Corporate Social Responsibility or Payoff Asymmetry?  
A Study of Endogenous Timing Game

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Abstract

We revisit an endogenous timing game by introducing corporate social responsibility into the firms’ payoffs. Pal (1998, Economics Letters) investigates an endogenous timing game in mixed oligopoly wherein one welfare-maximizing public firm competes against profit-maximizing private firms. He shows that the outcome is completely different from that of private oligopoly. We find that this change in payoff does not matter as long as the payoffs are symmetric, which is in contrast to the results of Pal. Our result indicates that not welfare-concerning objectives but asymmetry in objectives yields specific results in the literature on mixed oligopoly.

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Key words: observable delay, mixed oligopoly, partial privatization, welfare-concern objective

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1 Introduction

We introduce an aspect of corporate social responsibility into the firms’ payoffs (wherein firms care about both profits and social welfare). We consider a model wherein firms maximize the weighted sum of total social surplus and profits. This approach is widely adopted in the literature on mixed oligopolies wherein a partially privatized firm cares about both its own profits and social welfare (See Matsumura (1998) and Ghosh and Mitra (2010a)). Ghosh and Mitra (2010b) generalize this partial privatization approach and consider the situation where all firms care about both social welfare and their own profits (social responsibility approach). In this paper, we follow their approach. They provide a convincing rationale for this social responsibility approach. We investigate the observable delay game formulated by Hamilton and Slutsky (1990). We find that introducing social responsibility does not affect the timing choice as long as the payoffs are symmetric.

The Cournot, Bertrand, and Stackelberg models have occupied important positions in the literature on oligopoly theory. The Cournot and Bertrand models involve simultaneous moves, while the Stackelberg model involves sequential moves. Each of these models produces a different equilibrium outcome, that is, the equilibrium outcome crucially depends on the timing of each firm’s choice. In addition, firms usually choose their actions and the timing of their actions themselves. Thus, it is important to endogenize the timing. Many papers have already discussed this problem. We take a close look at the observable delay game, which is a popular model in the literature on the endogenous timing game.\(^1\)

It is known that under standard conditions, quantity competition yields simultaneous-move choice (Cournot) and price competition yields sequential-move choice (Stackelberg) when firms

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\(^1\)See Matsumura and Ogawa (2009) and works cited by them.
maximize their own profits. Pal (1998) revisits this problem by introducing a welfare-maximizing payoff into one firm (public firm) and keeping the other firm (private firm) as a profit-maximizer (i.e., a mixed market). He shows that in a mixed market, sequential-move choice (Stackelberg) appears in equilibrium, in contrast in a private market. Bárcena-Ruiz (2007) investigates price competition and shows that simultaneous-move choice (Bertrand) appears in equilibrium. In short, in mixed markets, results opposite to those in private markets are derived. Their results suggest that introducing a welfare-maximizing objective matters in timing choice.\textsuperscript{2}

However, they assume that only one firm maximizes welfare. Thus, it is unclear whether introducing a welfare-maximizing objective or asymmetry among the firms’ objectives matters. We investigate a symmetric non-profit-maximizing objective among firms. We assume that firms maximize the weighted sum of their own profits and total social surplus. We find that regardless of the weight of total social surplus, quantity competition yields simultaneous-move choice (Cournot) and price competition yields sequential-move choice (Stackelberg). Our results indicate that it is not a welfare-maximizing payoff but payoff asymmetry among firms that matters in the endogenous timing game.

\section{Model}

We adopt a standard differentiated duopoly with linear demand (Dixit, 1979). Firm 0 and firm 1 produce differentiated commodities for which the inverse demand function is given by

\[ p_i = \alpha - \beta q_i - \beta \delta q_j \quad (i = 1, 2, \ i \neq j), \]

where \( p_i \) and \( q_i \) are firm \( i \)’s price and quantity, respectively; \( \alpha, \beta \) are positive constants; and \( \delta \in (0, 1) \). The marginal production cost \( m \) is constant. We assume that \( \alpha > m \).

