Partial Privatization under Multimarket Price Competition

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Abstract

We investigate the effects of multimarket contacts on the privatization policy in mixed duopoly under price competition. There are two markets, one of which is served solely by the state-owned public firm, and the other is served by both public and private firms. Two markets are linked by the production technology of the public firm. With a fairly general model, we first show that privatization is never optimal in the absence of multimarket contacts, i.e., if there is only one monopoly or duopoly market. Then, using a linear-quadratic specification, we show that a positive degree of privatization can be optimal in the presence of multimarket contacts. The optimal degree of privatization is non-monotonically affected by relative sizes of the two markets and the degree of product differentiation.

JEL classification H42, L33

Keywords Multimarket contacts, partial privatization, state-owned public enterprise

1 Introduction

Since the early 1980s, we have observed a worldwide wave of privatization of state-owned public enterprises. Nevertheless, many public and semi-public enterprises (i.e., firms owned by both public and private sectors) are still active in planned and market economies in developed, developing, and transitional countries. While some public enterprises are traditional monopolists in natural monopoly markets, a considerable number of public (including semi-public) enterprises compete with private enterprises in a wide range of industries.[1] Optimal privatization policies in such mixed oligopolies have attracted extensive attention from economics researchers in fields such as industrial

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[1]Examples include United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication, Électricité de France, and Korea Investment Corporation.

organization, public economics, financial economics, international economics, development economics, and political economy.[2]

Specifically, drawing on the result of Matsumura (1998) that full nationalization is never optimal in Cournot mixed duopoly, many studies on mixed oligopolies investigate how economic environments affect the optimal degree of privatization.[3] In this way, most studies of privatization policies in mixed oligopoly use the quantity competition model to characterize the optimal privatization policies. However, there are many applications where it is more plausible to assume that firms compete in prices.[4] In addition, as shown by Matsumura and Ogawa (2012), when public and private enterprises can choose whether to compete in price or quantity, they choose to compete in price in the equilibrium. Therefore, discussing the optimal privatization policies under price competition is also important in both practical and theoretical perspectives. That said, there is a recognized fact in the literature of mixed oligopoly that the privatization policy, as a device to change the public firm's objective toward profit maximization, does not improve the welfare. The reason is that the privatization increases the public firm's price, and also prices of private firms through the strategic interaction, both of which harm welfare.

We argue that this rationale depends on the assumption that firms compete in one single market. If the public firm provides in multiple markets, the result changes. Multimarket contacts have a special reason to be addressed in realistic mixed oligopoly situations. That is a so called universal service obligation on a public sector. For example, in transportation industries, there are several situations where the public firm provides its services not only in urban but also rural regions probably due to universal service reasons, while private firms only provide services in urban areas (e.g., Amtrak in U.S., or Japan Railway Hokkaido). These cases can be seen as multimarket contacts in mixed oligopoly, where markets are geographically separated. Alternatively, firms may provide multiple products. For example government has a share of major electricity providers and recently deregualted electricity and gas markets so that electricity and gas providers can enter the other market.[5] For these types of environments, we show an opposite result to the recognized fact, that is a positive degree of privatization can be optimal. This result sheds light on an important aspect of privatization policy in mixed oligopoly. In the presence of multimarket contacts, privatization of the public firm can stimulate, rather than deter, the competition in urban districts through the improved production efficiency of the public firm.

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[2]The idea of mixed oligopoly dates at least to Merrill and Schneider (1966). Recently, the literature on mixed oligopoly has become richer and more diverse. For examples of mixed oligopolies and recent developments in this field, see Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), Colombo (2016), Chen (2017), Matsumura and Sunada (2013), and the papers cited therein.

[3]For example, see Lin and Matsumura, (2012) for the share of foreign investors who purchases the stock of public firm, Matsumura and Kanda (2005) for free entry, and Sato and Matsumura (2017) shadow cost of public funds

[4]For the anslyses of price competition in mixed oligopolies, see Bárcena-Ruiz (2007), Matsumura (2012), Cremer et al.(1991), and Andersion et al. (1997) for examples.

[5]For other examples of multi-market contacts in mixed oligopoly, see Kawasaki and Naito (2017).

Modeling the multimarket situation, the paper studies a variation of the model of Kawasaki and Naito (2017). It has two markets, one of which is served solely by the state-owned public firm, and the other is served by both public and private firms. Two markets are linked by the production cost of the public firm. As explained later, the optimality of privatization comes from the intra-firm production substitution of the public firm. An increase in the degree of privatization decreases the production of the public firm in the monopoly market. This decreases the marginal cost of production for the duopoly market, which raises the incentive to increase the production. When the degree of product differentiation between public and private firms are small, the latter effect tends to dominate the unilateral effect of privatization to decrease the production in the duopoly market. Under the price competition, this decreases the equilibrium price of the private firm through the strategic interaction and it improves the welfare. This is the mechanism that partial privatization can be optimal in the presence of multimarket contacts.

After showing that partial privatization can be optimal, the study proceeds to characterization of the optimal degree of privatization with respect to the relative sizes of markets and degree of product differentiation. It shows that the optimal degree of privatization exhibits non-monotonicity (inverted U-shape relation) in the relative size of the monopoly market and the degree of product substitution. Both of these parameters have two countervailing effects. On the one hand, an increase in the size of the monopoly market (or decrease in product differentiation) makes an increase in the degree of privatization desirable due to the competition-accelerating effect on the duopoly market. On the other hand, when the size of the monopoly market is too large (or the products in the duopoly market is almost homogeneous), the magnitudes of competition-accelerating effects become nil, since the duopoly market is tiny relatively to the overall economy (or the competition in the duopoly is already enough harsh) in the first place. Relative sizes of these two effects generates the non-monotonicity.

