Oligopoly Theory (2)
Quantity Competition

Aim of this lecture

(1) To understand the concept of quantity competition

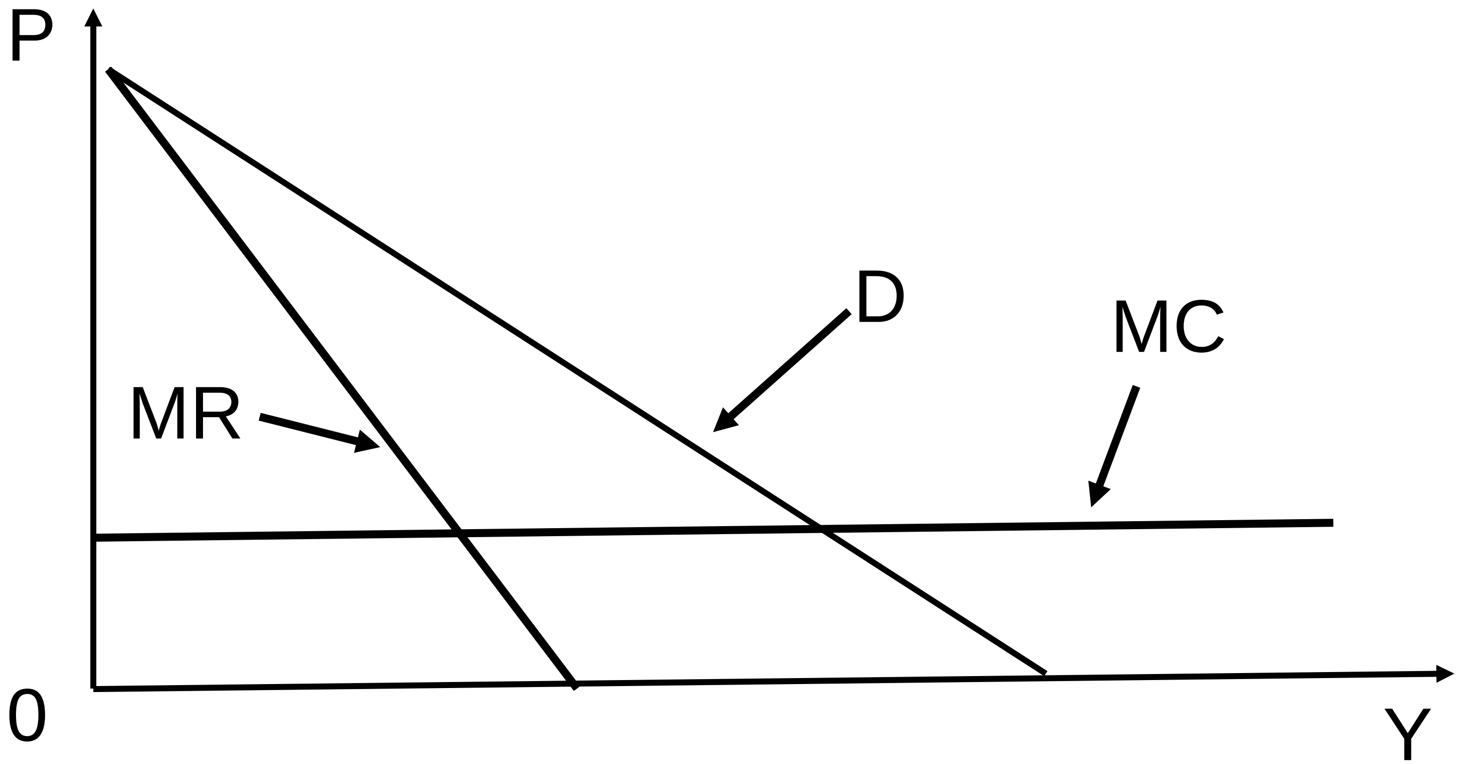
(2) To understand the ideas of strategic substitutes and complements

(3) To understand the relationship between the stability of Cournot equilibrium and comparative statistics

Outline of the Second Lecture

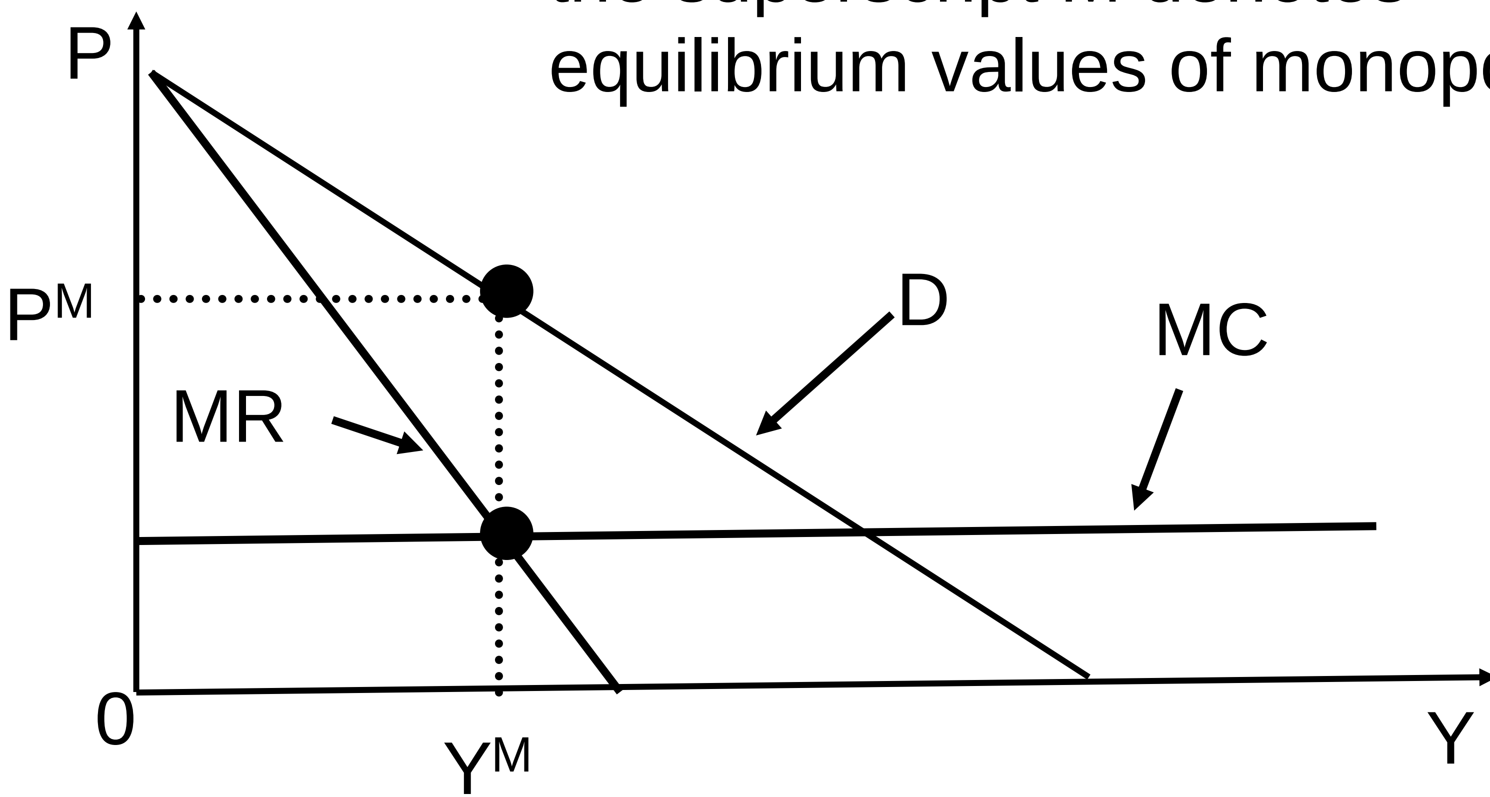
2-1 Monopoly
2-2 Price or Quantity Competition
2-3 Cournot Model
2-4 Strategic Complement and Strategic Substitute
2-5 Stability Condition
2-6 Stability Condition and Comparative Statistics
2-7 Stability Condition and Uniqueness of the Equilibrium
2-8 Cournot Limit Theorem and Perfect Competition

Monopoly Producer



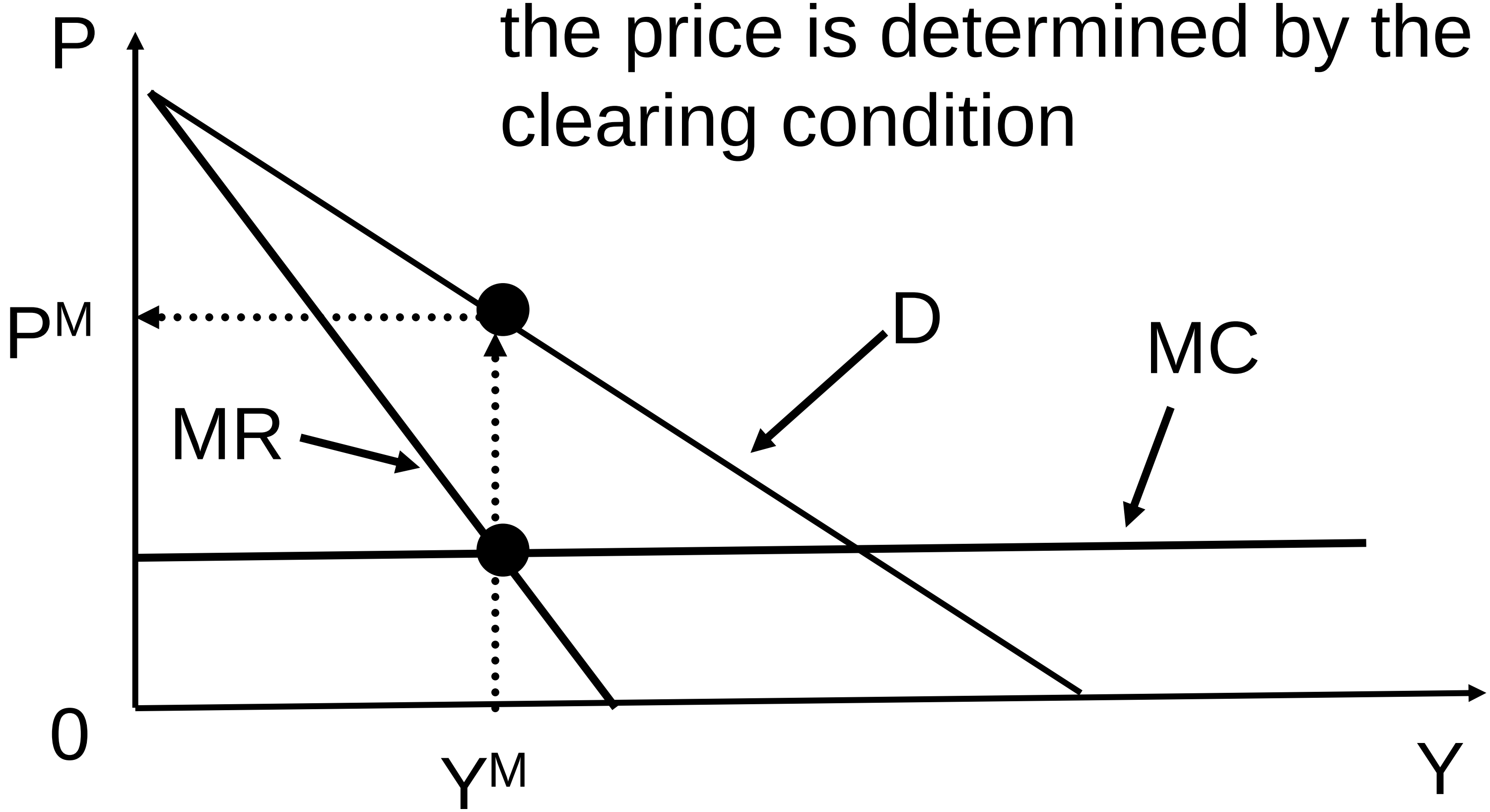
Equilibrium of Monopoly Producer

the superscript M denotes equilibrium values of monopoly



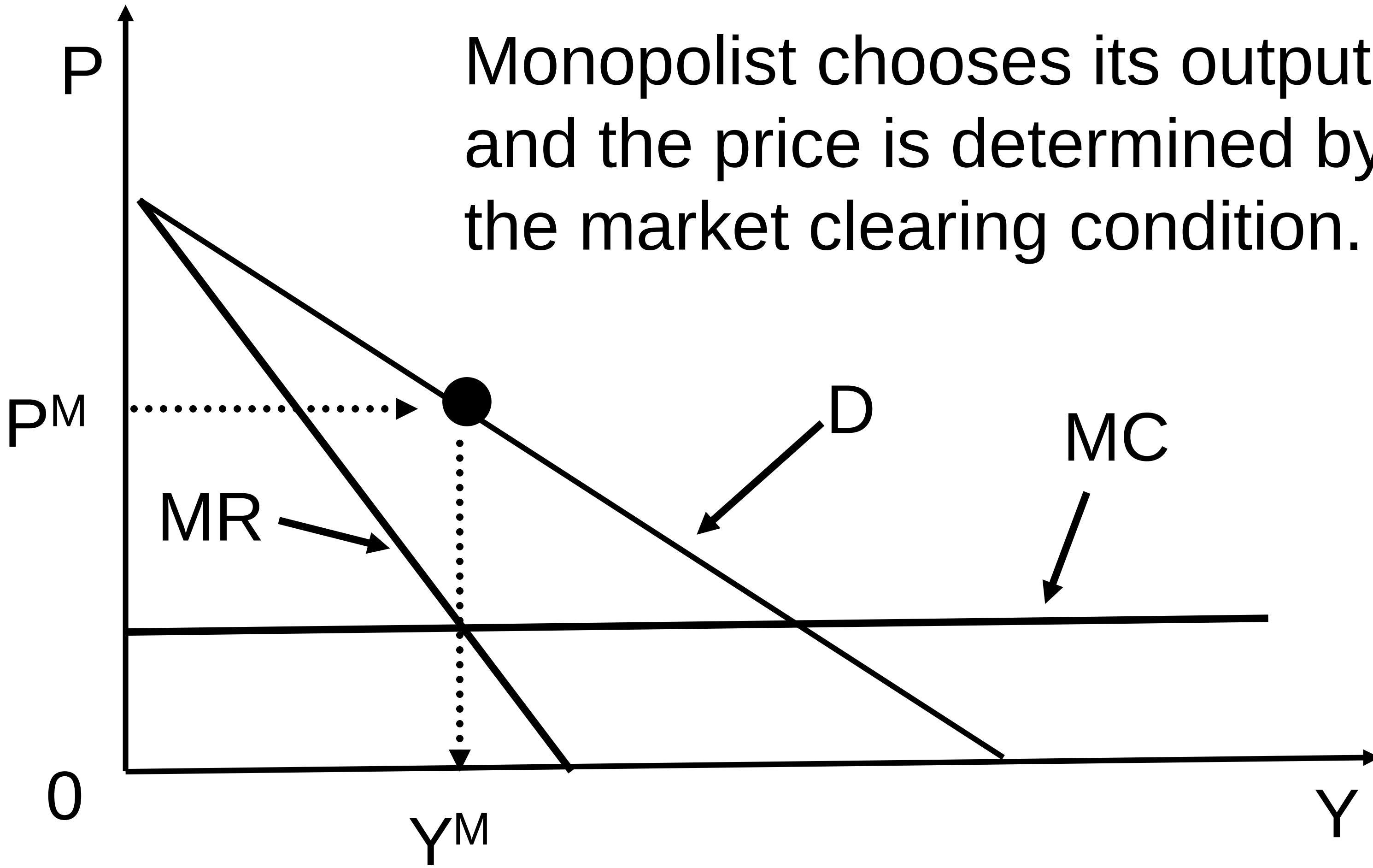
Marshallian View of the Market
Quantity → Price