\textsuperscript{2}In the literature on mixed oligopoly, the discussion on endogenous timing is important and so rich and diverse. See Bárcena-Ruiz and Garzón (2010), Bárcena-Ruiz and Sedano (2011), Capuano and De Feo (2010), Lu (2006), Matsumura (2003a,b), Matsumura and Ogawa (2010), Tomaru and Kiyono (2010), and Tomaru and Saito (2010).
Firm $i$’s payoff is $V_i = \theta SW + (1 - \theta)\pi_i$, where $\theta \in [0, 1)$, $SW$ is the total social surplus (sum of the firms’ profits and consumer surplus) and $\pi_i$ is firm $i$’s profit. $\theta$ indicates the weight of social responsibility into the payoff of each firm. $\pi_i (i = 1, 2)$ and $SW$ are given by

$$
\pi_i = (p_i - m)q_i,
$$
$$
SW = \pi_1 + \pi_2 + \left[ \alpha(q_1 + q_2) - \frac{\beta(q_1^2 + 2\delta q_1q_2 + q_2^2)}{2} - p_1q_1 - p_2q_2 \right].
$$

We consider two models involving quantity and price competition. We consider the observable delay game of Hamilton and Slutsky (1990), where firms first choose the timing of their actions. There are two possible time periods for action choice (price or quantity choice) and each firm chooses its action in only one of the two periods. In the first stage, firm $i (i = 1, 2)$ simultaneously chooses whether it likes to be the leader ($t_i = L$) or the follower ($t_i = F$). If the two firms’ choices are consistent, i.e., one chooses to be the leader and the other does to be the follower, they get the equilibrium payoffs of a sequential-move game under the agreed timing. Otherwise, i.e., if both want to be the leader or the follower, they receive the equilibrium payoffs of a simultaneous-move game. See Table 1 for the payoff matrix of the observable delay game in our environment, where $V_F (V_L)$ denotes firm $i$’s equilibrium payoff in the sequential-move game when it is the follower (leader), and $V_N$ denotes each firm’s equilibrium payoff in the simultaneous-move game (Nash).

$$
\begin{array}{c|cc}
1 \backslash 2 & L & F \\
L & (V_N, V_N) & (V_L, V_F) \\
F & (V_F, V_L) & (V_N, V_N) \\
\end{array}
$$

Table 1: Payoff matrix of the observable delay game.

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$^3$The quasi-linear utility function of a representative consumer, $U(q_1, q_2) = \alpha(q_1 + q_2) - \beta(q_1^2 + 2\delta q_1q_2 + q_2^2)/2 - (p_1q_1 + p_2q_2)$, provides the demand and social surplus (welfare) functions adopted in this paper.
3 Quantity competition

Let superscript “C” denote quantity competition.

3.1 Cournot competition

First, we consider the simultaneous-move game (Cournot competition). Each firm $i$ maximizes its payoff $V_i = \theta SW + (1 - \theta)\pi_i$ with respect to $q_i$. The first-order condition is

$$\frac{\partial V_i^C}{\partial q_i} = \alpha - m - (2 - \theta)\beta q_i - \beta\delta q_j = 0.$$  

(1)

The second-order condition is satisfied. From (1), we obtain the following reaction function.

$$R_i^C(q_j) = \frac{\alpha - m - \beta\delta q_j}{\beta(2 - \theta)}.$$  

(2)

Let subscript “N” denote the equilibrium outcome in this Cournot game. We have

$$q_N^C = \frac{\alpha - m}{\beta(\delta - \theta + 2)},$$

$$V_N^C = \frac{(\alpha - m)^2(\theta + \delta\theta - \theta^2 + 1)}{\beta(\delta - \theta + 2)^2}.$$  

3.2 Stackelberg competition

Second, we consider the sequential-move game. Without loss of generality, we assume that firm 1 chooses its output and then firm 2 chooses its output. Firm 2 chooses $q_2 = R_2^C(q_1)$ and firm 1 maximizes its payoff, $V_1^C(q_1, R_2^C(q_1))$. Let subscripts “L” and “F” denote the equilibrium outcome
of the leader and the follower in the Stackelberg game. We have

\[
q_L^C = \frac{(\alpha - m)(\theta^2 - 4\theta - 2\delta + \delta \theta^2 + 4)}{\beta(6\theta^2 - 4\delta^2 - 12\theta - \theta^3 + 3\delta^2 \theta + 8)},
\]

