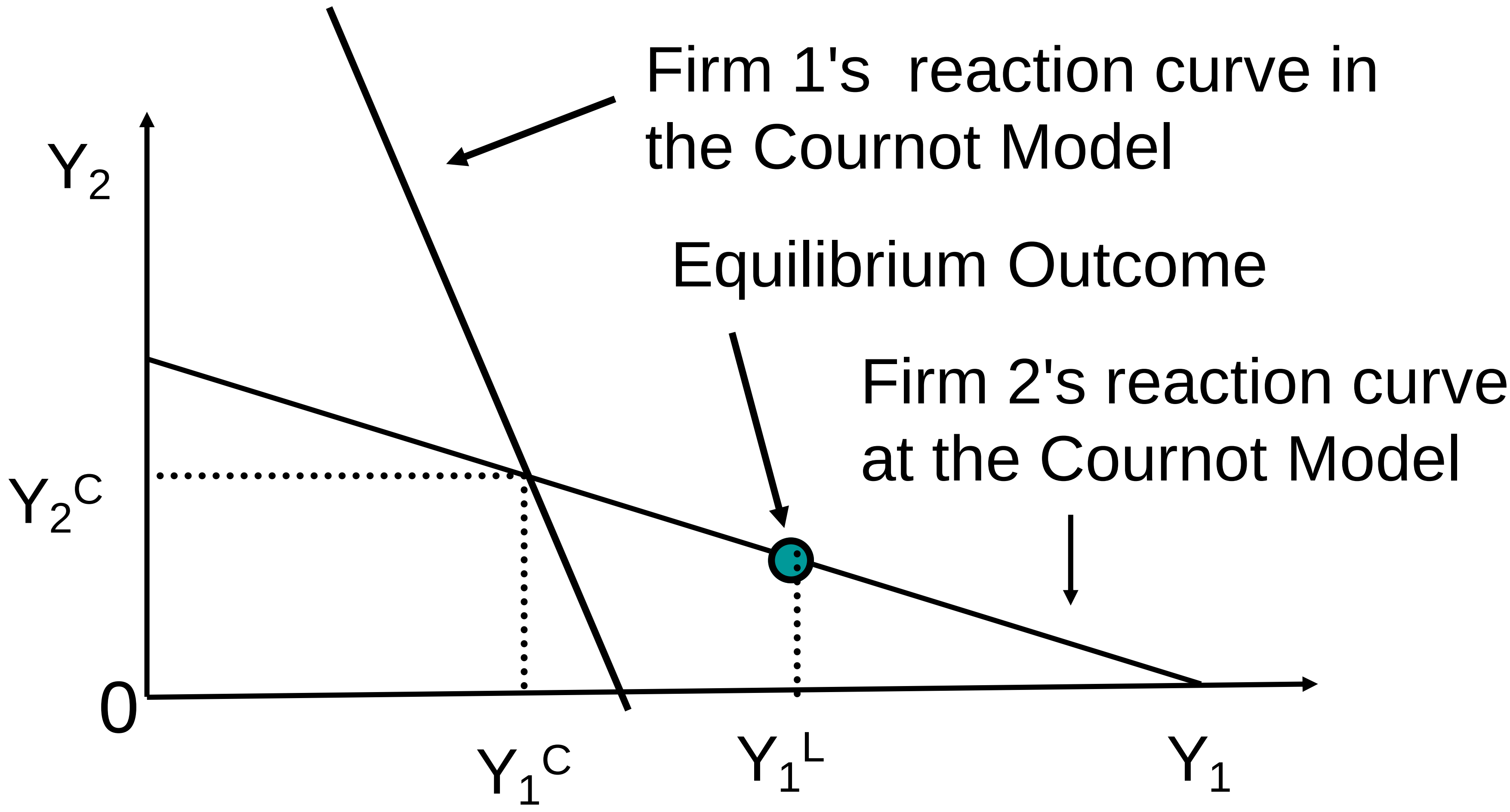
$Y_1(1) = Y' \geqq Y_1^C$, and $Y_2(1) = R_2(Y')$.

Next, we show that firm 1 can not improve its payoff by deviating the above strategy.

Suppose that firm 1 increases the first period production. Since firm 2's total output does not change and $Y' \geqq R_1(Y_2(1))$, the deviation never improves its payoff.

Suppose that firm 1 decreases the first period production. Then firm 2's total output becomes $R_2(Y_1(1))$. It reduces the profit of firm 1 since $Y' < Y_1^L$

Equilibrium Outcomes



Equilibrium

$Y_1(1) = Y_1^L$, and $Y_2(1) = 0$.

Since firm 1's total output does not depend on $Y_2(1)$, the deviation by firm 2 never improves its payoff.

Given that firm 2 produces in period 2 only, becoming the Stackelberg Leader is optimal for firm 1.

Inventory Costs

Suppose that there are some positive inventory costs.
Suppose that the inventory costs are sufficiently large. Then both firms produce in period 2 only and Cournot outcome appears in equilibrium.
Suppose that it costs ε if the first stage production is positive. It is positive and sufficiently small.

Question: Derive the equilibrium outcome.

Inventory Costs

Suppose that it costs ε if the first stage production is positive. It is positive and sufficiently small.

Question: Derive the equilibrium outcome

Answer：Two Stackelberg outcomes

Reason：Firms have no incentive to produce the output that is smaller than the Cournot output.

Equilibrium Outcomes with Small Inventory Costs