Market Structure and Privatization Policy under International Competition

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Abstract

We investigate the relationship among market structure, privatization and tax-subsidy policies. We find that if there is no foreign competitor, privatization does not matter under optimal tax-subsidy policy regardless of the number of firms. This is not true if there are foreign competitors, and privatization more likely improves welfare when the number of firms is larger even under optimal tax-subsidy policy. We also investigate two Stackelberg models, public leadership and private leadership. We find that private leadership yields larger (smaller) total social surplus than the private leadership when the presence of foreign firms among private firms is small (large).

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1 Introduction

The first purpose of this paper is to investigate the relationship among privatization, competition, and tax-subsidy policies in mixed oligopoly. The second purpose is to discuss a desirable role of public enterprise in mixed oligopoly under an optimal tax-subsidy policy.

Studies of mixed oligopoly involving both state-owned public enterprises and private enterprises are increasingly becoming popular. In recent financial crisis, many private enterprises facing financial problems are nationalized, either fully or partially. Studies of mixed oligopoly collect further attentions. Before the recent increase of public enterprises, mixed oligopoly existed in a range of industries such as transportation, telecommunications, energy, steel, automobile, and overnight-delivery industries, as well as in services such as banking, home loans, health care, life insurance, hospitals, broadcasting, and education and still has significant importance in many countries.\(^1\) Thus, studies of mixed oligopoly must continue to be important.

There are two lines of related articles in mixed oligopoly. The first is the literature on mixed oligopoly and market structure. Several works showed that privatization more likely improves welfare when the number of private firms is large (De Fraja and Delbono (1989), Han and Ogawa (2007, 2008), and Matsumura and Shimizu (2010)). In these papers, the governments control the public firms within the market (as an instrument of regulation), instead of using direct policies from outside the markets. However, governments often directly intervene by taxes and subsidies for many sectors such as medical care, education, energy, financial, and international trading industries.

The second line is the literature on the optimal subsidy in mixed oligopoly. White (1996) discussed a Cournot model in mixed and private oligopolies and showed the privatization of the public firm affects neither welfare level nor optimal subsidy policy (\textit{privatization neutrality theorem}). Poyago-Theotoky (2001) considered the public firm’s leadership in mixed oligopoly; Tomaru and Saito (2010) investigated

\(^1\) The pioneering work on mixed oligopoly was performed by Merrill and Schneider (1966). They as well as many other studies assumed that a public firm maximizes welfare (consumers surplus plus firms’ profits) while a private firm maximizes its own profits. For recent discussions on mixed oligopoly, see Gil-Moltó and Poyago-Theotoky (2008) and Ishida and Matsushima (2009).

First, we reexamine the relationship between competition and privatization policies. At the first glance, the second line literature mentioned above suggests that the result in the first line literature (privatization more likely improves welfare when the market is more competitive) does not hold under optimal tax-subsidy policy since privatization does not matters regardless of the market structure. We show, however, that under optimal tax-subsidy policy privatization in fact more likely improves welfare when the number of private firms is larger unless all private firms are domestic. If the foreign investors hold non-zero shares in private firms, privatization does matter and privatization policy and competition policy are not independent but complements.

Next, we discuss two Stackelberg models and again reexamine the privatization neutrality theorem. One is the model where the public firm produces and then the private firms produce (public leadership). Another is the model where the private firms produce and then the public firm produce (private leadership). Under private leadership the public firm plays a complementary role of the private sector (i.e., the public firm plays a passive role). Under the public leadership, the public firm plays a more positive role to control the behavior of private firms. Pal (1998) showed that when private firms are domestic, private leadership is better than the public leadership for social welfare. In other words, the public firm should take a role of potential competitor rather than directly controlling the behaviors of private firms as the Stackelberg leader. Matsumura (2003b) showed that when private firms are foreign, public leadership is better than private leadership for domestic social surplus.\(^3\) However, they

\(^2\) Fjell and Heywood (2004) derived a non-neutral result. The other works assume that private firms move simultaneously in private oligopoly, while they consider the asymmetric order of moves in private oligopoly. They find that the first-best is not achieved after privatization by a simple unit subsidy. In this paper, we do not assume any asymmetry among private firms before and after privatization.