Several recent researches focus on the privatization policy in multimarket mixed oligopoly settings. Bárcena-Ruiz and Garzón (2016) and Dong (forthcoming) consider the privatization policy of a state holding corporation which has plants operating in multiple markets and show that the demand interdependence of markets affects the optimal privatization policies. Haraguchi et al. (forthcoming) consider the privatization of a public enterprise in a mixed market in the presence of neighboring private markets. They show a nonmonotone relationship between the optimal degree of privatization and the number of firms in neighboring markets. The difference between these papers and ours is two-hold; while they consider quantity competition situations ours does price one; and the multimarket interaction is represented by demand interdependence between the markets on theirs, but in ours it is by the cost one within the firm serving both.

2 Model

Consider a model of multimarket mixed price competition. There are a state-owned public firm, firm 0, and a private firm, firm 1. Markets are two, A and B. Market A is solely provided by firm 0, while market B is provided by both firm 0 and firm 1. This means that the public firm serves two markets, in one of which it competes with the private firm.

The representative consumer in market A is characterized by its relative size $\phi \in [0,1]$ and the utility function $U^A(x_0^A) + y^A$, where x_0^A is the amount of the consumption of the products provided by firm 0 and y^A is the consumption of the composite goods. The representative consumer in market B is characterized by its relative size $(1-\phi)$ and the utility function $U^B(x_0^B, x_1^B) + y^B$, where x_0^B and x_1^B are the amount of the consumption of the products provided by firm 0 and firm 1, and y^B is the consumption of the composite goods. Assuming that the representative consumer in each market has enough income and U^A and U^B are concave, its consumption is derived from the first-order conditions

$$\frac{\partial U^A}{\partial x_0^A} = p_0^A,\ \frac{\partial U^B}{\partial x_0^B} = p_0^B,\ \text{and}\ \frac{\partial U^B}{\partial x_1^B} = p_1^B, \tag{1}$$

where p_0^A, p_0^B, and p_1^B are prices of products. We denote $D^A(p_0^A)$, $D_0^B(p_0^B, p_1^B)$, and $D_0^B(p_0^B, p_1^B)$ as the demand functions and $CS^A(p_0^A)$ and $CS^B(p_0^B, p_1^B)$ as the consumer surpluses. Note that, by the envelope theorem, $\partial CS^A/\partial p_0^A = -D_0^A$ and $\partial CS^B/\partial p_i^B = -D_i^B$, $i = 0, 1$ hold.

We assume that the products in market B are substitutes, i.e., $\partial D_i^B/\partial p_j^B < 0$ for $i \neq j$. We also assume that the demands are symmetric, that is, $D_0^B(x,x) = D_1^B(x,x)$. Further, we assume that the demand functions satisfy the following regularity condition

$$\frac{\partial D_i^B}{\partial p_i^B} + \frac{\partial D_i^B}{\partial p_j^B} < 0 \text{ for } i = 0, 1, j \neq i. \tag{2}$$

This condition means that if the prices of both firms simultaneously increase, the demands for both products decrease, which is natural to assume in many applications.

The production technologies of firms are given by cost functions $C_0(q_0^A, q_0^B)$ and $C_1(q_0^B)$. Then, the profit of each firm is given by

$$\Pi_0(p_0^A, p_0^B, p_1^B) = \phi D^A(p_0^A)p_0^A + (1-\phi)D_0^B(p_0^B, p_1^B)p_0^B - C_0(\phi D^A(p_0^A), (1-\phi)D_0^B(p_0^B, p_1^B)), \tag{3}$$

and

$$\Pi_1(p_0^B, p_1^B) = (1-\phi)D_0^B(p_0^B, p_1^B) - C_1((1-\phi)D_0^B(p_0^B, p_1^B)). \tag{4}$$

Social welfare SW is given by

$$SW = \phi CS^A(p_0^A) + (1-\phi)CS^B(p_0^B, p_1^B) + \Pi_0(p_0^A, p_0^B, p_1^B) + \Pi_1(p_0^B, p_1^B). \tag{5}$$

Firm 0 maximizes the weighted average of its own profit and social welfare

$$\Omega = \alpha\Pi_0 + (1-\alpha)SW, \tag{6}$$

where $\alpha \in [0,1]$ is the degree of privatization.

3 Equilibrium

We adopt subgame-perfect equilibrium as the solution and solve the model by backward induction. In the market stage, the first-order conditions for firm 0 are given by[6]

$$\begin{aligned}
\frac{\partial \Omega}{\partial p_0^A} &= \alpha \left(\frac{\partial D_0^A}{\partial p_0^A} \left(p_0^A - \frac{\partial C_0}{\partial q_0^A} \right) + D_0^A \right) + (1-\alpha) \frac{\partial D_0^A}{\partial p_0^A} \left(p_0^A - \frac{\partial C_0}{\partial q_0^A} \right) = 0, \\
\frac{\partial \Omega}{\partial p_0^B} &= \alpha \left(\frac{\partial D_0^B}{\partial p_0^B} \left(p_0^B - \frac{\partial C_0}{\partial q_0^B} \right) + D_0^B \right) + (1-\alpha) \left(\frac{\partial D_0^B}{\partial p_0^B} \left(p_0^B - \frac{\partial C_0}{\partial q_0^B} \right) + \frac{\partial D_1^B}{\partial p_0^B} \left(p_1^B - \frac{\partial C_1}{\partial q_1^B} \right) \right) = 0,
\end{aligned} \tag{7}$$

and the first-order condition for firm 1 is given by

$$\frac{\partial D_1^B}{\partial p_1^B} \left(p_1^B - \frac{\partial C_1}{\partial q_1^B} \right) + D_1^B = 0. \tag{8}$$

