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Randomized Strategy Equilibrium and Simultaneous-Move Outcome in the Action Commitment Game with a Small Cost of Leading

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Abstract

We introduce a small cost of leading (small gain from waiting) in the two-player action commitment game formulated by Hamilton and Slutsky (1990). We investigate a quantity competition model with linear demand and constant marginal costs. We find that there exists a unique randomized strategy equilibrium as long as the leading cost is positive and not too large. The randomized strategy equilibrium converges to the simultaneous-move equilibrium (Cournot equilibrium) as the cost of leading approaches zero. We also investigate a price competition model in differential product markets. We show that there exist randomized strategy equilibria, and any of them converge to the simultaneous-move equilibrium (Bertrand equilibrium).

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1 Introduction

In their pioneering work, Hamilton and Slutsky (1990) formulated two important models of the endogenous timing game: observable delay game and action commitment game. Both models have been intensively used as frameworks for analyzing the endogenous role in many subsequent works.

In the action commitment game, Hamilton and Slutsky (1990) showed that three pure strategy equilibria exist: two sequential-move outcomes (either player 1 or player 2 becomes the Stackelberg leader) and one simultaneous-move outcome (both players act in the first period such as in the Cournot model and the Bertrand model). They also emphasized that the simultaneous-move outcome is less plausible because the two sequential-move equilibria are the only pure strategy equilibria in undominated strategies and the simultaneous-move outcome is supported by weakly dominated strategies.

Further, many subsequent works have shown that the simultaneous-move outcome is vulnerable. Albæk (1992), Mailath (1993), Normann (1997), and Hirokawa and Sasaki (2000) investigated incomplete information games wherein waiting players obtain additional information on demand or cost. In other words, leading players lose some informational gain. They show that while a slight informational gain from waiting eliminates the simultaneous-move equilibrium where both players act in the first period, it does not eliminate the two sequential-move outcomes. Robson (1990) and Matsumura (1997, 1999) showed that a slight inventory cost yields the same result in complete information games. These results show that the set of equilibrium outcomes is not lower hemi-continuous with respect to the cost of leading and that the simultaneous-move outcome is an equilibrium only when the cost is strictly zero. This may indicate that the simultaneous-move outcome (such as Bertrand and Cournot) is less plausible in the endogenous timing game than the sequential-move outcomes (such as Stackelberg).

However, recent studies on laboratory experiments have suggested the opposite. Huck et al. (2002) and Fonseca et al. (2005) examined quantity competition setting and reported that the Stackelberg outcomes are rarely observed, while the Cournot outcome often appears.

In this paper, we bridge the gap between the contrasting theoretical and empirical results by investigating a randomized strategy equilibrium in the action commitment game.

In this paper, we introduce a small cost of leading into the action commitment game with quantity competition. In the action commitment game with small leading costs, there are two pure strategy equilibria (either player 1 or player 2 is the leader). Naturally, we expect that there is a randomized strategy equilibrium, too. In fact, we find that a randomized strategy equilibrium does exist and it converges to the simultaneous-move outcome when the cost of leading converges to zero. Our result indicates that the simultaneous-move outcome is not as vulnerable as the current literature insists. We can regard the simultaneous-move equilibrium as a degenerate randomized strategy equilibrium, which is the unique symmetric equilibrium.

We also show that our main result does not depend on the assumptions of strategic substitutes and/or the first-mover advantage, which typically appear in quantity competition models. We investigate a price competition model in differentiated product markets. We find that the model has randomized strategy equilibria and any of them converge to the Bertrand equilibrium under the standard assumptions in the literature on Bertrand competition.

2 Quantity Competition

We formulate a duopoly model with quantity competition. Firms produce perfectly substitutable commodities. Let y_i denote the output of firm i ($i = 1, 2$). Let each firm's marginal production cost be c. The market demand is given by $P(Y) = a - Y$ (price as a function of quantity), where $Y = y_i + y_j$ is the total output of the duopolists. We assume that $a > c$.

