Comparing Specific and Ad Valorem Taxes under Price-inelastic Demand with Quality Differentiation

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The Beginning of the Question- A Government Needs Tax Revenues

- As we know, in principle, imposing tax will lead to a deadweight loss if the market is in the absence of externalities and information asymmetry.

- But we need a government, and the government needs tax revenues to support government expenditure.

- So the question turns into that, if we need to impose a tax on goods or services, what kinds of tax system is better?

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Two Tax Types

Except for the lump-sum tax, there are two tax types, specific and ad valorem taxes, that usually we can see.

Receipts in Japan

Receipts in Taiwan

In the whole world, it is a popular phenomenon that a specific tax is imposed on cigarettes, alcoholic beverages, and motor fuels.

	Cigarettes	Alcoholic Beverages	Gasoline and Diesel
the U.S.	Specific Tax	Specific Tax	Specific Tax
Mainland China	Ad valorem Tax	Specific Tax	Specific Tax
Japan	Specific Tax	Specific Tax	Specific Tax
the U.K.	Specific Tax	Specific Tax	Specific Tax
Taiwan	Specific Tax	Specific Tax	Specific Tax

Cigarettes and alcoholic beverages are addictive products and motor fuels belongs to the necessary good for people.

The question we would like to ask in this paper is that, which one is better, a specific tax or an ad valorem tax, if the government needs tax revenues.

- **The result from the conventional wisdom– an ad valorem tax is better than a specific tax under imperfect competition**
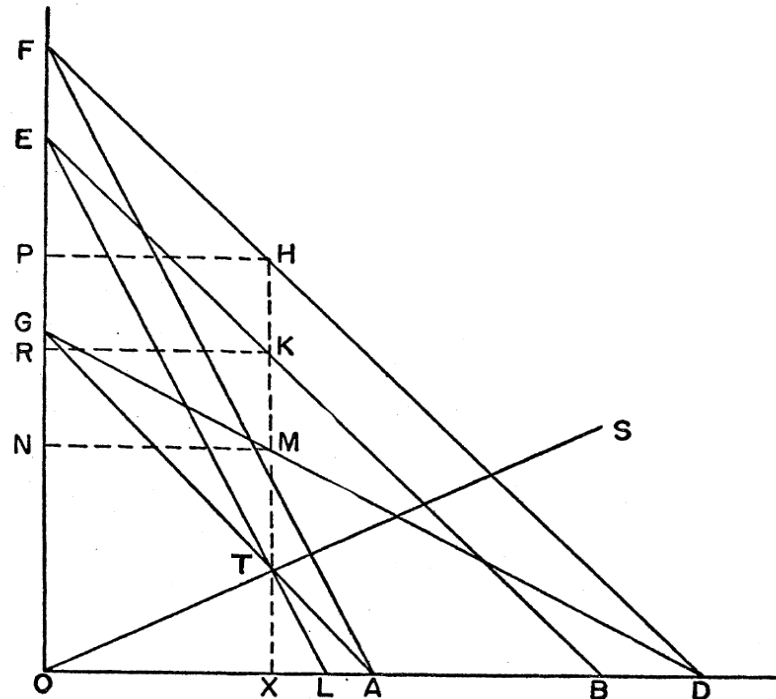
Line DF is the demand curve without any taxes. Line EB is the demand curve under a specific tax and line GD is the demand function under an ad valorem.

Intuitions: the demand elasticity under an ad valorem tax is larger than that under a specific tax. The larger elasticity firms face, the total P and Q there are.

Results in literature

- Two Criteria to compare these two taxes-Pareto Ranking and Welfare Ranking

If we impose a commodity tax on
a product, there are three
components, consumers surplus,
producers surplus and tax revenues,
in social welfare.

Welfare Ranking

Pareto Ranking

- If a component (usually tax revenues or consumers surplus)is fixed, the other two components under Tax A are both larger than the other two components under Tax B.-Pareto Ranking
- If a component (usually tax revenues or consumers surplus)is fixed, the total welfare under Tax A is larger than that under Tax B.-Welfare Ranking

Results in literature

An ad valorem tax is welfare-superior to a specific tax .
(Welfare Ranking, the traditional result)

Suits and Musgrave (1953), (**Pareto Ranking**)

Delipalla and Keen (1992), Skeath and Trandel (1994),

Schröder (2004)

A specific tax may be welfare-superior to an ad valorem tax.

