Prize Sharing in Collective Contests

Abstract

The characteristics of endogenously determined sharing rules and the group-size paradox are studied in a model of group contest with the following features: (i) The prize has mixed private-public good characteristics. (ii) Groups can differ in marginal cost of effort and their membership size. (iii) In each group the members decide how much effort to put without observing the sharing rules of the other groups. We provide simple characterizations of the relationship between group characteristics, performance of the competing groups (winning probability and per capita expected utility) and the type of sharing rules they select. Interestingly, richer and more efficient groups or groups with larger valuation of the prize tend to be more equalitarian. We also clarify under what circumstances such tendency is due to larger membership.

Keywords: collective contest, mixed public-good prize, endogenous sharing rules, the group-size paradox.

1. Introduction

This study considers collective contests for group-specific benefits. Examples of such contests include a competition by local governments for subsidies, an R&D race by research groups or a competition for resources by countries. Given the prevalence of collective contests and their significant efficiency and distributional implications, the question how various characteristics of competing groups affect the results of such contests has been of major concern in the relevant literature in economics and political science. The main objective of this study is to re-examine the question in the extended setting where unobservable group sharing rules are determined endogenously. It turns out that adding this new feature of endogenous unobservable sharing rules enables the derivation of new unequivocal results regarding the relationship between the contest parameters and the endogenous variables. In our model, m (≥ 2) competing groups can differ both in their membership and in their efficiency. The efficiency differences are represented by variable costs of effort made by the individual group members. So the model covers a rather wide range of *asymmetric group contests*, i.e., contests by groups with different characteristics.

A special feature that distinguishes this type of contests from others is that, as clearly argued by Olson (1965), the contestants have to cope with the collective-action or the free-rider problem. Olson also recognized the possibility that "selective incentives" within a group can mitigate the problem. In a contest, how to share the prize plays a critical role in providing such incentives. Studies of collective contests on private-good prizes do consider alternative ways of prize division among the members of the winning group. One possibility is that, as in Katz and Tokatlidu (1996), Wärneryd (1998) and Konrad (2004), the division of the prize is also determined non-cooperatively, subsequent to its award to the winning group[1]. Another possibility is that, prior to the contest on the private-good prize, members of a group agree on the sharing rule of the prize. Nitzan (1991) parameterizes the sharing rule applied by a group as a linear combination between the *equalitarian* and the *relative effort* sharing rules. Under such rules part of the prize is divided equally among the group members (according to the equalitarian rule) and the rest is divided proportionally to the members' efforts (according to the relative effort rule)[2]. Lee (1995) and Ueda (2002) examine the endogenous determination of such group sharing rules of the prize by adding a stage where each group selects the rule so that it maximizes its welfare.

On first glance, the decision on the group sharing rule might appear trivial. The larger the part of the prize divided according to the relative effort rule, the stronger the selective incentives are, and, in turn, the higher the winning probability of the group. Why not share, therefore, the whole prize using the relative effort rule[3]? The answer is based on the classical arguments of Sen (1966) who pointed out, in the context of producer cooperatives, that the relative effort sharing rule can induce too much effort from the members of a group to attain a Pareto-efficient outcome (for the

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[1] The studies following this line of research (see Hausken (2005) and Konrad (2006) for a comprehensive survey) treat explicitly the intra and inter- group conflicts. In this hierarchical structure of contests the private-good prize is not really shared by the members of the winning group. In fact, such two-stage competition implies that any pre-agreement regarding the prize sharing among the group members is impossible and the anarchic division of the prize within a group determines the scope of effective selective incentives.

[2] The class of group sharing rules has an alternative interesting interpretation. As argued by Baik (1994) and Baik and Lee (2001), it can be interpreted as a "winner-help-loser" agreement, or a self insurance device applied by the groups.

[3] Actually, the both of Lee and Ueda, who assume linear benefit functions of the prize and linear cost functions of effort, conclude that all competing groups usually choose to share the whole prize according to the relative effort rule.

group). The maximization of the group's winning probability differs from the maximization of the group's welfare, and, consequently, competing groups can choose different group sharing rules, depending on their characteristics. In particular, when competing groups are asymmetric in their efficiency or membership size, the group selecting the highest weight of the relative effort rule is not necessarily the one attaining the highest winning probability. The relationship between the winning probabilities, welfare, and the intensities of selective incentives of competing groups is by no means transparent. A valuable feature of our extended model is that it enables the derivation of several general properties of this relationship, that cannot be obtained in the less general models of collective contests examined in the existing literature.

Our analysis is based on the assumption of "unobservable group sharing rules," which has been recently introduced by Baik and Lee (2007). Former studies of collective contests with predetermined group sharing rules presume that each group's sharing rule is observable from outside. The decision on the group sharing rule is, however, made within a group. Even if a group announces its sharing rule to other competing groups before the contest, its members could be inclined to secretly change it if deemed desirable for all of them. To make the announcement credible, the outsiders must be assured that such a secret change is impossible. Without restrictive assumptions that decisions made within a group are transparent and detection of the changes is easy, a model of group contests with observable group sharing rules fails therefore to be persuasive. Furthermore, whether the group sharing rule is observable or not makes a critical difference for the decisions of the competing groups. The reason is that an irreversible choice of an observable sharing rule works as a strategic commitment. A group sharing rule determines the relationship between each member's effort and the share of the prize. Hence, if the sharing rule of a group is observable, then its change affects the effort of the individuals in other groups via the change in their expectations of the effort level of the group.

We will exclude such unreliable strategic effects by assuming that the agreed sharing rule in a group is unobservable by members of the other groups[4]. But this assumption has a price: the subgame-perfect equilibrium is no longer an effective solution concept[5]. When the sharing rule of a group is unobservable from outside, the form of the common utility function of its members becomes private information of the group. Outsiders cannot know the return a member of the group gets by increasing his effort. Furthermore, the group sharing rules must be determined in anticipation of the occurrence of such private information in the contest. As will be clarified below, the extensive form of such a game has no proper subgame. We therefore propose the pure-strategy perfect Bayesian equilibrium as a solution concept, with an innocuous regularity condition on the belief profiles. It will be shown that such equilibrium uniquely exists in our model. The relation between our approach and that of Baik and Lee will also be discussed later.

The last, but not the least, merit of our model is its comprehensive treatment of the nature of the contested prize. In the arguments on competing asymmetric groups, a prominent topic is the "group size paradox," i.e., a larger group is less effective in pursuing its interest because of the free-rider problem. Hence, the

[4] In the field of industrial organization, the strategic effects of internal contracts between competing vertical structures are often mentioned. Observability of the contracts is seen as an important factor there. See Kats (1991).