We assume that the second-order conditions are satisfied, i.e., the Hessian matrix of Ω is negative definite, and $\partial^2 \Pi_1 / \partial p_1^B < 0$. We also assume that the strategy of firm 1 exhibits strategic complementarity, that is,

$$\frac{\partial D_1^B}{\partial p_0^B} \left(1 - \frac{\partial D_1^B}{\partial p_1^B} \frac{\partial^2 C_1}{\partial {q_1^B}^2} \right) + \frac{\partial^2 D_1^B}{\partial p_1^B \partial p_0^B} \left(p_1^B - \frac{\partial C_1}{\partial q_1^B} \right) > 0. \tag{9}$$

A sufficient condition for the strategic complementarity is that $\partial^2 D_1^B / (\partial p_1^B \partial p_0^B) \geq 0$ and C_1 being weakly convex.

Further, to guarantee the uniqueness and the stability of the equilibrium, we put the following restriction. Let $R_0^A(p_1^B)$ and $R_0^B(p_1^B)$ be the best-response functions of firm 0 and $R_1^B(p_0^B)$ be the best-response function of firm 1. We assume that $|\partial R_0^A / \partial p_1^B| < 1$, $|\partial R_0^B / \partial p_1^B| < 1$, and $|\partial R_1^B / \partial p_0^B| < 1$.

Let $p_0^A(\alpha)$, $p_0^B(\alpha)$, and $p_1^B(\alpha)$ be the equilibrium prices given α.

Next, in the privatization stage the government chooses $\alpha \in [0, 1]$ to maximize SW. Let α^* be the welfare-maximizing value of α. In the case of interior solution, the first-order condition is given by

$$\begin{aligned}
\left. \frac{dSW}{d\alpha} \right|_{\alpha = \alpha^*} =& \frac{dp_0^A}{d\alpha} \phi \frac{\partial D_0^A}{\partial p_0^A} \left(p_0^A - \frac{\partial C_0}{\partial q_0^A} \right) + (1-\phi) \frac{dp_0^B}{d\alpha} \left(\frac{\partial D_0^B}{\partial p_0^B} \left(p_0^B - \frac{\partial C_0}{\partial q_0^B} \right) + \frac{\partial D_1^B}{\partial p_0^B} \left(p_1^B - \frac{\partial C_1}{\partial q_1^B} \right) \right) \\
&+ \frac{dp_1^B}{d\alpha} (1-\phi) \left(\frac{\partial D_0^B}{\partial p_1^B} \left(p_0^B - \frac{\partial C_0}{\partial q_0^B} \right) + \frac{\partial D_1^B}{\partial p_1^B} \left(p_1^B - \frac{\partial C_1}{\partial q_1^B} \right) \right) = 0.
\end{aligned} \tag{10}$$

[6] In the model of price competition with strictly convex costs, firms may have incentives not to serve all the amount demanded. We ignore such possibilities in this model since as shown by Matsumura (2012), if the public firm faces the universal service obligation, there are no such incentives.

We assume that the second-order condition is satisfied. In the case of corner solution, we have either $(dSW/d\alpha)|_{\alpha=0} \leq 0$ or $(dSW/d\alpha)|_{\alpha=1} \geq 0$.

As a conventional wisdom, in the public monopoly or mixed oligopoly with price competition, positive degree of privatization would never be optimal. The following lemma and proposition formalize this conventional wisdom.

Lemma 1. *If $\phi = 0$, $(dp_0^B/d\alpha)|_{\alpha=0} > 0$ and $(dp_1^B/d\alpha)|_{\alpha=0} > 0$.*

Proof. See Appendix. □

Proposition 1. *If $\phi = 0$ or $\phi = 1$, full nationalization is optimal.*

Proof. See Appendix. □

The reason for the above result is that an increase in the degree of privatization from full nationalization increases the public firm's price since it leans to its own profit, which also increases the price of private firms through the strategic interaction. The former change has negligible effect on the welfare since the public firm is a welfare maximizer (envelope theorem), but the latter harms the welfare.

4 Partial Privatization with Multimarket Contact

4.1 General Form

So far we have focused on single market price competition. Now let us turn to multimarket situation and optimal privatization policy under that which is drastically different than before. It is shown that the main driving force of the difference is a cost linkage of the public firm between markets.

With multimarkets, two major forms of the connection in the literature are demand interaction and cost linkage. We restrict our study to the cost one, because a prime example of interest is a so called universal service provided by a single public sector in which geographically separated multimarkets are covered. If two markets are far away like in urban and rural areas, then the demand interaction seem to play less role than the cost[7].