We consider the action commitment game of Hamilton and Slutsky (1990) where firms first choose the timing of their actions. The game is a complete information game. There are two possible time stages for production choice and each firm chooses its output in only one of the two stages. In the first stage, firm i independently chooses its output y_i, or waits to produce until the next stage. At the beginning of the second stage, each firm observes the rival's output produced in the first stage. In the second stage, only the firm i that has been waiting until the second stage produces its output y_i.

The payoff of firm i ($i, j = 1, 2, i \neq j$) is given by $U_i = (P(Y) - c)y_i - \varepsilon$ if firm i produces in the first period and $U_i = (P(Y) - c)y_i$ if it produces in the second period, where ε indicates the cost of leading. We assume that $\varepsilon \in [0, (a-c)^2/72)$.[1]

2.1 Fixed Timing Games

Before discussing the endogenous timing game formulated in the previous section, we investigate the fixed timing games (simultaneous-move and sequential-move games).

2.1.1 Cournot game

Consider the simultaneous-move game where firm i ($i = 1, 2$) independently chooses y_i. The first-order condition is

$$a - 2y_i - y_j - c = 0. \tag{1}$$

Let $R_i(y_j)$ be the reaction function of firm i in the Cournot game. It is given by

$$R_i(y_j) = \frac{1}{2}(a - y_j - c). \tag{2}$$

Let the superscript 'C' denote the equilibrium outcome in the Cournot game. We have

$$y_1^C = y_2^C = \frac{1}{3}(a-c). \tag{3}$$

2.1.2 Stackelberg game

Consider the sequential-move game where firm i chooses y_i first and then firm j chooses y_j after observing y_i ($i, j = 1, 2, i \neq j$). Firm j chooses $y_j = R_j(y_i)$ and firm i maximizes $P(y_i + R_j(y_i))y_i - cy_i - \varepsilon$. Let the superscript 'L' and 'F' denote the equilibrium outcome of the leader and the follower in the Stackelberg game, respectively. We have

$$y_i^L = \frac{1}{2}(a-c), \quad y_j^F = \frac{1}{4}(a-c). \tag{4}$$

[1] If $\varepsilon = (a-c)^2/72$, the three pure strategy equilibria again appear. One is a Cournot equilibrium wherein both firms choose the second stage and the other two are Stackelberg equilibria. If $\varepsilon > (a-c)^2/72$, the unique equilibrium exists and it is a Cournot equilibrium.

2.2 Results

We investigate the action commitment game. First, we present two results on the pure strategy equilibria in the general context of the action commitment game.

Result 1 (Hamilton and Slutsky 1990) *Suppose that ε is zero. There are three pure strategy equilibria. One is the Cournot equilibrium where both firms choose to produce in the first stage. The other two are Stackelberg equilibria where one firm chooses to produce in the first stage and the other chooses to produce in the second stage.*

Result 2 (Matsumura 1999) *Suppose that ε is positive and sufficiently small. There are two pure strategy equilibria and both are Stackelberg equilibria.*

We now present our result. We discuss the randomized strategy equilibrium.

Proposition 1 *Suppose that $\varepsilon \in (0, (a-c)^2/72)$. (i) There exists a unique randomized strategy equilibrium and it is symmetric where each firm chooses the first stage with probability $q^E(\varepsilon) \in (0,1)$. (ii) Let $q^E(\varepsilon) \in (0,1)$ be the probability with which each firm chooses to produce in the first stage in the unique randomized strategy equilibrium. Then $q^E(\varepsilon)$ is continuous and strictly decreasing with $\lim_{\varepsilon \to 0} q^E(\varepsilon) = 1$ and $\lim_{\varepsilon \to (a-c)^2/72} q^E(\varepsilon) = 0$.*

Proof See Appendix.

Three equilibria exist regardless of whether ε is zero or positive (as long as $\varepsilon < (a-c)^2/72$). When ε is zero, only pure strategy equilibria exist.[2] When ε is strictly positive and sufficiently small, two pure strategy equilibria and one randomized strategy equilibrium exist. Although the set of pure strategy equilibria is discontinuous at $\varepsilon = 0$, this discontinuity disappears if we consider randomized strategy equilibrium along with pure strategy equilibrium. We can regard the Cournot equilibrium when $\varepsilon = 0$ as a degenerate randomized strategy equilibrium. Our result indicates that the Cournot equilibrium is not so vulnerable if we pay attention to randomized strategy equilibrium. We believe that as is suggested in the empirical works such as

[2] Hamilton and Slutsky (1990) discussed that in the case with linear demand and equal constant marginal costs, no equilibrium, symmetric or asymmetric, with mixing over the timing of actions exists.

