Strategic Management of R&D Pipelines with Co-Specialized Investments and Technology Markets

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Abstract

The theoretical literature on managing R&D pipeline is largely based on real option theory making decisions about undertaking, continuing, or terminating projects. The theory typically assumes that each project or causally related set of projects is independent. Yet, casual observation suggests that firms expend much effort on "managing" and "balancing" its R&D pipeline, where managing appears to be related to the choice of R&D selection thresholds, project risk and whether to buy or sell projects to fill the pipeline. Not only do these policies appear to differ across firms, they also appear to vary over time for the same firm. Changes in management policies suggest that the choice of R&D selection thresholds is a time-varying strategic decision and there may be some type of vertical interdependency among R&D projects in different stages. In this paper we develop a model using dynamic programming techniques that explains why firms vary in their R&D project management policies. The novelty and value of our model derives from the central insight that some firms invest in downstream co-specialized activities that would incur substantial adjustment costs if R&D efforts are unsuccessful while other firms have no such investment. If transaction costs in technology markets are positive, which implies that accessing the market for projects is costly, these investments lead to state-contingent project selection rules that create a dynamic and vertical interdependency among R&D activities and product mix. We describe how choices of R&D selection thresholds, preferences over project risk, and use of technology markets for the buying and selling of projects differ by the state of the firm's pipeline, the magnitude of transaction costs in the adjustment market, and the magnitude of technology costs. These results yield interesting managerial implications and public policy.

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Introduction

How do firms strategically manage their research and development (R&D) pipelines? Consider Merck, which was in an enviable position in the mid-1980s. Several blockbuster drugs had been recently commercialized and its R&D pipeline was chalk-full of potential blockbusters. With this success Roy Vagelos, Merck's CEO, shifted his firm's policy and admonished scientists to drop projects in the pipeline that did not have blockbuster potential (Nichols 1994, 107). Envy diminished by 1993 as Merck's pipeline grew thin: the pipeline did not promise to replace revenue losses from drugs coming off patent later in the decade. The thinness of the R&D pipeline became critical by 1998 when blockbusters began coming off patent (Seiden, 1998). Internal development produced few blockbuster drugs, which led Merck to another shift in policy that increased its reliance on in-licensing, joint ventures, and collaborations for replenishing its pipeline.[1] With the pipeline remaining precariously thin (Barrett, 2001), CEO Raymond Gilmartin again shifted policy by reorganizing the firm in 2000 to greatly increase its ability to purchase projects from the technology market (Pisano, 2002). Merck's pipeline received unexpected additional pressure when in 2005, it withdrew Vioxx from the market. In response, Merck announced a new policy that would put an even greater emphasis on bringing in compounds from the outside as well as accelerating the development of several internally discovered compounds (Bellucci, 2005).

[1] For instance, Merck spent about $200 MM on third-party collaborations between 1998 and 2000 and, by 2000, one-third of Merck's products came from external research or licensing collaborations (Pisano, 2002).

Merck's policy shifts for managing its R&D pipeline hints at the potential of a complex interaction between the state of its pipeline, the financial thresholds it used to evaluate project advancement and termination, the nature of R&D projects it undertook, and its choice to invest in an internal R&D versus acquiring projects originating outside of the firm. This history of policy shifts suggests at a minimum that Merck's financial hurdles for advancing projects fluctuated over the past 20 years shifting from high thresholds selecting only to develop potential blockbuster drugs to lower thresholds that included Merck's willingness to commercialize its less valuable projects and to acquire other firms' discoveries. This history also may reflect changes in preference for projects—Merck may have shifted from high-risk high-return internal projects to lower-risk and lower-return projects, which might further explain Merck's difficulty in replacing revenue from products that came off patent.

Merck's experience is not unique suggesting that such interactions may be pervasive.[2] Indeed, any firm from any industry where the value of commercialized projects ultimately expires faces similar conundrums: What financial threshold should a firm use for evaluating projects and should these thresholds vary over time? Should a firm buy or sell innovations in technology markets or rely on internal development and commercialization? To what extent should a firm's risk preferences for projects vary over time? More generally, how should a firm's policies for strategically managing R&D change in response to the state of its pipeline?

A variety of academic literature considers some of these questions but does so in piecemeal and incomplete ways. The project finance literature, for instance, proposes that firms should advance any project whose expected value is greater than zero. This prescription does not

[2] For instance, in the early 1990s, Zeneca, with its most promising pipeline in many years, implicitly raised its project selection threshold and stopped several R&D projects (Pharmaceutical Business News, 1992). Relatedly, industry observers assert that many large pharmaceutical manufacturers lowered their R&D project selection thresholds in the late 1990s in response to weak pipelines (Tanouye and Langreth, 1997). Many large pharmaceutical firms merged or were acquired over the past 15 years ostensibly to fill thin pipelines.

seem to accord with actions taken by some firms like Merck that terminated apparently positive expected value projects, as well as changed thresholds over time, because it fails to account for the impact of a single project on the whole pipeline of projects.[3] Product development literature in marketing and operations that discusses R&D pipelines recommends a stage and gate system for assessing which projects to advance or terminate. This literature argues that "balancing" the portfolio is an important strategic goal (Cooper et al. 2001); yet, it provides no systematic analysis of how to balance (Cooper 1986) nor discusses the possibility that thresholds may change over time. This literature also does not provide a comprehensive framework for thinking about how technology markets play a role in strategically managing the R&D pipeline. A variety of literatures in economics, management, and organization theory discuss licensing, joint ventures, and partnerships[4]; but, these literatures rarely come in contact with issues concerning the managing of the R&D pipeline nor do they inform the financial thresholds and risk preferences used to make pipeline management decisions.

In this paper, we explore the strategic management of R&D pipelines in the sense that policies and their associated pattern of resource allocation decisions are critical to a firm's long run performance (Barney 1997). Unlike the previous literature, we do not model product and technology market competition or cooperation; but, do provide an analytic assessment of how to

[3] Since Roberts and Weitzman (1981), real option theory has been the theoretical workhorse for assessing R&D projects and making decisions about undertaking, continuing, or terminating projects. Based on option pricing theory (Black and Scholes 1973, Merton 1973), a voluminous theoretical literature on real options and investment under uncertainty has emerged that explores not only R&D project decisions but also any project requiring sunk investment with uncertain outcomes (e.g., Schwartz and Trigeorgis 2001).

[4] Previous economic literature has asked whether incumbents or entrants engage in more R&D investment (e.g. Schumpeter 1942; Arrow 1962; Reinganum 1983, 1989; Gans and Stern 2000). However, research does not directly address why incumbent or entrant or firms with different portfolios of R&D projects may choose different selection rules. One explanation from recent research by Gans and Stern (2000) implies biotechnology start-ups' have a preference for novel innovation in order to enhance their bargaining power in possible post-innovation negotiations for licensing and acquisitions. Our paper provides an alternative explanation. Kamien and Schwartz (1978) argue that liquidity constraints associated with ongoing operations may explain systematic differences among firms' decision rules for R&D project selection. Such constraints themselves, however, may be endogenous to these decision rules rather than the cause of them. For example, a firm may have difficulty in borrowing and face liquidity constraints where asymmetric information is significant because it targets only high-uncertainty projects.

dynamically manage the R&D pipeline. One of the key insights on which we build our theory is our observation that some firms invest in downstream co-specialized activities (Williamson 1985) such as manufacturing, distribution channels, and brands. We assume that firms would incur substantial adjustment costs if R&D efforts are unsuccessful causing the firm to run out of projects to commercialize that utilize these co-specialized assets. For instance, some pharmaceutical firms make co-specialized investments in "drug reps" who receive firm-specific training and invest in idiosyncratic relationships with the select physicians who are most likely to prescribe the pharmaceutical products the firm sells. Should a firm fail to commercialize new products, it would forego returns from these co-specialized investments and personnel would leave for better opportunities or would be laid off to avoid incurring their carrying cost. While these two types of costs differ, they both have the same economic impact in that the firm loses the opportunity of continued returns from its past investments; so, we refer to them collectively as adjustment costs. To include adjustment costs in a typical expected value analysis for each project is complicated by the fact that these costs are contingent on the state of the pipeline. For example, an incumbent firm's decision to terminate one project raises the probability of incurring future adjustment costs; however, this probability hinges on the performance of other projects in the pipeline now and in the future.[5] We argue that these adjustment costs can have substantial implications for innovative behavior and firm performance; hence, they should be evaluated through investigating the dynamic interdependencies among project decisions.

Another key insight is that firms in some instances may be able to avoid incurring adjustment costs if projects for commercialization can be purchased in the technology market. The benefit of this option depends on the magnitude of the costs of transacting in this market.

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[5]For another example, a start-up firm's decision of pushing an R&D project into the stage of commercialization implies that the firm in the future may incur adjustment costs for which the probability depends on the strength of its R&D pipeline.

Transferring technology can face substantial transaction costs (Caves, Crookell, and Skilling 1982). For example, project knowledge can be tacit (Teece, 1977, 1981), which can lead to costly transfer as well as adverse selection and moral hazard problems. If transaction costs are low, firms may be able to avoid adjustment costs by purchasing external R&D projects from the technology market. This option becomes increasingly less likely for higher transactions costs, which may impact the likelihood of incurring adjustment costs, thresholds, and risk preference.

Building on these insights and utilizing the stage-gate literature (for a summary see Calantone and de Benedetto, 1990; Cohen et al., 1998), we introduce a dynamic programming model that assumes the R&D pipeline is comprised of three stages—research, development, and commercialization—and two gates—advance-to-development and advance-to-commercialization. We assume the possibility of different pipeline states depending on the projects present in development and commercialization stages. We refer to different combinations of pipeline states, which symbolizes different types of incumbents and entrants, as different "types" of firms and explore how managers dynamically adjust their policies for selecting projects in these states. We parameterize both the possibility of adjustment costs, which are incurred if the firm has no project with which to replace an expired product, and transaction costs in the technology market, where firms can purchase projects to replenish their pipeline or sell projects to avoid investing in co-specialized assets. We then derive the corresponding optimal project selection thresholds and "buy or sell" price thresholds for each firm type. Our model complements recent research on R&D portfolio management. Cassiman and Ueda (2006), for instance, study why an established firm might choose not to commercialize an innovation while a start-up would chose to do so. While complementary to our approach, firms in their model face excess supply of R&D projects and their model assumes a frictionless technology market whereas our model studies a technology market with positive transaction costs and introduces the notion of adjustment costs to R&D pipeline management.