We begin with introducing notations. Given FOCs (7) and (8), define Hessian matrices H_3 and H_2 respectively as follows,

$$H_3 = \begin{pmatrix} \frac{\partial^2 \Omega}{\partial {p_0^A}^2} & \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} & \frac{\partial^2 \Omega}{\partial p_1^B \partial p_0^A} \\ \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} & \frac{\partial^2 \Omega}{\partial {p_0^B}^2} & \frac{\partial^2 \Omega}{\partial p_1^B \partial p_0^B} \\ \frac{\partial^2 \Pi_1}{\partial p_0^A \partial p_1^B} & \frac{\partial^2 \Pi_1}{\partial p_0^B \partial p_1^B} & \frac{\partial^2 \Pi_1}{\partial {p_1^B}^2} \end{pmatrix}, \; H_2 = \begin{pmatrix} \frac{\partial^2 \Omega}{\partial {p_0^A}^2} & \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} \\ \frac{\partial^2 \Omega}{\partial p_0^B \partial p_0^A} & \frac{\partial^2 \Omega}{\partial {p_0^B}^2} \end{pmatrix}. \quad (11)$$

[7] Conjecturally, effects of incorporating the demand interaction into the model depend on whether it is substitute or complement between markets. If two markets are in a substitutional relationship, privatization tends to be suboptimal more than in the present setting, while complement optimal. This is because price decreasing in the duopoly market caused by privatization in the monopoly market get weakened if substitutional and strengthened if complement.

Assume H_3 and H_2 are negative semidefinite. Note that $\partial^2\Omega/\partial p_1^B \partial p_0^A = \partial^2 \Pi_1/\partial p_0^A \partial p_1^B = 0$ because of no demand interaction. The next lemma sheds light on where to focus when checking necessity of (at least) partial privatization.

Lemma 2.

$$sgn\left(\left.\frac{dSW}{d\alpha}\right|_{\alpha=0} \right) = -sgn\left(\left.\frac{dp_1^B}{d\alpha}\right|_{\alpha=0} \right) = -sgn\left(\left.\frac{\partial R_0^B}{\partial \alpha}\right|_{\alpha=0} \right) \tag{12}$$

Proof. See the Appendix. □

Deviating from full nationalization to partial privatization, its effect appears through the private firm's price change in the duopoly market only, since the envelop theorem. And the direction of the price change fully depends on the strategic complementarity, thus eventually on the direction of an adjustment of the public firm's best response function in the market.On this lemma, we can clarify the critical role of the cost linkage.

Consider a small increase in α at $\alpha = 0$. We have

$$sgn\left(\frac{\partial R_0^B}{\partial \alpha} \right) = sgn\left(\frac{\partial p_0^B}{\partial \alpha} + \frac{dp_0^B}{dp_0^A}\frac{\partial p_0^A}{\partial \alpha} \right) \tag{13}$$

The first term and the second factor of the second term are the first order effects of privatizing onto public firm's prices. These are positive. The remaining factor consists of relative market sizes, a price effect on a demand in the monopoly market, cross derivative of the public firm's cost function, and pass-through effect. That is,

$$\frac{dp_0^B}{dp_0^A} = \phi(1-\phi)\frac{dD^A}{dp_0^A}\frac{\partial C_0}{\partial q_0^B \partial q_0^A}\frac{dp_0^B}{d(\partial C_0/\partial q_0^B)}. \tag{14}$$

If the public firm's cost function exhibits economies of scope, the sign of (12) is always positive, which implies that full nationalization is optimal by Lemma 2. We note this result as another lemma.

Lemma 3. *When the cost function of the state-owned firm exhibits (weakly) economies of scope, full nationalization is optimal.*

By contrast, if the public firm's cost function has a property of diseconomies of scope, the sign of (12) can be negative. If this happens, full nationalization is suboptimal by Lemma 2. This leads to the main result of our paper.

Proposition 2. *When a cost function of the state-owned firm exhibits diseconomies of scope, full nationalization is possibly suboptimal.*

It is worth discussing the plausibility of our diseconomies of scope condition. In our model, the state-owned firm exhibits diseconomies of scope if an increase in the production in one markets increases the marginal cost in the other market. This is plausible if there are several fixed inputs commonly used for production in both markets (e.g., managerial resources, factories for firm-specific inputs). In such a case, an increase in the production in one market causes a congestion in fixed inputs, increasing the marginal cost of production in the other market.

One can also argue that if there are diseconomies of scope between markets, the privatized firm may have incentive to divide itself into two. There are several factors deterring such an incentive. First, establishing another company may incur a fixed cost for recruiting managers, purchasing facilities, and so on. Second, if one market is so small that it goes bankrupt after the division, such activity may be forbidden by the government due to a distributional concern.

In the next subsection, we present a parametric example as a proof, where a nonempty set of parameter values in which partial privatization is optimal exists.

4.2 Parametric Form

To show an example of Proposition 2, that is, the case where (partial) privatization is optimal with the public's cost function which exhibits diseconomies of scope, we use a following quadratic utility and cost specification. $U^A(x_0^A) = x_0^A - (x_0^A)^2/2$, $U^B(x_0^B, x_1^B) = x_0^B + x_1^B - ((x_0^B)^2 + 2\gamma x_0^B x_1^B + (x_1^B)^2)/2$ for $\gamma \in (0,1)$, $C_0(q_0^A, q_0^B) = (q_0^A + q_0^B)^2/2$, and $C_1(q_1^B) = (q_1^B)^2/2$.[8] There is a potential issue of corner solution for consumer choice (i.e., $x_0^B = 0$) since firm 0 has cost disadvantage. To avoid this complication, we restrict our attention to the range of parameter values (γ, ϕ) such that the demand for each good is positive in the equilibrium.[9] Then we yield

$$\Pi_1 = p_1^B(1-\phi)\left(\frac{1-\gamma-p_1^B+\gamma * p_0^B}{1-\gamma^2}\right) - \frac{(1-\phi)^2}{2}\left(\frac{1-\gamma-p_1^B+\gamma p_0^B}{1-\gamma^2}\right)^2, \tag{15}$$