\(^3\) For the discussion on sequential move games and endogenous timing games in mixed oligopoly, see also Beato and Mas-Colell (1984), Matsumura (2003a), Lu (2006), Bárceña-Ruiz (2007) and Tomaru and Kiyono (2010).
ignore the tax-subsidy policy. As Poyago-Theotoky (2001) and Tomaru and Saito (2010) showed, the privatization neutrality theorem holds in these two Stackelberg models when private firms are domestic. Under privatization neutrality theorem, it seems nonsense to discuss whether or not private leadership is better for social welfare because both yields the same total social surplus. In this paper, we show that whether or not the public firm takes leadership matters unless the share of foreign investors in private firms is zero. We find that private leadership yields larger (smaller) total social surplus than the public leadership, when the share of foreign investors in private firms is positive and small (large).

The remainder of this paper is organized as follows. Section 2 formulates the model and derives the equilibrium outcomes in both mixed and private oligopolies. Section 3 compares the resulting welfare from these two oligopolies and discuss the welfare implication of privatization. Section 4 investigates public and private leaderships. Section 5 concludes this paper.

2 The Model

Consider an industry which consists of one state-owned public firm and \( n \) private firms producing a homogeneous commodity. All firms have identical and increasing marginal cost technologies. Let the cost function be \( C(q) = \frac{1}{2}q^2 \). Let \( q_0 \) be the output of the public firm and \( q_i \) be the private firm \( i \) \((i = 1, 2, \ldots, n)\). The inverse demand function is given by \( P = P(Q) = a - Q \), where \( Q = q_0 + \sum_{i=1}^{n} q_i \) is the total output of the market. The public firm’s objective is to maximize domestic social surplus and the private firm’s is to maximize its own profit, which are commonly assumed in the literature on mixed oligopoly. Each firm’s profit is given by

\[
\Pi_i = P(Q)q_i - C(q_i) + sq_i = \left[ a - \left( q_0 + \sum_{j=1}^{n} q_j \right) \right] q_i - \frac{1}{2}q_i^2 + sq_i, \tag{1}
\]

where \( s \) is a unit subsidy rate. We assume that all private firms are symmetric. Let \( \theta \) be the shares of foreign investors in private firms. Alternatively, we can assume that there are \( \theta n \) foreign private firms and \( (1 - \theta)n \) domestic private firms. These two sorts of formulation yield exactly the same equilibrium outcomes and welfare implications.
The domestic welfare is given as

\[ W = \int_0^Q P(z)dz - P(Q)Q + \Pi_0 + (1 - \theta) \sum_{i=1}^n \Pi_i - sQ. \]  (2)

The subsidies are included in the welfare expression as a component of profits and a state expenditure. Unlike works analyzing subsidized mixed oligopolies consisting of public and domestic private firms, the welfare is directly affected by subsidy rate \( s \), since a part of profits is transferred to the foreign investors.

The game runs as follows. In the first stage, the government sets the subsidy rate so as to maximize the domestic welfare. Observing the government’s choice, all the firms select their outputs simultaneously in the second stage. We solve this game by the backward induction as usual.

**2.1 Equilibrium of subsidized mixed oligopoly**

We first derive the second-stage Cournot Nash equilibrium outcomes. The public firm (firm 0) selects its output to maximize welfare(2), whereas each private firm selects its output to maximize its profits (1). Then, the first-order conditions are

\[ \frac{\partial W}{\partial q_0} = a - 2q_0 - (1 - \theta) \sum_{i=1}^n q_i = 0, \]  (3)

\[ \frac{\partial \Pi_i}{\partial q_i} = a - Q - 2q_i + s = 0, \quad (i = 1, 2, \cdots, n). \]  (4)

The second-order conditions are satisfied. As easily observed, the public firm’s reaction to the other private firms does not depend on \( s \), though the welfare itself is directly affected by \( s \). It implies that the public firm’s equilibrium output in the second stage is influenced by \( s \) only through strategic interaction.