Monopolist chooses its output and the price is determined by the market clearing condition

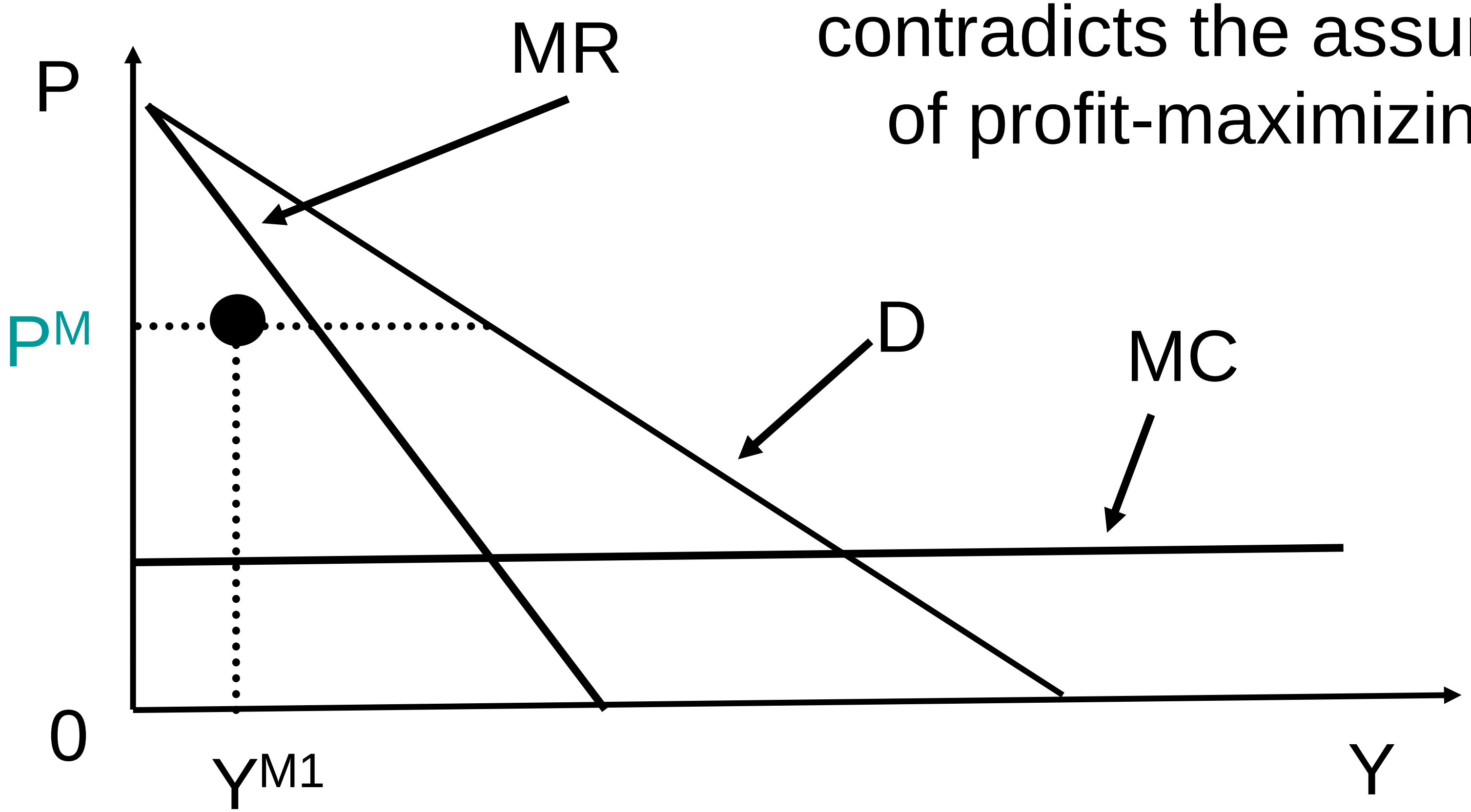


Walrasian View of the Market
Price → Quantity

Monopolist chooses its output and the price is determined by the market clearing condition.



Why does not the monopolist choose both quantity and price？



contradicts the assumption of profit-maximizing

Why does not the monopolist choose both quantity and price？

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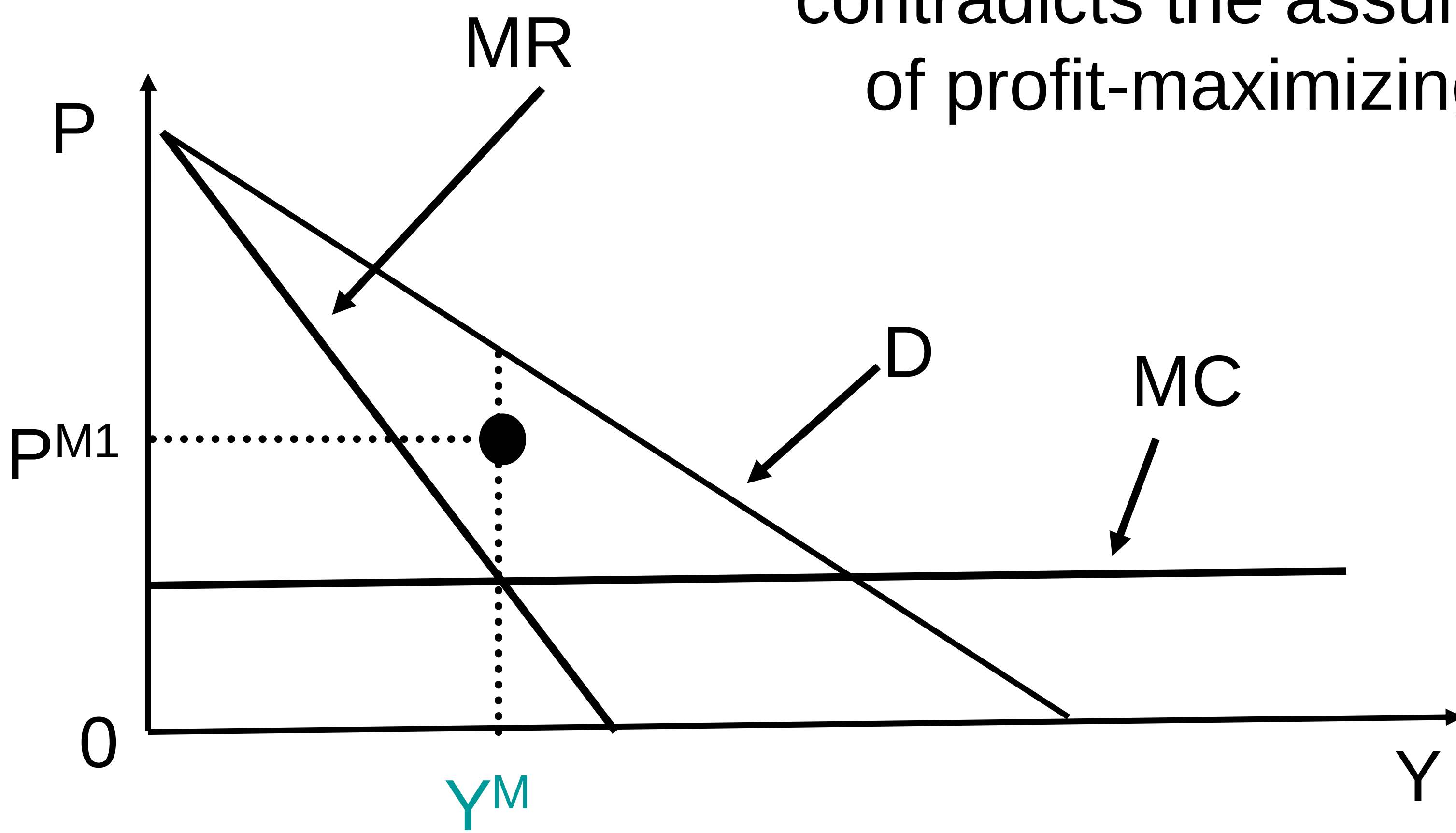


Duopoly

Suppose that there are two or more firms in the market

～The price depends on both its own output and the rivals' outputs.

～The output depends on both its own price and the rivals' prices.

⇒The competition structure depends on whether firms choose their outputs or prices.

Quantity Competition Model (The second lecture)

Price Competition Model (The third lecture)

Which model should we use？(The third lecture)

Cournot Duopoly

Firm 1 and firm 2 compete in a homogeneous product market (product differentiation is fully discussed in the 12th lecture and is also discussed briefly in the 4th lecture).

Each firm i independently chooses its output $Y_i \in [0, \infty)_.$

Each firm maximizes its own profit $\Pi_{i.}$

$\Pi_i = P(Y)Y_i - C_i(Y_i)$, P: Inverse demand function,

Y: Total output, Y_i: Firm i's output, C_i: Firm i's cost function

$P' < 0$, $C' > 0$, $C'' \geqq 0$ (Henceforth, I assume these unless I explicitly make contradicting assumptions.)

Reaction Function

Reaction function of firm 1 ~ $R_1(Y_2)$: Given the output of firm 2, Y_2, $Y_1 = R_1(Y_2)$ implies that Y_1 is the optimal (payoff-maximizing) output for firm 1.

The first-order condition is $P + P'Y_1 = C_1'$.

$\Rightarrow R_1(Y_2)$ is derived from this first-order condition.

The second-order condition is

$2P' + P''Y_1 - C_1'' < 0$

Henceforth, I assume the second-order conditions unless I explicitly make contradicting assumptions.