$$\begin{aligned}\Pi_o =& p_0^A\phi(1-p_0^A) + p_0^B(1-\phi)\left(\frac{1-\gamma-p_0^B+\gamma p_1^B}{1-\gamma^2}\right)\\ &- \frac{1}{2}\left(\frac{(1-\phi)(1-\gamma-p_0^B+\gamma p_1^B)}{1-\gamma^2} + \phi(1-p)\right)^2,\end{aligned}$$

$$CS^A = \frac{(1-p_0^A)^2}{2}, \tag{16}$$

and

$$CS^B = \frac{{p_0^B}^2 + {p_1^B}^2 + 2(1-p_0^B-p_1^B) - 2\gamma(1-p_1^B)(1-p_0^B)}{2(1-\gamma^2)}. \tag{17}$$

In this specification, we obtain the following lemma.

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[8]In this specification, all the assumptions put in the general model hold.

[9]A sufficient condition is $\gamma < T^{-1}(\phi)$, where $T(\gamma)$ is defined by $T(\gamma) = \frac{\gamma^3-\gamma^2-2\gamma+3}{-\gamma^3+2\gamma+1}$. We can see that $T(\gamma)$ is decreasing.

Lemma 4. *If $\phi \in (0,1)$ then,*

$$\exists \gamma^*(\phi) \quad s.t. \quad \forall \gamma \in (\gamma^*, T^{-1}(\phi)] \quad \left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} < 0 \text{ and } \left.\frac{dp_1^B}{d\alpha}\right|_{\alpha=0} < 0, \tag{18}$$

$$\text{and } \forall \gamma \in (0, \gamma^*] \quad \left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} \geq 0 \text{ and } \left.\frac{dp_1^B}{d\alpha}\right|_{\alpha=0} \geq 0. \tag{19}$$

Proof. See Appendix. □

The mechanism behind Lemma 4 is the following. Departing from full nationalization to partial privatization makes a public enterprise lean to own profit, and that basically pulls up its prices in both markets. In a market solely supplied by the public firm especially, it leads to less production. Because of the less production in the one market, the public firm can have a room in its cost function to cut down the price in the other market. This pass-through effect gets stronger as their products being similar, and beyond some threshold it dominates the first pulling up effect. Finally, the dominating pass-through effect pulls down the competitor's price as well through strategic complement relationship.

Lemma 4 immediately yields our main proposition stating an optimality of the partial privatization in price competition situation, which never be optimal without multimarket contacts.

Proposition 3. *If $\phi \in (0,1)$ then for $\gamma^*(\phi)$ defined in Lemma 4,*

$$\forall \gamma \in (\gamma^*(\phi), T^{-1}(\phi)] \quad \left.\frac{dSW}{d\alpha}\right|_{\alpha=0} > 0 \tag{20}$$

Proof. See Appendix. □

Figure 1 shows the set of parameter values (γ, ϕ) where partial privatization is optimal. From this figure, we can see that partial privatization is optimal for a wide range of parameter values.

4.3 Comparative Statics

Though calculating the optimal degree of privatization in an explicit form is complicated, still there are somethings to imply its non-monotonicity with respect to two parameters, γ and ϕ.

Proposition 4. *The relationship of the optimal degree of privatization and the degree of product differentiation is non-monotonic. Precisely,*

$$\forall \phi \in (0, \lim_{\gamma \to 1} T(\gamma)) \quad \left.\frac{d^2SW}{d\gamma d\alpha}\right|_{\alpha=0,\gamma=\gamma^*} > 0, \lim_{\gamma\to 1}\left(\left.\frac{d^2SW}{d\gamma d\alpha}\right|_{\alpha=0}\right) < 0.$$

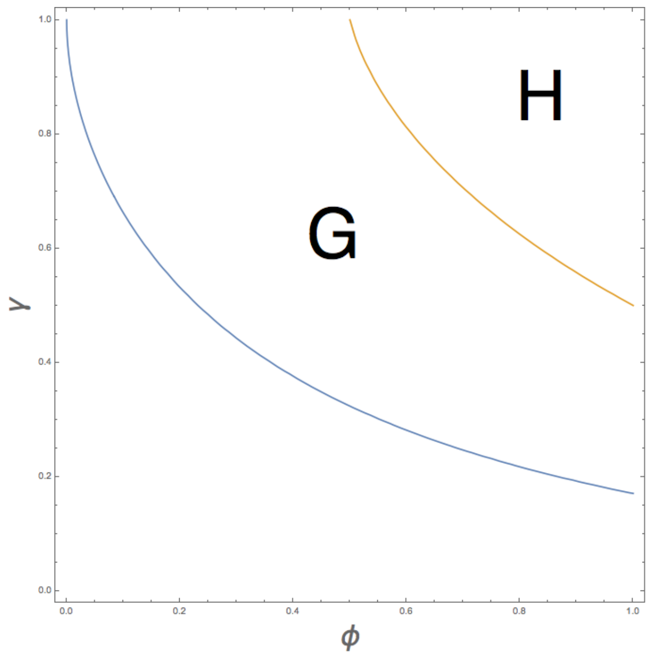

Figure 1: Area G is the area where (partial) privatization is optimal and the equilibrium is at interior. Area H is where the equilibrium quantity of the public firm in the duopoly market is zero with full nationalization.The border between two is $T(\gamma)$.