Hamilton (1999) (monopsony)

Anderson *et al*. (2001) (Bertrand competition with differential products and production costs)

Pirttila (2002), and Droge and Schroder (2009) (a negative externality)

Wang and Zhao (2009) (a horizontally differentiated duopoly with asymmetric costs, regardless of Cournot or Bertrand duopoly)

Wang, Chou and Liang (2018) (a vertically differentiated oligopoly with asymmetric costs, regardless of Cournot or Bertrand oligopoly)

Basic model setting in this paper

- Corresponding to addictive products (cigarettes or alcohol) or necessary goods (fuel oil), we use a fully covered model with vertical differentiation to describe a market in which the demand elasticity is inelastic, and firms' products are vertically differentiated.

- Given the tax type, two firms can choose only their product price in the short run, and choose both their product quality and price in the long run.

$$(s_1, p_1) \qquad (s_2, p_2)$$

$$\underline{\theta} \qquad\qquad \theta_{12} \qquad\qquad \bar{\theta} = \underline{\theta} + 1$$
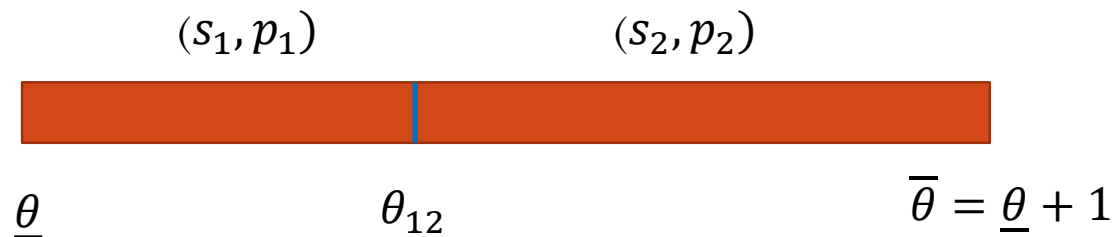
Main results in this paper

- We obtain several results.
- (1) Regardless of the situations in the short run or long run, a specific tax can be superior to an ad valorem tax.

  In the short run, the ad valorem tax affects the output distribution between high-quality and low-quality firms. (under two cases, when the marginal cost of the high-quality product is relatively low or when the marginal cost of the high-quality product is relatively high but the ad valorem tax rate is high.

  In the long run, the ad valorem tax affects the quality distribution between high-quality and low-quality firms. (when the ad valorem tax rate is higher)

- (2) We also show that a specific tax may not only be welfare superior to, but may also Pareto dominate an ad valorem tax in the long run, when the ad valorem tax rate is high enough.

Model setting in details

- 2 firms, firm i's production cost is $TC_i = c_i q_i = s_i^2 q_i$, q_i and p_i are product i's quality and price respectively. $s_2 > s_1$

- Consumers' utility is $U = \theta s_i - p_i$,

- θ is consumers' tastes for quality, and is a uniform distribution, $\theta \in [\underline{\theta}, \underline{\theta} + 1]$.

- Marginal consumer locates at $\theta_{12} = \underline{\theta} + 1 - (p_2 - p_1)/(s_2 - s_1)$.

- $q_2^D = \underline{\theta} + 1 - \theta_{12} = \underline{\theta} + 1 - (p_2 - p_1)/(s_2 - s_1)$

- $q_1^D = \theta_{12} - \underline{\theta} = (p_2 - p_1)/(s_2 - s_1) - \underline{\theta}$

The welfare superiority of the two tax schemes in the short run

- In the short run, the profit-maximizing problem under the specific tax scheme is

$$\underset{p_i^t}{Max}\ \pi_i^t = (p_i^t - c_i - t)q_i^t, i = 1,2, \quad (1)$$

$$p_1^{t*} = \frac{2c_1 + c_2 + (1 - \underline{\theta})(s_2 - s_1)}{3} + t;$$

$$p_2^{t*} = \frac{2c_2 + c_1 + (2 + \underline{\theta})(s_2 - s_1)}{3} + t$$

$$\theta_{12}^{t*} = \frac{(c_2 - c_1)}{3(s_2 - s_1)} + \frac{(1 + 2\underline{\theta})}{3}$$

The specific tax is completely transferred to the consumers.