From the first-order conditions, we find that

\[ Q^{SM}(s, \theta) = \frac{[2 + n(1 + \theta)]a + n(1 + \theta)s}{4 + n(1 + \theta)}, \]  (5)

\[ q_i^{SM}(s, \theta) = \frac{a + 2s}{4 + n(1 + \theta)}, \quad (i = 1, 2, \cdots, n), \]  (6)

\[ q_0^{SM}(s, \theta) = \frac{(2 + n\theta)a - (1 - \theta)ns}{4 + n(1 + \theta)}, \]  (7)
where the superscript SM denote the equilibrium outcome at the second stage (given s) in the mixed oligopoly.

The output of firm 1 is increasing in s (subsidy rate) and that of firm 0 is decreasing in it. The production subsidy directly stimulates production by the private firm, and it reduces the resulting output of the public firm through strategic interaction. Firm 0 expands $q_0$ as $\theta$ increases. This is because in the presence of foreign private firms, the expansion of production by the public firm improves the terms of trade.\(^{4}\)

We now solve the first stage of the game. Substituting (5)–(7) into the welfare function (2) and maximizing it with respect to $s$, we obtain

$$s^M(\theta) = \frac{a \left[ 2 + (n - 6)\theta - n\theta^2 \right]}{2(2 + n + 6\theta + n\theta^2)},$$

(8)

where the superscript M denote the equilibrium outcome of the full game in the mixed oligopoly.

From (8), we have $s^M(0) > 0$ and $s^M(1) < 0$; therefore, $s^M(\theta)$ can be negative (tax-regime). We also note that the optimal subsidy would be either increasing or decreasing in $\theta$. Differentiate $s^M$ with respect to $\theta$ yields

$$s^M'(\theta) = -\frac{a \left[ 24 + 4n(1 + 2\theta) + n^2(-1 + 2\theta + \theta^2) \right]}{2(2 + n + 6\theta + n\theta^2)^2}.$$  

(9)

Thus, the sign of it depends on $n$ and $\theta$. Suppose that $n$ is relatively small. In this case, the numerator in the right-hand side is positive, thus $s^M'(\theta)$ is negative. When $n$ is large, it is ambiguous. $s^M'(\theta)$ would be positive if $\theta$ is small, whereas it would be negative if $\theta$ is sufficiently large. We will find that this is a contrasting result to that of the private oligopoly. For a convenience of exposition and an emphasis on such a difference between mixed and private oligopolies, the intuition behind the result obtained here is relegated to the next section.

\(^{4}\) Fjell and Pal (1996) showed that a public firm increases its output as the number of private firms owned by only foreigners becomes large. This is because in the presence of foreign private firms, expansion of public firm’s production improves the terms of trade through the strategic substitution and it enhances the domestic welfare. An effect analogous to Fjell and Pal (1996) works out even in our model. For the discussion on mixed oligopoly with international competition, see also Corneo and Jeanne (1994), Pal and White (1998), Mukherjee and Suetrong (2009), Wang et al (2009) and Inoue et al (2009).
Substituting (8) into (5)–(7), we obtain

\[
\begin{align*}
Q^M(\theta) &= \frac{a[2 + 6\theta + n(2 + \theta + \theta^2)]}{2(2 + n + 6\theta + n\theta^2)}, \\
q_0^M(\theta) &= \frac{a[2 + (6 + n)\theta + n\theta^2]}{2(2 + n + 6\theta + n\theta^2)}, \\
q_i^M(\theta) &= \frac{a}{2 + n + 6\theta + n\theta^2} \\
\Pi_0^M(\theta) &= -\frac{a^2[-6 + (6 + n)\theta + n\theta^2][2 + (6 + n)\theta + n\theta^2]}{8(2 + n + 6\theta + n\theta^2)^2}, \\
\Pi_i^M(\theta) &= \frac{3a^2}{2(2 + n + 6\theta + n\theta^2)^2}, \\
W^M(\theta) &= \frac{a^2[2 + 6\theta + n(2 + \theta^2)]}{4(2 + n + 6\theta + n\theta^2)}.
\end{align*}
\]