Proof. See Appendix. □

When γ is near γ^*, competition in the duopoly market is relatively not so harsh that a room for welfare improvement is enough large. Thus, as γ gets higher, or products similar, the pass-through effect from the monopoly market to the duopoly market caused by privatization becomes larger and simply leads bigger welfare improvement. When γ is near 1, however the competition is already cut-throat enough that privatization cannot make significant welfare gain.

The next proposition is on the relative market size ϕ.

Proposition 5. *The relationship of the optimal degree of privatization and the relative sizes of markets is non-monotonic.Precisely,*

$$\forall \gamma \in [\gamma^*(1), T^{-1}(1)) \quad \left.\frac{d^2 SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=\phi^*} > 0, \left.\frac{d^2 SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=1} < 0$$

where $f(\gamma^*(1), 1) = 0$ *and* $f(\gamma, \phi) \gtreqless 0 \Leftrightarrow \phi \gtreqless \phi^*$.

Proof. See Appendix. □

Increasing ϕ has two effects on the welfare result of privatization. One is since the monopoly market swells, the amount of marginal cost reduction induced by privatization heightens. The other is since the duopoly market shrinks, welfare gain there by the reduction do as well. The net result of these two flips as ϕ comes close to 1.

From Proposition 4 and Proposition 5, it is natural to expect that the optimal degree of privatization has inverted U-shape, that is, increasing in γ and ϕ as long as these are below some threshold and decreasing above them. We confirm this conjecture by numerical examples shown in Figure 2.

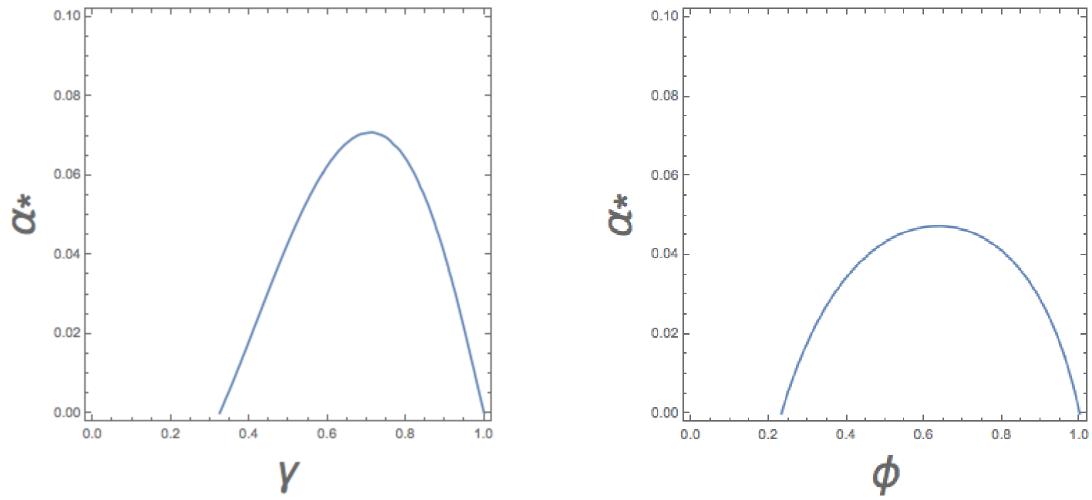

Figure 2: Numerical examples of an optimal degree of privatization(Left:$\phi = 1/2$, Right:$\gamma = 1/2$).

5 Conclusion

In this paper, we have formally shown the well recognized fact that under the price competition, privatization of public enterprises never improves welfare if they serve to a single market. Then we have shown that the partial privatization can be optimal if the public firm faces multimarket contacts. These results shed lights on the importance of taking multimarket interactions into account for the analysis of optimal privatization policies. In addition, these results have policy implications for the privatization policy in sectors such as transportation, in which public enterprises often solely serve to rural areas and compete with private enterprises in urban areas.

Appendix

Equilibrium Prices for Section 4

The equilibrium prices given α under the specification in Section 4 are as follows:

$$p_0^A(\alpha) = \frac{1}{\delta}[\alpha^2\left(\gamma^2-1\right)(\phi-3)+\alpha\left(\gamma\left(\gamma\left(\gamma^2+\gamma(\phi-1)+(\phi-2)\phi-4\right)-\phi+1\right)-2\phi+6\right) +(\gamma-1)\gamma\left(\left(\gamma^2-2\right)\phi+\gamma\right)-2\gamma-\phi+3] \quad (21)$$

$$p_0^B(\alpha) = \frac{1}{\delta}[\alpha^2(\gamma^2-1)(2\gamma+\phi-3)+\alpha(\gamma(\gamma(\gamma^2+2\gamma\phi+\gamma+(\phi-1)\phi-5)-\phi-2)-2\phi+6) +\gamma((\gamma-2)\gamma(\gamma+1)(\phi+1)+2\phi)-\phi+3] \quad (22)$$

$$p_1^B(\alpha) = \frac{\left(\gamma^2+\phi-2\right)\left(\alpha^2\left(\gamma^2-1\right)+\alpha\left(\gamma^2(\phi+1)+\gamma-3\right)+\gamma-2\right)}{\delta}, \quad (23)$$

where

$$\delta \equiv \alpha^2\left(\gamma^2-1\right)(\phi-3)+\alpha\left(\gamma^4+\gamma^2\left(\phi^2-\phi-7\right)-3\phi+9\right)+\gamma^4(\phi+1)-2\gamma^2(\phi+2)-2\phi+6. \quad (24)$$