[5] In the group contest with endogenous and *observable* group sharing rules, we can naturally apply the subgame-perfect equilibrium as the solution concept. After the determination of the group sharing rules, every member of every group enters a contest with full knowledge of all the sharing rules. For any configuration of group sharing rules, the corresponding contest is thus a proper subgame.

existence of group-size advantage in a collective contest has been questioned. Inspired by the analysis of Chamberlin (1974), some researchers have examined the possibility of the group-size advantage relating to the degree of rivalry in the consumption of the prize. Katz, Nitzan and Rosenberg (1990) and Riaz, Shogren and Johnson (1995) examine the problem in the case where the prize is a pure public good for each group[6]. It has been shown that, in this setting, a group with larger membership attains a winning probability larger than or equal to that of a smaller group. In terms of winning probabilities, therefore, a larger group is not less effective in group contests for pure public prizes. The group size paradox is not valid in such cases.

As an important extension, Esteban and Ray (2001) study collective contests with a mixed private-public-good prize; part of the prize is a public good and part of the prize is a private good. They have been able to derive a simple sufficient condition relating to the elasticity of the marginal cost of effort that ensures that a larger group attains a higher winning probability. The validity of the group-size paradox has been thus restricted in a wide range of cases, even if the prize contains a private good part. We could argue, however, that their approach is not thorough, because they postulate that the private part of the prize is equally divided in the winning group. As we have already pointed out, a group could adapt its sharing rule on the private part of the prize, to enhance its advantage in the contest. One of our chief concerns is to examine how endogenous group sharing rules affect the validity of the group-size paradox, in the Esteban and Ray - type contests for a mixed private-public-good prize. Actually we treat more general cases by permitting imperfect

[6] Ursprung (1990) provides an interesting application of this kind of collective contests to a two-candidate electoral competition. For another approach which applies all-pay auction to a contest for a public-good prize, see Baik, Kim and Na (2001).

substitutability between the public and the private components of the contested prize. Our setting thus provides a very rich and flexible basis to analyze the effect of group and prize characteristics on the performance of groups in collective contests and on their unobservable, endogenously chosen sharing rules.

We have been able to establish a simple relationship, in equilibrium, between the characteristics of a group, its selected sharing rule, its winning probability and per capita expected utility. Our first main result is that *a group attaining a higher winning probability chooses a more equalitarian sharing rule* that divides a larger part of the private-good component of the prize equally, independent of each member's performance in the contest. We then derive several clear-cut results on the effect of asymmetry in the efficiency or in the membership size of the contestants. Specifically, it turns out that a larger group always attains a higher winning probability, unless the prize is purely private. In this sense, *the group-size paradox never occurs*, provided that *the sharing rules are allowed to be endogenous and the prize includes some public part*. In all the results, the elasticity of the benefit from the private part of the prize, which is denoted by η, plays an important role.

The next section presents our extended model of collective contests. Section 3 contains the equilibrium analysis and the first main result. Section 4 considers general asymmetric group contests, and presents the basic relationship between the characteristics of a group, its selected sharing rule, its winning probability and per capita expected utility, including the results on the group-size paradox. Some concluding remarks appear in Section 5. All proofs are given in Section 6.

2. The Extended Group Contest

(a) Prize, Benefit and Cost.

Let us consider a contest in which m groups compete for a prize. The membership of group i is denoted by N_i ($i = 1, \dots, m$). We assume that the prize is a mixture of public and private goods. That is, a winning group gets some group-specific public goods and private goods that can be shared among its members[7].

For simplicity, we assume that every member of every group applies the same benefit function $B(q, G)$ to evaluate the prize, where q is the amount of the private good distributed to the individual and G is the amount of group-specific public good provided to the group to which the individual belongs. This function is twice differentiable, and $B(q, G) > 0$ unless $(q, G) = (0, 0)$. Furthermore, $B_q > 0$, $B_G \geq 0$, and $B_{qq} \leq 0$ hold for all $q > 0$, $G > 0$. The CES benefit function $B(q,G) = \left(b_1 q^{\rho} + b_2 G^{\rho}\right)^{\frac{1}{\rho}}$ with $0 < b_1 < 1$, $0 < b_2 < 1$ and $\rho \leq 1$, satisfies these conditions. We will refer to this useful special case later.

We normalize the total prize to unity, and denote the ratio of the private-good part by γ $(0 < \gamma \leq 1)$. That is, the model covers all prize compositions but the pure

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[7] Such a mixed prize can be found, for example, in R&D contests. In such contests, the prize won by one of the competing groups consists of improved reputation (the status and recognition associated with winning the R&D race, which can be equally shared by all members of the winning firm) and of monetary benefits (the profit associated with winning the contest, that can be shared equally or not-equally by some or by all group members). In regional, community or government division contests the prize is often some budget, part of which can take the form of monetary transfers while the rest must be used to supply some local public goods, see Nitzan (1994). When a local government wins a contested subsidy earmarked for some public undertaking, part of it can be provided as an extra margin for the employed local people. Even an electoral competition can be conceived as a contest on a prize with mixed private-public good components, because a winning candidate is typically committed to the provision of both public and private benefits to his supporters. Finally, if members of the winning group jointly taste the delight of victory, any group contest for a private-good prize is actually that for a mixed prize.

public-good case. The ratio is given exogenously. We assume that, prior to the contest, the 'planner' of each group determines the rule applied for sharing the private part of the prize. This rule is assumed to be chosen from the class of sharing rules that are linear combinations of the equalitarian and the relative-effort sharing rules. Denote the weight of the relative-effort rule in group *i* by δ_i. Then, if group *i* wins the contest, a member of the group having put effort $a \geq 0$ receives the benefit

$$B\left(\gamma \cdot (\delta_i \cdot \frac{a}{A_i} + (1-\delta_i) \cdot \frac{1}{N_i}), 1-\gamma\right),$$

where A_i is the aggregate amount of effort put by the members of group *i*.

A member of group *i* incurs the cost $v_i(a)$ when making an effort equal to *a* while trying to win the prize. The cost function is symmetric within a group, but it can differ across the competing groups. For every *i*, let $v_i(0) = 0$, $v_i'(a) > 0$ and $v_i''(a) \geq 0$ for all $a > 0$. To guarantee that every individual chooses a positive effort in equilibrium, we also assume that $\lim_{a \to 0} v_i'(a) = 0$[8].

(b) The Structure of the Contest.

The extended group contest proceeds in two stages. In the first stage, the decisions on the value of δ_i , that is, the group sharing rules of the private-good part of the prize, are made simultaneously by the planners of the groups. The objective of each planner is to maximize some group welfare function which is strictly increasing with respect to the utility of every member of the group. Such maximization ensures the selection of a Pareto-efficient group outcome. It should be pointed out that the introduction of

[8] This assumption rules out linear cost functions. The main reason for this assumption is to avoid cumbersome cases of "oligopolization," i.e., some groups put no effort in the contest. While many researches ignore such cases, they usually appear in models of group contests with linear costs. See Ueda (2002).

the planners is a purely expository device to rigorously fit our description of the equilibrium to the standard non-cooperative game form. The results of the model would not change, if the group sharing rule is cooperatively committed by the members who wish to attain a (group) Pareto optimum.