Cournot Equilibrium

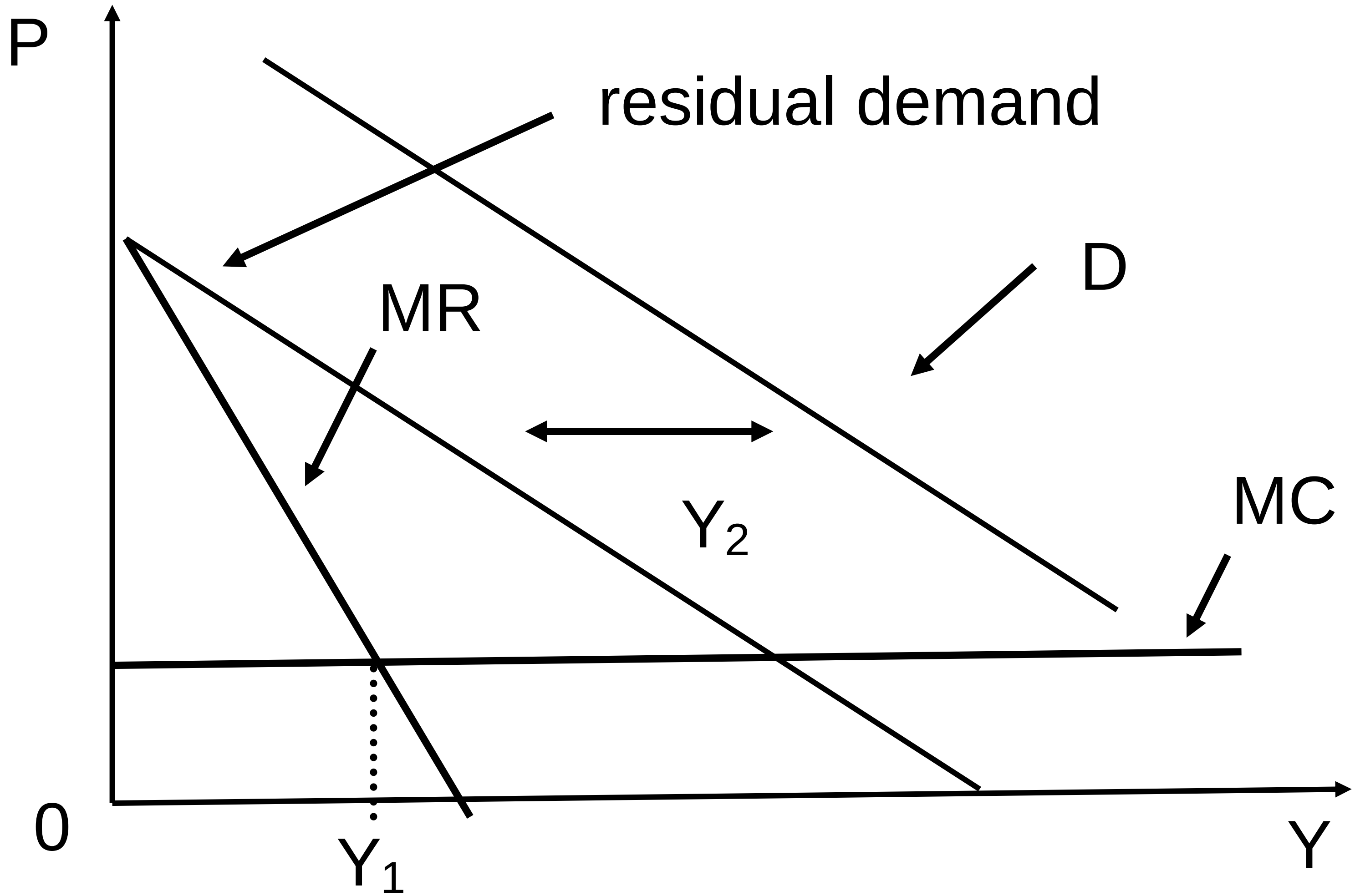
Nash Equilibrium of the Cournot Model

~Cournot Equilibrium

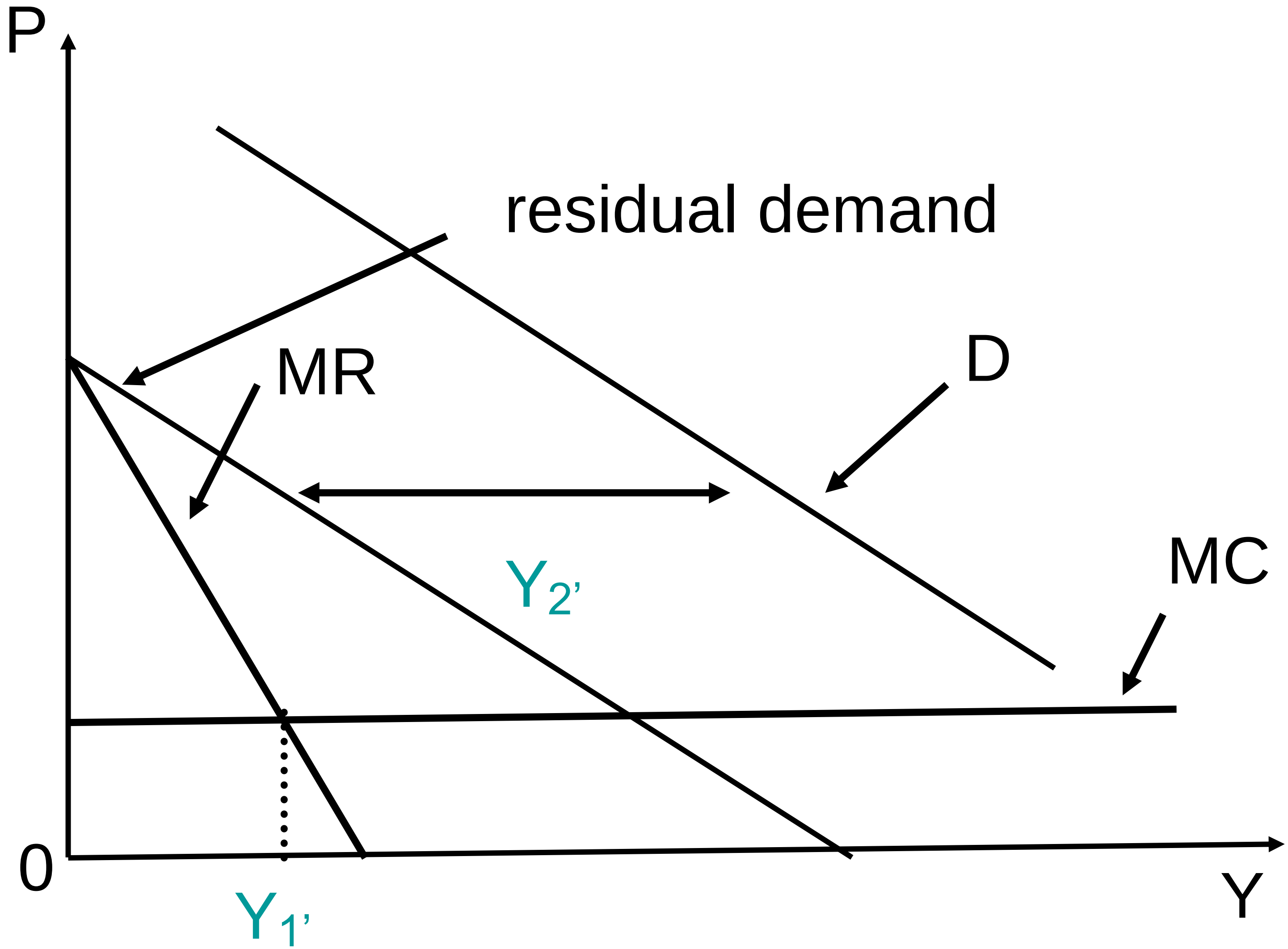
Derivation of the Cournot Equilibrium

Solving $P + P'Y_1 = C_1'$, $P + P'Y_2 = C_2'$

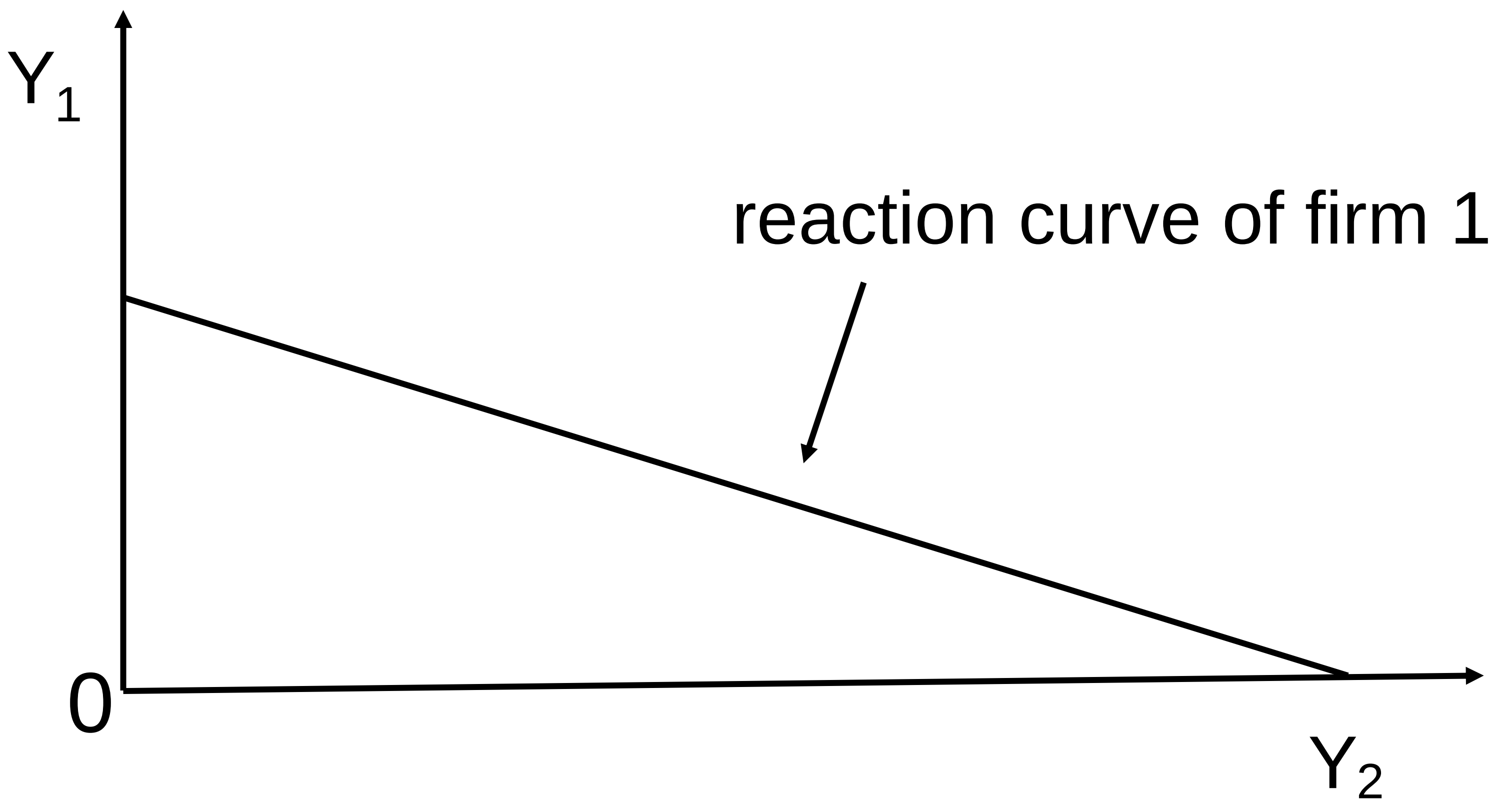
Residual Demand



Derivation of Reaction Function



Reaction Curve of Firm 1



slope of the reaction curve

$P+P'Y_1 - C_1' = 0 \rightarrow dY_1/dY_2 = -(P'+P''Y_1)/(2P'+P''Y_1 - C_1'')$

$P'+P''Y_1 > 0 \Rightarrow$ upward sloping of the reaction curve (strategic complements)

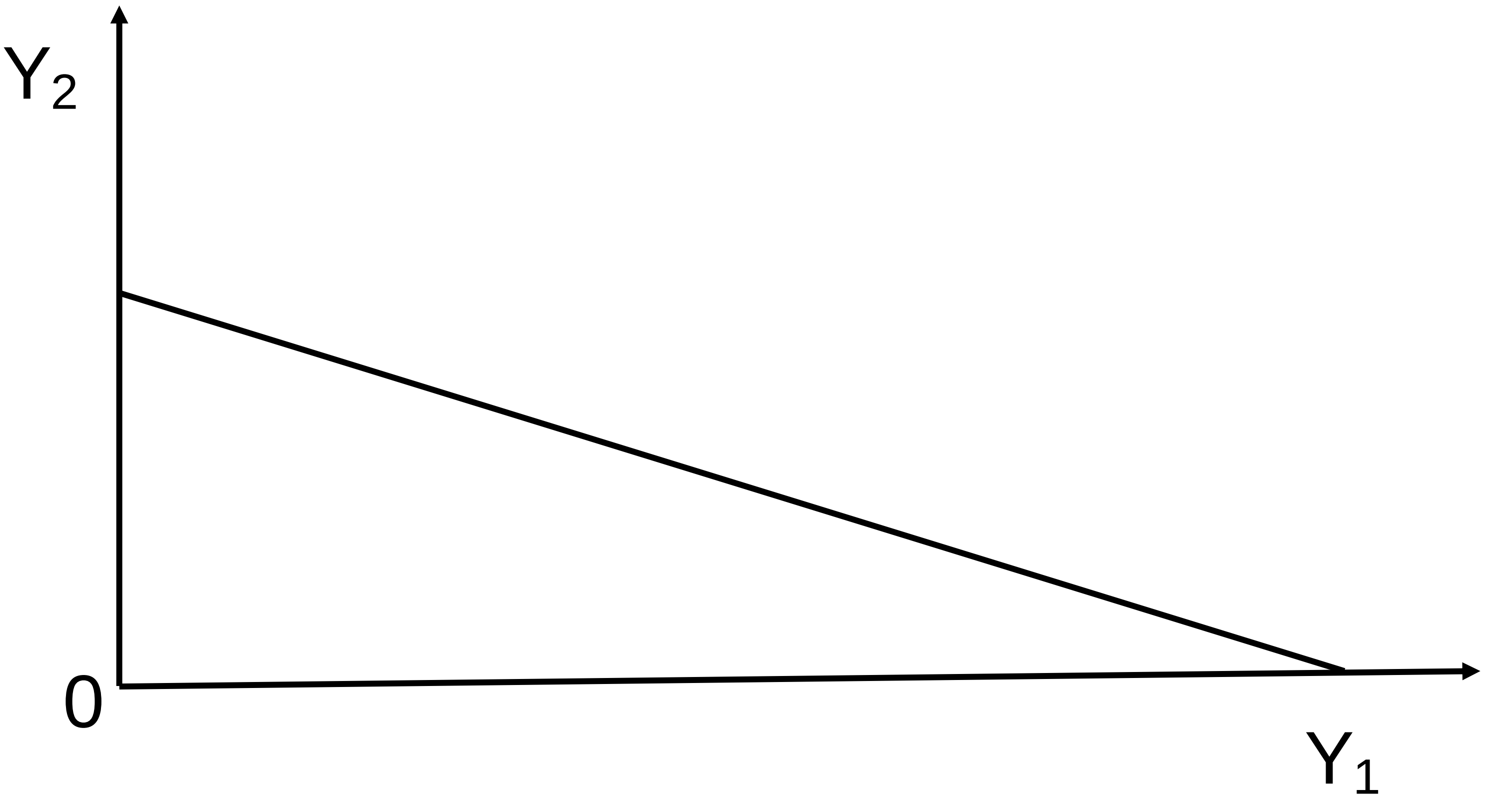
～an increase in the rival's output increases the marginal revenue of the firm: unnatural in the context of Cournot competition, but it is possible.

$P'+P''Y_1 < 0 \Rightarrow$ downward sloping of the reaction curve (strategic substitutes)

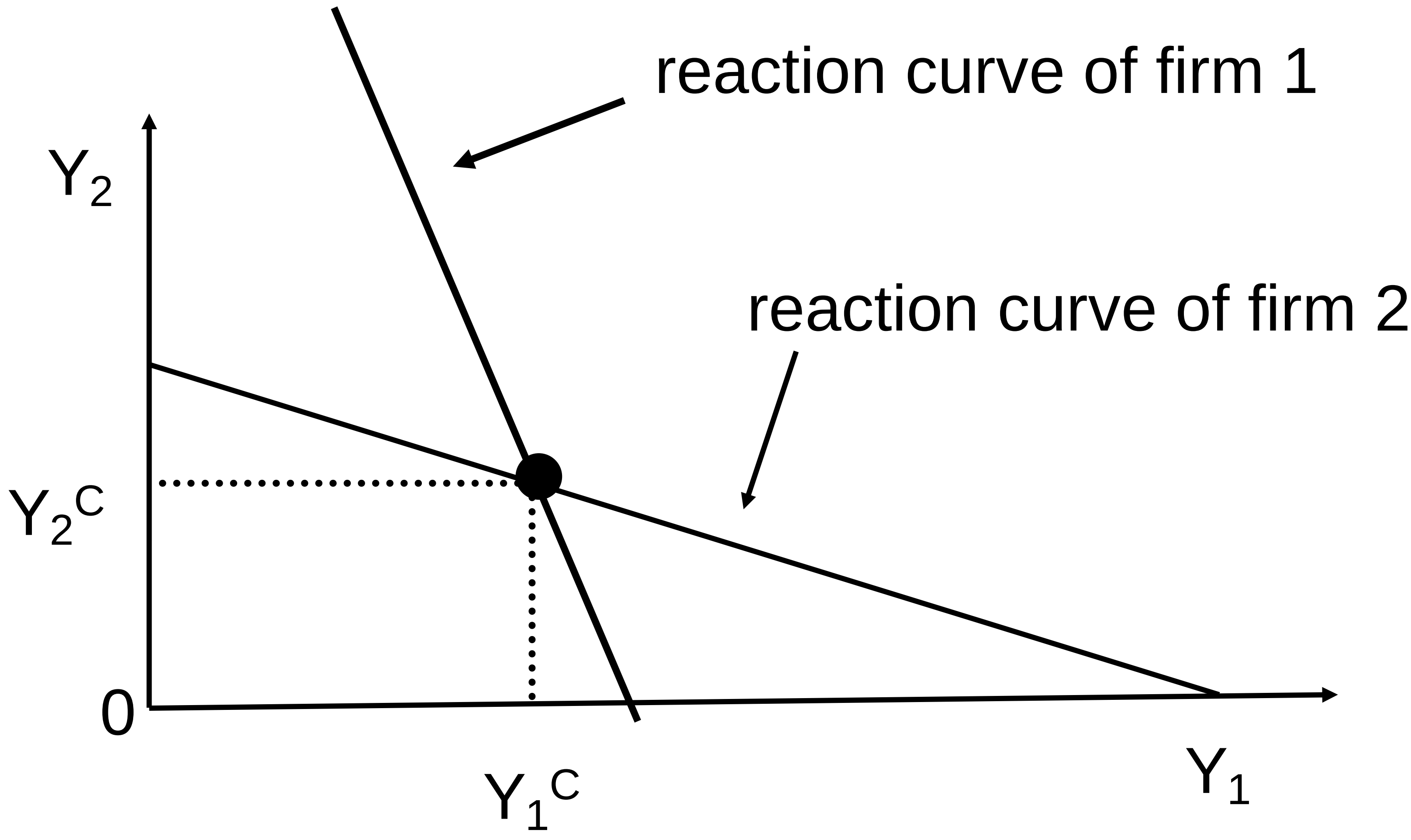
～ an increase in the rival's output reduces the marginal revenue of the firm

In this course, I assume $P'+P''Y_1 < 0$ unless I make explicit contradicting assumptions.