Huck et al. (2002) and Fonseca et al. (2005), the Cournot equilibrium is plausible in endogenous timing contexts.

3 Price Competition

In the previous section we discussed quantity competition and showed that the randomized strategy equilibrium converges to the Cournot equilibrium. In this section, we examine price competition in differentiated product markets and show that the equilibrium also converges to the Bertrand equilibrium.

3.1 Fixed Timing Games

Before discussing the endogenous timing game, we investigate the fixed timing game (simultaneous-move and sequential-move games). We assume that the following fixed timing games have a unique pure strategy equilibrium and that is stable. We also assume the interior solution.

3.1.1 Bertrand game

Consider the simultaneous-move game where firm i $(i = 1, 2)$ independently chooses p_i. The payoff of firm i is given by $V_i(p_i, p_j)$ $(i = 1, 2,\ i \neq j)$. We assume that $V_i(p_i, p_j)$ is strictly concave, continuously differentiable and increasing in p_j. We also assume that $|\partial^2 V_i(p_i, p_j)/\partial^2 p_i| > |\partial^2 V_i(p_i, p_j)/\partial p_i \partial p_j|$. This is a so called stability condition.[3] These assumptions are standard in the field.

The first-order condition is

$$\frac{\partial V_i(p_i, p_j)}{\partial p_i} = 0.$$

Let (p_i^B, p_j^B) be the Bertrand equilibrium prices.

3.1.2 Stackelberg game

Consider the sequential-move game where firm i chooses p_i first and then player j chooses p_j after observing p_i $(i, j = 1, 2, i \neq j)$. Let $R_j(p_i)$ be the reaction function of firm j. We assume

[3] See, among others, Vives (1985).

that $R_j(p_i)$ is continuously differentiable and upward-slopping, i.e., $R_j' > 0$. Firm i maximizes $V_i(p_i, R_j(p_i)) - \varepsilon_i$ with respect to p_i. The first-order condition is

$$\frac{\partial V_i(p_i, p_j)}{\partial p_i} + \frac{\partial V_i(p_i, p_j)}{\partial p_j}\frac{dR_j(p_i)}{dp_i} = 0.$$

We assume that $V_i(p_i, R_j(p_i))$ is strictly concave. Let superscript 'L' and 'F' denote the equilibrium outcome of the leader and the follower in the Stackelberg game. Let (p_i^L, p_j^F) be the equilibrium actions in the sequential-move game. We assume that $p_i^L > p_i^F$. This always holds under the assumptions made above if two firms are symmetric. Needless to say, the symmetry of the firms is not a necessary condition.

3.2 Results

We discuss the equilibria of the action commitment game. As in quantity competition, three pure strategy equilibria exist when $\varepsilon = 0$, and two pure strategy equilibria (Stackelberg equilibria) exist when ε is positive and small.

We now present our results on randomized strategy equilibria. Define $\bar{\varepsilon}_i := V_i(p_i^L, p_j^F) - V_i(p_i^B, p_j^B)$. We can show that $\bar{\varepsilon}_i$ is strictly positive.

Proposition 2 *Suppose that* $0 < \varepsilon_1 < \bar{\varepsilon}_1$ *and* $0 < \varepsilon_2 < \bar{\varepsilon}_2$. *(i) For any* $\varepsilon_1, \varepsilon_2$*, a randomized strategy equilibrium exists. (ii) Take any sequence of randomized strategy equilibria where firm* i*'s cost of leading is* ε_i^n *and firm* i *chooses the first stage with probability* q_i^n*. If* $\lim_{n\to\infty} \varepsilon_i^n = 0$*, then* $\lim_{n\to\infty} q_j^n = 1$*. If* $\lim_{n\to\infty} \varepsilon_i^n = \bar{\varepsilon}_i$*, then* $\lim_{n\to\infty} q_j^n = 0$.