In the second stage, each participant in the contest described in the previous subsection, who is a member of some group, chooses his/her effort level individually, given the group sharing rule determined in the first stage. But we introduce here the assumption that the value of δ_i is the private information for the individuals belonging to group i. *A group sharing rule is not observable from outside.* Consequently, the second stage under any configuration of group sharing rules cannot be a proper subgame: each player, an individual in a competing group, cannot specify the payoff functions of the other groups' members. (Notice that the relation of their benefits and efforts are not determined if their groups' sharing rules are not specified.) Every individual is required to infer the sharing rules in the other groups, when he/she makes a decision on effort.

Given the effort levels put by all individuals, the contest winning probability of group i is determined by

$$\pi_i = \frac{A_i}{\sum_{k=1}^{m} A_k}, \tag{1}$$

where A_k is the total amount of effort made by the members of group k. Although we apply the common simple lottery contest success function, notice that our model allows heterogeneity in the contestants' effectiveness by allowing differences in the cost functions of the groups.

3. Equilibrium.

As we have argued in the last section, a contest under any configuration of group sharing rules cannot be a proper subgame of our model. At the beginning of a contest, members of a competing group observe their group sharing rule, and also perceive that similar observations are made in the other groups. Everyone knows that the contest will take place under some configuration of the group sharing rules, but no one completely knows which configuration is realized. Reflecting such ignorance, every individual's information set already contains multiple nodes at the outset of the contest. Absence of a proper subgame makes futile the refinement by the notion of subgame perfect equilibrium. This is the reason why we use the perfect Bayesian equilibrium notion, even-though our model is not an incomplete information game.

More precisely, we will use the pure-strategy perfect Bayesian equilibrium as a solution concept, with a regularity condition on the belief profiles. For simplicity, we assume that only pure strategies are available for each player. That is, we do not consider the possibility of randomization in each information set.

To define a perfect Bayesian equilibrium, we need to consider an assessment, that is, a specification of a pair of an action and a belief, for each information set. The decisions on the group sharing rules by the planners are made simultaneously at the beginning of the game. For their information sets, therefore, the belief is trivial. In equilibrium, each group's sharing rule is chosen to maximize the planner's group welfare function, given the decisions on the sharing rules in the other groups and the succeeding actions of the individual contestants.

So let us turn to the decision on effort by a member, say the kth member of group i. Being told the sharing rule in his/her own group, the member faces the effort decision in the possible contests. So an individual's information set can be indexed by a value of δ_i, that corresponds to his/her group sharing rule to which this person becomes aware at the beginning of the contest. It is impossible to distinguish among the nodes at which different sharing rules are chosen in some other groups. A strategy of the member is, therefore, described as a function of δ_i, which is denoted by $a_{ik}(\delta_i)$. Also, his/her belief μ_{ki} with respect to the sharing rules of other groups can be constructed, depending only on the value of δ_i. For each δ_i, the value $\mu_{ki}(\delta_i)$ is a probability measure defined on $[0, 1]^{m-1}$, the space of possible configurations of the sharing rules in the other groups,

$$\delta_{-i} = (\delta_1, \cdots, \delta_{i-1}, \delta_{i+1}, \cdots, \delta_m) .$$

Now, pick a profile of the sharing rules δ_1^*, …, δ_m^* and individual decisions on effort, $(a_{jh}^*(\delta_j), \mu_{jh}^*(\delta_j))$, for all $j = 1, \ldots, m$; $h = 1, \ldots, N_j$, and $\delta_j \in [0, 1]$. Let us specify the necessary conditions for the profile to be a perfect Bayesian equilibrium. In a general representation, the expected utility of the kth member of group i at the information set indexed by δ_i' is calculated as

$$\int \frac{A_i^*(\delta_i')}{\sum_{j \neq i} A_j^*(\delta_j) + A_i^*(\delta_i')} B\left[\gamma \cdot \left(\delta_i' \cdot \frac{a_{ik}^*(\delta_i')}{A_i^*(\delta_i')} + \frac{1-\delta_i'}{N_i}\right), 1-\gamma\right] \mu_{ik}^*(\delta_i')(d\delta_{-i}) - v_i(a_{ik}^*(\delta_i')), \qquad (2)$$

where $A_j^*(\delta_j) = \sum_{h=1}^{N_j} a_{jh}^*(\delta_j)$. At the information set indexed by δ_i^*, which lies on the equilibrium path, however, the requirement of consistency considerably simplifies the belief, which has to satisfy

$$\mu_{ik}^*(\delta_i^*)\left(\left\{\delta_{-i} = \delta_{-i}^*\right\}\right) = 1 .$$

That is, the individuals must correctly infer the sharing rules in the other groups, given the strategies of the planners.

Now, let us introduce a regularity condition with respect to the beliefs at the un-reached information sets. We say that the belief by the kth member of group i, μ_{ik}^*, in a perfect Bayesian equilibrium is *stable* if

$$\mu_{ik}^{*}(\delta_i)\left(\left\{\delta_{-i}=\delta_{-i}^{*}\right\}\right)=1\text{, for all }\ \delta_i\neq\delta_i^{*},$$

i.e., the individual believes that any deviation from the sharing rule in his own group is irrelevant to the decisions made in the other groups. In our model, it is very natural to require that μ_{jh}^{*} is stable for all j and h, because the determination of the group sharing rule is not observable from outside. Even if an individual is told an out-of-equilibrium sharing rule by the planner, there is no reason for him/her to believe that such a deviation affects decisions by outside groups. Henceforth, we take such *a pure-strategy perfect Bayesian equilibrium with stable belief profiles* as the solution concept of our model. Then, we have the following basic result:

Lemma 1. *A profile of strategies* $\delta_1^{*}, \ldots, \delta_m^{*}$, *and* $a_{jh}^{*}(\delta_j)$, *for all* $j = 1, \ldots, m$; $h = 1, \ldots, N_j$, *and* $\delta_j\in[0,1]$ *constitute a pure strategy perfect Bayesian equilibrium with stable belief profiles, if and only if the following two conditions hold:*

Condition 1. For all $i = 1, \ldots, m$; $k = 1, \ldots, N_i$, *and* $\delta_i\in[0,1]$, $a_{ik}^{*}(\delta_i)=\dfrac{A_i^{*}(\delta_i)}{N_i}$ *with*

$$(1-\frac{A_i^{*}(\delta_i)}{A})\cdot B\left(\frac{\gamma}{N_i},1-\gamma\right)+\gamma\delta_i\cdot\frac{\partial B\left(\frac{\gamma}{N_i},1-\gamma\right)}{\partial q}\cdot(1-\frac{1}{N_i})-v_i'\left(\frac{A_i^{*}(\delta_i)}{N_i}\right)\cdot A=0, \tag{3}$$

where $A=\sum_{j\neq i}A_j^{*}(\delta_j^{*})+A_i^{*}(\delta_i)$.