Our model leads us to three sets of results. First, our model's analytic results demonstrate that the thresholds for advancing (as well as for buying and selling) projects differ by the state of the firm's pipeline, the magnitude of transaction costs in the technology market, and the magnitude of adjustment costs. Incumbent firms—those firm types already possessing co-specialized assets—choose projects with positive expected value, whether developed internally or purchased in the market, so long as transaction costs for buying and selling in the technology market are negligible. This policy changes dramatically as transaction costs increase in the technology market. Incumbent firms may choose internal projects—even those with negative expected value—in order to avoid incurring either adjustment or transaction costs. We simulate these policies in the context of pharmaceuticals to estimate the relative size of effects.

Second, simulations based on our model show that preferences over project risk differs by the state of the firm's pipeline, the magnitude of transaction costs in the technology market, and the magnitude of adjustment costs. We show that, in general, firms are risk-loving in their R&D activities because managers can limit loss by terminating unprofitable projects. But entrants— those firm types not possessing co-specialized assets—are comparatively more risk-loving than incumbents implying differences in project risk and hence portfolio management. We also find that changes in adjustment costs and transaction costs can effect preferences for project risk; both incumbents and entrants are increasingly less risk-loving the greater are adjustment costs or transaction costs. Interestingly, efforts to lower adjustment costs and to reduce transaction costs may be substitutes: relatively low adjustment costs neutralize the effect of greater transaction costs (low adjustment costs reduce the needs to trade in the market) while relatively low transaction costs neutralize the impact of greater adjustment cost (the existence of technology market helps firms to avoid adjustment costs).

Third, our model implies that entrants are more likely to sell their projects to incumbents as transaction costs decline. For sufficiently high adjustment costs or sufficiently low transaction costs, entrants specialize in R&D activities and never commercialize projects. Our model predicts that incumbents and entrants pursue R&D projects with more similar risk characteristics as either adjustment costs or transaction costs decline; but, in the former case entrants are increasingly more likely to commercialize. These results yield interesting public policy implications as well as generate incentives for managers to shape both transaction costs in the market and adjustment costs within their firms.

The paper proceeds by introducing our dynamic R&D management model and summarizing its main analytic results. Our discussion section uses this model as well as several simulations to present our three sets of implications for the strategic management of R&D pipelines. We discuss the extent to which our model informs Merck's behavior and we identify the strengths and weaknesses of our model, which helps us to discuss its generality to industries beyond pharmaceuticals, and identify several additional research questions associated with R&D portfolio management that flow from our implications.

Model

We introduce a dynamic model in which the focal firm, in each period, initiates a project that goes through two stages (in two periods) before generating a possible new product. Stage I is the *research* stage in which a potential product is first identified while stage II is the *development* stage in which the full characteristics of the product and its demand in the market are examined. To simplify our analysis, we assume that, at any point of time, the focal firm has at most one project in each stage, and can introduce at most one product to the market. Also, for simplicity we assume that R&D capability is homogeneous among firms in the industry, which implies that the prior distributions for the market value of potential products are identical. We further assume that the technology market for projects exists only in stage II; but, lack of a market for stage I projects does not affect our results because no firm would trade projects at the end of stage I, as long as transaction costs in the stage are not significantly smaller than those in stage II.[6] Therefore, the firm makes decisions on (1) whether to continue or terminate the existing stage I project, and (2) whether to commercialize, sell, terminate the stage II project if it exists, or to buy a project developed by another firm.

We assume that the demand for the product once it is introduced to the market lasts only for two periods. The firm can have at most one new and/or old product in the market. When the product expires in two periods, the firm will incur an *adjustment cost c*, unless it can introduce a new product in the market during this period because it is at this point in time that co-specialized assets would lose their value. Alternatively, we can assume that the cost c has to be borne when the firm first launches a product in the market where it has neither new nor old product. All results we present later hold under this alternative assumption except for Lemma 1 in the appendix, which determines the ordering of value functions among different firm types.

Let x_t be the value of the product generated from the project started in period t, which we assume to follow a continuous distribution function $f(\cdot)$ and that $f(x_t) > 0$ for all $x_t \in (-\infty, +\infty)$. By the end of stage I, the firm receives a signal, s_t, about the products value. We assume that $s_t = x_t + \varepsilon_t$ where ε_t is a noise and distributed continuously with $E[\varepsilon_t] = 0$. The posterior expected

[6] In a setting where some buyers have complementary technology for the seller, or the market is not thick enough to make technology available anytime, which we rule out in this paper, there will be trades for early stage projects.

mean is given by $z_t = E(x_t \mid s_t)$. We write $f(x_t \mid s_t)$ as the density function of x_t conditional on signal s_t. By abusing notation, we also denote $f(x_t \mid z_t) \equiv f(x_t \mid s_t)$.

The firm then decides whether or not to advance the product to stage II based on the signal s_t. The research cost in stage I is assumed to be sunk and hence neglected throughout this analysis. If the firm decides to advance a project to stage II, the firm incurs the development cost $d > 0$[7]. By the end of the stage, we assume that x_t is fully revealed and the firm may launch the product into the market. For simplicity, we assume that the firm earns the profit of x_t in the first year of the sale and none in the second year.[8] The firm earns no revenue from its investment in development if it decides to terminate the project.

Instead of commercializing its own R&D project, the firm may sell or buy a project in each period in the technology market. We assume that this market is competitive, namely, there are many firms and each firm takes the price as given. When firms trade projects in the market, a *transaction cost* τ will be incurred. In order to keep the consideration of transaction costs consistent with our perfect competition assumption, we assume that the value of a project x is verifiable and a buyer and a seller can sign a contract that is contingent on x.[9] We further define ε as market premium such that buyers pay $x + \varepsilon$ in the technology market and sellers receive $x + \varepsilon - \tau$ for selling the projects.[10] Depending on the demand and supply in the technology market, ε can be positive or negative. For example, if there are only a few projects available with many

[7] Development costs can involve a wide array of expenditures. For instance, in the pharmaceutical industry, undertaking animal and human trials and getting regulatory approval through the FDA and agencies in other countries can consume upward of a billion dollars and take as long as a decade. While other industries may not face such extreme costs, the design, prototyping, and testing of new products and processes nonetheless can represent a substantial cost.

[8] By making this assumption, we avoid taking into account the substitution effect between new and old products when the firm launches a new one before the old one expires.

[9] Alternatively, we may assume that transaction costs are incurred after the asymmetric information problem is resolved and two parties sign a contract for trading the technology.

[10] Alternatively, we can assume that buyers pay $x + \varepsilon + \tau_B$ in the technology market and sellers receive $x + \varepsilon - \tau_S$ for selling the projects where $\tau_B + \tau_S = \tau$. How the transaction cost is shared between the buyers and the sellers does not affect our results.

buyers, ε is likely to be larger than τ while, if there is a large supply of projects buyers may enjoy a discount, i.e., ε may be negative. We assume that ε does not change over time, which implies that we only consider steady states where the distribution of individual firm types remains unchanged. These assumptions may be unrealistic if there is only a limited number of firms in the market or technology changes over time. However, we believe it is acceptable for our purpose of illustrating how R&D management policies differ among firms with different downstream assets. Figure 1 graphically presents our three stages and the decision options for each stage.

The following four state variables describe our system.

$i_t =$	1	if the firm has a project in stage II in period t.
	0	otherwise.

$j_t =$	-1	if the firm has no product in the market in period t.
	0	if the firm has only an old product in the market in period t.
	1	if the firm has a new product in the market in period t.

x_{t-1}: true market value of the project in stage II revealed to the firm in period t. The subscript denotes that the project starts at the beginning of period $t-1$.

z_t: posterior expected value of the product in stage I after observing the signal s_t at the end of period t. The subscript denotes that the project starts at the beginning of period t.

For notational simplicity, we often drop subscript t. Note that (i, j) indicate how urgent it is to develop or launch a product to avoid incurring the adjustment cost. We call the firm whose pipeline is described by (i, j) "firm (i, j)". For example, firm (0,-1) and firm (1,-1), which we collectively refer to as entrants, do not have a product in the market and thus face no imminent threat of incurring an adjustment cost. This example corresponds to the case of a firm that has not yet made co-specialized investments in downstream activities. In contrast, firm (0,0) and firm (1,0) will incur the adjustment cost unless it launches a new product by either commercializing its current project (feasible only for firm (1,0)) or buying technology from another firm in the current period. Finally, firms (0,1) and (1,1) have a new product in the market, which gives them two periods to introduce a new product before incurring the adjustment costs. We refer to these latter four firm types as incumbents because given a new or old product in the market they already possess co-specialized investments.

Let $V(z_t, x_{t-1}, i_t, j_{3,t} : \varepsilon, \tau)$ be the firm's value function evaluated at the end of period t and parameterized by the market premium ε and the transaction cost τ. We often omit subscript t and parameter ε and τ, and use shorter notation $V(z, x, i, j)$ or $V_{i,j}(z, x)$. Note that $V_{i,j}$ does not depend on x when $i = 0$ because there is no project in the second stage, thus we omit x (i.e., $V_{0,j}(z)$).

Our model's three decision variables are:

$e_{1,t} =$	1	if the firm continues a project to stage II at the end of period t.
	0	otherwise.
$e_{2,t} =$	1	if the firm launches a product to the market at the end of period t.
	0	otherwise.
$f_t =$	1	if the firm sells its product to another firm at the end of period t.
	-1	if the firm buys a product from another firm at the end of period t.
	0	otherwise.