Proof. See Appendix.

Under this general setting, the uniqueness of the randomized strategy equilibrium is not guaranteed. However, any randomized strategy equilibria converge to the Bertrand equilibrium as the cost of leading approaches zero. Thus, our main result still holds under strategic complements.

4 Concluding Remarks

In their pioneering work on endogenous timing, Hamilton and Slutsky (1990) showed that three pure strategy equilibria exist: one simultaneous-move equilibrium (Cournot or Bertrand) and two sequential-move equilibria (Stackelberg). Many works pointed out that the Cournot or Bertrand equilibrium is not robust since it does not constitute a pure strategy equilibrium if a small gain from waiting (such as avoiding inventory costs or additional informational gain) is introduced. On the contrary, empirical works using laboratory experiments showed that in the quantity competition, Cournot behavior often appears while Stackelberg behavior is rarely seen. We bridge the gap between the two contrasting results by investigating randomized strategy equilibrium. We find that there exists a randomized strategy equilibrium and it converges to the Cournot or the Bertrand equilibrium as the gain from waiting approaches zero. Thus, we show that the pure strategy simultaneous-move equilibrium in the action commitment game is a degenerate randomized strategy equilibrium.

If we restrict our attention to pure strategy equilibria, we get that the set of equilibrium outcomes is not lower hemi-continuous with respect to the gain from waiting, and that the Cournot or Bertrand outcome is an equilibrium only when the gain from waiting is exactly zero. However, if we extend our attention to randomized strategy equilibria, the set of equilibria is continuous. The equilibrium outcome and equilibrium welfare which are very close to those of the Cournot or the Bertrand equilibrium appear in the randomized strategy equilibrium when the gain from waiting is sufficiently small. Thus, the Cournot or the Bertrand equilibrium may be as robust as the other two Stackelberg equilibria; this result is consistent with the results of Huck et al. (2002) and Fonseca et al. (2005).

Although we show the result under fairly general conditions in strategic complement cases, we failed to show it under general conditions in strategic substitute cases. This remains for future research.

Appendix

Proof of Proposition 1(i) Suppose that firm i chooses to produce in the second stage. At the beginning of the second stage, firm i observes whether or not firm j has produced in the first stage. When firm j chooses to produce in the first stage, firm i chooses $y_i = R_i(y_j) = (a - c - y_j)/2$. When firm j chooses to produce in the second stage, both firms face Cournot competition. As a result, firms choose $y_1 = y_2 = (a - c)/3$.

Suppose that firm i chooses to produce in the first stage. Let superscript '1' implies the first period production. For example, y_i^1 is the output of firm i when it chooses to produce in the first stage. Let q_j be the probability that firm j chooses to produce in the first stage. Firm i chooses y_i^1 so as to maximize $q_j(a - y_i^1 - y_j^1 - c)y_i^1 + (1 - q_j)(a - y_i^1 - R_j(y_i^1) - c)y_i^1 - \varepsilon$. The first-order condition is

$$q_j(a - c - y_j^1 - 2y_i^1) + (1 - q_j)((a - c)/2 - y_i^1) = 0. \tag{5}$$

The best reply of firm i when it chooses to produce in the first stage is

$$y_i^1 = \frac{1}{1 + q_j}\left[\frac{(1 + q_j)(a - c)}{2} - q_j y_j^1\right]. \tag{6}$$

In the randomized strategy equilibrium, the expected payoff of firm i when it chooses to produce in the first stage must be equal to its expected payoff when it chooses to produce in and the second stage. Thus,

$$\begin{aligned} &q_j(a - y_i^1 - y_j^1 - c)y_i^1 + (1 - q_j)(a - y_i^1 - R_j(y_i^1) - c)y_i^1 - \varepsilon \\ &= q_j(a - y_j^1 - R_i(y_j^1) - c)R_i(y_j^1) + (1 - q_j)(a - y_i^C - y_j^C - c)y_i^C. \end{aligned} \tag{7}$$