As expected, the output of the public firm exceeds that of each private firm. The public firm’s output is increasing in \(\theta\), whereas the private firm’s output is decreasing in it. Note that the public firm’s profits could be negative when \(\theta\) is sufficiently large. In the case of sufficiently large \(\theta\), the optimal subsidy rate \(s^M\) becomes negative, that is, a production tax is imposed to all the firms. Recall that the public firm becomes more aggressive to improve the terms of trade if \(\theta\) increases. This aggressive behavior under the high tax rate makes the after tax profits of the public firm negative. This is a new result in the literature on mixed oligopoly under optimal tax-subsidy policies.

### 2.2 Private Oligopoly

We now proceed to the analysis of post-privatization. Following previous literature, we regard privatization as a change in the objective of firm 0 from maximizing welfare to profit. Furthermore, we assume that the privatized firm is owned only by domestic residents. This assumption is imposed only to allow us to directly compare mixed and private oligopoly, without the need to consider how the profits of the public and privatized firms will be included in welfare.\(^5\)

All the firms select their outputs to maximize their profits, which results in the following first-order conditions:

\[
\frac{\partial \Pi_i}{\partial q_i} = a - Q - 2q_i + s = 0, \quad (i = 0, 1, \cdots, n).
\]

\(^5\) We can consider a situation wherein foreigners hold shares in the privatized firm \(\theta\), similar to other private firms. We obtain similar results in this model formulation as well.
Immediately, we find that the equilibrium outputs as $s$ given are as follows.

$$q_i^{SP}(s) = \frac{a + s}{3 + n}, \quad (i = 0, 1, \ldots, n), \quad Q^{SP} = \frac{(n + 1)(a + s)}{3 + n},$$

(10)

where the superscript $SP$ denote the equilibrium outcome at the second stage (given $s$) in the private oligopoly.

Substituting (10) into the welfare function (2) and maximizing it with respect to $s$, we obtain

$$s^P(\theta) = \frac{a[1 + n(1 - 3\theta)]}{2 + n^2 + 3n(1 + \theta)},$$

(11)

where the superscript $P$ denote the equilibrium outcome of the full game in the private oligopoly.

Like the optimal subsidy in the mixed oligopoly, $s^P$ goes either way of positive or negative. More interestingly, $s^P$ is always decreasing in $\theta$, which is not true in the mixed oligopoly. The intuition is as follows. In both mixed and private oligopolies, the production level of each private firm is too low for domestic welfare. Thus, the government has an incentive for raising $s$ so as to stimulate the production of private firms. On the other hand, a raise of $s$ increases the outflow of surplus to foreign investors and it reduces domestic welfare. The latter effect becomes more significant when $\theta$ is large. Thus, $s^P$ is decreasing in $\theta$.

In the mixed oligopoly, an additional distortion effect exists. The public firm produces more aggressively when $\theta$ is larger; resulting in a further reduction of the production in private firms. So as to mitigate welfare loss by under-production of private firms, the government has an incentive for raising $s$. The first and the second effect discussed in the previous paragraph is weak when $n$ is large and the third effect discussed in this paragraph can dominate these. It yields the non-monotone result in the mixed oligopoly (i.e., $s^M$ can be either increasing or decreasing in $\theta$).