$$q_1^{t*} = \frac{\left(c_2 - c_1 + (1 - \underline{\theta})(s_2 - s_1)\right)}{3(s_2 - s_1)}; q_2^{t*} = \frac{\left(c_1 - c_2 + (2 + \underline{\theta})(s_2 - s_1)\right)}{3(s_2 - s_1)}$$

The welfare superiority of the two tax schemes in the short run

- In the short run, the profit-maximizing problem under the ad valorem tax scheme is

$$\underset{p_i^\tau}{Max}\, \pi_i^\tau = ((1-\tau)p_i^\tau - c_i)q_i^\tau, i = 1,2, \qquad (7)$$

- $p_1^{\tau*} = \frac{2c_1 + c_2}{3(1-\tau)} + \frac{(1-\underline{\theta})(s_2 - s_1)}{3}$;

$p_2^{\tau*} = \frac{2c_2 + c_1}{3(1-\tau)} + \frac{(2+\underline{\theta})(s_2 - s_1)}{3}$. $\theta_{12}^{\tau*} = \frac{(c_2 - c_1)}{3(1-\tau)(s_2 - s_1)} + \frac{(1+2\underline{\theta})}{3}$

- $\frac{\partial \theta_{12}^{\tau*}}{\partial \tau} > 0,\ \ Why?\ \ \tau p_2^\tau > \tau p_1^\tau$

- It means that an ad valorem tax will affect the output distribution between the high and low quality firms.

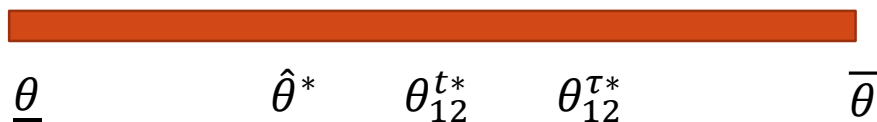- On the other hand, $\theta_{12}^{\tau*} - \theta_{12}^{t*} > 0$

The welfare superiority of the two tax schemes in the short run

- We need a standard to compare the superiority (the efficient output distribution) between these two tax schemes.
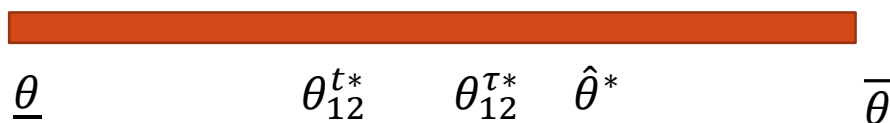
- $Max_{\widehat{\theta}} \; SW = \int_{\underline{\theta}}^{\widehat{\theta}} (\theta s_1 - c_1) d\theta + \int_{\widehat{\theta}}^{1+\underline{\theta}} (\theta s_2 - c_2) d\theta$

$$\widehat{\theta}^* = \frac{c_2 - c_1}{s_2 - s_1}$$

Two cases.

| $\underline{\theta}$ | $\widehat{\theta}^*$ | θ_{12}^{t*} | $\theta_{12}^{\tau*}$ | $\overline{\theta}$ |

The specific tax is better.

| $\underline{\theta}$ | θ_{12}^{t*} | $\theta_{12}^{\tau*}$ | $\widehat{\theta}^*$ | $\overline{\theta}$ |

If τ is smaller (larger),
the ad valorem is better (worse).

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The welfare superiority of the two tax schemes in the short run

- **Proposition 1.** *Provided that the quality levels are fixed in the short run*, we can propose:

- *A specific tax is always welfare superior to an ad valorem tax when the marginal cost of the high-quality product is small, i.e.,* $c_2 < (1 + 2\underline{\theta})(s_2 - s_1)/2 + c_1$.

- *Given that the marginal cost of the high-quality product is large, i.e.,* $c_2 > (1 + 2\underline{\theta})(s_2 - s_1)/2 + c_1$, *a specific tax is welfare superior (inferior) to an ad valorem tax when* $\tau > (<)\hat{\tau}$.

The superiority of the two tax schemes in the long run

- A two-stage game. Given the tax scheme, two firms simultaneously decide the quality level in the first stage, and then simultaneously decide the product price in the second stage.