Condition 2. $\delta_i^{*}\in\underset{0\leq\delta_i\leq 1}{\arg\max}\,\dfrac{A_i^{*}(\delta_i)}{A}\cdot B\left(\dfrac{\gamma}{N_i},1-\gamma\right)-v_i\left(\dfrac{A_i^{*}(\delta_i)}{N_i}\right)$, *for all* $i = 1, \ldots, m$.

Remark 1: Condition 1 requires that every individual belonging to the same group chooses a symmetric equilibrium effort (of course, those belonging to different groups can choose different effort levels), and therefore, attains a symmetric utility level. As long as a planner seeks to maximize a group welfare function which is strictly increasing with respect to the utility of every member, he/she must maximize the per capita utility. Hence we need not be concerned about the identity of the planner if he/she is one of the group members.

Remark 2: One can see that the conditions given in Lemma 1 correspond to the equilibrium conditions presented in Baik and Lee (2007). Although their proposed definition of equilibrium for a group contest with unobservable group sharing rules is simpler than ours, it involves some difficulties. Lemma 1 provides a theoretical foundation that rationalizes their proposal; it implies that their equilibrium conditions can be conceived as an appropriate "short-cut" for those obtained by a pure-strategy perfect Bayesian equilibrium[9].

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[9] As equilibrium conditions, Baik and Lee require that (1) in every group i, member h chooses his effort level under δ_i, denoted by $a_{ih}^*(\delta_i)$, to maximize the expected utility, given all other individuals' efforts and the group sharing rules of the other competing groups; and also, (2) in each group i, the equilibrium group sharing rule δ_i^* maximizes the group welfare given $a_{ih}^*(\delta_i)$ for $h = 1, \ldots, N_i$, and the efforts by all individuals in all other groups. These requirements seem natural and one might find no ambiguity in them at first glance. To exclude incredible threats, each member's effort in a contest should be restricted to maximize his/her utility, given the observed sharing rule in his/her own group and the strategies of members of the other groups. However, that given *strategy* of each member in other groups cannot be a single *value* of effort level, because everyone can change the effort, depending on the sharing rule observed in his/her own group. A member of group i must optimize his effort level for each $a_{jh}^*(\delta_j)$ ($j \neq i$, $h = 1, \ldots, N_j$), which is a *function* of the unobservable group sharing rule. It is ultimately necessary, therefore, to introduce the belief of every group member on group sharing rules in the other groups, even though his/her payoff does not directly depend on them.

Thus Baik and Lees's equilibrium conditions require an explicit explanation why the best responses correspond to fixed actions and not to the other players' strategies. Otherwise, one can reasonably claim that, in fact, each member in a group makes a decision knowing the group sharing rules in the other groups, which is incompatible with the basic assumption of the model. Our regularity condition of the "stable belief" rationalizes the equilibrium conditions of Baik and Lee, and we argue that each individual's belief on the sharing rules in the other groups would usually be independent of the sharing rule chosen in his/her own group. Using the perfect Bayesian equilibrium as a solution concept for the model is not a redundant detour, but an essential procedure.

Remark 3: Let the vector $\delta = (\delta_1, \cdots, \delta_m)$ denote the actual profile of the sharing rules given prior to the contest. Condition 1 implies that the effort levels of the members of the *i*th group do not depend on δ, but only on δ_i. A change of the sharing rule in other groups is not detected by the members of the group and has no effect on their behavior. This is the main implication of the unobservable group sharing rules[10].

By equation (3), $A_i^*(\delta_i)$ is differentiable with respect to δ_i and $\frac{\partial A_i^*}{\partial \delta_i} > 0$. Then, as a necessary and sufficient condition of Condition 2 for group *i*, we have:

$$\frac{A - A_i^*(\delta_i^*)}{A} \cdot B\left(\frac{\gamma}{N_i}, 1-\gamma\right) - v_i'\left(\frac{A_i^*(\delta_i^*)}{N_i}\right) \cdot \frac{A}{N_i} \geq (\leq)\ 0 \text{ if } \quad \delta_I^* > 0\ (< 1). \qquad (4)$$

By equation (3), we can see that $\delta_i^* = 0$ is impossible. Notice that the left-hand side of (4) is strictly decreasing with respect to the value of A_i^*. Therefore, if (4) holds as an equality for some $\delta_i < 1$, it must be the unique solution and equal to δ_i^*. Otherwise, the per capita utility is strictly increasing with respect to δ_i on [0,1], and $\delta_i^* = 1$ must hold.

Suppose, firstly, that group *i* has an *interior* group sharing rule $0 < \delta_i^* < 1$ in equilibrium. Denoting the winning probability of group *i* by $\pi_i = \frac{A_i^*(\delta_i^*)}{A}$, equation (4) has the form:

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[10] If the sharing rules were observable from outside, the effort levels by members of a group would depend on the whole vector δ. The total amount of effort made by the members of group *j* could then be written as $A_j(\delta)$. In turn, a change of a sharing rule in one group would directly affect the effort levels in the other groups. By such a strategic effect, condition 2 would have the form

$$\delta_i^* \in \arg\max_{0 \leq \delta_i \leq 1} \frac{A_i(\delta_{-i}^*, \delta_i)}{A} \cdot B\left(\frac{\gamma}{N_i}, 1-\gamma\right) - v_i\left(\frac{A_i(\delta_{-i}^*, \delta_i)}{N_i}\right), \textit{ for all } i = 1, \ldots, m.$$

where $A = \sum_{j=1}^{m} A_j(\delta_{-i}^*, \delta_i)$.

$$(1-\pi_i)\cdot B\left(\frac{\gamma}{N_i},1-\gamma\right)-v_i'\left(\frac{A}{N_i}\cdot\pi_i\right)\cdot\frac{A}{N_i}=0\cdot \tag{5}$$

Substitution of (5) into (3) yields our first main result.

Proposition 1. *An interior equilibrium sharing rule of the private-good component of the prize among the N_i members of group i is given by*

$$\delta_i^* = \frac{1-\pi_i}{\eta(N_i,\gamma)}, \tag{6}$$

where

$$\eta(N_i,\gamma)=\frac{\partial}{\partial q}B\left(\frac{\gamma}{N_i},1-\gamma\right)\cdot\frac{\frac{\gamma}{N_i}}{B\left(\frac{\gamma}{N_i},1-\gamma\right)} \tag{7}$$

is the elasticity of the benefit from the private part of the prize.

The proposition establishes that there exists a direct relationship between the winning probability and the endogenously determined share of the private part of the prize that is equally distributed among the group members, as long as the sharing rule is interior. Notice that since the benefit function is concave with respect to the private part of the prize, we get that $\eta(N_i,\gamma)\le 1$, with strict inequality unless the prize is purely private ($\gamma=1$) and the benefit function is linear with respect to the private part of the prize. Using Lemma 1 and Proposition 1, we can confirm the following result.