Substituting (11) into (10), we obtain

$$Q^P(\theta) = \frac{a(1 + n)^2}{2 + n^2 + 3n(1 + \theta)}, \quad q^P_i(\theta) = \frac{a(1 + n)}{2 + n^2 + 3n(1 + \theta)}, \quad (i = 0, 1, \ldots, n),$$

$$\Pi^P_i(\theta) = \frac{3a^2(1 + n)^2}{2[2 + n^2 + 3n(1 + \theta)]^2}, \quad (i = 0, 1, \ldots, n), \quad W^P(\theta) = \frac{a^2(1 + n)^2}{2[2 + n^2 + 3n(1 + \theta)]^2}.$$

As expected, equilibrium outputs, profits, and welfare are decreasing in $\theta$. 

8
3 Comparisons

In this section we discuss the effect of privatization by comparing the equilibrium outcomes in mixed and private oligopolies. In particular, we examine whether the privatization neutrality theorem propounded by White (1996) holds even when foreign investors launch the purchases of shares in domestic private firms. The upshot is presented as Proposition 1.

**Proposition 1:** (i) If $\theta = 0$, then $W^M = W^P$. (ii) Suppose that $\theta > 0$. Then

\[
\begin{cases}
(a) & W^M(\theta) > W^P(\theta), & \text{if } 1 \leq n < \frac{\sqrt{73} - 1}{2}, \\
(b) & W^M(\theta) \lesssim W^P(\theta) \iff \theta \gtrless \frac{n^2 + n - 18}{3n}, & \text{if } \frac{\sqrt{73} - 1}{2} \leq n \leq 1 + \sqrt{19}, \\
(c) & W^M(\theta) < W^P(\theta), & \text{if } n > 1 + \sqrt{19}.
\end{cases}
\]

**Proof:** Simple calculation yields

\[
W^M(\theta, n) - W^P(\theta, n) = \frac{a^2 n \theta^2 (18 - n - n^2 + 3n\theta)}{4(2 + n + 6\theta + n\theta^2)(2 + n^2 + 3n(1 + \theta))}.
\]

Substituting $\theta = 0$ into it yields Proposition 1(i).

When $\theta > 0$, $\text{sign}(W^M - W^P) = \text{sign}(18 - n - n^2 + 3n\theta)$. Straightforwardly, we obtain

\[
W^M(\theta) \gtrless W^P(\theta) \iff \theta \gtrless \frac{n^2 + n - 18}{3n}.
\]

$(n^2 + n - 18)/(3n)$ is negative for all $\theta \in (0, 1]$ if $1 \leq n < (\sqrt{73} - 1)/2$. $(n^2 + n - 18)/(3n)$ is larger than 1 for all $\theta \in (0, 1]$ if $n > 1 + \sqrt{19}$. Thus, we obtain Proposition 1(ii). ■

Proposition 1(i) is the privatization neutrality theorem which is intensively discussed in the literature on mixed oligopoly. Proposition 1 (ii) indicates that the theorem does not hold when $\theta > 0$. Case (a) of Proposition 1(ii) states that privatization does not improve welfare when $n$ is small, whereas Case (c) of it states that privatization improves welfare when $n$ is large. Case (b) of it indicates that the result is ambiguous when $n$ is intermediate. However, since $(n^2 + n - 18)/(3n)$ is increasing in $n$, given $\theta$, $W^M < W^P$ is more likely satisfied when $n$ is larger. In short, Proposition 1 indicates that privatization is more likely improve welfare when $n$ is larger.
4 Public leadership and private leadership

We now turn to the second topic. We consider two Stackelberg models. One is the model where the public firm produces and then the private firms produce (public leadership). The other is the model where the private firms produce and then the public firm produces (private leadership). We discuss whether public or private leadership yields a larger domestic welfare. In other words, we investigate whether or not the public firm should take a role of potential competitor (private leadership) rather than directly controlling the behaviors of private firms as the Stackelberg leader (public leadership).