Substituting $i = 1, j = 2$ and $i = 2, j = 1$ into (6) and (7), we have four equations. Solving these four equations with respect to y_1^1, y_2^1, q_1, and q_2, we have the equilibrium outcome. From the two equations in (6), we have

$$y_i^1 = \frac{1 + q_i}{2(1 + q_i + q_j)}(a - c) \quad (i, j = 1, 2, i \neq j). \tag{8}$$

Substituting (8) into (7), we have

$$\frac{(1 - q_2)(2 + 4q_1 - 5q_2 + 4q_1q_2 + 2q_1^2 - 7q_2^2)(a - c)^2}{144(1 + q_1 + q_2)^2} = \varepsilon. \tag{9}$$

$$\frac{(1-q_1)(2+4q_2-5q_1+4q_2q_1+2q_2^2-7q_1^2)(a-c)^2}{144(1+q_2+q_1)^2}=\varepsilon. \tag{10}$$

Subtracting (10) from (9), we obtain

$$\frac{(q_1-q_2)(11+4q_1+4q_2-5q_1q_2-7q_1^2-7q_2^2)(a-c)^2}{144(1+q_1+q_2)^2}=0. \tag{11}$$

We can show that $11+4q_1+4q_2-5q_1q_2-7q_1^2-7q_2^2=0$ if and only if $q_1=q_2=1$. Thus, at the randomized strategy equilibrium, $q_1=q_2$ must hold.

Substituting $q_1=q_2=q$ into (9), we have

$$\frac{(1-q)(2-q-q^2)(a-c)^2}{144(1+2q)^2}=\frac{(1-q)^2(2+q)(a-c)^2}{144(1+2q)^2}=\varepsilon. \tag{12}$$

Let

$$h(q):=\frac{(1-q)^2(2+q)(a-c)^2}{144(1+2q)^2}.$$

We have

$$h(0)=\frac{(a-c)^2}{72}>0,\ h(1)=0,\ \text{and}\ \frac{dh(q)}{dq}=-\frac{(1-q)(11+5q+2q^2)(a-c)^2}{144(1+2q)^3}<0\ \forall q\in[0,1).$$

Thus, (12) has a unique solution $q^E\in(0,1)$ for all $\varepsilon\in(0,(a-c)^2/72)$—a randomized strategy equilibrium. □

Proof of Proposition 1(ii) By the implicit function theorem,

$$\frac{dq^E(\varepsilon)}{d\varepsilon}=\frac{1}{\frac{dh(q)}{dq}}<0$$

holds for all $q\in[0,1)$. Thus $q^E(\varepsilon)$ is decreasing in ε.

$q^E(\varepsilon)$ is the solution of $h(q)=\varepsilon$. Since $h(1)=0$, we have $q^E(0)=1$. Since $h(0)=(a-c)^2/72$, we have $q^E((a-c)^2/72)=0$. We can show that $q^E(\varepsilon)$ is continuous for all $\varepsilon\in[0,(a-c)^2/72)$, so $q^E(\varepsilon)\to 1$ as $\varepsilon\to 0$ and $q^E(\varepsilon)\to 0$ as $\varepsilon\to(a-c)^2/72$. □

Proof of Proposition 2(i) Suppose that firm i chooses to name its price in the second stage. At the beginning of the second stage, firm i observes whether or not firm j named its price in the first stage. When firm j chooses to name its price in the first stage, firm i chooses $p_i=R_i(p_j)$.

When firm j chooses to name its price in the second stage, firms face Bertrand competition. As a result, firms choose $(p_i, p_j) = (p_i^B, p_j^B)$ in equilibrium.