Reaction Curve of Firm 2

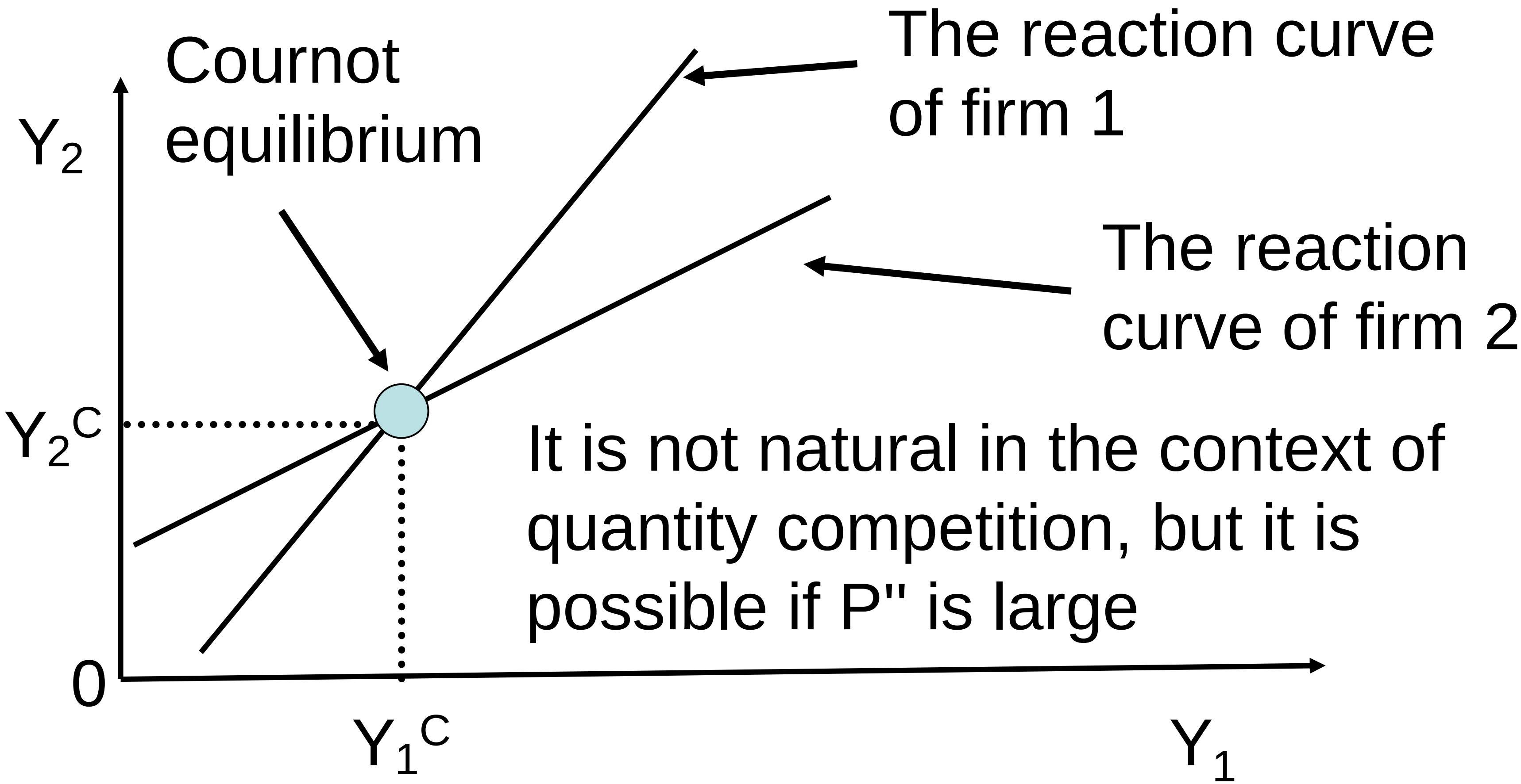


Cournot Equilibrium



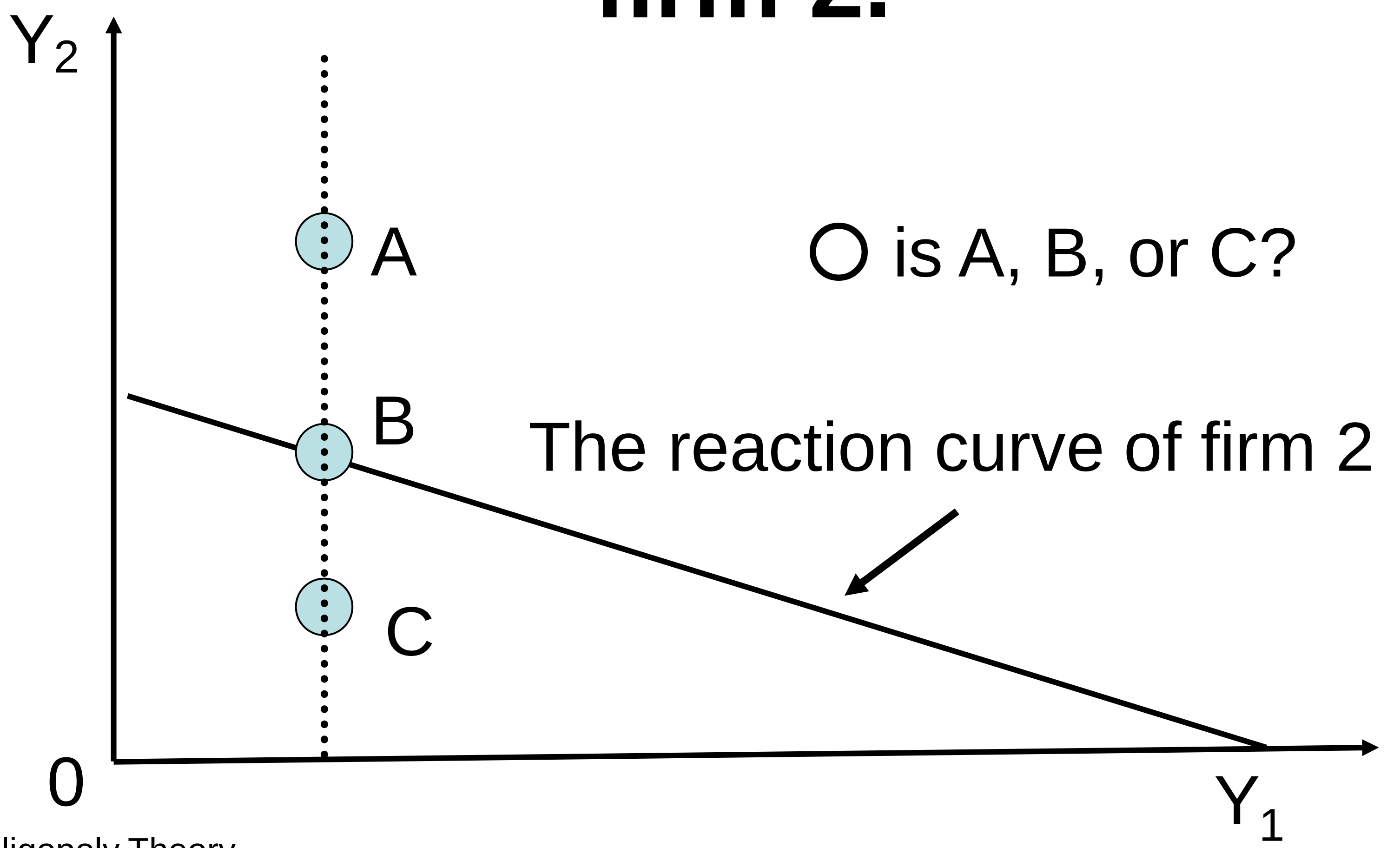
the superscript C denotes Cournot Equilibrium

A case of strategic complements

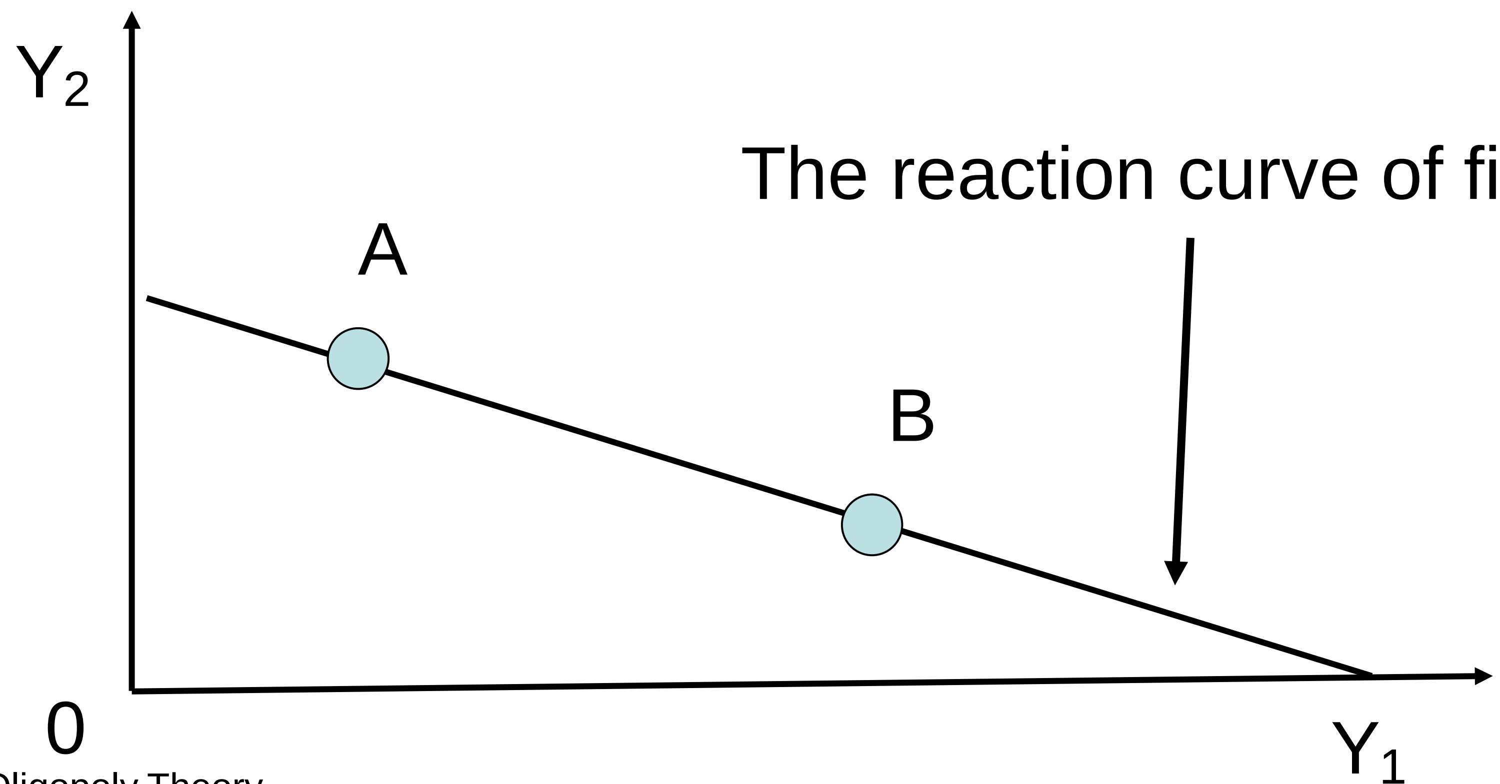


It is not natural in the context of quantity competition, but it is possible if P'' is large

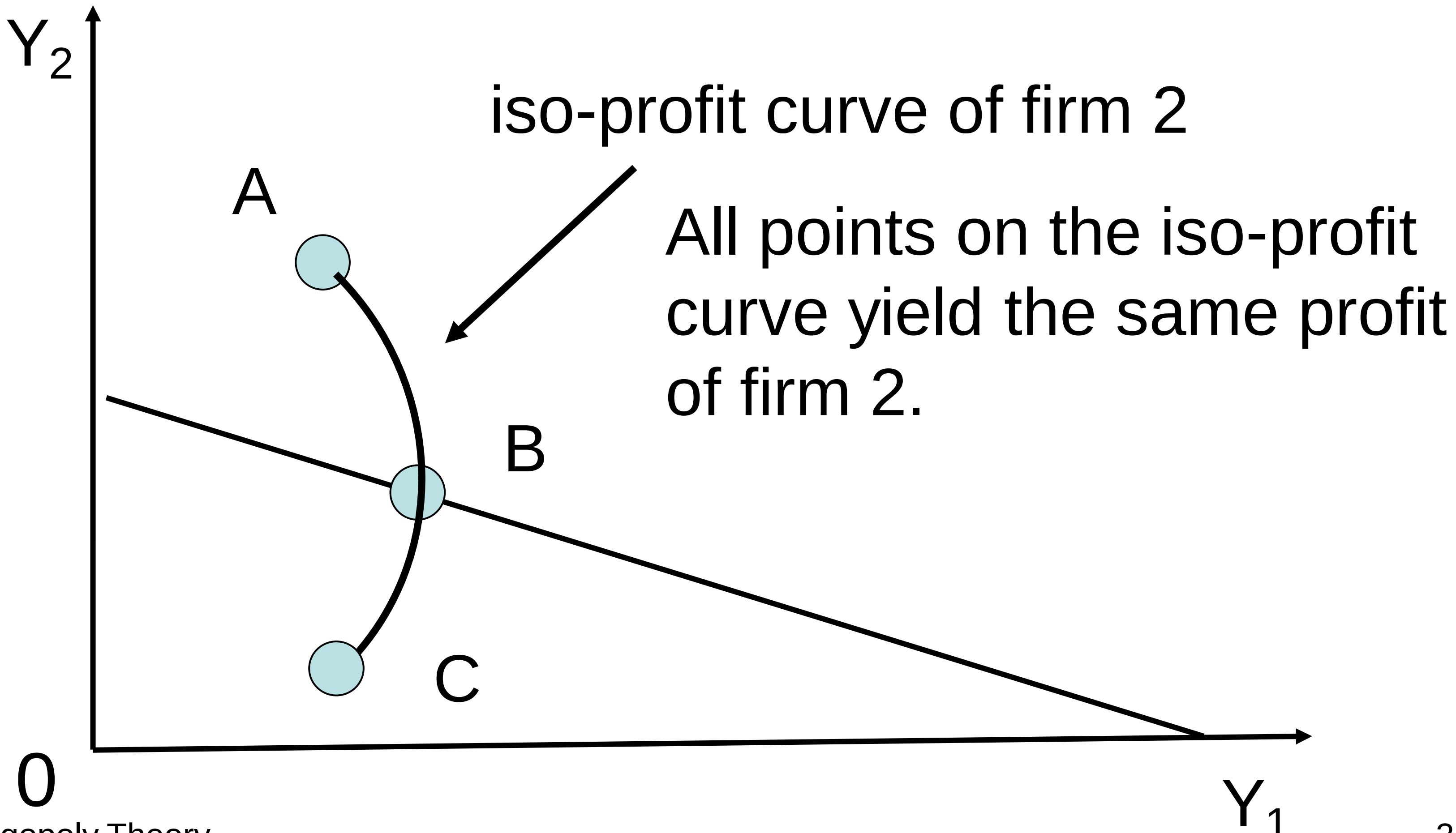
Question: Among three points A, B, and C, ○ yields the largest profit of firm 2.