4.1 Public leadership

The government chooses \( s \) so as to maximize welfare. After observing \( s \) firm 0 chooses \( q_0 \). After observing \( s \) and \( q_0 \) each private firm chooses its output. Expecting private firms’ reactions derived from (4), firm 0 determines its output. Thus, denoting the reduced welfare which firm 0 confronts at its decision stage as \( \hat{W}(q_0, s, \theta) \), the first-order condition for firm 0 is

\[
\frac{\partial \hat{W}}{\partial q_0} = \frac{ns(-1 + 3\theta) + a(4 + n + 3n\theta) - q_0(8 + 5n + n^2 + 3n\theta)}{(n + 2)^2} = 0.
\]

The second-order condition is satisfied. The equilibrium outcomes given \( s \) are:

\[
q_{SL}^i(s, \theta) = \frac{a(2 + n) + [4 + n] s}{8 + n^2 + n(5 + 3\theta)} \quad (i = 0, 1, \ldots, n),
\]

\[
q_{SL}^0(s, \theta) = \frac{ns(-1 + 3\theta) + a(4 + n + 3n\theta)}{8 + n^2 + n(5 + 3\theta)},
\]

where the superscript \( SL \) denote the equilibrium outcome in the second stage (given \( s \)) of the Stackelberg model with public leadership.

In the Cournot mixed oligopoly, the output of the public firm is decreasing in \( s \). In the Stackelberg mode with public leadership, it holds only when \( \theta < 1/3 \). When \( \theta > 1/3 \), the output of the public firm is increasing in \( s \).

Substituting (12)–(13) into the welfare function (2) and maximizing it with respect to \( s \), we obtain

\[
s_{SL}^L(\theta) = \frac{a(1 - 3\theta)}{2 + n + 6\theta}.
\]
where the superscript L denote the equilibrium outcome in the full game of the Stackelberg model with public leadership.

Similar to the case of Cournot mixed oligopoly, (14) could be negative. The government chooses a positive subsidy if $\theta < 1/3$, and otherwise, it imposes the production tax. The critical difference between the optimal subsidies under Cournot and Stackelberg competition is the effects of $\theta$ on them. In the Stackelberg with public leadership, an increase in $\theta$ always lowers the subsidy rate, while in the Cournot mixed duopoly it can raise the optimal subsidy rate.

In the Cournot mixed oligopoly, the public firm chooses its output given the outputs of the private firms, while the government chooses $s$ taking into account that $s$ affects the outputs of the private firms. In other words, the government and the public firm solves the different problem even when both are welfare maximizers. The government must consider the effects on the output of the public firm as well as on the outputs of the private firms. On the contrary, under the public leadership both government and the public firm choose their actions taking into account that their behaviors affect the outputs of the private firms. Thus, even if we formulate the model where the government chooses both $s$ and $q_0$, we obtain exactly the same results as those in the Stackelberg with public leadership. The government can reduce $s$ without bothering that it distorts the output of the public firm. Thus, we obtain a similar result in the private oligopoly.

Substituting (14) into (10), we obtain

$$q^L_0(\theta) = \frac{a(1 + 3\theta)}{2 + n + 6\theta}, \quad q^L_i(\theta) = \frac{a}{2 + n + 6\theta} \quad (i = 0, 1, \cdots, n),$$

$$\Pi^L_0(\theta) = \frac{3a^2(1 - \theta)(1 + 3\theta)}{2(2 + n + 6\theta)^2}, \quad \Pi^L_i(\theta) = \frac{3a^2}{2(2 + n + 6\theta)^2} \quad (i = 0, 1, \cdots, n), \quad W^L(\theta) = \frac{a^2(3\theta + 1 + n)}{2(2 + n + 6\theta)}.$$