Proposition 2. *In an extended group contest, there exists a unique pure strategy perfect Bayesian equilibrium with stable belief profiles.*

4. Group Characteristics and Performance

Henceforth we concentrate on interior equilibria in which every group chooses a "mixed" sharing rule of the private component of the prize, i.e., $0 < \delta_i^* < 1$, for all $i = 1, \ldots, m$. We first discuss the implications of Proposition 1 regarding the relationship between the group equalitarianism and its winning probability. We then examine asymmetric contests and show how differences in the cost function and group membership affect the winning probabilities, per capita utility and the selected group sharing rules.

(a) The Selected Sharing Rule, Performance and Benefit Elasticity

By Proposition 1, an interior equilibrium sharing rule must satisfy equation (6). From this surprisingly simple relationship, we can conclude that a group attaining a higher winning probability chooses a lower δ_i, or a more equalitarian sharing rule, other things being equal. It also makes clear that δ_i is negatively related to $\eta(N_i, \gamma)$. That is, a group with a higher elasticity of benefit from the private part of the prize tends to be more equalitarian.

Equation (3) implies that a higher weight on the relative effort rule used by a group induces more effort from its members. So Proposition 1 might seem paradoxical. Let us clarify the point by resorting to the following proposition.

Proposition 3. *If two groups k and l are symmetric, i.e.* $N_k = N_l$ *and* $v_k = v_l$*, then* $\delta_k^* = \delta_l^*$ *holds and these groups share the same winning probability and per-capita utility. Furthermore, if all competing groups are symmetric, then their common weight of the relative effort rule* δ^* *is given by*

$$\delta^* = \frac{1 - \frac{1}{m}}{\eta(N, \gamma)}. \tag{8}$$

The proposition establishes that if two groups choose different values of the weight, they must be different either in their costs or in their membership size. In such a case, the group with the larger weight of the relative effort rule could put a lower total effort and attain a lower winning probability because of its efficiency or size advantage. There is no dubiety here. Moreover, we can interpret equation (8) as a special case of equation (6) that implies the understandable direct relationship between δ^* and m (δ^* is increasing with respect to m). When the number of contestants is large, each group uses more of the private prize to give selective incentives to its members.

We may interpret equation (6) as follows. Complete reliance on the relative effort rule induces the members to make excessive efforts that prohibit the attainment of Pareto optimum, while reliance just on the equalitarian rule also results in an inefficient outcome, because the individuals are induced to make insufficient efforts. A group that can secure a higher winning probability has room to loosen up its members' incentive to make efforts, still providing enough utility gain to compensate for the reduction in the winning probability. Also, a higher $\eta(N_i, \gamma)$ implies that individuals of a group are more "hungry" for the private good prize, and work harder to get their share. This provides a group with a room to reduce δ_i, that is, to apply a more equalitarian sharing rule.

(b) Different Efficiency.

Assuming that all the groups have the same number of members, say N, we now allow variability in the efficiency of the groups that takes the form of different marginal costs of effort.

Proposition 4. *Let the members of group k have lower marginal costs than those of members of group l. That is, $v_k'(a) < v_l'(a)$ for all $a > 0$. Then, in equilibrium, $\pi_k > \pi_l$ and $\delta_k^* < \delta_l^*$. Also, per capita utility is larger in group k than in group l.*

By Proposition 4, a group with more efficient contestants attains a higher winning probability, applies a more equalitarian group sharing rule and secures a higher per capita utility. Baik and Lee (2007) have shown, in their specific model of two-group contest with equal group membership and a pure private-good prize, that a more efficient group chooses a more equalitarian rule. Their result can be considered as a corollary of our proposition. Actually, in addition to the above straightforward implications, Proposition 4 can be easily applied to shed light on the role of differences in the valuation of the prize, in income and in lobbying capability[11].

Differences in valuation of the prize

To study the effect of variability in the evaluation of the prize, we modify the model by letting members of group i have the benefit function $w_iB(q, G)$, where $w_i > 0$ is the augmenting factor. If $w_k > w_l$, members of group k value the contested prize more than members of group l, without affecting the value of the elasticity $\eta(N, \gamma)$. Letting a member's cost function have the same form v in all groups, equation (4) takes the form:

[11] The early arguments on the role of different valuations of the prize in the usual contests are found in Hillman and Riley (1989).

$$(1-\pi_i)\cdot B\left(\frac{\gamma}{N},1-\gamma\right)-\frac{1}{w_i}\cdot v'\left(\frac{A}{N}\cdot\pi_i\right)\cdot\frac{A}{N}=0,$$

In this case, $\frac{v'(a)}{w_i}$ can be conceived as the marginal cost in group i. By applying Proposition 4, we can conclude that $w_k > w_l$ implies that $\pi_k > \pi_l$, $\delta_k^* < \delta_l^*$, and $u_k > u_l$. That is, high valuation of the prize and high efficiency that takes the form of low marginal cost have the same effect on the group winning probability, sharing rule and per capita utility.

Differences in income.

To study the effect of income variability, we modify the model by assuming that an individual's preferences are represented by an additively separable utility function of his benefit from the prize and of his income I. Specifically, the individual's utility is

$$B(q, G) + V(I),$$

where $V' > 0$ and $V'' < 0$ for all $I > 0$. Interpreting the effort level a as money expenditure, we can define the cost function of a member in group i as

$$v_i(a) = -V(I_i - a),$$

where I_i is the common income of the members of group i.[12] Let $I_k > I_l$, that is, the members of group k are richer than those of group l. Then, by assumption,

$$v_k'(a) = V'(I_k - a) < V'(I_l - a) = v_l'(a),$$

and so, by Proposition 4 we get that $\pi_k > \pi_l$, $\delta_k^* < \delta_l^*$, and $u_k > u_l$.

That is, high income can be interpreted as low marginal cost of effort and therefore it has the same effect on the group winning probability, sharing rule and per capita

[12] Rigorously speaking, this cost function does not satisfy the assumption $\lim_{a\to 0} v_i'(a) = 0$, which assures that in equilibrium every group makes a positive effort. In this version of our extended contest, therefore, our arguments are valid, provided that the equilibrium effort of every group is positive.

utility.

Differences in political influence

To study the effect of variability in the political influence or lobbying power of the individuals, we have to modify the model by introducing asymmetry into the contest success function. Namely, allow members of different groups to differently affect their group winning probability. Essentially, in such asymmetric version of the contest it is possible to formulate differences in political capabilities such that they are equivalent to differences in the marginal cost of effort across groups. In turn, we could establish, again by applying Proposition 4, that increased political power, which is interpreted as reduced marginal cost of effort, increases the group winning probability, increases the group equalitarianism and increases the group per capita utility.