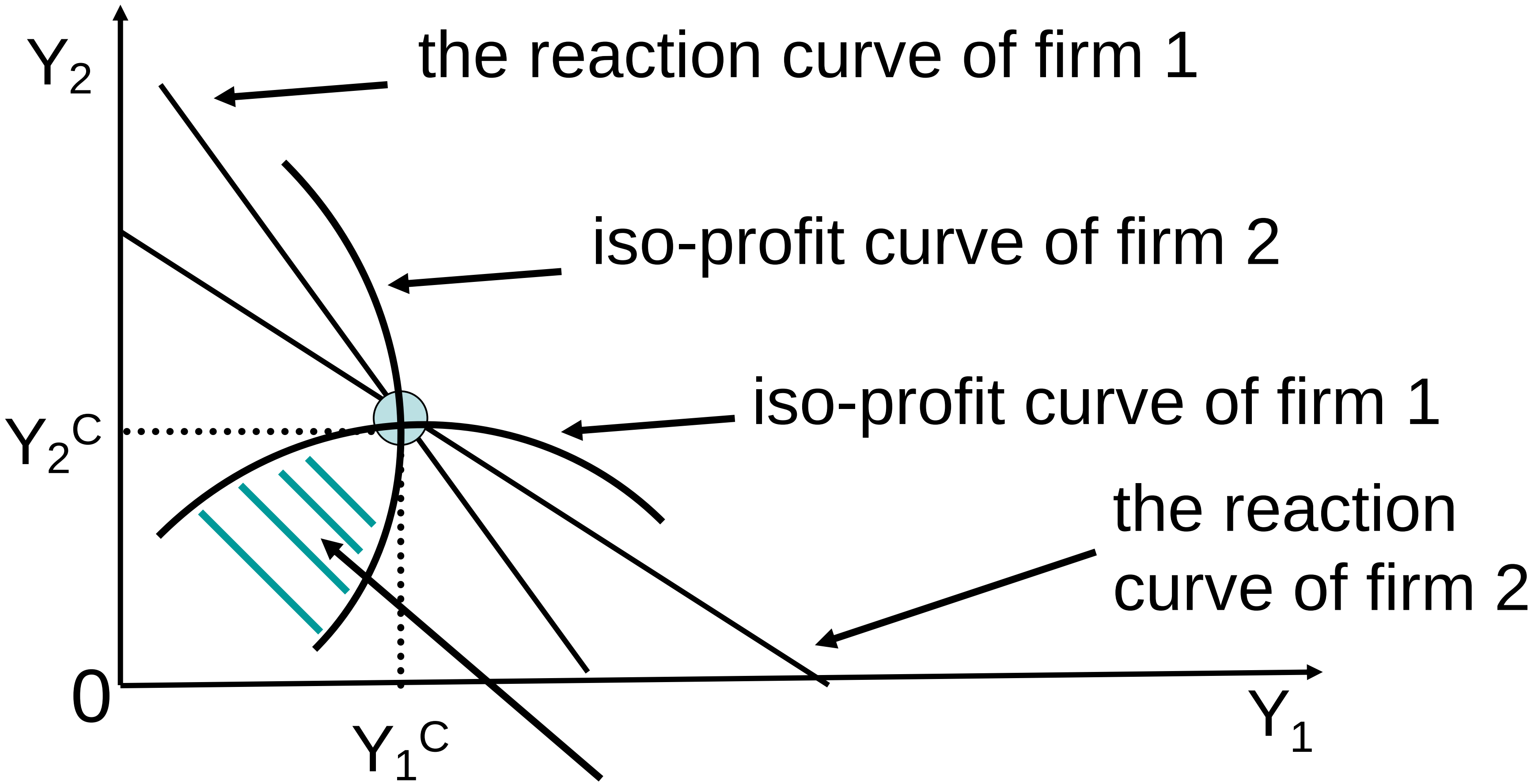
Question: Which yields larger profit of firm 2, A or B?



Firm 2's iso-profit curve profit



Cournot Equilibrium and Efficiency



Moving from the Cournot equilibrium to this point improves both firms' payoffs (profits)

Welfare Implications

Each firm maximizes its own profit with respect to its output, without considering the negative effect on the rival.

→The output at the Cournot equilibrium is excessive from the viewpoint of total profits maximization.

However, at the Cournot equilibrium, $P+P'Y_1 - C_1'=0$, so $P - C_1' > 0$.

→The output at the Cournot equilibrium is insufficient from the viewpoint of total social surplus maximization (total social surplus maximization is achieved when $P = C_1' = C_2'$).

Cournot Oligopoly

Firm 1, firm 2, ..., firm n compete in a homogeneous product market.

Each firm i independently chooses its output

$Y_i \in [0, \infty)$.

Each firm maximizes its own profit Π_i.

$\Pi_i = P(Y)Y_i - C_i(Y_i)$, P: Inverse demand function,

Y: Total output, Y_i: Firm i's output, C_i: Firm i's cost function

$P' < 0$, $C' > 0$, $C'' \geqq 0$

The same model except for the number of the firms

Cournot Equilibrium

Derivation of the Cournot equilibrium

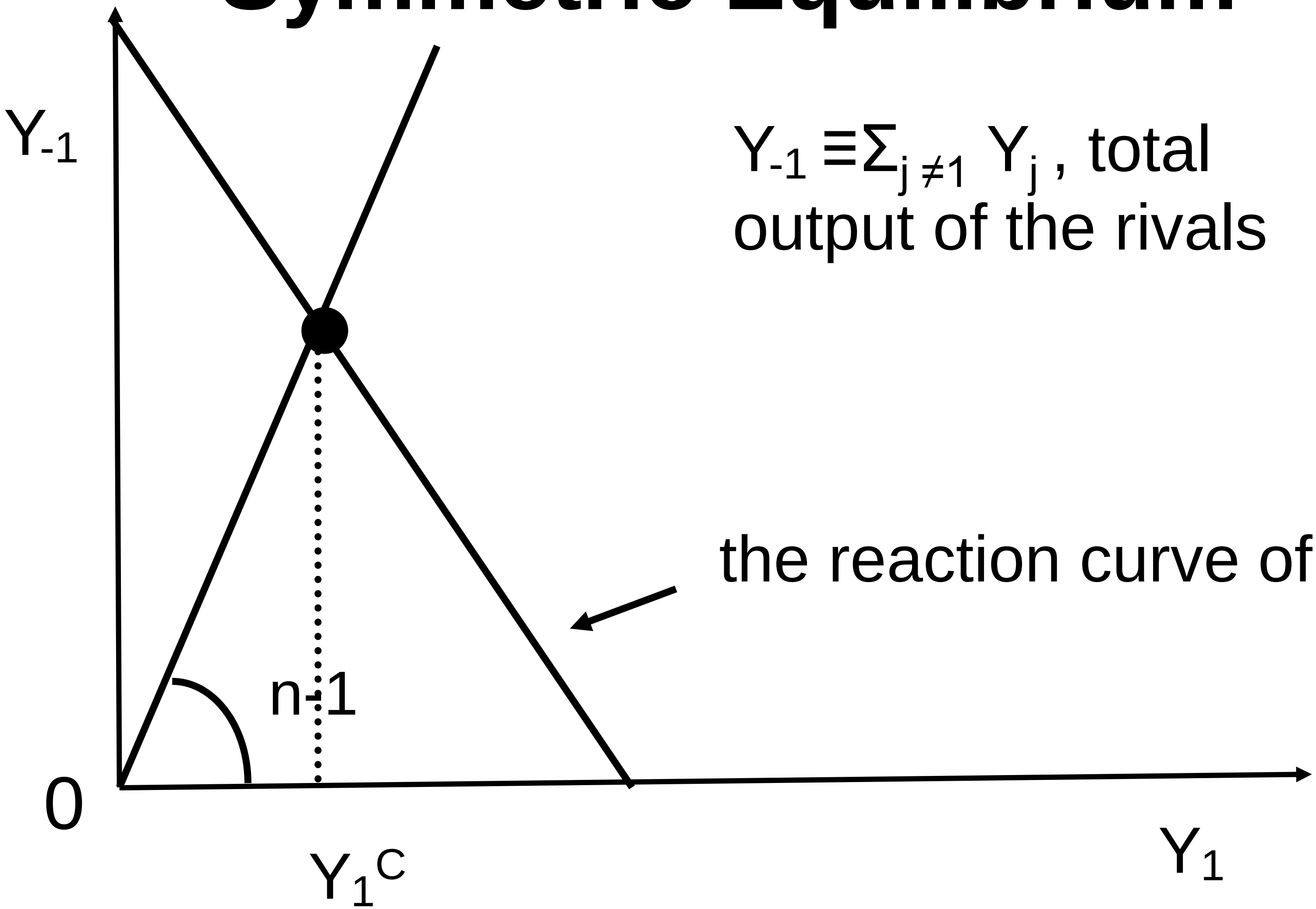
Solving the system of equations

$P+P'Y_1 = C_1'$, $P+P'Y_2 = C_2'$,…, $P+P'Y_n = C_n'$

If firms are symmetric (all firm have the same cost function), the symmetric equilibrium is derived from

$P+P'Y_1 =C_1'$, $Y=nY_1$ (or equivalently $Y_{-1}=(n-1)Y_1$ where $Y_{-1} \equiv \Sigma_{j \neq 1} Y_j$, total output of the rivals)

Symmetric Equilibrium



$Y_{-1} \equiv \Sigma_{j \neq 1} Y_j$, total output of the rivals

the reaction curve of firm 1

Cournot Limit Theorem

The first-order condition for firm 1 is

$P+P'Y_1=C_1'$. From it, we obtain

$P(1+(P'Y/P)(Y_1/Y))=C_1'$

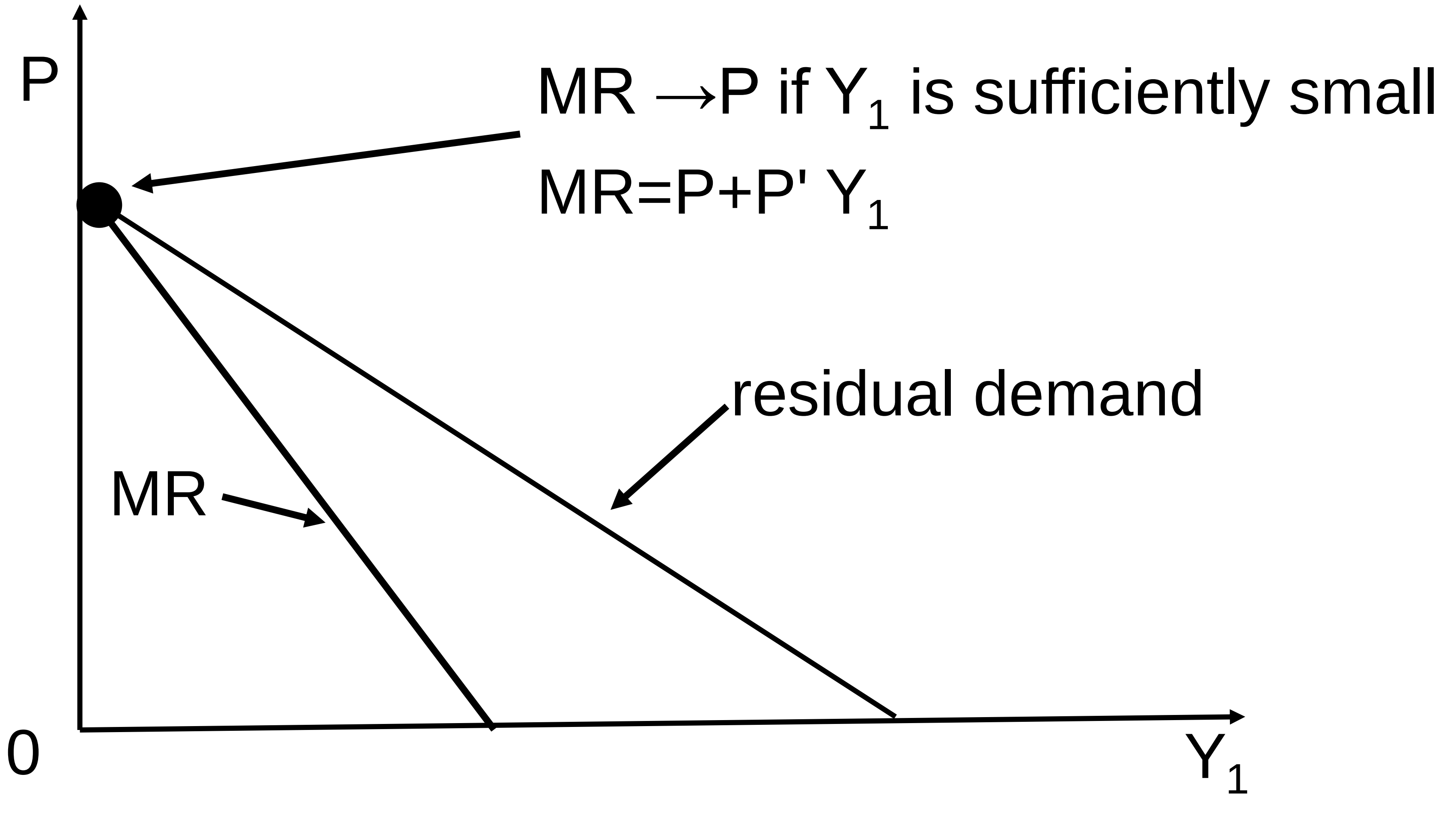
$\Rightarrow P(1-\eta^{-1}(Y_1/Y))=C_1'$ (η: price elasticity of the demand)

$\eta\rightarrow\infty$ $P\rightarrow C_1'$ (the world of price taker)

$Y_1/Y\rightarrow 0$ $P\rightarrow C_1'$(the world of Cournot Limit theorem)

Cournot Limit Theorem~ If the number of firms is sufficiently large (if the market share of each firm is sufficiently small), the price is sufficiently close to the marginal cost of each firm.

Marginal Revenue for Small Firms



perfect competition

Price Taker: The player who chooses his/her behavior given the price exogenously.

In the context of quantity competition, the firm is a price taker if it thinks that the price remains unchanged even if it increases the output.

In fact, unless the price elasticity of the demand is infinity, an increase in the output of each player reduces the price, whether the player is small or large.

The explanation that a firm is a price taker when its size is too small to affect the price seems ridiculous.

Micro Foundation of Perfect Competition

In the Cournot model, all firms are price makers (they recognize that $P'<0$).

However, if the number of the firms is sufficiently large, the equilibrium price is approximately equal to the perfectly competitive equilibrium price.

Perfect competition equilibrium ≒Cournot equilibrium in the large economy ~ Perfect competition model is an approximation of the real world when the number of firms is sufficiently large.

General Discussions on the Slope of Reaction Curve

Payoff function of player 1 $U_1(a_1,a_2)$, where $a_i \in [0,a^u]$, $\partial U_1/\partial a_2 \geqq 0$ and the equality holds only if $a_1=0$.

The first-order condition is $\partial U_1/\partial a_1=0$

The second-order condition is $\partial^2 U_1/\partial a_1^2<0$. We assume that the second-order condition is satisfied.

$R_1' = - \partial^2 U_1/\partial a_1 \partial a_2 / \partial^2 U_1/\partial a_1^2$

Whether strategies are strategic complement or substitute depends on the sign of the cross derivative $\partial^2 U_1/\partial a_1 \partial a_2$.