### 4.2 Private leadership

The government chooses $s$ so as to maximizes welfare. After observing $s$ private firms choose their outputs simultaneously. After observing $s$ and the outputs of private firms, the public firm chooses its output. Expecting the public firm’s reaction derived from (3), each private firm selects its output. Let
profits of private firm $i$ be $\hat{\Pi}_i$. Its first-order condition is given by

$$\frac{\partial \hat{\Pi}_i}{\partial q_i} = \frac{a + 2s - (1 + \theta) \sum_{j \neq i} q_j - 2(2 + \theta)q_i}{2} = 0, \quad (i = 1, 2, \cdots, n).$$

The second-order condition is satisfied. From the first-order conditions of all the private firms, we obtain the equilibrium outcomes given $s$:

$$q_i^{SF}(s, \theta) = \frac{a + 2s}{3 + n + (1 + n)\theta} \quad (i = 0, 1, \cdots, n), \quad (15)$$

$$q_0^{SF}(s, \theta) = \frac{(3 + \theta + 2n\theta)a - 2s(1 - \theta)s}{2(3 + n + (1 + n)\theta)}, \quad (16)$$

where the superscript SF denote the equilibrium outcome in the second stage (given $s$) of the Stackelberg model with public followership.

Substituting (15)–(16) into the welfare function (2) and maximizing it with respect to $s$, we obtain

$$s^F(\theta) = \frac{a \left[ 1 + (n - 3)\theta - (2 + n)\theta^2 \right]}{2 \left[ 2(1 + \theta)^2 + n(1 + \theta^2) \right]}, \quad (17)$$

where the superscript F denote the equilibrium outcome in the full game of the Stackelberg model with public followership.

Substituting (17) into (15)–(16) yields

$$q_0^F(\theta) = \frac{a(1 + \theta)[2 + (2 + n)\theta]}{2 \left[ 2(1 + \theta)^2 + n(1 + \theta^2) \right]}, \quad q_i^F(\theta) = \frac{a}{2(1 + \theta)^2 + n(1 + \theta^2)} \quad (i = 0, 1, \cdots, n),$$

$$\Pi_0^F(\theta) = \frac{a^2(1 + \theta)[2 + (2 + n)\theta][4 - (2 + n)\theta - (2 + n)\theta^2]}{8 \left[ 2(1 + \theta)^2 + n(1 + \theta^2) \right]^2},$$

$$\Pi_i^F(\theta) = \frac{a^2(2 + \theta)}{2 \left[ 2(1 + \theta)^2 + n(1 + \theta^2) \right]^2} \quad (i = 0, 1, \cdots, n), \quad W^F(\theta) = \frac{a^2 \left[ 2(1 + \theta)^2 + n(2 + \theta^2) \right]}{4 \left[ 2(1 + \theta)^2 + n(1 + \theta^2) \right]}.$$

### 4.3 Comparison

We now present results on whether public or private leadership yields a larger domestic surplus under the optimal tax-subsidy policy.

**Proposition 2:** If $\theta = 0$, $W^L = W^F = W^M = W^P$.

**Proposition 3:** Suppose that $\theta > 0$. Then (i) $W^L > \max\{W^P, W^M\}$ and (ii) $W^L > W^F$ if and only
if \( \theta > \frac{2}{(n+2)} \).

**Proofs:** Simple calculations reveal that

\[
W^L(\theta) - W^P(\theta) = \frac{9a^2n\theta^2}{2(2+n+6\theta)[2+n^2+3n(1+\theta)]} \geq 0,
\]

\[
W^L(\theta) - W^M(\theta) = \frac{a^2n^2\theta^2}{4(2+n+6\theta)(2+n+6\theta+n\theta^2)} \geq 0,
\]

and the equalities hold if and only if \( \theta = 0 \). This implies Proposition 3(i). We also have

\[
W^L(\theta) - W^F(\theta) = \frac{a^2n\theta[-2+(2+n)\theta]}{4(2+n+6\theta)[2(1+\theta)^2+n(1+\theta^2)]},
\]

where \( W^L \) is equal to \( W^F \) if \( \theta = 0 \). These imply Proposition 2.