- Under the specific scheme,

- $s_1^{t*} = \frac{4\underline{\theta}-1}{8}, s_2^{t*} = \frac{5+4\underline{\theta}}{8}$

- $p_1^{t*} = \frac{25-8\underline{\theta}+16\underline{\theta}^2}{64} + t, p_2^{t*} = \frac{49+40\underline{\theta}+16\underline{\theta}^2}{64} + t$

- $q_i^{t*} = \frac{1}{2}, \theta_{12}^{t*} = \frac{1}{2} + \underline{\theta}$

- **Proposition 2.** *The specific tax is **tax neutral** in the long-run equilibrium.*

The superiority of the two tax schemes in the long run

- Under the ad valorem tax scheme,

- $s_1^{\tau *} = \dfrac{(4\underline{\theta}-1)(1-\tau)}{8}, s_2^{\tau *} = \dfrac{(5+4\underline{\theta})(1-\tau)}{8}$

- $p_1^{\tau *} = \dfrac{(25-8\underline{\theta}+16\underline{\theta}^2)(1-\tau)}{64}, p_2^{\tau *} = \dfrac{(49+40\underline{\theta}+16\underline{\theta}^2)(1-\tau)}{64}$

- $q_i^{\tau *} = \dfrac{1}{2}, \ \theta_{12}^{\tau *} = \dfrac{1}{2} + \underline{\theta}$

- We can find that

- $(1). s_1^{t *} > s_1^{\tau *}, s_2^{t *} > s_2^{\tau *}$
  $|\partial s_1^{\tau *}/\partial \tau| < |\partial s_2^{\tau *}/\partial \tau|.$ Why ? $\textcolor{red}{\tau p_2 > \tau p_1}$

  $(2).$ The ad valorem tax affects the quality distribution between the high and low quality firms, but the specific tax will not.

The superiority of the two tax schemes in the long run

- We need a standard to help us to compare the superiority (the efficient quality distribution) between these two tax schemes, again.

- $\hat{\theta}^{s*} = \frac{1}{2} + \underline{\theta}, s_1^{s*} = \frac{1+4\underline{\theta}}{8}, s_2^{s*} = \frac{3+4\underline{\theta}}{8}$

- And, $s_1^{s*} > s_1^{t*} > s_1^{\tau*}, s_2^{t*} > s_2^{s*}, and\ s_2^{t*} > s_2^{\tau*}$

- An increase in τ (from τ=0) will let

- $s_1^{s*} \longrightarrow s_1^{\tau*}$ is bad for social welfare (enhancing distortion);

- $s_2^{\tau*} \longrightarrow s_2^{s*}$ is good for social welfare if τ is smaller (improving distortion);

- $s_2^{s*} \longrightarrow s_2^{\tau*}$ is bad for social welfare if τ is larger (enhancing distortion).

- (s_i^{s*} is fixed)

The superiority of the two tax schemes in the long run

- **Proposition 3.** *Supposing that the market is fully covered and that the quality levels are endogenously determined in the long run, a specific tax is welfare superior (inferior) to an ad valorem tax, when* $\tau \geq (\leq)12/(13 + 16\underline{\theta} + 16\underline{\theta}^2)$.

We also prove that

- **Proposition 4.** *Supposing that the market is fully covered and that the quality levels are endogenously determined in the long run, a specific tax can Pareto dominate an ad valorem tax, when the ad valorem tax rate is high, i.e.,* $\tau \geq$ $60/(37 + 16\underline{\theta} + 16\underline{\theta}^2) \equiv \tau^p$.

Results and Contributions

- Main Results

- (1) A specific tax is superior to an ad valorem tax in the short run, when the marginal cost of the high-quality product is relatively low, or when the marginal cost is relatively high but the ad valorem tax rate is high.

- (2).A specific tax may not only be welfare superior to, but may also Pareto dominate an ad valorem tax in the long run, when the ad valorem tax rate is high.

Results and Contributions

- Contributions

- (1) Reverse the traditional results (welfare ranking and Pareto ranking)

  (a). Suits and Musgrave (1953), Delipalla and Keen (1992), Skeath and Trandel (1994), Schröder (2004), Denicolò and Matteuzzi (2000) and Anderson *et al*. (2001)

  (b). Skeath and Trandel (1994)

- (2) We prove that for inelastic demand, a specific tax is welfare superior to an ad valorem tax when governments would like to collect larger tax revenues, regardless of whether the imposition of the tax scheme can or cannot influence the quality levels.

Thank you for your attention!