Existence of the Equilibrium

From the definitions of the reaction function and the Nash equilibrium, we have

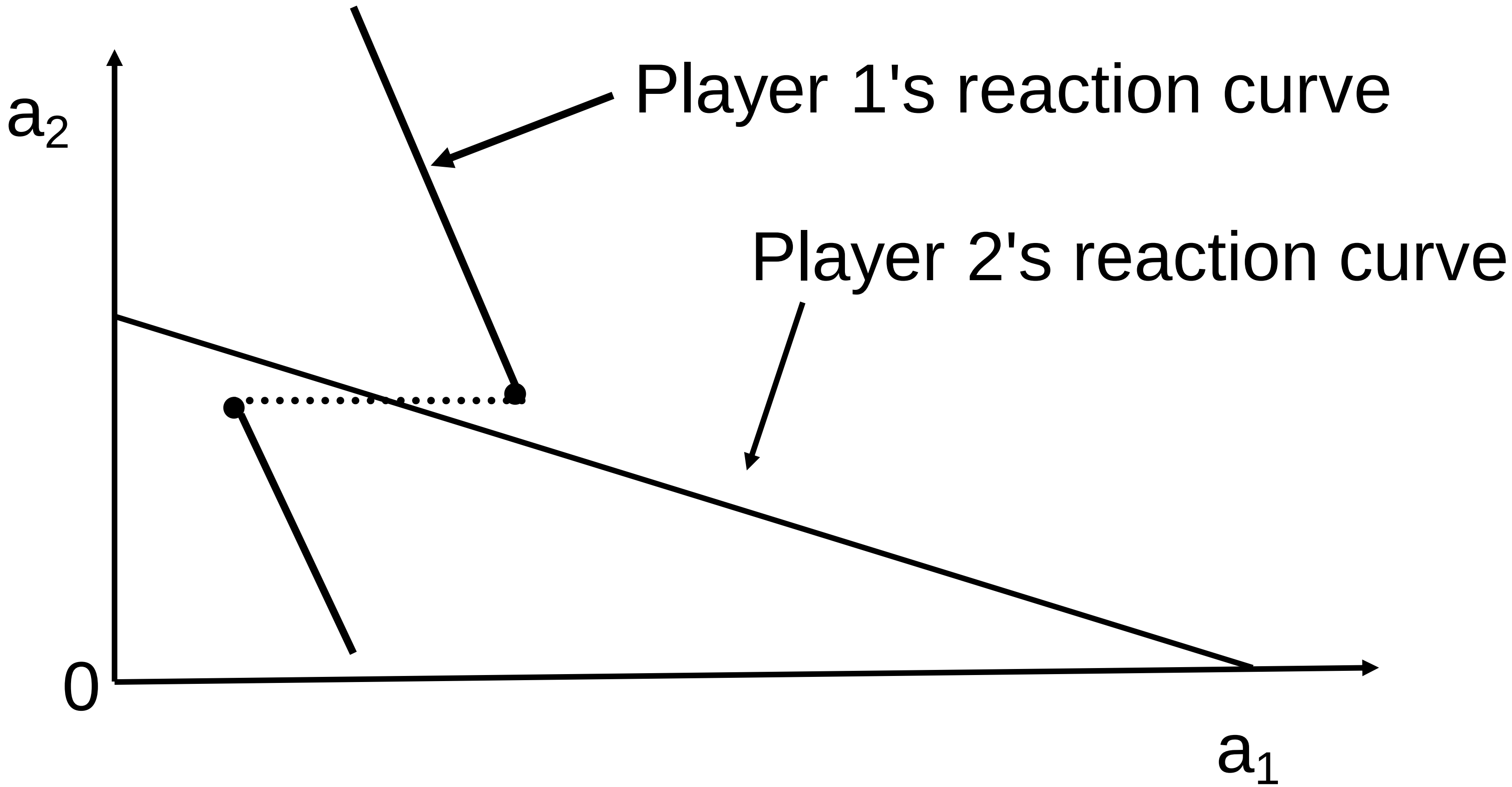
$R_1(R_2(a_1^N)) = a_1^N$, $R_2(R_1(a_2^N)) = a_2^N$

We can use the fixed-point theorem to show the existence of the equilibrium.

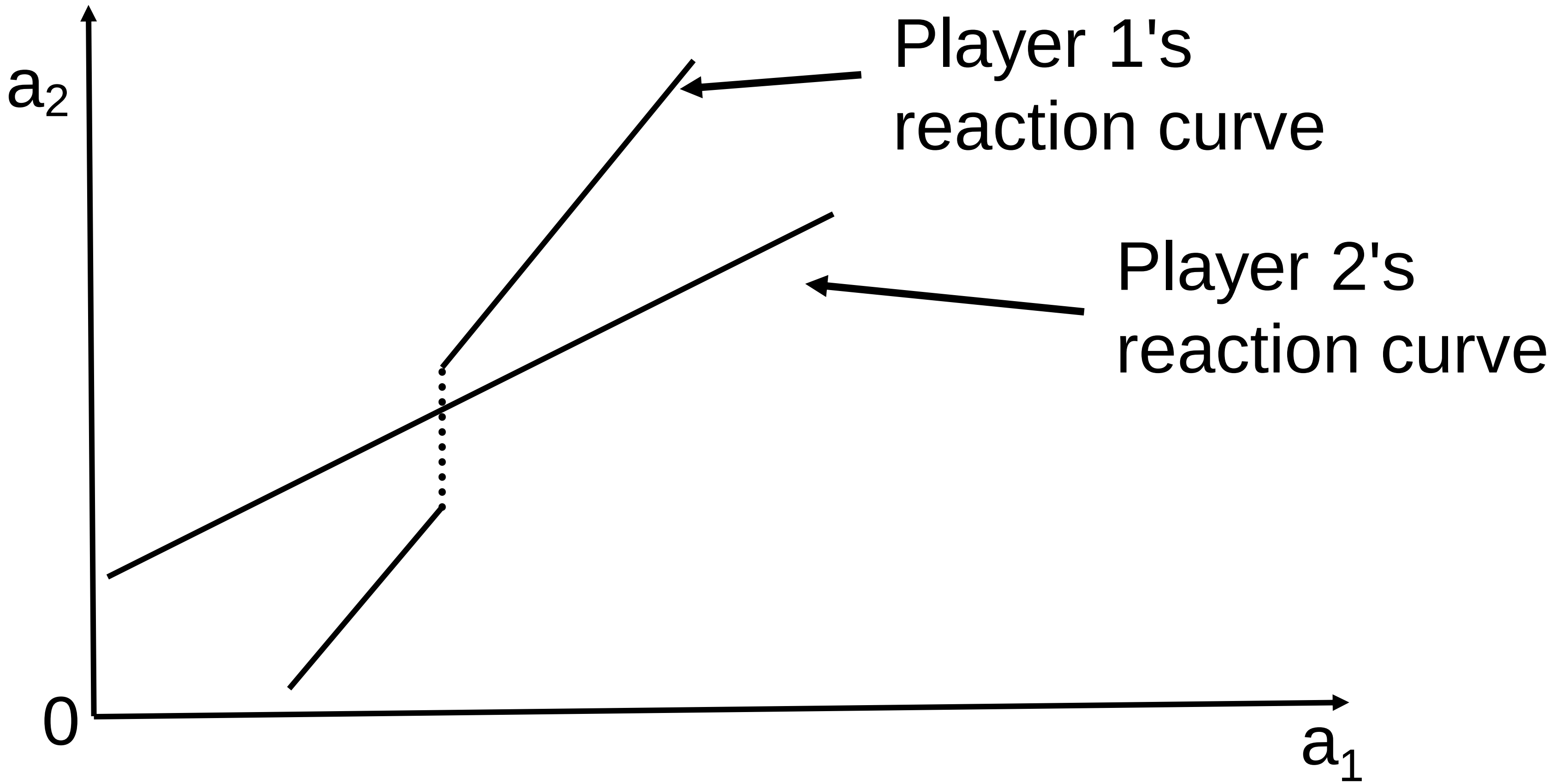
There exists an equilibrium under moderate conditions, either under strategic substitutes or complements.

A key property is continuity of the reaction function.

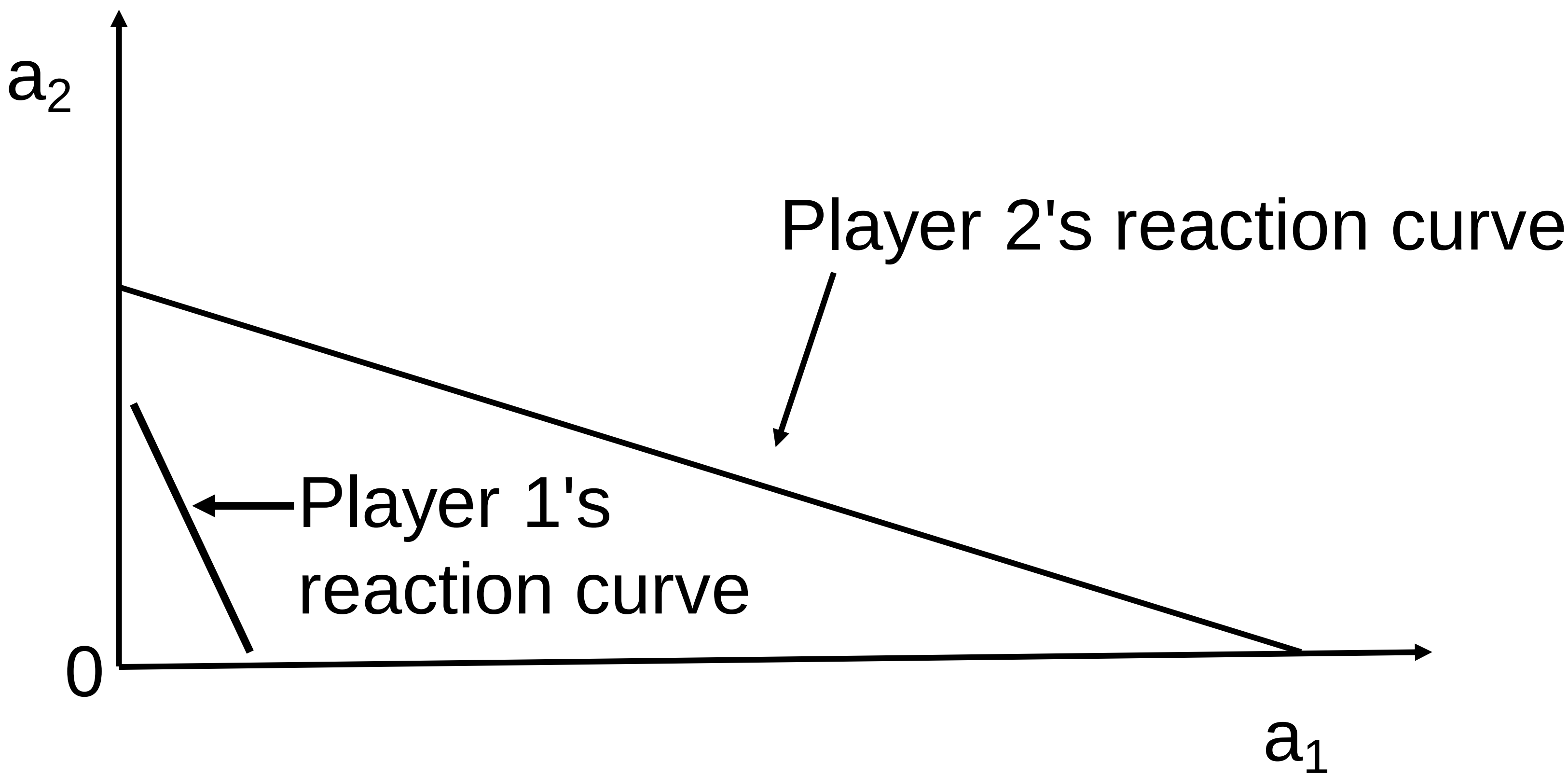
Non-existence of the pure strategy equilibrium



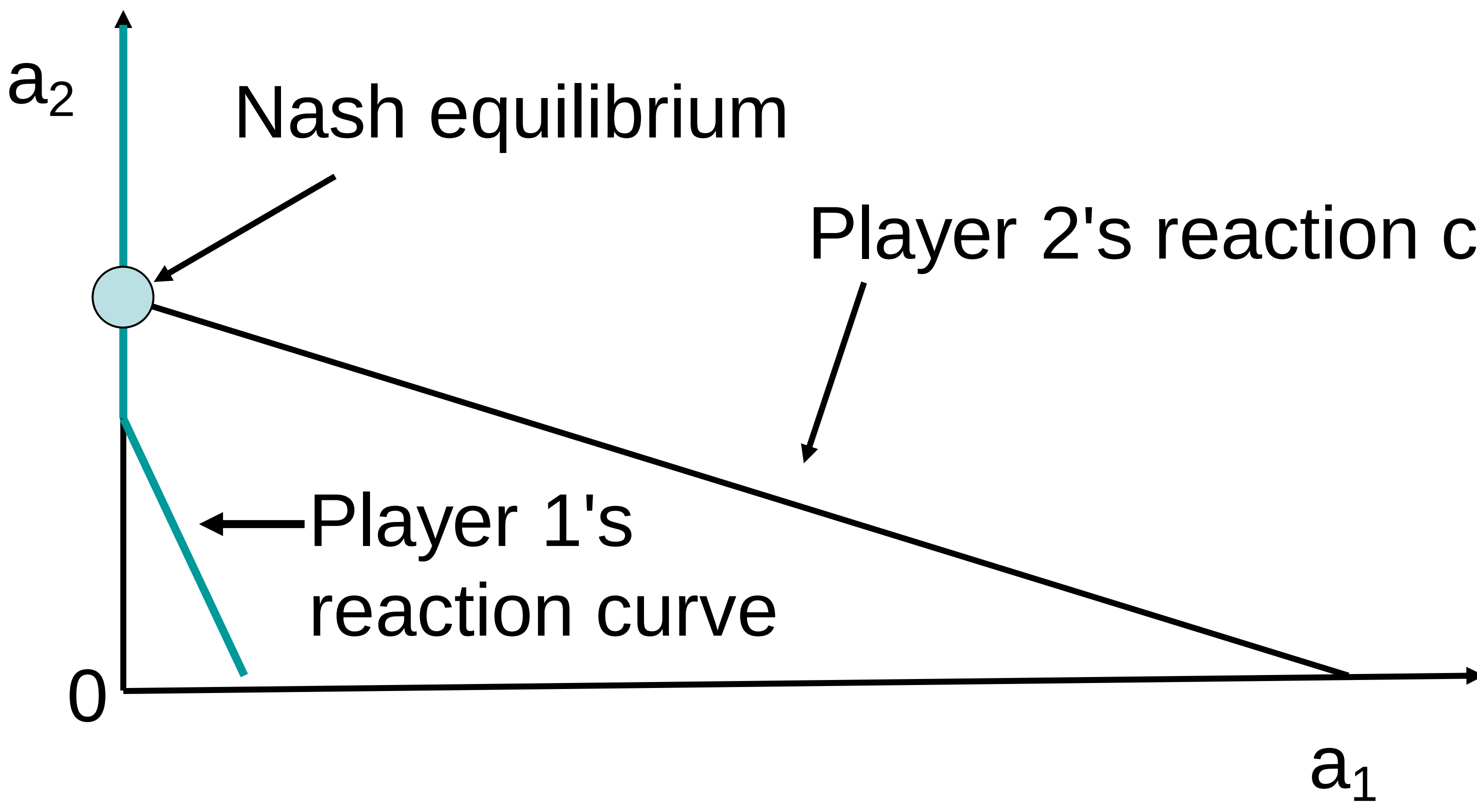
Non-existence of the pure strategy equilibrium



Non-existence of the pure strategy equilibrium?