The options of launching a product ($e_{2,t} = 1$) to the market include: (1) launch own product ($f_t = 0$); and (2) buy product in the technology market ($f_t = -1$). Or, the firm may not launch a new product in the market ($e_{2,t} = 0$) with the options: (3) terminate the project in the second stage, if there is one ($f_t = 0$); and (4) sell the project in the second stage in the technology market ($f_t = 1$).[11] Therefore, a firm faces the constraint $e_{2,t} + f_t = \{0,1\}$. A dynamic linkage

[11] Note that it is suboptimal to buy a technology from the market and simultaneously sell its own project to the market. Suppose a firm sells its own technology project whose value is x and buys another whose value is x' in the market. The firm earns a profit $x + \varepsilon - \tau$ by selling its own project and receives - ε by buying a project in the market.

exists between the firm's current decisions of ($e_{1,t}$, $e_{2,t}$, f_t) and its future profits. For example, an entrant's decision of launching a product in the market implies that it may face in the future the possibility of incurring c. State variables (z_t, x_{t-1}, i_t, j_t) indicate the *strength* of the pipeline. Intuitively, the strength of the pipeline will affect decisions in R&D activities and technology markets. We formally model how these decisions are made by using the dynamic programming approach.

First, the firm has to pay d if $e_{1,t} = 1$ (*i.e.,* advances a project into stage II), and c if $j_t = 0$ and $e_{2,t} = 0$ (*i.e.,* has an old product in the current period and does not launch new product in the market). Then it earns $x_{t-1} + \varepsilon - \tau$ if $f_t = 1$ (i.e., it sells project in the technology market), x_{t-1} if $e_{2,t} = 1$ and $f_t = 0$ (i.e., launches own project in the market), and $-\varepsilon$ if $f_t = -1$ (i.e., launches a new product in the market by buying a project from the technology market). Therefore, the static profit function can be written as follows:

$$\pi_t = -de_{1,t} - c1_{j_t=0}(1-e_{2,t}) + x_{t-1}1_{e_{2,t}+f_t=1} + \varepsilon f_t - \tau 1_{f_t=1}$$

where 1_* is an indicator function, *i.e.*, $1_* = 1$ when * is true but 0 otherwise. Let δ be the discount factor. The Bellman equation takes the following form:

$$\begin{aligned} V(z_t, x_{t-1}, i_t, j_t; \varepsilon, \tau) = \max_{e_{1,t}, e_{2,t}, f_t} [&-de_{1,t} - c1_{j_t=0}(1-e_{2,t}) + x_{t-1}1_{e_{2,t}+f_t=1} + \varepsilon f_t - \tau 1_{f_t=1} \\ &+ \delta E_{z_{t+1}, x_t | z_t} V(z_{t+1}, x_t, i_{t+1}, j_{t+1}; \varepsilon, \tau)] \end{aligned} \quad (1)$$

The transition of states from period t to $t+1$ is the following: If $e_{1,t} = 1$, the firm will have a project in stage II in the next period, and *vice versa* if $e_{1,t} = 0$; hence, $i_{t+1} = e_{1,t}$. If $e_{2,t} = 1$, the firm will have a new product in the market in the next period (*i.e.,* $j_{t+1} = 1$); if $j_t = 1$ and $e_{2,t} = 0$, the firm will have an old product in the market next period (*i.e.,* $j_{t+1} = 0$); if $j_t = 0$ or -1, and $e_{2,t} =$

Therefore, the net income is $x + \varepsilon - \tau - \varepsilon = x - \tau$. The firm, however, should have earned x by commercializing its own project without incurring any transaction costs. Firms should never simultaneously buy and sell in the technology market.

0, the firm will have no product in the market next period (*i.e.*, j_{t+1} = -1). Hence, $j_{t+1} = e_{2,t} - 1_{j_t \leq 0}(1 - e_{2,t})$. Also, as we discussed above the prior distribution for z_{t+1} is $f(x)$, and the posterior distribution for x_t is $f(x \mid z_t)$ in period *t*.

Our dynamic programming approach transforms the optimization problem into one of computing the value function $V(\cdot)$. Under some general conditions that are all satisfied here, Blackwell's theorem shows that the value function can be identified uniquely and there are some well-known and quite effective algorithms for computing the function $V(\cdot)$.[12] Furthermore, the solutions to the Bellman equation specify the optimal project selection rules for the firm. Key to our analysis is the dynamic interdependency among project selection decisions which, as we will explain, arises when adjustment and transaction costs are positive: the firm's decisions depend on the strength of its R&D pipeline, which is a function of (z_t, x_{t-1}, i_t, j_t), and such decisions are time-varying as state variables change over time.

Analytical Results

1. Zero Adjustment Costs

We first consider a baseline case in which the adjustment cost *c* is zero. In this case, all firm types will have the same project valuations, which implies that there will be no trade in the technology market and no difference in project selection rules.

> **Proposition 1.** When $c = 0$, no firm profitably trades in the technology market. Let $\hat{z}$ be such that $\delta \int_0^{+\infty} x f(x \mid \hat{z}) dx = d$. Then, any firm will advance (terminate) its project in stage I to stage II if $z > \hat{z}$ ($z < \hat{z}$) and it will commercialize (terminate) its final-stage R&D project if $x > 0$ ($x < 0$) for all types of firms.
>
> **Proof in the appendix.**

[12] For the conditions required for Blackwell's theorem, see, for example, Stokey and Lucas (1989).

Proposition 1 shows that absence of adjustment cost no dynamic interdependency among projects is present and that the firm evaluates each project separately. Given our assumption of homogeneous R&D capability, all firms value the same project in the same way, which makes transaction costs associated with the technology market immaterial. Therefore, a product should be introduced to the market as long as its expected value is positive and the project should be advanced to development as long as its option value exceeds the cost of development, which is the independent project evaluation prescription found in Roberts and Weitzman (1981) and, more generally, in the project finance literature (e.g., see Brealey and Myers 2002).

2. No Transaction Cost Case

We now consider the case when τ is zero. The adjustment costs become irrelevant because, so long as there are many firms that engage in R&D activities and the equilibrium premium stays at the level where supply equals demand in the technology market, no firm incurs adjustment costs because they can purchase projects in the technology market.

Proposition 2. If $c > 0$ but $\tau = 0$, all firms will have the identical R&D selection policies. That is, firms will commercialize their projects or sell them if $x > -\varepsilon$ and will advance projects from stage I to stage II if $z > \hat{z}$ where $\hat{z}$ is determined by $\delta\int_{-\varepsilon}^{+\infty} xf(x \mid \hat{z})dx = d$.

Proof in the Appendix.

Proposition 2 shows that technology market access eliminates differences among firm types in their R&D selection rules for both early-stage late-stage projects regardless of the level of the adjustment cost. Compared with the case of zero adjustment cost in Proposition 1, the values of thresholds for advancing projects in both stage I and stage II are generally different. When $\varepsilon > 0$, the threshold for commercialization is lower that zero and, as a result, the advance-to-development threshold is higher than in the case of zero adjustment cost because the possibility of commercializing or selling negative-value projects lowers the return to the development effort.

3. Positive Adjustment and Transaction Costs

Now we present our main findings for the more general case, where $c > 0$ and $\tau > 0$, in the following three propositions. In this case dynamic interdependencies among projects in the pipeline emerge and the market price for projects affects optimal R&D thresholds. We begin by focusing on advance-to-development thresholds for the various firm types.

> **Proposition 3.1.** There are thresholds $\hat{z}_{0j}(\varepsilon,\tau)$ and $\hat{z}_{1j}(x,\varepsilon,\tau)$ such that the firm will advance the project from stage I to II only if $z \geq \hat{z}_{i,j}$, for i = 0, 1 ,and j = 1, 0, –1. When $\tau < c$, the thresholds can be ranked as $\hat{z}_{01}(\varepsilon,\tau) \leq \hat{z}_{11}(x,\varepsilon,\tau) \leq \hat{z}_{10}(x,\varepsilon,\tau) \leq \hat{z}_{00}(\varepsilon,\tau) \leq \hat{z}_{1-1}(x,\varepsilon,\tau) \leq \hat{z}_{0-1}(\varepsilon,\tau)$.[13] $\hat{z}_{11}(x,\varepsilon,\tau)$ is non-decreasing in x while $\hat{z}_{10}(x,\varepsilon,\tau)$ and $\hat{z}_{1-1}(x,\varepsilon,\tau)$ are non-increasing in x.

Proof in the appendix.

The above proposition describes how the optimal advance-to-development threshold, $\hat{z}$, differs across firm types. The threshold depends on ε, the market premium for purchasing technology. In the discussion that follows, we emphasize relative comparisons of thresholds of firms types without direct reference to ε. For instance, firm (0,1) has the most urgent need to advance the project in stage I because it has a new product in the market but its R&D pipeline is empty (it has no project currently in stage II) and thus it is eager to fill the pipeline, which causes it to have a low threshold compared to other firm types. In contrast, firm (0,-1) has the least urgent need because with no commercialized product it faces no adjustment costs and thus has the highest threshold.

[13] When $\tau \geq c$, under which assumption the technology market does not function, we can still prove that $\hat{z}_{01}(\varepsilon,\tau) \leq \hat{z}_{11}(x,\varepsilon,\tau) \leq \hat{z}_{10}(x,\varepsilon,\tau) \leq \hat{z}_{00}(\varepsilon,\tau)$ and $\hat{z}_{10}(x,\varepsilon,\tau) \leq \hat{z}_{1-1}(x,\varepsilon,\tau) \leq \hat{z}_{0-1}(\varepsilon,\tau)$.

As $\hat{z}_{01}(\varepsilon,\tau) \leq \hat{z}_{11}(x,\varepsilon,\tau)$ for any *x*, it implies that having a project that can be commercialized may reduce the urgency to advancing a project to development. The strict inequality holds within a range of the market premium *ε*, in which the price is too high for firm (0,1) to buy but too low for firm (1, 1) to sell a project in the technology market. Hence, in this range, firm (1,1) will launch a new product if *x* is sufficiently high and thus can be more selective in advancing the next project to stage II. The relationship between $\hat{z}_{0-1}(\varepsilon,\tau)$ and $\hat{z}_{1-1}(x,\varepsilon,\tau)$ is the opposite because having a successful project for entrants in stage II could increase the urgency to fill its pipeline if the firm actually invests in co-specialized assets. This is also the case for firm (0,0) and firm (1,0). To illustrate, consider a range of market premium in which the price is too high for firm (0,0) to buy but too low for firm (1,0) to sell a project. Firm (0,0) will give up launching a new product (i.e. will become firm (i,-1) in the next period) and, thus, will set the threshold high for advancing a project to stage II; but, firm (1,0) may commercialize its project and keep the threshold for development low. Hence $\hat{z}_{1j}(x,\varepsilon,\tau) \leq \hat{z}_{0j}(\varepsilon,\tau)$ when $j = 0$ *or* -1.