Proof of Lemma 1

In the case where $\phi=0$, the equilibrium prices given α is characterized by $\partial\Omega/\partial p_0^B=0$ and $\partial\Pi_1/\partial p_1^B=0$. Using the implicit function theorem, we have

$$H\begin{pmatrix}\frac{dp_0^B}{d\alpha}\\ \frac{dp_1^B}{d\alpha}\end{pmatrix} = -\begin{pmatrix}D_0^B-\frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B-\frac{\partial C_1}{\partial q_1^B}\right)\\ 0\end{pmatrix} \quad (25)$$

where

$$H=\begin{pmatrix}\frac{\partial^2\Omega}{\partial {p_0^B}^2} & \frac{\partial^2\Omega}{\partial p_0^B\partial p_1^B}\\ \frac{\partial^2\Pi_1}{\partial p_1^B\partial p_0^B} & \frac{\partial^2\Pi_1}{\partial {p_1^B}^2}\end{pmatrix} \quad (26)$$

At $\alpha = 0$, we have

$$
\begin{aligned}
&D_0^B - \frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right) \\
=&D_0^B + \frac{\partial D_1^B/\partial p_0^B}{\partial D_1^B/\partial p_1^B} D_1^B > 0,
\end{aligned} \tag{27}
$$

which follows from the regularity condition, $p_0^B < p_1^B$, and the symmetry of demand function.

Then, using Cramer's rule, we have

$$
\left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} = \frac{-\left(D_0^B + \frac{\partial D_1^B/\partial p_0^B}{\partial D_1^B/\partial p_1^B} D_1^B\right)\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}}{\det H} > 0
$$

since

$$
\begin{aligned}
\det H &= \frac{\partial^2 \Omega}{\partial {p_0^B}^2}\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2} - \frac{\partial^2 \Omega}{\partial p_0^B \partial p_1^B}\frac{\partial^2 \Pi_1}{\partial p_1^B \partial p_0^B} \\
&= \frac{\partial^2 \Omega}{\partial {p_0^B}^2}\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}\left(1 - \frac{\partial R_0^B}{\partial p_1^B}\frac{\partial R_1^B}{\partial p_0^B}\right) > 0
\end{aligned} \tag{28}
$$

from the stability condition.

Finally, the equation

$$
\frac{\partial^2 \Pi_1}{\partial p_1^B \partial p_0^B}\frac{dp_0^B}{d\alpha} + \frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}\frac{dp_1^B}{d\alpha} = 0
$$

implies that $dp_1^B/d\alpha > 0$. This completes the proof. Q.E.D.

Proof of Proposition 1

For any $\phi \in [0, 1]$, we have

$$
\left.\frac{dSW}{d\alpha}\right|_{\alpha=0} = (1-\phi)\frac{dp_1^B}{d\alpha}\frac{1}{\partial D_0^B/\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right)\left(\frac{\partial D_1^B}{\partial p_1^B}\frac{\partial D_0^B}{\partial p_0^B} - \frac{\partial D_1^B}{\partial p_0^B}\frac{\partial D_0^B}{\partial p_1^B}\right). \tag{29}
$$

When $\phi = 1$, this equals zero, which implies $\alpha^* = 0$. When $\phi = 0$, Lemma 1 implies that $dp_1^B/d\alpha|_{\alpha=0} > 0$. Since the term other than $dp_1^B/d\alpha|_{\alpha=0} > 0$, say dSW/dp_1^B, is negative from the stability condition and the first-order condition of firm 1, we have $(dp_1^B/d\alpha|_{\alpha=0})(dSW/dp_1^B) < 0$. Thus, we have $\alpha^* = 0$ in both cases. Q.E.D.

Proof of Lemma 2

First equality comes from the envelop theorem and $(dSW/dp_1^B)|_{\alpha=0} < 0$. For the second one, by Cramer's rule,

$$\frac{dp_1^B}{d\alpha} = \frac{dp_0^B}{d\alpha}\frac{dR_1^B}{dp_0^B}, \tag{30}$$

and the second factor is positive since strategic complementarity. Furthermore,

$$\frac{dp_0^B}{d\alpha} = \frac{det(H_2)}{det(H_3)}\frac{\partial^2\Pi_1}{\partial p_1^{B2}}\frac{\partial R_0^B}{\partial\alpha} \tag{31}$$

where others than the last factor are negative. Q.E.D.

Proof of Lemma 4

$$\left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} = \frac{(\gamma+1)\left(2\gamma^2+\phi-3\right)f(\gamma,\phi)}{\left(\gamma^4(\phi+1)-2\gamma^2(\phi+2)-2\phi+6\right)^2} \tag{32}$$

$$\left.\frac{dp_1^B}{d\alpha}\right|_{\alpha=0} = \frac{\gamma(\gamma+1)\left(\gamma^2+\phi-2\right)f(\gamma,\phi)}{\left(\gamma^4(\phi+1)-2\gamma^2(\phi+2)-2\phi+6\right)^2} \tag{33}$$

where

$$f(\gamma,\phi) \equiv \gamma^4(\phi+1)^2 - \gamma^3\left(\phi^2+\phi+1\right) - \gamma^2\left(\phi^2+8\phi+4\right) + 7\gamma(\phi+1) + \phi - 3. \tag{34}$$

Since $\gamma,\ \phi \in (0,1)$, the signs of the derivatives are opposite to the one of f. We have $f(0,\phi) < 0,\ f(1,\phi) > 0$ and $f(\gamma,\phi)$ belongs to C^∞ class. Then showing $f(\cdot,\phi)$ has at most one extremum in $\gamma \in (0,1)$ for any $\phi \in (0,1)$ proves Lemma 2.[10]