Suppose that firm i chooses to name its price in the first stage. Let p_i^1 be the price of firm i when it chooses to name its price in the first stage. Further, let p_j^2 be the price of firm j when firm i chooses p_i^1 in the first stage. Let q_j be the probability that firm j chooses to name its price in the first stage. Firm i chooses p_i^1 so as to maximize $q_j V_i(p_i^1, p_j^1) + (1-q_j)V_i(p_i^1, R_j(p_i^1)) - \varepsilon_i$. The first-order condition is

$$\begin{aligned}
&\frac{\partial \left[q_j V_i(p_i^1, p_j^1) + (1-q_j)V_i(p_i^1, R_j(p_i^1)) \right]}{\partial p_i^1} \\
=& q_j \frac{\partial V_i(p_i^1, p_j^1)}{\partial p_i^1} + (1-q_j)\left[\frac{\partial V_i(p_i^1, p_j^2)}{\partial p_i^1} + \frac{\partial V_i(p_i^1, p_j^2)}{\partial p_j^2} \cdot \frac{dR_j(p_i^1)}{dp_i^1} \right] = 0.
\end{aligned} \tag{13}$$

In the randomized strategy equilibrium, the expected payoff of firm i when it chooses to name its price in the first stage must be equal to its expected payoff when it chooses to name its price in the second stage. Thus,

$$q_j V_i(p_i^1, p_j^1) + (1-q_j)V_i(p_i^1, R_j(p_i^1)) - \varepsilon_i = q_j V_i(R_i(p_j^1), p_j^1) + (1-q_j)V_i(p_i^B, p_i^B). \tag{14}$$

Substituting $i = 1, j = 2$ and $i = 2, j = 1$ into (13) and (14), we have four equations. Solving these four equations with respect to p_1^1, p_2^1, q_1, and q_2, we have the equilibrium outcome.

Since we assume $|\partial^2 V_i(p_i, p_j)/\partial^2 p_i| > |\partial^2 V_i(p_i, p_j)/\partial p_i \partial p_j|$, we can apply the implicit function theorem to the two equations in (13). From the two equations in (13), we have the solution $p_i^1(q_i, q_j), p_j^1(q_i, q_j)$, which is continuously differentiable for all $q_i, q_j \in [0, 1]$.

Substituting $p_i^1(q_i, q_j), p_j^1(q_i, q_j)$ into (14), we have

$$\begin{aligned}
&q_j[V_i(p_i^1(q_i, q_j), p_j^1(q_i, q_j)) - V_i(R_i(p_j^1(q_i, q_j)), p_j^1(q_i, q_j))] \\
&\quad +(1-q_j)[V_i(p_i^1(q_i, q_j), R_j(p_i^1(q_i, q_j))) - V_i(p_i^B, p_i^B)] = \varepsilon_i.
\end{aligned} \tag{15}$$

Note that if $q_j = 0$, then $p_i^1(q_i, q_j) = p_i^L$ and the left-hand side of (15) equals $\bar{\varepsilon}_i$. Note also that if $q_j = 1$, then $p_i^1(q_i, q_j) = R_i(p_j^1)$ and the left-hand side of (15) equals zero. Hence, the left-hand side of (15) is greater (smaller) than the right-hand side of (15) when $q_j = 0$ (1). Thus, we can apply the fixed point theorem shown in Garcia and Zangwill (1979). Under these conditions, for any $\varepsilon_i, \varepsilon_j$ there exists (q_i^*, q_j^*) that comprises a randomized strategy equilibrium. $\square$

Proof of Proposition 2(ii). First, we prove that $\lim_{n\to\infty} q_j^n = 1$ if $\lim_{n\to\infty} \varepsilon_i^n = 0$. We prove it by contradiction. Suppose not. Then there exists a sequence of randomized strategy equilibrium such that $\{\varepsilon_i^n\}_{n=1}^{\infty}$ converges to 0 and $\{q_j^n\}_{n=1}^{\infty}$ does not converge to 1.

Let q_j^0 be $\liminf_{n\to\infty} q_j^n$. Since $q_j^n \leq 1$, $q_j^0 < 1$. Since $p_i^1(q_i, q_j) = p_i^L$ when $q_j = 0$, $q_j^0 > 0$ holds from (15).

Let $\{q_j^k\}_{k=1}^{\infty}$ be a subsequence of $\{q_j^n\}_{n=1}^{\infty}$ which converges to q_j^0. Note that $\{\varepsilon_i^k\}_{k=1}^{\infty}$ converges to 0.