The proposition implies policy rules for balancing the R&D pipeline portfolio. Firm (1,0) is more likely to raise the threshold for a project in stage I the more valuable is the project in stage II. Projects in stages I and II are *substitutes*, in the sense that the firm is in less need for a new product in the next period once the product in stage II is commercialized. In contrast, projects in stages I and II are *complements* for the (1,0) and (1,-1) type firms. Once the project in stage II is launched, the firm is eager to advance the project in stage I to lower the likelihood of incurring the adjustment costs in the future.

The next proposition considers the advance-to-commercialization decision. Firms with a project in stage II may choose to launch, abandon, sell its project, or buy a project in the technology market, while firms that have no project in stage II can only choose to buy one in the technology market.

Proposition 3.2. Consider firms that have a project in stage II. There is a threshold $\hat{x}_j(z,\varepsilon,\tau)$ such that the firm will either launch a product or sell its technology if $x > \hat{x}_j(z,\varepsilon,\tau)$, and it will either terminate the project in stage II or buy one in the market if $x < \hat{x}_j(z,\varepsilon,\tau)$ for *j* = 1, 0 and -1 and any ε and *z*. $\hat{x}_j(z,\varepsilon,\tau)$ satisfies $-\varepsilon \le \hat{x}_0(z,\varepsilon,\tau) \le \hat{x}_1(z,\varepsilon,\tau) \le \hat{x}_{-1}(z,\varepsilon,\tau) \le \tau - \varepsilon$. $\hat{x}_1(z,\varepsilon,\tau)$ is non-decreasing in *z* and $\hat{x}_0(z,\varepsilon,\tau)$ and $\hat{x}_{-1}(z,\varepsilon,\tau)$ are non-increasing in *z*.
For firms with no project in stage II (*i.e.* firm (0, *j*)), their threshold for commercialization is $\hat{x}_j(z,\varepsilon,\tau)$ defined above where *z* is sufficient low.

Proof in the appendix.

The inequalities in Proposition 3.2, $\hat{x}_{-1}(z,\varepsilon,\tau) \ge \hat{x}_1(z,\varepsilon,\tau) \ge \hat{x}_0(z,\varepsilon,\tau)$, imply that firm (1,0) is least likely to terminate its own project because it has the most urgent need to introduce a new product to avoid incurring adjustment costs whereas firm (1,-1) has the least urgent need to introduce a new product because it has no downstream co-specialized assets to maintain.

This proposition is simply the flip side of Proposition 3.1 and indicates that decisions on trade, development, and commercialization are all interdependent. For example, firm (1,-1) that does not have co-specialized investments is least likely to buy and it is especially so when the new project in the first stage is expected to be terminated. $\hat{x}_1(z,\varepsilon,\tau)$, which is non-decreasing in *z*, offers another example. Namely, firm (1,1) will *increase* the threshold for commercialization if there is the prospect of a successful product being introduced in the future (high *z*). This outcome arises because such a firm can choose to terminate an unprofitable product without the fear of incurring the adjustment cost.

Proposition 3.2 is not very informative about firms' decisions to buy or sell projects. Therefore, proposition 3.3 offers conditions under which firms buy or sell projects.

Proposition 3.3. Given any distribution of firm types, and τ and *c*:

i. Firms (0,1) and (1,1) will buy projects only if ε is at a level such that $\delta E(V_{00} - V_{01} \mid \tau, \varepsilon) < -\varepsilon$.[14] When this inequality holds, at least those with $z < \hat{z}_{01}(\varepsilon, \tau)$ and $x < -\varepsilon$ will buy projects. When $\delta E(V_{10} - V_{11} \mid z, \tau, \varepsilon) < -\varepsilon$ holds for all z, all (0,1) and (1,1) type firms with $x < -\varepsilon$ will buy projects.

ii. Firms (0,0) and (1,0) will buy projects only if ε is at a level such that $\delta E(V_{1-1} - V_{11} \mid z, \tau, \varepsilon) - c < -\varepsilon$. When this inequality holds for some $z > \hat{z}_{00}(\varepsilon, \tau)$, at least those with the corresponding *z* and $x < -\varepsilon$ will buy projects. When $\delta E(V_{0-1} - V_{01} \mid \tau, \varepsilon) - c < -\varepsilon$ holds, all (0,0) and (1,0) type firms with $x < -\varepsilon$ will buy projects.

iii. Firms (0,-1) and (1,-1) will buy projects only if ε is at a level such that $\delta E(V_{1-1} - V_{11} \mid z, \tau, \varepsilon) < -\varepsilon$. When this inequality holds for some $z > \hat{z}_{0-1}(\varepsilon, \tau)$, at least those with the corresponding *z* and $x < -\varepsilon$ will buy projects. When $\delta E(V_{0-1} - V_{01} \mid \tau, \varepsilon) < -\varepsilon$ holds, all (0,-1) and (1,-1) type firms with $x < -\varepsilon$ will buy projects.

iv. Firms (1,1) will sell their projects only if ε is at a level such that $\delta E(V_{10} - V_{11} \mid z, \tau, \varepsilon) > -\varepsilon + \tau$. When this inequality holds for some $z > \hat{z}_{11}(x, \varepsilon, \tau)$, at least those with the corresponding *z* and $x > -\varepsilon + \tau$ will sell their projects. When $\delta E(V_{00} - V_{01} \mid \tau, \varepsilon) > -\varepsilon + \tau$ holds, all (1,1) type firms with $x > -\varepsilon + \tau$ will sell their projects.

v. Firms (1,0) will sell their projects only if ε is at a level such that $\delta E(V_{0-1} - V_{01} \mid \tau, \varepsilon) - c > -\varepsilon + \tau$. When this inequality holds, at least those with $z < \hat{z}_{10}(x, \varepsilon, \tau)$ and $x > -\varepsilon + \tau$ will sell their projects. When $\delta E(V_{1-1} - V_{11} \mid z, \tau, \varepsilon) - c > -\varepsilon + \tau$ holds for all *z*, all (1,0) type firms with $x > -\varepsilon + \tau$ will sell their projects.

vi. Firms (1,-1) will sell their projects only if ε is at a level such that $\delta E(V_{0-1} - V_{01} \mid \tau, \varepsilon) > -\varepsilon + \tau$. When this inequality holds, at least those with $z < \hat{z}_{1-1}(x, \varepsilon, \tau)$ and $x > -\varepsilon + \tau$ will sell their projects. When $\delta E(V_{1-1} - V_{11} \mid z, \tau, \varepsilon) > -\varepsilon + \tau$ holds for all *z*, all (1,-1) type firms with $x > -\varepsilon + \tau$ will sell their projects.

Proof in the appendix.

From Lemma 1 in the appendix we can show that

$$\delta E(V_{1-1} - V_{11} \mid z, \tau, \varepsilon) > \delta E(V_{00} - V_{01} \mid z, \tau, \varepsilon) > \delta E(V_{1-1} - V_{11} \mid z, \tau, \varepsilon) - c \qquad (2)$$

$$\delta E(V_{0-1} - V_{01} \mid z, \tau, \varepsilon) > \delta E(V_{10} - V_{11} \mid z, \tau, \varepsilon) > \delta E(V_{0-1} - V_{01} \mid z, \tau, \varepsilon) - c \qquad (3)$$

[14] Since firm (0,*j*) does not have a project in stage II, *z* is not needed to calculate the expected value function in contrast to that for firm (1,*j*).

Proposition 3.3 together with inequalities (2) and (3) imply that firm (0,0) and firm (1,0) are most likely to buy projects while firm (0,-1) and firm (1,-1) are least likely to buy. Similarly, they imply that firm (1,-1) is most likely to sell its project while firm (1,0) is least likely to sell one. Figure 2 illustrates the differences among firms in the thresholds of market premium for buying and selling projects in the technology market assuming a fixed composition of firm types and value of τ.[15] On one hand, at a low level of ε (left side of the diagram) only (1,-1) type firms with $x > \tau - \varepsilon$ are sellers and, as ε increases, (1,1) firms with $x > \tau - \varepsilon$ might join the market selling their projects. On the other hand, at a high level of ε (right side of the diagram) only firm (0,0) and (1,0) with $x < -\varepsilon$ are buyers and, as ε decreases, other incumbent firms, firm (0,1) and (1,1) with $x < -\varepsilon$ could participate in buying. Figure 2 depicts equilibrium ε^* where only entrants (1,-1) with $x > \tau - \varepsilon$ are sellers and only incumbents with an old product in market (0,0) and (1,0) with $x < -\varepsilon$ are buyers, while incumbents (1,1) and (0,1) will not buy or sell.

Proposition 3.3 provides us with the first-order effect of a decline in the transaction cost τ. When τ declines, the conditions regarding both the level of ε and the range of (z, x) in propositions 3.3.iv-vi are all more or equally likely to be satisfied, implying that more firms will sell their projects. In this case, the number of sellers in Figure 2 will shift up. Although the diagram does not reflect the second-order effect through the change in the composition of firm types, the effect will reinforce the first-order effect (see footnote 14). When the number of

[15] Figure 2 does not depict demand and supply curves in the usual sense. The actual demand and supply curves will be steeper because: (1) the higher is the market premium, the greater is the number of sellers, generating more entrants who are most likely to sell projects; and (2) the lower is the market premium, the greater is the number of buyers, generating more (i,1) type incumbents who are most likely to buy projects. We do not attempt to draw such "correct" demand and supply curves because we don't know how the composition will change over time in response to a change in ε. We use the chart only to illustrate the number of buyers and sellers as a function of current market premium given the value functions in a steady state, and to discuss who are buyers and sellers in the market.

sellers rises, the market premium $\varepsilon*$ will drop; hence, there will also be more buyers. We conclude that more trades take place in the technology market as transaction cost decline.