(c) Different Membership Size

The advantage of membership size is the main topic of the work by Esteban and Ray. They provide a sufficient condition for a group with larger membership to attain a higher winning probability in equilibrium. They also prove that per capita utility increases (decreases) with membership size when the prize is purely public (private). In this sub-section we generalize and sharpen their results by allowing the endogenous determination of the group sharing rules and imperfect substitution between the private and the public components of the prize.

Let all members of the competing groups share the same cost function v. Following Esteban and Ray, we denote by $\alpha(a)$ the elasticity of the marginal cost,

$$\alpha(a)=\frac{a\cdot v''(a)}{v'(a)}.$$

Also, let us pretend that the membership N_i in (7) is a continuous variable and view the benefit elasticity η as its continuous function. The membership size viewed as a continuous variable will be denoted by *n*. Our next proposition generalizes Esteban and Ray's result on the relationship between the size of the competing groups and their winning probability.

Proposition 5. *Suppose that all group members share the same cost function v. Then the winning probability of an N'-member group is larger than that of an N-member group, N < N', if*

$$1 - \max_{n \in [N, N']} \eta(n, \gamma) + \inf_{a \geq 0} \alpha(a) > 0. \tag{9}$$

Proposition 5 establishes that in the extended contest, a larger group will have a higher winning probability, if the difference between $\max_{n \in [N, N']} \eta(n, \gamma)$ and $\inf_{a \geq 0} \alpha(a)$ is smaller than 1. Economically, this condition makes sense. A larger membership implies a smaller per capita private-good component of the prize. Confronting the smaller benefit, each member puts less effort. The extent of this incentive can be measured by $\eta(N_i, \gamma)$. On the other hand, the larger membership also implies lower individual's marginal costs at a given level of group effort, which induces more effort from each member. The extent of this second incentive can be measured by $\alpha(\frac{A_i}{N_i})$. If the extent of the difference between these two incentives, which can be measured by $(\max_{n \in [N, N']} \eta(n, \gamma) - \inf_{a \geq 0} \alpha(a))$, is sufficiently small, then the inequality $N < N'$ will imply that the effort made by the larger group (the N'-member group) is larger and, consequently, its winning probability is higher.

As we have already pointed out, $\eta(n,\gamma)$ is always less than 1, unless the prize is purely private. By Propositions 5 and 1, this implies

Corollary 1.

(a) *In a contest for a mixed private-public good where* $\gamma \neq 1$*, a larger group always attains a higher winning probability.*

(b) *Assume that* $\eta(n,\gamma)$ *is non-decreasing with respect to n. Then, in a contest for a mixed private-public good, the sharing rule applied by a larger group is more equalitarian.*

Part (a) of the Corollary implies that in our extended contest, allowing the endogenous determination of group sharing rules eliminates the ambiguity regarding the effect of group size on its winning probability; a larger size always increases the winning probability of the group, provided that the prize is not a pure private good. This result considerably strengthens Esteban and Ray's main claim that Olson's group size paradox does not necessarily matter. Part (b) shows that if $\eta(n,\gamma)$ is non-decreasing with respect to *n*, a larger size also results in increased equalitarianism.

The last proposition reveals that the elasticity of the benefit from the private-good component of the prize is also relevant to the relationship between membership size and per capita utility.

Proposition 6. *Suppose that all group members share the same cost function v. Then the per capita (expected) utility in an N'-member group is larger than that in an N-member group, N < N', if*

$$\max_{n \in [N, N']} \eta(n, \gamma) \leq \frac{1}{2}, \tag{10}$$

Esteban and Ray have proved that, when the prize is purely public, the per capita utility increases with membership size. We have shown that such group-size advantage in terms of per-capita utility can hold in cases of mixed public-private good prize, provided that the competing groups can choose the sharing rule of the private part of the prize.

At this stage, it is instructive to illustrate the results by examining the CES family of benefit functions, $B(q, G) = \left(b_1 q^{\rho} + b_2 G^{\rho}\right)^{\frac{1}{\rho}}$, with $0 < b_1 < 1$, $0 < b_2 < 1$ and $\rho \leq 1$. [13] With the CES form, we get that

$$\eta(n, \gamma) = \frac{b_1 \cdot \left(\frac{\gamma}{n}\right)^{\rho}}{b_1 \cdot \left(\frac{\gamma}{n}\right)^{\rho} + b_2 \cdot (1-\gamma)^{\rho}}, \tag{11}$$

which is always less than 1. That is, condition (9) is always satisfied, so a larger group indeed has the advantage of acquiring a higher winning probability. Also, note that when $\rho < 0$, $\eta(n, \gamma)$ becomes non-decreasing with respect to n, so we can apply Corollary 1(b). As to the use of Proposition 6, we can see that with the CES specification, the condition $\eta(n, \gamma) \leq \frac{1}{2}$ can be written as $\left(\frac{b_2}{b_1}\right)^{-\frac{1}{\rho}} \cdot \frac{\gamma}{1-\gamma} \leq n$. If $\left(\frac{b_2}{b_1}\right)^{-\frac{1}{\rho}} \cdot \frac{\gamma}{1-\gamma} \leq N_{\min}$ or $\gamma \leq \frac{N_{\min}}{N_{\min} + \left(\frac{b_2}{b_1}\right)^{-\frac{1}{\rho}}}$ holds, where $N_{\min} = \min\{N_1, \ldots, N_m\}$, then per capita utility is increasing with respect to membership size. That is, increased

[13] Notice that the linear specification of the benefit function adopted by Esteban and Ray, i.e., $B(q, G) = Mq + PG$, is a special case of the CES family.

membership is advantageous not only because it increases the group winning probability, but also because it increases the utility of its members. This result holds true in the cases where the public part of the prize is sufficiently large[14].

In their arguments against the "group-size paradox," Esteban and Ray have assumed that each competing group can commit to a pure equalitarian distribution rule prior to the contest. Propositions 5 and 6 have strengthened their results, by permitting the competing groups to commit to non-equalitarian group sharing rules. They strongly suggest that the group-size advantage in a contest would be enhanced as the range of group sharing rules available for competing groups gets wider.

5. Conclusion

We have examined an *m*-group contest for a mixed private-public-good prize, in which the private part is distributed by an endogenous, unobservable sharing rule that is applied by the group winning the contest. In our setting, asymmetry among the competing groups is allowed in terms of both efficiency and membership size. Imperfect substitutability between the public and the private components of the prize is also permitted. The group contest has a unique pure strategy perfect Bayesian equilibrium with stable belief profiles. Its characterization enabled the derivation of simple fundamental formulas that are most useful for analyzing the relationship between the characteristics of a group, its selected sharing rule, its winning

[14] With the notation of Esteban and Ray (2001) and their linear specification $B(q, G) = Mq + PG$,

$\theta_{\min} \geq \frac{1}{2}$,

or $\frac{\frac{M}{N_{\min}}}{P + \frac{M}{N_{\min}}} \leq \lambda$ *ισ α συφφιχιεντ χονδιτιον τηατ α λαργερ γρουπ ατταινσ α ηιγηερ περ χαπιτα υτ ιλιτψ, ωηερε* $\theta_{min} = 1 - \eta(N_{min}, \gamma)$, and $\lambda = 1 - \gamma$ (the ratio of the public part of the prize).

probability and the per capita expected utility. In those formulas, the elasticity of the benefit from the private part of the prize plays a key role. Our main findings are the following:

(i) A group securing a higher winning probability is more equalitarian. The extent of equalitarianism is positively related to the elasticity of the benefit from the private component of the contested prize (Proposition 1).