Let $f_\gamma(\gamma,\phi)$ and $f_{\gamma\gamma}(\gamma,\phi)$ be the first and second partial derivatives with respect to γ. Then,

$$f_\gamma(\gamma,\phi) = 4\gamma^3(1+\phi)^2 - 3\gamma^2(1+\phi+\phi^2) - 2\gamma(4+8\phi+\phi^2) + 7(1+\phi) \tag{35}$$

$$f_{\gamma\gamma}(\gamma,\phi) = 12\gamma^2(1+\phi)^2 - 6\gamma(1+\phi+\phi^2) - 2(4+8\phi+\phi^2)\ . \tag{36}$$

Since $f_{\gamma\gamma}(0,\phi) < 0$ and $f_{\gamma\gamma}$ is convex, f_γ has at most one extremum. In addition to it, $f_\gamma(0,\phi) > 0$ and $f_\gamma(1,\phi) < 0$ together show the solution of $f_\gamma(\cdot,\phi) = 0$ with respect to γ is unique, which implies so is the extremum of $f(\cdot)$. Q.E.D.

[10] Suppose that $f(\cdot,\phi)$ has at most one minimum or maximum. If the extremum is minimum at $\underline{\gamma}$, then $f_\gamma(\gamma,\phi) < 0$ for all $\gamma < \underline{\gamma}$, since otherwise there is some point $\gamma' \in (0,\underline{\gamma})$ such that $f_\gamma(\gamma',\phi) = 0$, contradicting the assumption that $f(\cdot,\phi)$ has at most one extremum. Similarly, $f_\gamma(\gamma,\phi) > 0$ for all $\gamma \in (\underline{\gamma},1)$. These imply that there exists γ^* such that $f(\gamma,\phi) < 0$ for any $\gamma \in [0,\gamma^*)$, $f(\gamma^*,\phi) = 0$, and $f(\gamma,\phi) > 0$ for any $\gamma \in (\gamma^*,1]$. The case where the extremum is maximum is analogous.

Proof of Proposition 3

Suppose that $T^{-1}(\phi) \geq \gamma > \gamma^*$.

$$\frac{dSW}{d\alpha} = \frac{dp_0^A}{d\alpha}\frac{dSW}{dp_0^A} + \frac{dp_0^B}{d\alpha}\frac{dSW}{dp_0^B} + \frac{dp_1^B}{d\alpha}\frac{dSW}{dp_1^B} \tag{37}$$

At $\alpha = 0$, the first and second term of the right hand side are zero from the envelope theorem. The first factor of the third term is negative from Lemma 2, and the second factor is negative as shown in the proof of Proposition 1 in the Appendix. Thus the sign of the whole derivative is positive. Q.E.D.

Proof of Proposition 4

When $\phi \in (0, \lim_{\gamma\to 1} T(\gamma))$, the equilibrium demand for each product is positive for all $\gamma \in (0,1)$, and we can conduct comparative statics by differentiation. First, for the case of $\gamma = \gamma^*$, from the envelope theorem and the definition of γ^* we have,

$$sgn\left(\left.\frac{d^2SW}{d\gamma d\alpha}\right|_{\alpha=0,\gamma=\gamma^*}\right) = -sgn\left(\left.\frac{d^2p_1^B}{d\gamma d\alpha}\right|_{\alpha=0,\gamma=\gamma^*}\right) = sgn\left(f_\gamma(\gamma^*,\phi)\right) \tag{38}$$

where f_γ is defined in the proof of Lemma 2. By the definition of γ^*, $f_\gamma(\gamma^*,\phi) \geq 0$. From the facts shown in the proof of Lemma 2, that $f_\gamma(0,\phi) >, f_\gamma(1,\phi) < 0$, and $f_\gamma = 0$ has a unique solution in terms of γ in $[0,1]$ show $f_\gamma(\gamma^*,\phi) \neq 0$.
Next, when $\gamma = 1$, note full-nationalization is optimal, since,

$$\left.\frac{dSW}{d\alpha}\right|_{\gamma=1} = \frac{-4\alpha(3-\phi)\phi}{(3+\alpha(3-\phi))^3}.$$

Then,

$$\lim_{\gamma\to 1}\left(\left.\frac{d^2SW}{d\gamma d\alpha}\right|_{\alpha=0}\right) = \frac{-4\phi}{27} < 0.$$

Q.E.D.

Proof of Proposition 5

When $\gamma \in (0, T^{-1}(1))$, the equilibrium demand for each product is positive for all $\phi \in (0,1)$, and we can conduct comparative statics by differentiation. For the case of $\phi = \phi^*$, as in the γ case, we have,

$$sgn\left(\left.\frac{d^2SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=\phi^*}\right) = sgn\left(f_\phi(\gamma,\phi^*)\right) \tag{39}$$

where f_ϕ is a partial derivative with respect to ϕ. Since $f(\gamma, 0) < 0$ and $\forall \gamma > \gamma^*(1)$, $f(\gamma, 1) > 0$ and $f(\gamma, \phi)$ is strictly increasing in ϕ, $f_\phi(\gamma, \phi^*) > 0$. For the other case, we have

$$\left.\frac{d^2 SW}{d\phi d\alpha}\right|_{\alpha=0,\phi=1} = \frac{-\gamma(2-\gamma)f(\gamma,1)}{8(1-\gamma)(2-\gamma^2)^3}.$$

This is negative $\forall \gamma \in [\gamma^*(1), 1)$. Q.E.D.

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