From (18), we obtain that \( \text{sign}(W^L - W^F) = \text{sign}(-2+(2+n)\theta) \). This yields Proposition 3 (ii). \( \blacksquare \)

Proposition 2 is the privatization neutrality theorem shown by Poyago-Theotoky (2001) and Tomaru and Saito (2010). Proposition 3 suggests that non-neutrality results again appear when \( \theta > 0 \).

De Fraja and Delbono (1989) discussed the case where \( s = \theta = 0 \) and derived a similar result to Proposition 3(i). Pal (1998) also discussed that the case where \( s = \theta = 0 \) and showed that the private leadership yields larger surplus than the public leadership. Matsumura (2003b) discussed that the case where \( s = 0 \) and \( \theta = 1 \) and showed that the private leadership yields smaller surplus than the public leadership. These are similar to Proposition 3(ii). If privatization neutrality theorem holds true for all \( \theta \), it is nonsense to discuss whether public or private leadership yields a larger domestic surplus. However, Proposition 3 says that discussing welfare implication in mixed oligopoly is still important even under the optimal tax-subsidy policy. Our result indicates that the above three results discussing the case without tax-subsidy still hold under the optimal tax-subsidy policy unless all private firms are domestic.

Proposition 3 (ii) has another implication. It states that public leadership more likely yields a larger surplus when \( n \) is large. In other words, if the market is competitive, the public firm should take the leadership. Under the public leadership, the total output of private firms becomes more sensitive to the output of public firm when \( n \) is larger. In other words, the strategic value for controlling the output
of the public firm increases. This is why the public leadership more likely improves domestic welfare when \( n \) is larger.

5 Concluding remarks

In this paper, we investigate two problems. One is the relationship among competition, privatization, and subsidy policies. Privatization neutrality theorem suggests that there is no relationship between privatization and competition policies under the optimal subsidy policy. Privatization does not matter regardless of the number of private firms. We find that this is true only when there is no foreign private firm. Unless the case where all private firms are domestic, privatization matters and privatization improves welfare more likely when the number of private firms is larger. In other words, privatization policy and competition policy are mutually complement.

Another is the value of public leadership. Privatization neutrality theorem suggests that public leadership and private leadership yield the same surplus under the optimal subsidy policy. We again showed that this is true only when there is no foreign private firm. Public leadership more likely yields a larger domestic surplus when the foreign presence in the product market is larger and the number of private firms is larger.

Our results also shed light on another aspect of the importance of the nationality of private firms in mixed oligopoly. Many works have already showed that nationality of private firms plays a crucial role. This paper shows that under optimal tax-subsidy policy, this has further importance. Welfare implications drastically change if we drop the assumption that all private firms are domestic.

Our non-neutrality result is fairly robust. If we introduce cost difference between public and private firms, the non-neutrality result is strengthened (in this case privatization is non-neutral even when all private firms are domestic). If we introduce product differentiation and/or multiple public firms discussed in Matsumura and Shimizu (2010), non-neutrality result still holds unless all private firms are domestic.\(^6\) If we introduce partial privatization discussed by Matsumura (1998) we can show that

\(^6\) For the discussion of endogenous cost difference between public and private firms, see Matsumura and Matsushima (2004) and Ishibashi and Matsumura (2006).
optimal degree of privatization is increasing in \( n \) and is decreasing in \( \theta \) as long as \( \theta > 0 \). If we consider free entries of private firms, the optimal subsidy can be negative and privatization matters even when \( \theta = 0 \).\(^7\) We believe that privatization does matter even under optimal tax-subsidy policy in broad situations.

\(^7\) Both in mixed and private oligopolies, the number of entering firms at free entry equilibrium is excessive and this yields these results. See Mankiw and Whinston (1986), Suzumura and Kiyono (1987), Ohkawa et al (2005), Matsumura and Kanda (2005), and Wang and Chen (2010).
References