(ii) A group with more efficient members applies a more equalitarian sharing rule and attains a higher per capita utility (Proposition 4).

(iii) Permitting the endogenous determination of group sharing rules considerably strengthens the advantage of membership size; the larger group always attains the higher winning probability, unless the prize is purely private (Proposition 5). The larger group can even attain the higher per capita utility, if the elasticity of benefit from the private part of the contested prize is large enough (Proposition 6).

Since most of the results are valid in the case of the pure private-good prize, the significance of our findings is preserved even if we disregard the nature of the prize in the model. However, a significant merit of the proposed model is its ability to treat mixed public-private-good characteristics of the prize in a very general way. In many group contests the prize contains both public and private factors. This is typically the case when the nature of the prize is determined by a commitment to allocate the fixed budget won by a group to the provision of some mix of private and public goods. Such a commitment may be voluntarily chosen by the winning group. It may also be enforced by the government that grants the prize. Our model would provide a useful tool for studying such contests.

One noticeable weak feature of our model, as well as of other models that study group contests, is the symmetry assumption regarding the members that belong to the same group. Without this assumption it is not clear how the arguments supporting the advantage of larger groups are amended. Investigation of the effect of asymmetry among members of a group on the analysis is a worthwhile undertaking for future research. Also, notice that our strong denial of the "group-size paradox" presumes that competing groups can commit to a distribution rule prior to the contest. Conversely, we could have conjectured that the group-size paradox in group contests is apt to happen, if the groups cannot commit to any distribution rule, and the prize sharing is determined anarchically. This is another important topic that deserves further study.

Finally, in our setting the extent of equalitarianism (equal sharing of the prize) is not determined by moral values, religious commitments or social ideology. It is the outcome of rational strategic incentives that arise in the contest environment. Interestingly, we find that in this competitive environment, more efficient groups (groups with lower marginal cost of effort) or groups with higher valuation of the prize, higher income or larger lobbying capabilities tend to be more equalitarian. That is, share equally a larger part of the private-good component of the prize. In addition, under the sufficient conditions we have stated, larger groups also tend to be more equalitarian. Testing empirically these predictions is a very interesting task which is beyond the scope of our study.

6. Proofs

Proof of Lemma 1.

Only-if-part: Under a stable belief profile, the kth member of group i must choose effort for an arbitrary group sharing rule δ_i that satisfies

$$a_{ik}^{*}(\delta_i) \in \arg\max_{a\geq 0} \frac{\sum_{h\neq k} a_{ih}^{*}(\delta_i) + a}{\sum_{j\neq i} A_j^{*}(\delta_j^{*}) + \sum_{h\neq k} a_{ih}^{*}(\delta_i) + a} B\left[\gamma \cdot \left(\frac{a\cdot\delta_i}{\sum_{h\neq k} a_{ih}^{*}(\delta_i) + a} + \frac{1-\delta_i}{N_i}\right), 1-\gamma\right] - v_i(a)$$

.

Since $\lim_{a\to 0} v_i'(a) = 0$, the maximization problem requires $a_{ik}^{*}(\delta_i^{*}) > 0$ and the fulfillment of the first order condition:

$$\frac{A - A_i^{*}(\delta_i)}{A^2} \cdot B\left(\gamma \cdot (\delta_i \cdot \frac{a_{ik}^{*}(\delta_i)}{A_i^{*}(\delta_i)} + (1-\delta_i)\cdot\frac{1}{N_i}), 1-\gamma\right)$$

$$+\frac{1}{A}\cdot\gamma\delta_i\cdot\frac{\partial B\left(\gamma\cdot(\delta_i\cdot\frac{a_{ik}^{*}(\delta_i)}{A_i^{*}(\delta_i)} + (1-\delta_i)\cdot\frac{1}{N_i}), 1-\gamma\right)}{\partial q}\cdot\frac{A_i^{*}(\delta_i) - a_{ik}^{*}(\delta_i)}{A_i^{*}(\delta_i)} - v_i'(a_{ik}^{*}(\delta_i)) = 0$$

As B is concave with respect to the private part of the prize, the left-hand-side of the equation is strictly decreasing with respect to a_{ik}. Hence we can confirm that every member of group i chooses a symmetric effort level. This yields equation (3).

Condition 1 implies that $A_i^{*}(\delta_i)$ is strictly increasing with respect to δ_i. Because of the stability of the belief profile, the planner of group i can change the aggregate effort of the group by a unilateral deviation from the equilibrium group sharing rule δ_i^{*}, keeping the effort by the other groups constant. Condition 1 also implies that every member attains a symmetric per capita utility at each value of δ_i. The criterion for choosing a group sharing rule is reduced to the maximization of per capita utility, as long as the planner wishes to ensure the selection of a Pareto efficient outcome. Hence Condition 2 is necessary to prevent deviations.

If-part: Let Conditions 1 and 2 hold. By Condition 1, each member of each group maximizes the expected utility over all of his/her information sets under the stable belief profile. By symmetry and Condition 2, each planner chooses a group sharing rule that maximizes the welfare function of his group, given the sharing rules in the other groups and the succeeding actions of the individual contestants. Q.E.D.

Proof of Proposition 1. Equation (6) can be derived from equations (3) and (5), by straightforward manipulations.

Proof of Proposition 2.

We will use the same technique as in Esteban and Ray (2001) and Ueda (2002). Consider, hypothetically, equation (5) as the condition implicitly defining π_i as a function of A, γ, and the membership N_i. Then, π_i is continuous and strictly decreasing in A. Also, $\lim_{A\to 0}\pi_i = 1$ and $\lim_{A\to\infty}\pi_i = 0$. As A increases, the value of δ_i derived from equation (6), which is required from Condition 1 and Condition 2 of Lemma 1 as long as $0 < \delta_i^* < 1$, approaches 1. If $\eta(N_i, \gamma)$ is less than 1, δ_i satisfying (6) can exceed 1 for the value of A larger than some level, say A_R. If the total effort A actually attains such a value in an equilibrium, group i must have $\delta_i^* = 1$, and then, π_i satisfies the equation

$$(1-\pi_i)\cdot B\left(\frac{\gamma}{N_i}, 1-\gamma\right) + \gamma\cdot\frac{\partial B\left(\frac{\gamma}{N_i}, 1-\gamma\right)}{\partial q}\cdot(1-\frac{1}{N_i}) - v_i'\left(\frac{A}{N_i}\cdot\pi_i\right)\cdot A = 0, \tag{12}$$

which is derived from (3), setting $\delta_i = 1$. Confirm that π_i is again continuous and strictly decreasing in A and $\lim_{A\to\infty}\pi_i = 0$, when equation (12) is hypothetically seen as the condition defining π_i as an implicit function.