Discussion

We use the analytical results of our model to draw three sets of implications. Our first set is that advance-to-development and advance-to-commercialization thresholds are contingent on the state of a firm's pipeline and on the magnitude of adjustment and transaction costs. Put differently, managing R&D pipelines becomes increasingly strategic whenever transaction costs and adjustment costs are greater than zero—which we maintain is a common occurrence. Our second set of implications is that preferences for project risk depend on the state of a firm's pipeline and on the magnitude of adjustment and transaction costs, which means that firm types vary in the riskiness of projects chosen and advanced. Our final set of implications is that the extent to which entrants commercialize versus sell their projects in the technology market depends on the magnitudes of transaction costs and adjustment costs, which has significance not only for social welfare but also for the managerial incentives of incumbents to shape these costs. Below we describe each set of implications in detail. As a follow-up to the illustration that motivated our research, we also discuss the extent to which our model informs Merck's behavior and we identify the strengths and weaknesses of our model, which helps us to discuss its generality to industries beyond pharmaceuticals. Finally, we discuss opportunities for future research and offer suggestions on empirical approaches to empirically evaluating our model's implications.

Thresholds

Our analytic results indicate that adjustment costs and transaction costs play critical roles in determining optimal advance-to-development and advance-to-commercialization thresholds. In the absence of adjustment costs or transaction costs, we find that each project is evaluated independently—without consideration of portfolio effects—and that the standard option result of Roberts and Weitzman (1981) obtains. The relationship among thresholds for the various firm types becomes more complicated when both adjustment costs and transaction costs are greater than zero. In general, thresholds for both advance-to-development and advance-to-commercialization are increasingly different among firm types the higher are both costs. At a sufficiently high transaction cost, the opportunity to buy and sell projects will foreclose. For higher magnitudes of transactions costs and adjustment costs, we also find that incumbents may adopt a policy with negative thresholds either for advance-to-development or advance-to-commercialization in order to keep from incurring adjustment costs.

One way to get a better sense of the importance of the different thresholds is to evaluate the relative magnitudes of these effects. To do so we tuned our model to characteristic features of the pharmaceutical industry and simulated thresholds for an incumbent with a new product in the market and a project ready to be commercialized (firm (1,1)) and an incumbent with an old product in the market and a project ready to be commercialized (firm (1,0)), which may represent the change of the R&D pipeline status of Merck from the 80s to 90s. Given the approximate time of four to five years of each R&D stage in pharmaceuticals, and the annual capital cost of about 15 percent, we assume a discount factor $\delta = 0.5$. We standardize x so that $x \sim$ normal(μ_o,σ_o) such that $\mu_o = 1$ and $\sigma_o = 1$. We assume $s = x + \varepsilon$, $\varepsilon \sim$ normal(0, 0.75)[16] and development cost is $d = 0.35$, which means that development costs are assumed to be approximately one-third the value of an average project and is not unusual for the pharmaceutical industry. We then chose different values for adjustment cost c and transaction cost τ. For

[16] The standard deviation of ε is set to be smaller than 1 because we assume the uncertainty caused by the signal in stage II should be lower than σ_o.

simplicity we assume that buyers and sellers evenly split the transaction cost, i.e., $\varepsilon = \tau/2$.[17] Using our dynamic optimization model, we simulate the optimal policies (e_{it}, e_{2t}, f_t) for each firm type. We use value iteration techniques developed in the dynamic programming literature[18] to compute the value functions and optimal policies under different values of c and τ.

Figure 3 presents the advance-to-commercialization threshold for three different magnitudes of adjustment costs (c = 0.75, 1, 1.25)[19] over the range of transaction costs from zero to three. For simplicity we only show the results when the value of stage I project z is low but results are similar for different levels of z. When transaction costs are zero the thresholds for firm (1,0) and firm (1,1) are identical and equal to zero for all values of c. As transaction costs take on higher values, thresholds for both firm (1,0) and firm (1,1) are lower. Notice that the thresholds for firm (1,0) are substantially more negative than the ones for firm (1,1). While both firm types lower their threshold to avoid incurring adjustment cost, firm (1,0) is more desperate for new product and therefore has a lower threshold. To put the scale we use into perspective, consider the firm (1,0) at a transaction cost of 1.5 and an adjustment cost of 1. In this case, the threshold is approximately -0.85, which implies that if the average potential drug is worth \$100 MM, the adjustment cost was \$100 MM, and the cost incurred when buying from the technology market is \$75 MM (as we assume $\varepsilon = \tau/2$), then firm (1,0) would be willing to advance to commercialization a drug with an expected value of -\$85 MM.

Also notice that, when transaction costs are low (consider the left side of Figure 3), thresholds for commercialization are lower for increasingly higher levels of transaction costs. These thresholds do not vary with changes in adjustment costs so long as transaction costs

[17] This value of ε may not be an equilibrium price for all levels of τ. We tried the cases of $\varepsilon = \tau/3$ and $\varepsilon = 2\tau/3$ and found same qualitative results.
[18] For example, see a detailed explanation of different solution methods in Rust (1994).
[19] Our purpose of setting the values for c is to illustrate the comparison especially with transaction costs τ. While somewhat arbitrary, varying the values of c and τ reveal similar patterns.

remain relatively low. For higher transaction costs (consider the right side of Figure 3) the thresholds level off and do not change with respect to further increases in transaction costs; yet, these thresholds now are sensitive to the magnitude of adjustment costs. Thresholds level off when transaction costs are too great for the technology market to form. Overall, the advance-to-commercialization threshold for firm (1,0) is approximately twice as negative as the thresholds for firm (1,1).

Figure 4 presents the advance-two-development thresholds for the same three magnitudes of adjustment costs over a similar range of transaction costs. This diagram represents results when firms have a low value for stage II project *x* so that it will not be commercialized. Here again, the thresholds for firm (1,1) and firm (1,0) are identical when transaction costs are zero. As transaction costs increase, the thresholds are increasingly different. Notably, the threshold for firm (1,0) remains positive because advancing a project to development will have no impact on whether or not it incurs adjustment costs in the next period. In contrast, with a new product in the market, advancing a project to development will impact whether or not firm (1,1) incurs an adjustment cost in the next period. Thus, the threshold for firm (1,1) is lower and becomes negative the greater are transaction costs. Here again we see that adjustment costs impact the threshold only when transaction costs are not low. The threshold for firm (1,1) levels off when transaction costs are too great relative to adjustment costs for the technology market to function. In contrast, adjustment costs do not impact the thresholds for firm (1,0) much because the value of *x* is so low that the firm will not commercialize the project in stage II, which means that the firm will incur the adjustment cost anyway regardless of what it does with the project in stage I. When transaction costs are so high that the technology market no longer functions, incumbent firm (1,0) with sufficiently low *x* will behave exactly the same as entrant firm (1,-1) (the right half of Figure 4). When x is high enough so that firms will commercialize their own projects, we find that thresholds for firms (1,1) and (1,0) are identical—both firms have good replacements for their current products so will become the same type of firm (·,1) in the next period. Also, thresholds will be lower the higher are adjustment costs, as is the case for firm (1,1) above.

Risk Preference

Managers must not only set thresholds for advancing and terminating projects but also make decisions about which project to advance when they have multiple projects at the same R&D stage. Such decisions depend on risk preferences – will managers choose high risk and potentially high return or low risk and low return projects? We can show that all value functions are convex in the (z,x) space for a fixed ε and thus all firm types are always risk-loving, i.e., firms prefer a project with higher variance as long as the expected return is the same. This is because of the option value of projects – firms may always terminate a project to avoid further loss in case its value is proved to be negative. As in the case of thresholds, the absence of adjustment and transactions costs implies that all firm types have identical risk preferences. Our model suggests that risk preferences for projects differ when adjustment and transactions costs are positive.

The extent to which firm types might advance more or less risky projects is not intuitively obvious from our model. To investigate the possibility that firm types may display different preferences for project risk we simulated iso-profit curves for each firm type and for different values of adjustment and transaction costs.

We began our analysis by supposing that firms can choose in the research stage from the set of multiple potential projects, each of which has a different expected value and level of uncertainty. For ease of interpretation, assume a standardized distribution for project value: $x \sim$

normal(μ_o,σ_o), and μ_o = 1.0, σ_o = 1.0. All other settings are the same as the simulation exercise we conducted above. We use (μ_o,σ_o) to first compute the expected value for each type of firm, *i.e.,* EV(0,-1), EV(0,0), and EV(0,1) respectively.[20, 21] We next assume a set of discrete values μ_1 that are either larger or smaller than μ_o, and then compute the set of standard deviations σ_1 corresponding to μ_1 that will give the same expected value to the firms at mean μ_o = 1.0 and standard deviation σ_o = 1.0. One may treat this set-up as different R&D projects, each with different expected values and spread in distribution, that bring equivalent (prior) expected value function to different types of firms. In this way we are able to assess how risk-averse or risk-loving a firm is conditional on its type, and make this comparison for different types of firms. Figure 5 graphs the simulated iso-profit curves where adjustment costs are set to 0.75 and transaction costs equal 1.0. Several implications can be deduced from this graph.

In general, the negative slope of the iso-profit curves indicates that all firms are risk-loving (under our assumed parameters). That is, all firm types are willing to accept greater variance for lower expected returns. A second result is that entrant—firm (0,-1) —which had a research project but has not advanced any project to either development or commercialization is more risk loving than incumbents who are about to incur adjustment costs (0,0) or incumbents who have a new product in the market but an empty pipeline (0,1). As an illustration, Figure 5 displays point A, which is a hypothetical project representing a particular (μ_1,σ_1) pair. Compared with the base project (μ_o,σ_o), firms (0,0) and (0,1) would choose project A to advance over the base project whereas firm (0,-1) would choose the base project.

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[20] *EV* is an expectation operator taken with respect to random variable *s* whose definition and (unconditional) distribution is given above. Note that the (conditional) distribution of *x* that is defined above is irrelevant because the firms do not have any project in stage II.