\[
q_F^C = \frac{(\alpha - m)(\delta \theta - 4\theta - 2\delta - \delta^2 + \theta^2 + \delta^2 \theta + 4)}{\beta(6\theta^2 - 4\delta^2 - 12\theta - \theta^3 + 3\delta^2 \theta + 8)},
\]

\[
V_L^C = \frac{(\alpha - m)^2(2\theta - 4\delta + \delta^2 - 6\theta^2 + 2\theta^3 + 2\delta \theta^2 - 2\delta^2 \theta + \delta^2 \theta^2 + 4)}{2\beta(6\theta^2 - 4\delta^2 - 12\theta - \theta^3 + 3\delta^2 \theta + 8)},
\]

\[
V_F^C = \frac{(\alpha - m)^2 z_1}{2\beta(6\theta^2 - 4\delta^2 - 12\theta - \theta^3 + 3\delta^2 \theta + 8)},
\]

where \( z_1 = -32\delta - 32\theta + 48\delta \theta - 8\delta^2 + 8\delta^3 + 2\delta - 48\theta^2 + 96\theta^3 - 62\theta^4 + 18\theta^5 - 2\theta^6 - 16\delta \theta^2 - 8\delta^2 \theta + 4\delta \theta^3 - 14\delta^4 \theta + 10\delta^5 \theta - 2\delta \theta^2 + 26\delta^2 \theta^2 - 15\delta^3 \theta^2 - 2\delta^3 \theta^2 + 4\delta^2 \theta^4 + 10\delta^3 \theta^3 + 4\delta^3 \theta^2 - 3\delta^2 \theta^5 + 6\delta^3 \theta^4 - 4\delta \theta^3 + 32 \).

### 3.3 Endogenous timing game

We now discuss the endogenous timing game.

**Proposition 1** (i) If \( \delta = \theta \), then \( V_L^C = V_N^C = V_F^C \). Thus, any \( (t_1, t_2) \) constitutes an equilibrium (both Cournot and Stackelberg outcomes can appear in equilibrium). (ii) If \( \delta \neq \theta \), only \( t_1 = t_2 = L \) constitutes an equilibrium (only Cournot outcome arises in equilibrium).

**Proof** We have

\[
V_L^C - V_N^C = \frac{(\alpha - m)^2(\theta - \delta)^2(1 - \theta)^2 \delta^2}{2\beta(\theta - \delta - 2)^2 z_2} \geq 0,
\]

\[
V_N^C - V_F^C = \frac{(\alpha - m)^2(\theta - \delta)^2 z_3(\theta - 1)^2(2 - \theta)\delta}{2\beta(\theta - \delta + 2)^2} \geq 0,
\]

where \( z_2 = 6\theta^2 - 4\delta^2 - 12\theta - \theta^3 + 3\delta^2 \theta + 8 \) and \( z_3 = -24\theta + 2\delta \theta - 8\delta^2 - \delta^3 + 12\theta^2 - 2\theta^3 - \delta^2 \theta^2 + 6\delta^3 \theta + 16 \).

The equalities in (3) and (4) hold if and only if \( \delta = \theta \). Thus, any \( t_i \) is optimal if \( \delta = \theta \), and otherwise, only \( t_1, t_2 = (L, L) \) constitutes an equilibrium. Note that \( z_2 > 0 \) for \( \delta \in (0, 1) \) and \( \theta \in [0, 1) \) because \( z_2 \) is decreasing in both \( \theta \) and \( \delta \) for \( (\delta, \theta) \in [0, 1]^2 \), and \( z_2 = 0 \) when \( \delta = \theta = 1 \). \( z_3 > 0 \) also holds because \( z_3 \) is decreasing in \( \theta \in [0, 1) \), and \( z_3 = 2 + \delta - 2\delta^2 - \delta^3 > 0 \) for \( \delta \in (0, 1) \) when \( \theta = 1 \). Q.E.D.
Except for the measure-zero event case (the case in which $\delta = \theta$), the unique equilibrium outcome is Cournot. This result indicates that introducing a positive $\theta$ does not matter for generic cases as long as the payoff is symmetric, which is in contrast with Pal’s (1998) result.\footnote{Although Pal’s model (homogeneous product and cost difference between firms) is different from ours, we can show that Pal’s result holds in our setting if $\theta_2 = 0$ and $\theta_1$ is close to 1.}

### 4 Price competition

Let superscript “B” denote price competition.