Now, consider *the pseudo winning probability function* of group i that depends on A, $\pi_i^P(A)$: $(0\ \infty) \to \mathbb{R}$, which is defined as follows: for any A in $(0, A_R]$, this function assigns the value of π_i given by equation (5), and for any A larger than

A_R, it assigns the value of π_i determined by equation (12). The derived function is continuous and strictly decreasing, with $\lim_{A\to 0}\pi_i^P(A) = 1$ and $\lim_{A\to\infty}\pi_i^P(A) = 0$.

Let us consider the value A^* with $\sum_{i=1}^{m} \pi_i^P(A^*) = 1$. Such a value certainly exists and is unique. It can be viewed as a candidate of the total equilibrium effort put by all the competing groups. Then, $\pi_i^P(A^*)A^*$ must be the aggregate effort put by group i in equilibrium. By using the definition of the pseudo winning probability function and equation (6), we can uniquely specify the group sharing rule δ_i^*. We can confirm that this rule δ_i^* and the aggregate group effort $A_i^*(\delta_i^*) = \pi_i^P(A^*)A^*$ satisfy the conditions of Lemma 1. The existence of equilibrium has thus been confirmed.

On the other hand, if we have an equilibrium with the total equilibrium effort A^*, Lemma 1 requires that the aggregate effort by group i in equilibrium satisfies $A_i^*(\delta_i^*) = \pi_i^P(A^*)A^*$. In equilibrium, however, the sum of the winning probabilities of the m groups must be equal to 1, and $\sum_{i=1}^{m} \pi_i^P(A^*) = 1$ has to be satisfied. This implies the uniqueness of the equilibrium. Q.E.D.

Proof of Proposition 3.

If two groups are symmetric, they have the same schedule of the pseudo winning probability function. At the unique equilibrium total effort A^*, therefore, such groups attains the same values of aggregate group effort. This implies that they have the same weight of the relative effort rule, the winning probability, and the per-capita utility. When all groups are symmetric, every group attains the winning probability $\frac{1}{m}$, and the equation (8) holds trivially. Q.E.D.

Proof of Proposition 4.

Equation (5) implies that

$$\pi_k \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) \cdot \frac{N}{A^*} + v_k'\left(\frac{A^*}{N} \cdot \pi_k\right) = \pi_l \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) \cdot \frac{N}{A^*} + v_l'\left(\frac{A^*}{N} \cdot \pi_l\right).$$

Suppose that $\pi_k \le \pi_l$. We therefore obtain the strict inequality: $v_k'\left(\frac{A^*}{N} \cdot \pi_k\right) < v_l'\left(\frac{A^*}{N} \cdot \pi_l\right)$,

which makes the above equation impossible. This means that $\pi_k > \pi_l$. Since $\eta(N_k, \gamma) = \eta(N_l, \gamma) = \eta(N, \gamma)$, we get that $\delta_k < \delta_l$. Finally, notice that $A_k^*(\delta_k^*)$ which maximizes the per capita utility of group k, is larger than $A_l^*(\delta_l^*)$. Denoting the per capita utility of group i by u_i, and noticing that $v_k(a) < v_l(a)$ for all $a > 0$, we get that

$$u_k = \frac{A_k^*(\delta_k^*)}{A^*} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_k^*(\delta_k^*)}{N} \cdot\right) \ge \frac{A_l^*(\delta_l^*)}{\sum_{j\neq k} A_j^*(\delta_j^*) + A_l^*(\delta_l^*)} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_l^*(\delta_l^*)}{N}\right)$$

$$> \frac{A_l^*(\delta_l^*)}{A^*} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_k\left(\frac{A_l^*(\delta_l^*)}{N}\right) > \frac{A_l^*(\delta_l^*)}{A^*} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) - v_l\left(\frac{A_l^*(\delta_l^*)}{N}\right) = u_l.$$

Q.E.D.

Proof of Proposition 5.

The basic idea of the proof is similar to the one applied by Esteban and Ray in the proof of their Proposition 1. Keeping the total effort unchanged at its equilibrium value A^* and examining the behavior of π_i, while pretending that in equation (5) N_i is a continuous variable, we obtain that

$$\left.\frac{\partial \pi_i}{\partial n}\right|_{n=N} = \frac{N}{\pi_i} \cdot \frac{1-\eta(N,\gamma)+\alpha\left(\frac{A^*}{N}\cdot\pi_i\right)}{\pi_i + \alpha\left(\frac{A^*}{N}\cdot\pi_i\right)}. \tag{13}$$

If inequality (9) holds, this derivative is positive at all values of n in the closed interval $[N, N']$. This establishes the validity of Proposition 5. Q.E.D.

Proof of Corollary 1. Directly obtained from Propositions 5 and 1.

Proof of Proposition 6.

Again, keeping the total effort unchanged at its equilibrium value A^* and viewing N_i as a continuous variable, we can examine the behavior of the per capita utility

$$u_i = \pi\left(A^*, N_i\right) \cdot B\left(\frac{\gamma}{N_i}, 1-\gamma\right) - v_i\left(\frac{A^*}{N_i} \cdot \pi\left(A^*, N_i\right)\right),$$

where $\pi(A^*, N_i) = \pi_i$ is the value of the winning probability given by equation (5). By using equations (5) and (13), we get that

$$\left.\frac{\partial u_i}{\partial n}\right|_{n=N} = \frac{\pi_i}{N} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) \cdot \left\{1 - \eta(N, \gamma) + \frac{\pi_i}{\pi_i + \alpha\left(\frac{A^*}{N} \cdot \pi_i\right)}\left(1 - \eta(N, \gamma) - \pi_i\right)\right\} \cdot$$

(14)

If $1 - \eta(N, \gamma) - \pi_i \geq 0$, then the right hand side of the equation is positive.

If $1 - \eta(N, \gamma) - \pi_i < 0$, then

$$\left.\frac{\partial u_i}{\partial n}\right|_{n=N} > \frac{\pi_i}{N} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) \cdot \left\{1 - \eta(N, \gamma) + \left(1 - \eta(N, \gamma) - \pi_i\right)\right\} = \frac{\pi_i}{N} \cdot B\left(\frac{\gamma}{N}, 1-\gamma\right) \cdot \left(2 - 2\eta(N, \gamma) - \pi_i\right) \cdot$$

Thus, $\max_{n \in [N, N']} \eta(n, \gamma) \leq \frac{1}{2}$ implies that the per capita utility is increasing with respect to membership size on the interval [N, N']. Q.E.D.

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