[21] In general, risk preferences will change given different values of *x*. Hence, for the simplicity of illustration we choose firms without stage II projects, i.e., firms (0,-1), (0,0) and (0,1), instead of other firms as we have chosen in the prior simulation exercise.

In order to better understand how preferences for project risk change with respect to transaction costs, we returned to our simulation and estimated the slope of the iso-profit curve at $\mu_o = 1.0$, $\sigma_o = 1.0$ for different values of transaction costs assuming an adjustment cost of 0.75. Figure 6 displays how risk preference for firms (0,-1), (0,0) and (0,1) varies with transaction costs. The patterns are very similar to the thresholds results above. With transaction costs equal to zero all firms have the same risk preference and are risk-loving. Firm (0,1) becomes increasingly less risk-loving (a steeper albeit negative slope) the greater is transaction costs, at least up to the point where the firm will no longer purchase in the technology market. On the other extreme, the entrant's risk preference also becomes less risk-loving but minimally so, until it can no longer sell projects in the technology market. Firm (0,0)'s risk preference falls in between these two cases until it can no longer purchase projects in the technology market because of high transaction costs. When this happens, firm (0,0) will incur adjustment costs with certainty and therefore its risk preference converges to that of the potential entrant. Although not presented in Figure 6, we also find that risk preference is lower when adjustment costs increase from 0.75 to 1.25. The impact is especially large for firm (1,1).

To summarize our simulation results, entrant firms with no co-specialized assets will tend to pursue high-uncertainty projects while incumbent firms with co-specialized assets will prefer relatively moderate-value low-uncertainty projects. In general, firms will be less risk seeking the greater are either adjustment costs or transaction costs.

The differences in risk preferences our model finds may provide at least one explanation for the enigma of R&D productivity, in which R&D productivity apparently declines with firm size and age (for example, see Cohen and Klepper 1996). So long as the investment in co-specialized downstream assets is positively correlated with firm size, incumbent large firms will be less risk-loving in research and development activities than small entrants, which, given the exit of unsuccessful small firms, should lead to the observed pattern. Our model provide an alternative explanation for Gans and Stern (2000) (see footnote 4) and also resonates with Williamson's (1985) arguments that specific investments have substantial implications for innovative behavior and firm performance. Unlike Williamson's discussion, however, we provide a formal model of how downstream co-specialized assets and transaction costs in the technology market affect investment decisions.

Buying and Selling in the Technology Market

As presented in the section of analytical results, our model implies that entrants engaged in R&D are most likely to sell their projects to incumbents and incumbents with expiring products are most likely to buy such projects. For sufficiently low transaction costs, the following patterns will appear: (1) Entrants specialize in R&D activities and rarely commercialize their projects.. Firm (1,-1) will likely sell its project so long as the value of its project at the end of the development stage is greater than the transaction costs less the market premium; (2) Incumbent firm (0,0) is most likely to buy but so will firm (1,0) if the value of its own stage II project is less than the negative of the market premium ε, the cost incurred if it buys projects from the technology market; (3) Incumbent firm (1,1) will likely focus on advancing its own projects unless the market premium is great at which point it might sell projects. Figure 2 illustrates an example where, under equilibrium ε^*, incumbents (0,0) and (1,0) with $x<-\varepsilon^*$ will buy while entrants (1,-1) with $x>\tau-\varepsilon^*$ will sell projects. Higher transaction costs will increasingly reduce the likelihood of buying and selling projects and the market fails when transaction costs are high.

These implications are interesting from a policy standpoint. For instance, from a welfare economics point of view, a decline in transaction costs in the technology market might improve efficiency through three channels: (1) more projects which would otherwise be terminated by new start-ups will be commercialized by incumbents; (2) incumbents will commercialize fewer unprofitable projects (just simply to avoid incurring adjustment costs) as they can purchase more profitable ones in the market; and (3) the prospect of being able to sell projects in the market will encourage entrepreneurs to advance more R&D projects to the development stage.

The technology market also is of interest to managers. For instance, we conclude that incumbents have an incentive to encourage the formation of institutions that lower transaction costs in technology markets. Transaction costs might be reduced by encouraging the formation of third parties such as technology consulting and transfer firms that can lower adverse selection and moral hazard problems (numerous kinds of these firms can be found in the pharmaceutical industry) or by encouraging refinements in laws to strengthen intellectual property protections. Doing so allows incumbents to retain their vertically integrated position and eliminates potential for investment of co-specialized assets by entrants who might otherwise commercialize projects.

If incumbents cannot lower transaction costs in the technology market, an indirect way to generate a similar industry structure is to raise the level of adjustment costs. For instance, an incumbent pharmaceutical firm may increase its own co-specialized investments in training sales representatives hence other incumbents have to follow in order to compete. Analogous to the well-known IO literature on the effect of increases in sunk investment on entry/competition, here the effect of an increase in co-specialized investments is to encourage start-ups to sell in the technology market instead of commercialize their own projects. Though to explore the impact on competition is out of the scope of the current paper, we believe it is an interesting area for future research based on our model framework.

Face Validity and Future Research

The face validity of our model perhaps can be assessed by viewing the Merck illustration through the lens of our theory. Our propositions indicate that when a pipeline is full—that is when products have been recently commercialized and new products that can potentially replace commercialized products once they expire are in development—the (1,1) firm adopts high thresholds for advance-to-commercialize. If the firm has a high valued project in later R&D stages that is expected to be introduced into the market soon, it will choose riskier projects in the early R&D stages. Such was the case in the mid-1980s when Vagelos admonished scientists to drop projects that did not have blockbuster potential when Merck's pipeline was full and it had successful new products. Nonetheless, if its projects in later R&D stages are less successful, the (1,1) firm keeps the threshold for advance-to-development relatively low because potential products may replace the current blockbuster product in time or redundant technology can be sold in the market (see Figure 4) . In other words, managers choose to commercialize only those projects with blockbuster potential but continue to invest in keeping the pipeline full because projects in early development stage have high option values for such a firm. So the firm is more likely to choose less risky project in the early R&D stages (e.g., firm (0,1) is the least risk-loving firm type in our simulation results). This may have been the case for Merck in the early 90s when its R&D pipeline started to thin out with fewer drugs with blockbuster potential. As commercialized products grow old but the firm has projects in development—a (1,0) firm—managers lower the advance-to-market threshold, which makes it more likely to advance lower value projects in the firm and more desperate to look for outside alternatives from the technology market. Merck's actions in the late 1990s and 2005 to replenish it pipeline by purchasing projects from the technology market and lowering its advance-to-development threshold are consistent with our model's predictions. The parallels between our propositions and Merck's actions suggest that our model does offer face validity at least with respect to Merck's experience.

While the pharmaceutical industry motivated our model, its propositions are not unique to Merck or to that industry. Our model can be applied to any firm in any industry where R&D pipelines, transaction costs in the technology market, and adjustment costs are important. Our model also suggests a new insight for managing R&D portfolios. Whereas the literature on managing R&D portfolios typically either treats projects as independent or considers the portfolio of all projects in its entirety (often only those projects at a single stage), our model indicates that projects should be partitioned vertically and managed with respect to the co-specialized assets needed for their commercialization. Incumbents may possess several clusters of complementary assets co-specialized to the commercialization of different types of projects. In the pharmaceutical industry, for instance, a firm may have one sales force co-specialized for the distribution of cardiovascular pharmaceuticals and another sales force co-specialized for the distribution of oncology products. Such a firm's R&D portfolio should be partitioned so that cardiovascular and oncology projects are managed as separate portfolios. No other research has identified and motivated this type of vertical portfolio partitioning and management. Furthermore, while most of the previous literature focus on the substitutability or complementarity relationship among projects at same R&D stages or products in the market, our study provides a new perspective at looking at the vertical dynamic relationship between projects in different R&D stages, which we find to be critical in industries where transaction costs in the technology market and adjustment costs are important.

Future research needs to bring our predictions to data. Empirically operationalizing our predictions is difficult but possible. Many firms, especially high technology firms, develop estimates of project risk as well as formally state thresholds (for an accounting of different methodologies see Cooper and Edgett, 2001). For instance, Merck has used risk value estimates along with Monte Carlo modeling to make advancement and termination decisions for projects since the 1980s (see Lewent and Nichols, 1994). Single firm case studies using historical panel data would provide an important first step for empirically evaluating our predictions of both thresholds and risk preferences in response to changes in the state of a firm's portfolio.

The buying and selling of projects in technology market provides other opportunities for empirical research. For instance, recent research in the pharmaceutical industry (Higgins and Rodriguez, 2006) assessed the state of firms' R&D portfolios and found that firms with thin R&D pipelines are more likely to purchase technology projects through acquisitions, licensing, and joint ventures. These findings resonate with our model. However, further research is warranted because co-specialized assets were not identified. Nonetheless, Higgins and Rodriguez lay out an empirical approach for assessing firm pipelines, which could be augmented to account for adjustment costs by examining each firm's co-specialized assets. So, in the pharmaceutical industry, we anticipate that those established firms that end up with few candidate drugs in their R&D pipeline will be more likely to form joint ventures with biotechnology firms whose drug discoveries are well matched with the incumbent's co-specialized distribution channels. Relatedly, we anticipate that mergers and acquisitions may be made for similar reasons, which imply both weakness in a firm's pipeline and the match between development pipelines and distribution channels are relevant factors. Also, the pharmaceutical industry offers sufficient transaction data to assess to what extent price premia correlate with our theory. Our model also provides predictions of which firm types are more likely to buy, which more likely to sell, and which more likely to rely on own R&D development as transaction costs change. Exogenous changes in legislative regulations such as the US Antitrust Guidelines for the Licensing of Intellectual Property (1995) and the emergence of the third party information creators and aggregators all have impacts on transaction costs. Recognizing these shifts in transaction costs, we can examine our propositions with technology market transaction data.

Another area of future study involves the number of research projects initiated and which project among multiple potential candidates should be chosen to advance to later stages. Our model focuses on the vertical management of projects in different R&D stages but does not explicitly consider the horizontal management – managing multiple projects in each stage, which is an important element of managing and balancing R&D portfolios. A more complete understanding of portfolio management could be derived from a model in which multiple projects per stage are considered.