#### 4.1 Bertrand competition

First, we consider the simultaneous-move game (Bertrand competition). Each firm $i$ maximizes its payoff $V^B_i = \theta SW + (1 - \theta)\pi_i$ with respect to $p_i$. The first-order condition is

$$\frac{\partial V^B_i}{\partial p_i} = \frac{m - 2p_i + \alpha + p_j \delta - \delta \alpha + p_i \theta - \alpha \theta - m \delta \theta + \delta \alpha \theta}{\beta (1 + \delta)(1 - \delta)} = 0. \tag{5}$$

The second-order condition is satisfied. From (5), we obtain the following reaction function.

$$R^B_i(p_j) = \frac{m + \alpha + p_j \delta - \delta \alpha - \alpha \theta - m \delta \theta + \delta \alpha \theta}{2 - \theta}. \tag{6}$$

Let subscript “N” denote the equilibrium outcome in this Bertrand game. We have

$$p^B_N = \frac{m + \alpha - \delta \alpha - \alpha \theta - m \delta \theta + \delta \alpha \theta}{2 - \theta},$$

$$V^B_N = \frac{(\alpha - m)^2 (1 - \delta + \theta - \theta^2)(1 - \delta \theta)}{\beta (2 - \delta - \theta)^2 (\delta + 1)}. \tag{2}$$

#### 4.2 Stackelberg competition

Second, we consider the sequential-move game. Without loss of generality, we assume that firm 1 chooses its price and then firm 2 chooses its price. Firm 2 chooses $p_2 = R^B_2(p_1)$ and firm
Proposition 2. Let \( p_1^B(p_1), p_2^B(p_1) \). Let subscripts “L” and “F” denote the equilibrium outcome of the leader and the follower in this Stackelberg game. We have

\[
\begin{align*}
p_L^B &= \frac{z_4}{6\theta^2 - 4\delta^2 - 2\theta - 3\delta^2\theta + 8}, \\
p_F^B &= \frac{z_5}{6\theta^2 - 4\delta^2 - 2\theta - 3\delta^2\theta + 8}, \\
V_L^B &= \frac{(\alpha - m)^2 z_6}{2\beta(12\theta + 4\delta^2 - 6\theta^2 + \theta^3 - 3\delta^2\theta + 8)(\delta + 1)}, \\
V_F^B &= \frac{(\alpha - m)^2 z_7}{2\beta(6\theta^2 - 4\delta^2 - 12\theta - 3\delta^2\theta + 8)(\delta + 1)},
\end{align*}
\]

where \( z_4 = 4m + 4\alpha + 2m\delta - 4m\theta - 2\delta\alpha - 8\alpha\theta - 4m\delta\theta + 4\delta\alpha\theta - 2m\delta^2 + m\theta^2 - 2\delta^2\alpha + 5\alpha\theta^2 - \alpha\theta^3 + 3m\delta\theta^2 - m\delta^2\theta - m\delta^3\theta + 3\delta\alpha\theta^2 + 4\delta^2\alpha\theta + \delta\alpha\theta^3 + 2m\delta^2\theta^2 - 2\delta^2\alpha\theta^2, \)

\( z_5 = 4m + 4\alpha + 2m\delta - 4m\theta - 2\delta\alpha - 8\alpha\theta - 5m\delta\theta + 5\delta\alpha\theta - m\delta^2 - m\delta^3 + m\theta^2 - 3\delta^2\alpha + \delta^3\alpha + 5m\theta^2 - 3m\delta^2\theta - m\delta^3\theta + m\delta^3\theta - 4\delta\alpha\theta^2 + 4\delta^2\alpha\theta + \delta\alpha\theta^3 - \delta^3\alpha + m\delta^2\theta^2 - \delta^2\alpha\theta^2, \)

\( z_6 = 3\delta^2 - 2\delta\theta - 2\theta + \delta^3 + 6\theta^2 - 2\theta^3 + 6\delta^2 - 2\delta^2\theta - 6\delta^3 - 2\delta^3\theta + 2\delta^4 + 5\delta^2\theta^2 - 4\delta^2\theta^3 + 3\delta^3\theta^2 - 4, \)