Non-existence of the pure strategy equilibrium?



Existence of the Equilibrium

$R_1(R_2(a_1^{(1)})) = a_1^{(2)}$, $R_2(R_1(a_2^{(1)})) = a_2^{(2)}$

Substituting $a_i^{(1)} = a_i^C$ into the above system yields $a_i^{(2)} = a_i^C$.

What happens if we substitute $a_i^{(1)} \neq a_i^C$ into the above system?

→The discussions on the stability and the uniqueness of the equilibrium.

Stability

$|R_1(R_2(a_1)) - a_1^C| < |a_1 - a_1^C|$

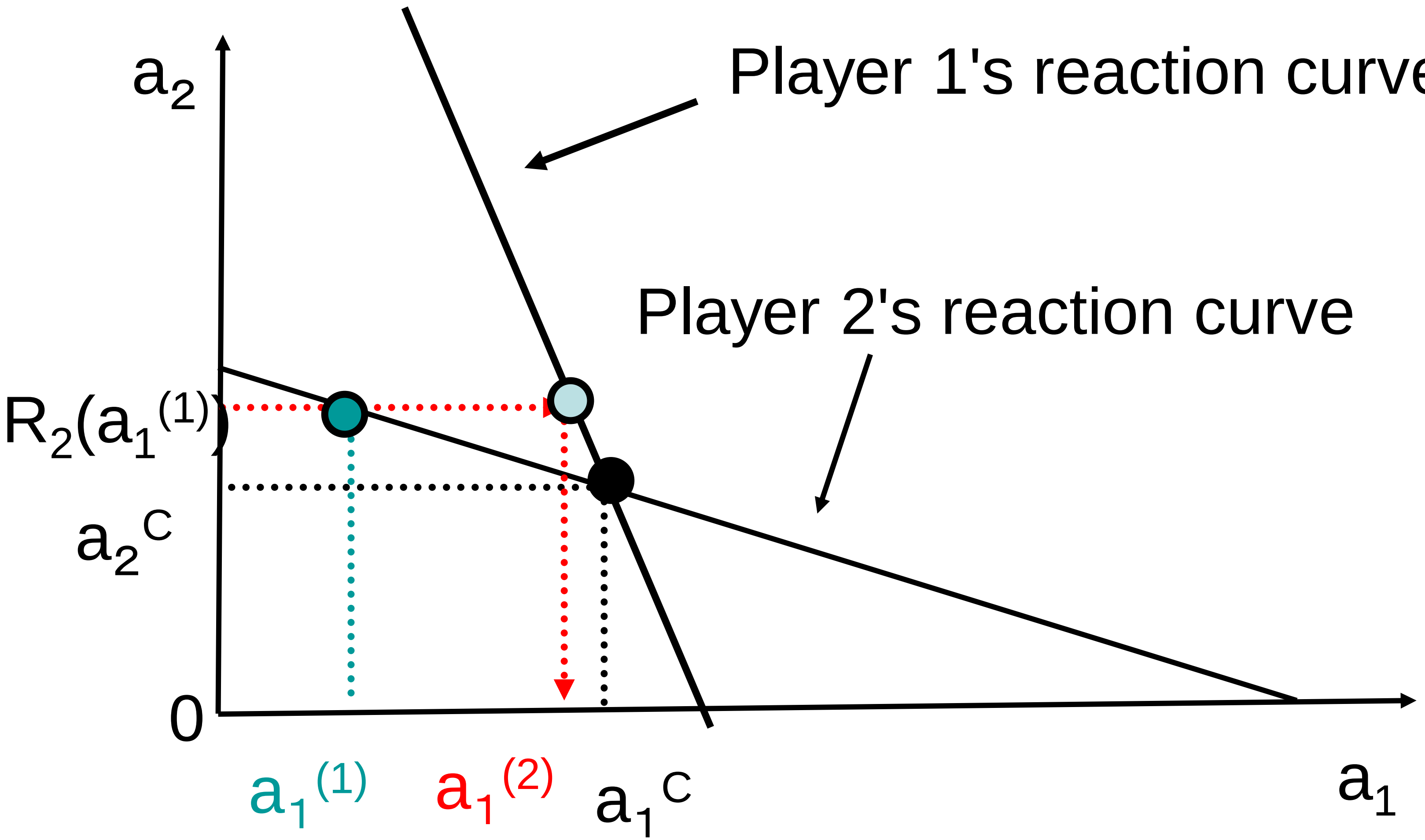
$|R_2(R_1(a_2)) - a_2^C| < |a_2 - a_2^C|$

~Starting from the non-equilibrium point and consider the best reply dynamics

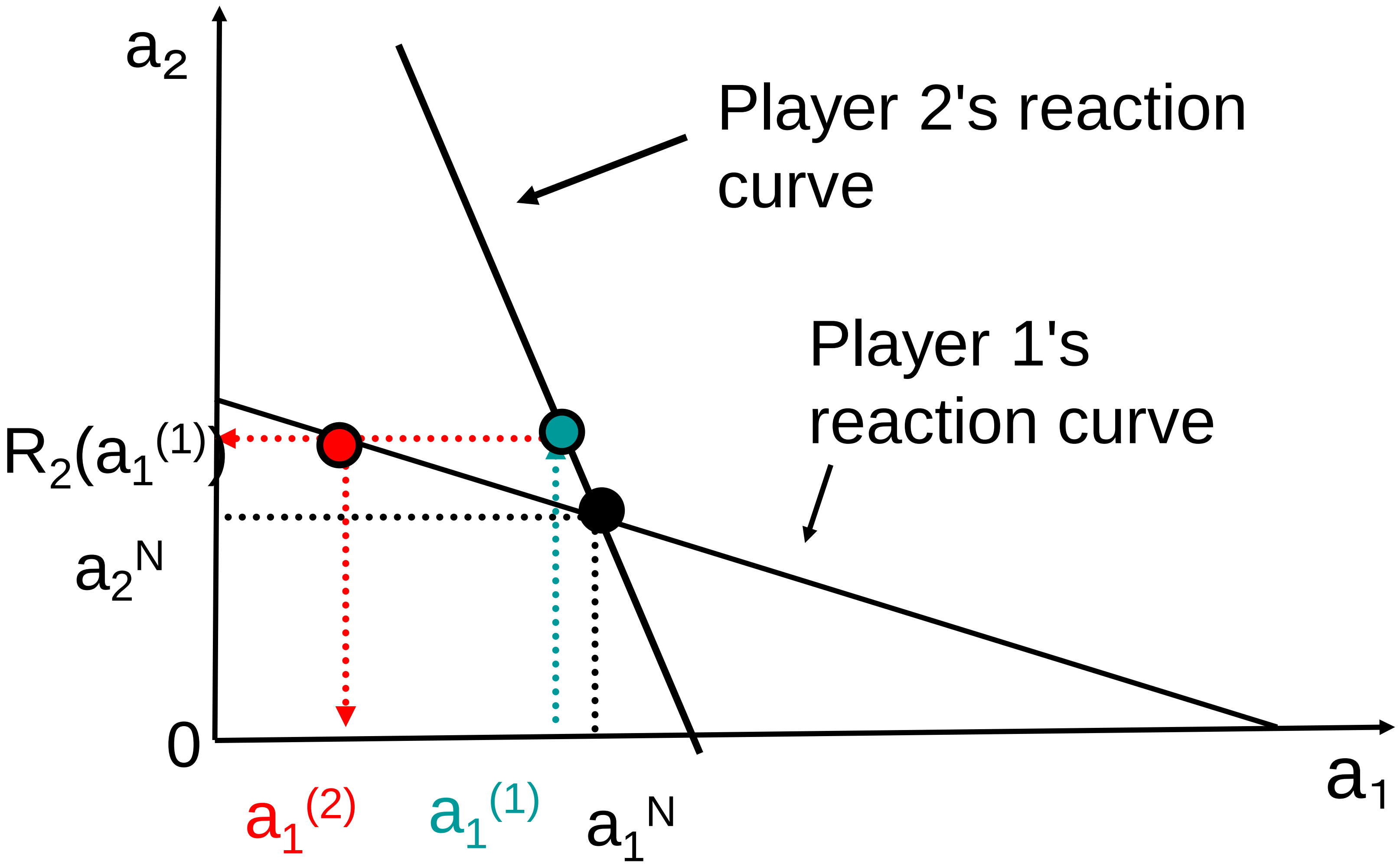
→The distance from the equilibrium point is decreasing

⇒This Nash equilibrium is stable.

Stable Equilibrium



Unstable Equilibrium



A sufficient condition for the stability of the equilibrium

$|R_i'| < 1$~The absolute value of the reaction curve is smaller than one.

~One unit increase of the rival's output changes the optimal output of the firm less than one unit.

e.g., $|R_1'| < 1 \Rightarrow$

Frim2's output is 10→Firm 1's optimal output is 5

Frim2's output is 5→Firm 1's optimal output is 7

e.g., $|R_1'| > 1 \Rightarrow$

Frim2's output is 10→Firm 1's optimal output is 5

Frim2's output is 5→Firm 1's optimal output is 11

Why do we often assume the stability condition?

Cournot Model is a One-Shot Game. It seems nonsense to discuss the dynamic adjustment.

However, most IO papers assume this condition.

Why?

(1) This condition is plausible since it is satisfied under standard settings of cost and demand conditions.

(2) evolution, learning

(3) for comparative statistics

(4) uniqueness

Stability condition is satisfied under moderate conditions

(1) $|R_i'| < 1$ is satisfied under the assumptions of strategic substitutes, non-decreasing marginal cost, and decreasing demand function.

From the first-order condition $P+P'Y_1 - C_1' = 0$, we have

$R_1' = dY_1/dY_2 = -(P'+P''Y_1)/(2P'+P''Y_1 - C_1'')$

strategic substitutes $(P'+P''Y_1<0)$, $C_1'' \geqq 0$, $P' < 0$

$\rightarrow -1 < R_1' < 0$

It is quite natural to assume the stability condition.

Stability condition is satisfied under moderate conditions

(1) $|R_i'| < 1$ is satisfied under the plausible assumptions.

Payoff function of player 1 is $U_1(a_1,a_2)$.

The first-order condition is $\partial U_1/\partial a_1=0$

$R_1' = -\partial^2 U_1/\partial a_1 \partial a_2 / \partial^2 U_1/\partial a_1^2$

$|R_i'| < 1 \rightarrow$ cross effect is weaker than its own effect

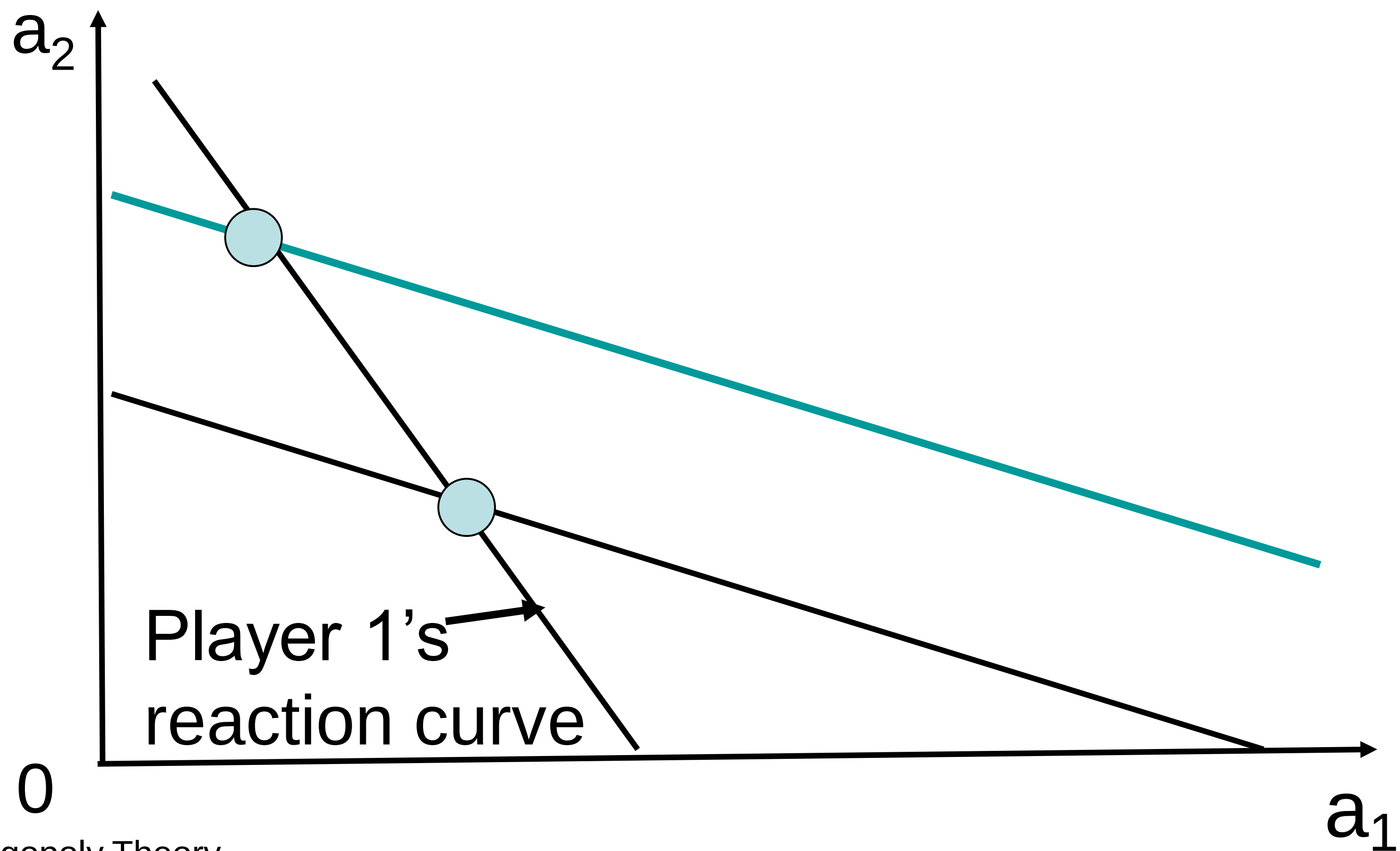
~ naturally satisfied.