Finally, one fascinating question we could ask in the future is how the technology market will look like in the stead-state equilibrium. How industry characteristics affect the market premium of technology and market turnovers? In our model, we simply assume that the market is in the equilibrium and derive the optimal R&D management policies without solving the market clearing condition. If we could add the process of calculating the market composition of firm types and price adjustment to clear the market into our simulation, we could analyze truly dynamic interdependencies among industry characteristics, R&D activities and trades in technology market in the long-term equilibrium. Such a model, if successfully developed, presents a substantial opportunity for future research.

Conclusion

The most important implication of our model is that managers strategically manage their vertical R&D projects in different stages by dynamically changing policies to balance their portfolios.

This dynamic management strategy arises when firms do, or might in the future make, co-specialized investments in downstream activities and when transaction costs in technology markets are positive. Our theory proposes how these policies change depending on the magnitude of co-specialized investments and transaction costs as well as the state of the pipeline. Put differently, our model suggests that managing R&D pipelines becomes increasingly strategic whenever transaction costs and adjustment costs are greater than zero. Our model provides a general formulation in the sense that the standard prescription of independently advancing projects with a positive expected value is a special case of our model in which adjustment costs are zero.

Our model led us to three sets of results. First, our model's analytic results demonstrated that the thresholds for advancing (as well as for buying and selling) projects differ by the state of the firm's pipeline, the magnitude of transaction costs in the technology market, and the magnitude of adjustment costs. Incumbent firms choose projects with positive expected value, whether developed internally or purchased in the market, so long as transaction costs for buying and selling in the technology market are negligible. As transaction costs increase in the technology market, they may choose internal projects—even those with negative expected value—in order to avoid incurring either adjustment or transaction costs. Second, firms are in general risk-loving in their R&D activities, but entrants are comparatively more risk-loving than incumbents implying differences in project risk and hence portfolio management. Changes in adjustment costs and transaction costs can effect preferences for project risk – both incumbents and entrants are increasingly less risk-loving the greater are adjustment costs or transaction costs. Third, our model implied that entrants are more likely to sell their projects to incumbents as transaction costs decline. For sufficiently high adjustment costs or sufficiently low transaction costs, entrants specialize in R&D activities and rarely commercialize projects. These results yield interesting public policy implications as well as generate incentives for managers to shape both transaction costs in the market and adjustment costs within their firms.

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Figure 1: Modeling Assumptions with Technology Market and Adjustment Costs

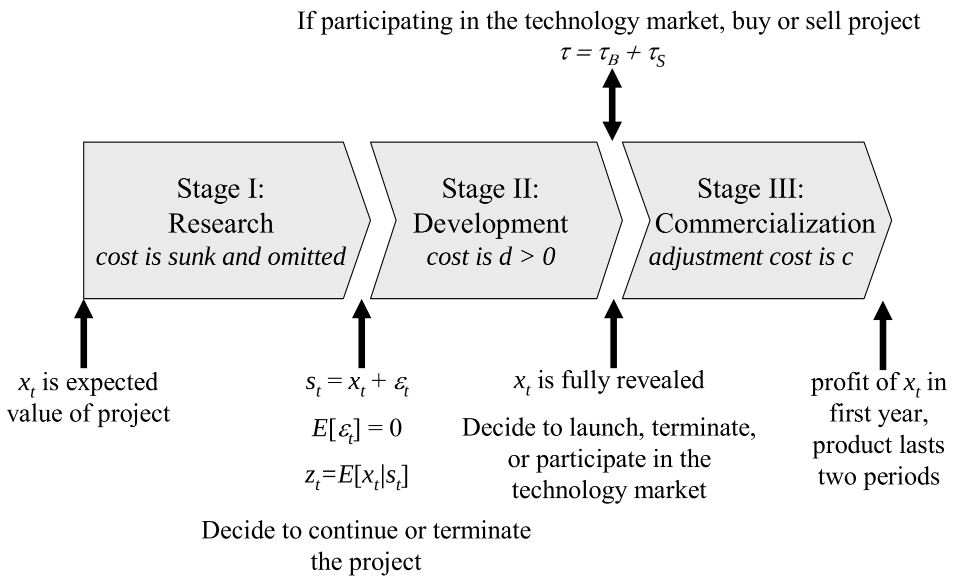


Figure 2: Numbers of Buyers and Sellers in the Technology Market

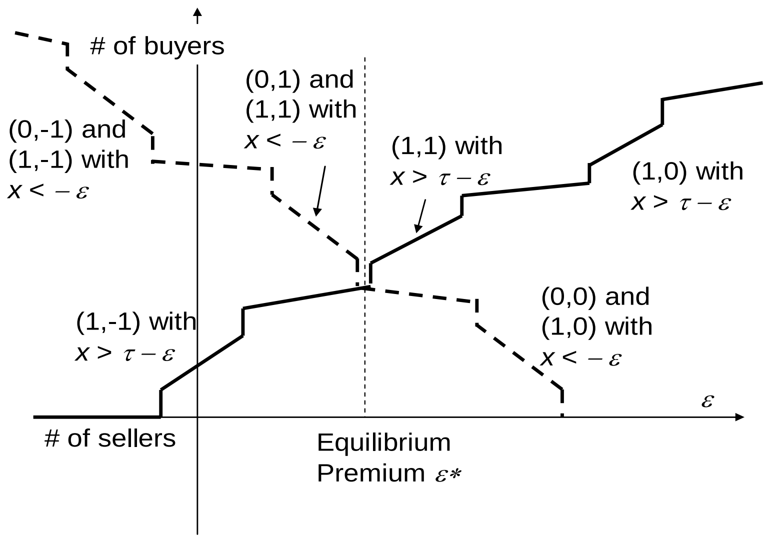
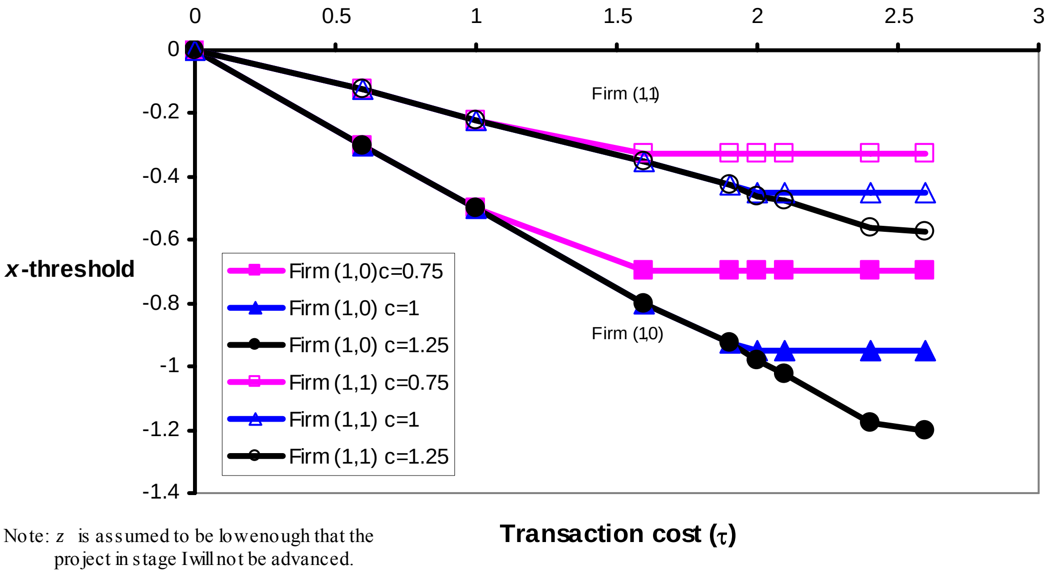
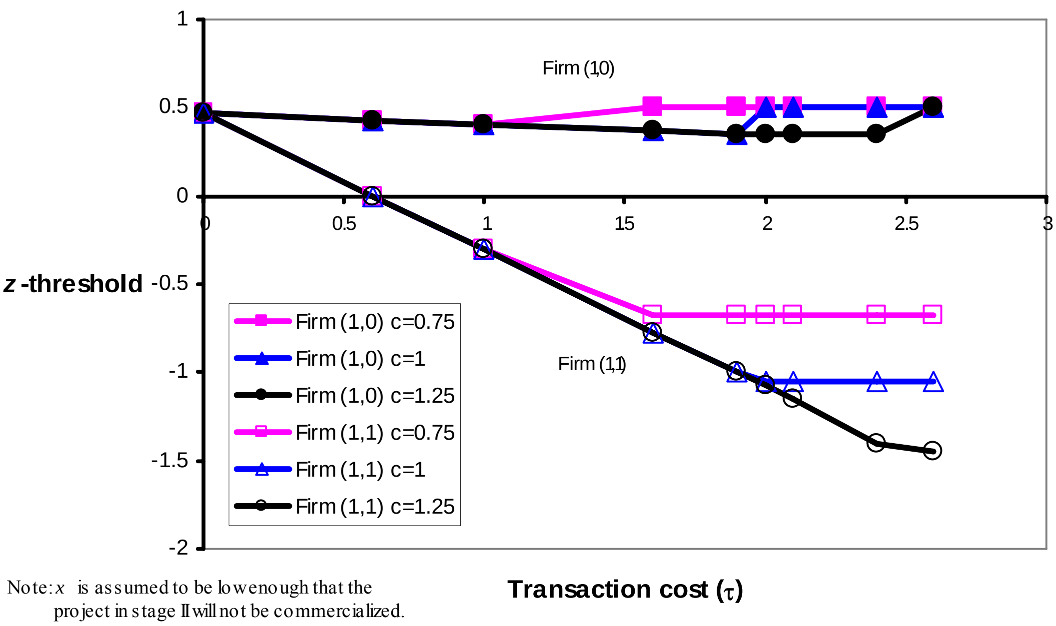


Figure 3: Change of $\hat{x}$ of Incumbents



Note: z is assumed to be low enough that the project in stage I will not be advanced.

Figure 4: Change of $\hat{z}$ of Incumbents



Note: x is assumed to be low enough that the project in stage II will not be commercialized.