\( z_7 = 16\delta\theta - 32\theta - 40\delta^2 + 10\delta^4 - 2\delta^5 - 48\theta^2 + 96\theta^3 - 62\theta^4 + 18\delta^5 - 2\theta^6 - 64\delta\theta^2 + 40\delta^2\theta + 108\delta^3\theta - 100\delta^3 + 3\delta^4\theta + 54\delta^5\theta + 5\delta^5\theta - 16\delta^6 - 2\delta^7 - 6\delta^2\theta^2 + 37\delta^2\theta^3 - 4\delta^3\theta^2 - 62\delta^2\theta^4 + 11\delta^3\theta^3 - 18\delta^4\theta^2 + 33\delta^2\theta^5 - 10\delta^3\theta^4 + 5\delta^4\theta^3 - 4\delta^5\theta^2 - 6\delta^2\theta^6 + 3\delta^3\theta^5 + 2\delta^4\theta^4 + 4\delta^5\theta^3 + 32. \)

4.3 Endogenous timing game

We now discuss the endogenous timing game.

**Proposition 2** (i) If \( \delta = \theta \), then \( V_L^B = V_N^B = V_F^B \). Thus, any \((t_1, t_2)\) constitutes an equilibrium (both Bertrand and Stackelberg outcomes can appear in equilibrium). (ii) If \( \delta \neq \theta \), then \( t_1 \neq t_2 \) (only Stackelberg outcome arises in equilibrium).

**Proof** We have

\[
\begin{align*}
V_L^B - V_N^B &= \frac{(\alpha - m)^2(\delta - \theta)^2(1 - \delta)(1 - \theta)^2\delta^2}{2\beta(\delta + \theta - 2)^2 z_2(\delta + 1)} \geq 0, \quad (7) \\
V_F^B - V_N^B &= \frac{(\alpha - m)^2(\delta - \theta)^2 z_8}{2\beta(\delta + \theta - 2)^2 z_2^2(\delta + 1)} \geq 0, \quad (8)
\end{align*}
\]
where \( z_8 = -24\theta - 2\delta\theta - 8\delta^2 + \delta^3 + 12\theta^2 - 2\theta^3 + \delta\theta^2 + 6\delta^2\theta + 16 \). The equalities in (8) and (8) hold if and only if \( \delta = \theta \). Thus, any \( t_i \) is optimal if \( \delta = \theta \), and otherwise, only \((t_1, t_2) = (L, F)\) and \((t_1, t_2) = (F, L)\) constitute equilibria. Note that \( z_2 > 0 \) (see the proof of Proposition 1). Note also that \( z_8 > 0 \) for \( \delta \in (0, 1) \) and \( \theta \in [0, 1) \) because \( z_8 \) is decreasing in both \( \theta \) and \( \delta \) for \((\delta, \theta) \in [0, 1]^2\) and \( z_8 = 0 \) when \( \delta = \theta = 1 \). Q.E.D.

Except for the measure-zero event case (the case in which \( \delta = \theta \)), only Stackelberg outcomes arise in equilibrium. Again, this result indicates that introducing a positive \( \theta \) does not matter for generic cases, which is in contrast to Bárcena-Ruiz’s (2007) result.

5 Concluding remarks

In this paper, we introduce the effect of corporate social responsibility formulated by Ghosh and Mitra (2010b) into an endogenous timing game. We obtain results in contrast to those of Pal (1998) and Bárcena-Ruiz (2007) who discussed mixed oligopolies. We find that introducing the welfare-concern objective does not matter as long as the payoff functions are symmetric.

A similar principle can be applied to the price-quantity choice discussed by Singh and Vives (1984). They show that choosing the quantity contract is a dominant strategy as long as the products are substitutes in a private duopoly in which both firms are profit-maximizers, whereas Matsumura and Ogawa (2012) show that choosing the price contract is a dominant strategy in a mixed duopoly. We can again show that payoff asymmetry, (not introducing the welfare-maximizing payoff), matters and Singh and Vives (1984)’s result holds as long as the payoff is symmetric. These two results suggest that introducing symmetric welfare-concerns objective has quite contrasting implications to asymmetric welfare-concern objective discussed in the literature on mixed oligopoly.

9
References