By the continuity of p_i^1, p_j^1, R_j, and V_i,

$$\begin{aligned}
&\lim_{k\to\infty}[q_j^k(V_i(p_i^1(q_i^k, q_j^k), p_j^1(q_i^k, q_j^k)) - V_i(R_i(p_j^1(q_i^k, q_j^k)), p_j^1(q_i^k, q_j^k)))\\
&\quad + (1-q_j^k)(V_i(p_i^1(q_i^k, q_j^k), R_j(p_i^1(q_i^k, q_j^k))) - V_i(p_i^B, p_j^B))]\\
=&q_j^0(V_i(p_i^1(q_i^0, q_j^0), p_j^1(q_i^0, q_j^0)) - V_i(R_i(p_j^1(q_i^0, q_j^0)), p_j^1(q_i^0, q_j^0))) \qquad (16)\\
&\quad + (1-q_j^0)(V_i(p_i^1(q_i^0, q_j^0), R_j(p_i^1(q_i^0, q_j^0))) - V_i(p_i^B, p_j^B))\\
=&\lim_{k\to\infty} \varepsilon_i^k = 0
\end{aligned}$$

holds from (15). Thus, $q_j^0 \in (0,1)$ and $\varepsilon_i^0 = 0$ comprise the randomized strategy equilibrium.

On the other hand, for any $q_j \in (0,1)$ $p_i^1(q_i, q_j)$ must be in (p_i^B, p_i^L).[4] Consider firm i's payoff when i chooses $p_i^1 = R_i(p_j^1(q_i^0, q_j^0))$ in the first period:

$$q_j^0 V_i\Big(R_i(p_j^1(q_i^0, q_j^0)), p_j^1(q_i^0, q_j^0)\Big) + (1-q_j^0)V_i\Big(R_i(p_j^1(q_i^0, q_j^0)), R_j(R_i(p_j^1(q_i^0, q_j^0)))\Big). \qquad (17)$$

Since $p_i^B < p_i^1(q_i^0, q_j^0)$ and R_j is upward slopping, $p_j^B < R_j(p_i^1(q_i^0, q_j^0))$. Because $p_j^F = R_j(p_i^L)$ and $p_i^1(q_i^0, q_j^0) < p_i^L$, $R_j(p_i^1(q_i^0, q_j^0)) < p_j^F$. By the assumption of $p_i^F < p_i^L$, we have $R_j(p_i^1(q_i^0, q_j^0)) < p_j^L$. Since $V_i(p_i, R_j(p_i))$ is strictly concave, $V_i(p_i, R_j(p_i))$ increases in p_i for $p_i \in (p_i^B, p_i^L)$. Because we have shown that $R_j(p_i^1(q_i^0, q_j^0)) \in (p_i^B, p_i^L)$, $V_i(R_i(p_j^1(q_i^0, q_j^0)), R_j(R_i(p_j^1(q_i^0, q_j^0)))) > V_i(p_i^B, R_j(p_i^B)) = V_i(p_i^B, p_j^B)$. Thus, (17) is strictly larger than $V_i(p_i^1(q_i^0, q_j^0), R_j(p_i^1(q_i^0, q_j^0))) -$

[4] This was suggested in Hamilton and Slutsky (1990), and was shown in Pastine and Pastine (2004).

$V_i(p_i^B, p_j^B)$. Since firm i's price when it names its price in the first stage is $p_i^1(q_i, q_j)$, (17) is equal to or smaller than $V_i(p_i^1(q_i^0, q_j^0), p_j^1(q_i^0, q_j^0)) - V_i(R_i(p_j^1(q_i^0, q_j^0)), p_j^1(q_i^0, q_j^0))$. Thus, the equality in (16) does not hold—a contradiction.

Next, we prove that $\lim_{n\to\infty} q_j^n = 0$ if $\lim_{n\to\infty} \varepsilon_i^n = \bar{\varepsilon}_i$. The first term in (15) is non-positive and the second term in (15) is at largest $(1 - q_j)\bar{\varepsilon}_i$. Thus q_j must be close to zero when ε_i is close to $\bar{\varepsilon}_i$. $\square$

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