The stability condition and comparative statistics

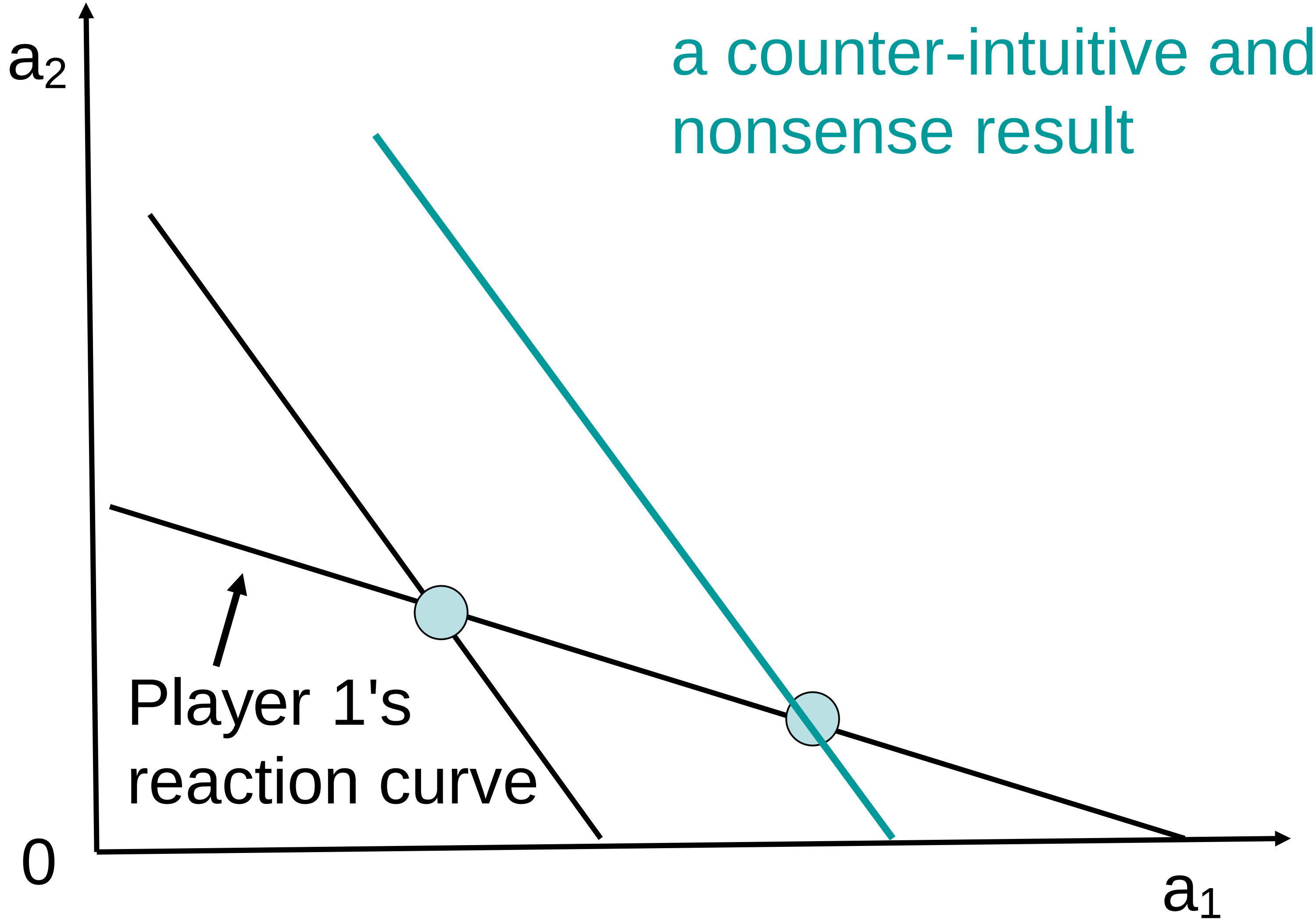
(3)So as to obtain clear results of comparative statistics, we usually assume the stability condition.

- Without the stability condition, the results of comparative statistics often become ambiguous.
- It is nonsense to derive counter-intuitive results under the assumption $|R_i'| > 1$, which is not satisfied under plausible cost and demand conditions.

Player 2's reaction curve and the Nash equilibrium



Unstable Nash Equilibrium



Caution

When you write a theoretical paper and find a counter-intuitive result, you should check whether or not the problem you formulate satisfies the stability condition. If not, the result is not a surprising result and most referees may think that it is obvious.

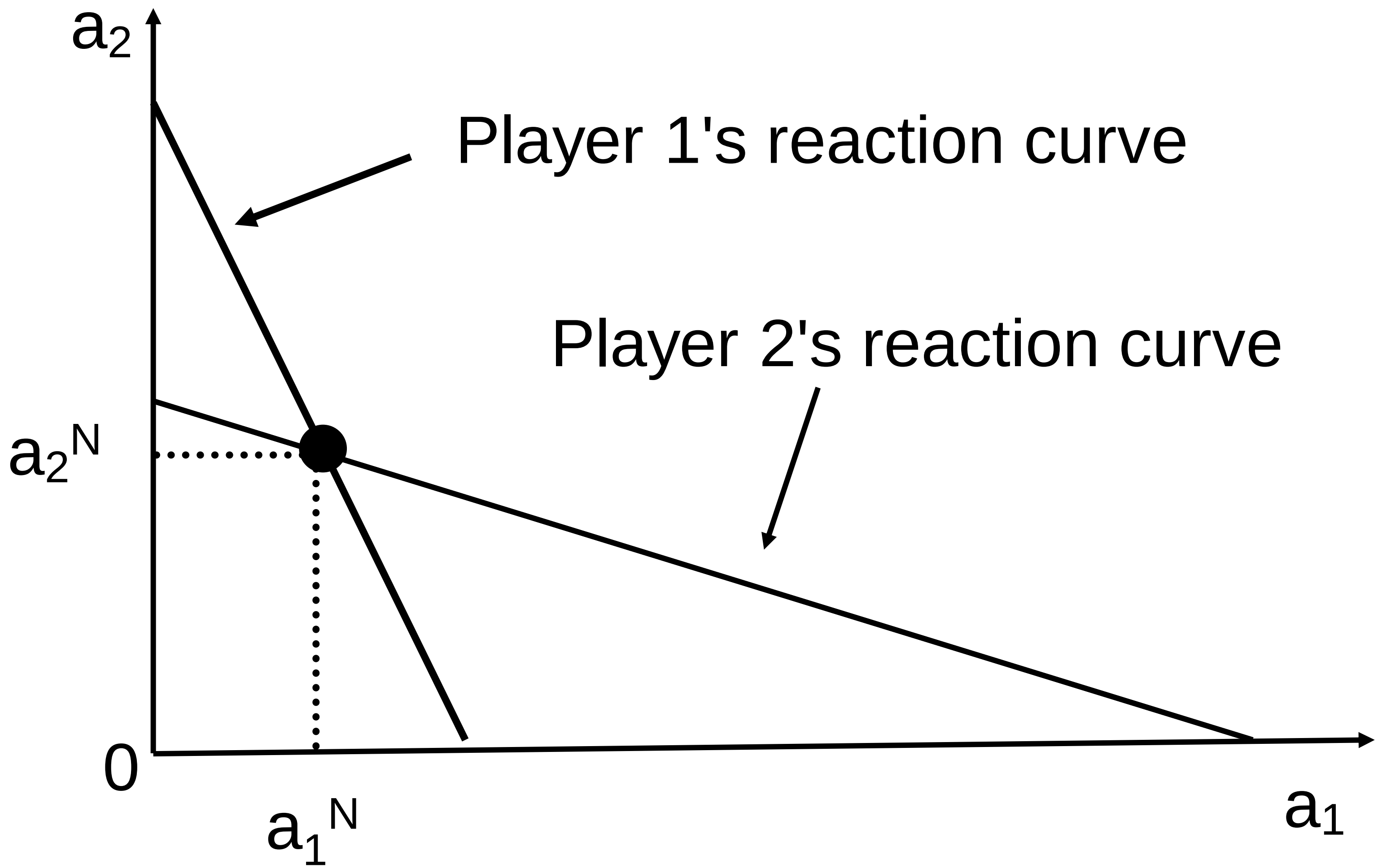
The uniqueness of the equilibrium and the stability condition

(4) If the stability condition is satisfied globally, the equilibrium is unique (only one equilibrium exists).

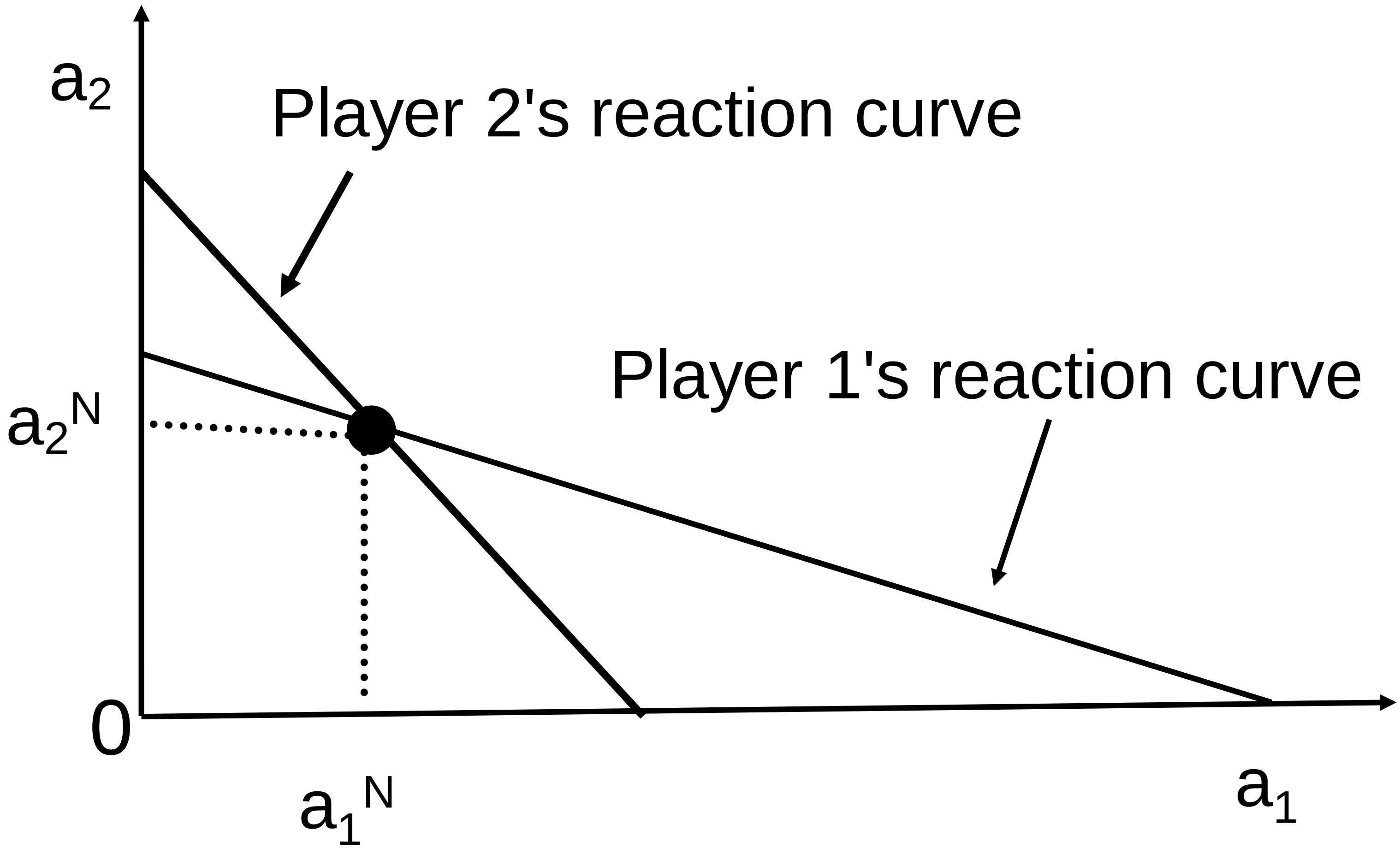
We can show it by using Contraction Mapping Theorem.

(Remark) The stability condition is sufficient, but not necessarily condition for the uniqueness of the equilibrium.

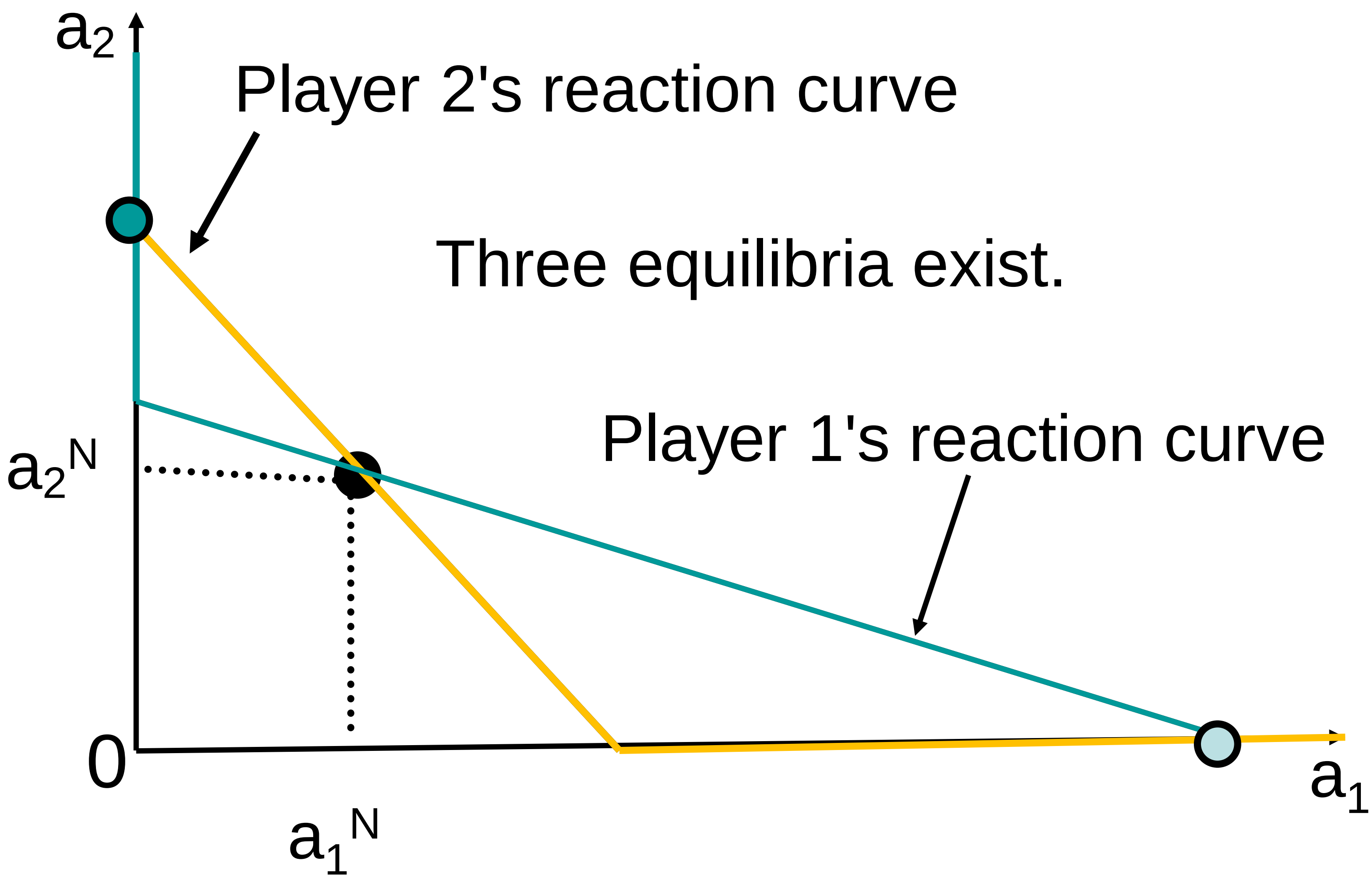
Stable Nash Equilibrium



Does unstable case also yield the unique Nash equilibrium?



Unstable Nash Equilibrium



Stable Nash Equilibrium