Figure 5: Iso-Profit Curves (c = 0.75, τ = 1)

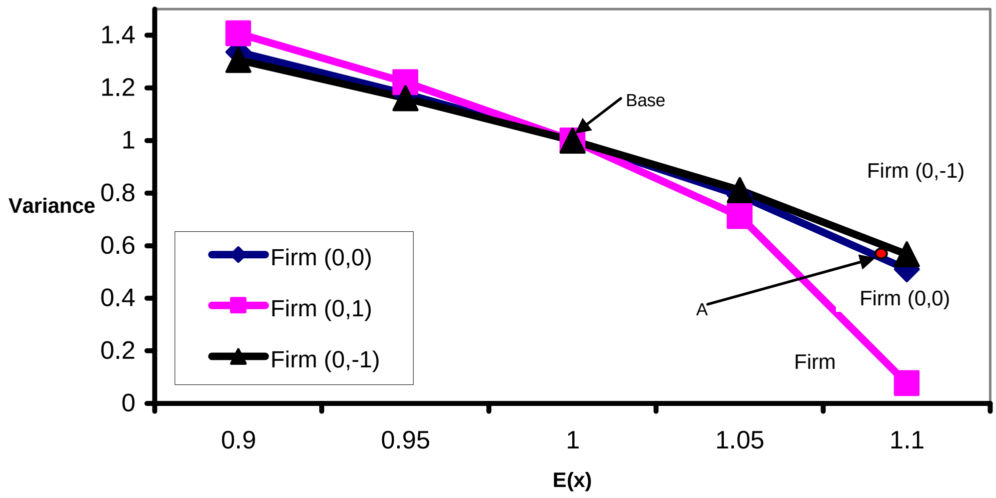
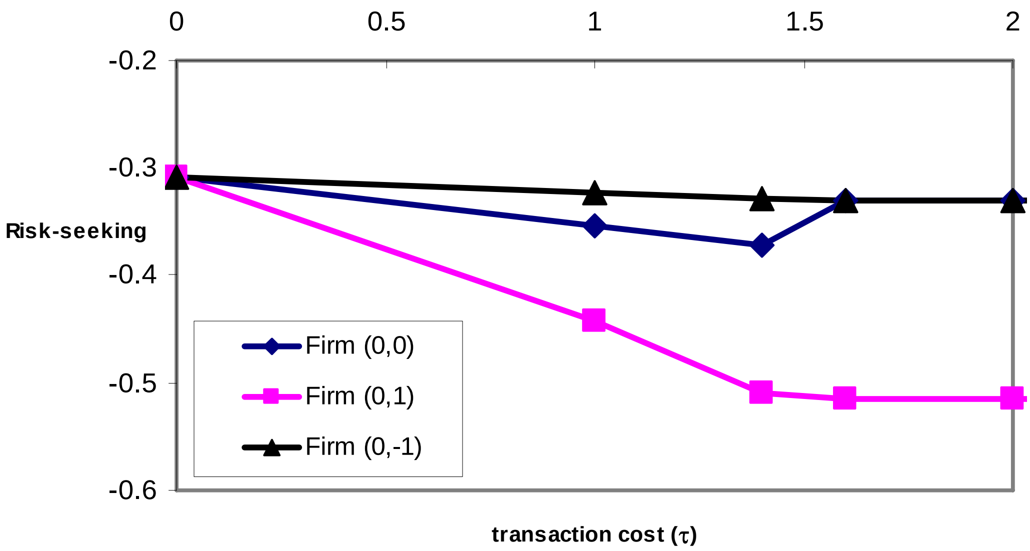


Figure 6: Risk-Seeking (c = 0.75)



Appendix

Before proceeding with our analysis, we set up some model preliminaries that will be useful to understand our theoretical results. Let V_{ij} denote the value function in (1). Given two values of *i* and three values of *j*, we have altogether six types of firms. Suppose that the firm has a project in stage II, the value of which is *x*, and the firm is choosing between selling the project in the technology market and abandoning it. It is straightforward that the firm will sell only if the revenue it obtains is positive, *i.e.*, $x+\varepsilon-\tau \geq 0$. Suppose that the firm has a project in stage II and has decided to launch a new product in the market, and the firm is choosing between launching its own project with value *x* and buying another project in the technology market, where it has to pay the premium of ε. Again, it is straightforward that the firm will launch its own project only if $x \geq -\varepsilon$. Define $|X|_+ \equiv \max\{X,0\}$ and $E(V \mid z_t) \equiv E[V(z_{t+1},x_t) \mid z_t]$. We can break down the value function of the six types of firms from (1) as follows:

$$V_{01}(z)=\max\{\delta EV_{00}; -d+\delta E(V_{10} \mid z); -\varepsilon+\delta EV_{01}; -d-\varepsilon+\delta E(V_{11} \mid z)\},$$

$$V_{11}(z,x)=\max\{|x+\varepsilon-\tau|_+ +\delta EV_{00};\ -d+|x+\varepsilon-\tau|_+ +\delta E(V_{10} \mid z);\\ |x+\varepsilon|_+ -\varepsilon+\delta EV_{01};\ -d+|x+\varepsilon|_+ -\varepsilon+\delta E(V_{11} \mid z)\},$$

$$V_{00}(z)=\max\{-c+\delta EV_{0-1}; -d-c+\delta E(V_{1-1} \mid z);\ -\varepsilon+\delta EV_{01}; -d-\varepsilon+\delta E(V_{11} \mid z)\},$$

$$V_{10}(z,x)=\max\{-c+|x+\varepsilon-\tau|_+ +\delta EV_{0-1};\ -d-c+|x+\varepsilon-\tau|_+ +\delta E(V_{1-1} \mid z);\\ |x+\varepsilon|_+ -\varepsilon+\delta EV_{01};\ -d+|x+\varepsilon|_+ -\varepsilon+\delta E(V_{11} \mid z)\},$$

$$V_{0-1}(z)=\max\{\delta EV_{0-1}; -d+\delta E(V_{1-1} \mid z);\ -\varepsilon+\delta EV_{01}; -d-\varepsilon+\delta E(V_{11} \mid z)\},$$

$$V_{1-1}(z,x)=\max\{|x+\varepsilon-\tau|_+ +\delta EV_{0-1};\ -d+|x+\varepsilon-\tau|_+ +\delta E(V_{1-1} \mid z);\\ |x+\varepsilon|_+ -\varepsilon+\delta EV_{01};\ -d+|x+\varepsilon|_+ -\varepsilon+\delta E(V_{11} \mid z)\},$$

We use the following notation in the proofs of propositions:
Define

$$V_1(z)=\max\{\delta EV_{00}; -d+\delta E(V_{10} \mid z)\}$$
$$V_2(z)=\max\{\delta EV_{01}; -d+\delta E(V_{11} \mid z)\},$$
$$V_3(z)=\max\{\delta EV_{0-1}; -d+\delta E(V_{1-1} \mid z)\}.$$

We can reduce the value functions of all six firm types into the following:

$$V_{01}(z)=\max\{V_1(z);\ -\varepsilon+V_2(z)\}, \tag{A.4}$$
$$V_{11}(z,x)=\max\{|x+\varepsilon-\tau|_+ +V_1(z);\ |x+\varepsilon|_+ -\varepsilon+V_2(z)\}, \tag{A.5}$$
$$V_{00}(z)=\max\{-c+V_3(z);\ -\varepsilon+V_2(z)\}, \tag{A.6}$$
$$V_{10}(z,x)=\max\{-c+|x+\varepsilon-\tau|_+ +V_3(z);\ |x+\varepsilon|_+ -\varepsilon+V_2(z)\}, \tag{A.7}$$
$$V_{0-1}(z)=\max\{V_3(z);\ -\varepsilon+V_2(z)\}, \tag{A.8}$$
$$V_{1-1}(z,x)=\max\{|x+\varepsilon-\tau|_+ +V_3(z);\ |x+\varepsilon|_+ -\varepsilon+V_2(z)\}. \tag{A.9}$$

Based on the above preliminaries, we have the following results which are frequently used to prove propositions:

Lemma 1. Compare the values of different types of firms, we have $V_{00}(z) \le V_{01}(z) \le V_{0-1}(z)$ and $V_{10}(z,x) \le V_{11}(z,x) \le V_{1-1}(z,x)$. In addition, $V_{10}(z,x) - V_{00}(z) \ge V_{11}(z,x) - V_{01}(z) \ge V_{1-1}(z,x) - V_{0-1}(z)$. All inequalities are strict for some sets of (*z, x*) with positive measure, implying that, for any z_t,

$$E[V_{10}(z_{t+1}, x_t) - V_{00}(z_{t+1}) \mid z_t] > E[V_{11}(z_{t+1}, x_t) - V_{01}(z_{t+1}) \mid z_t] > E[V_{1-1}(z_{t+1}, x_t) - V_{0-1}(z_{t+1}) \mid z_t]$$

Proof of Lemma 1

By comparing (A.6) with (A.8), and (A.7) with (A.9), it is immediate that

$$V_{0-1}(z) - c \le V_{00}(z) \le V_{0-1}(z),$$

and

$$V_{1-1}(z,x) - c \le V_{10}(z,x) \le V_{1-1}(z,x).$$

Hence, it can be derived that,

$$V_3(z) - c \le V_1(z) \le V_3(z). \tag{A.10}$$

Then, by definition we have the first result:

$$V_{0-1}(z) - c \le V_{00}(z) \le V_{01}(z) \le V_{0-1}(z) \tag{A.11}$$

and

$$V_{1-1}(z,x) - c \le V_{10}(z,x) \le V_{11}(z,x) \le V_{1-1}(z,x). \tag{A.12}$$

From the above inequalities and by definition,

$$V_1(z) \le V_2(z) \le V_3(z). \tag{A.13}$$

Next, define

$$Z_1 \equiv \{z \in R \mid V_1(z) \ge -\varepsilon + V_2(z)\},$$

$$Z_0 \equiv \{z \in R \mid -c + V_3(z) \ge -\varepsilon + V_2(z)\},$$

$$Z_{-1} \equiv \{z \in R \mid V_3(z) \ge -\varepsilon + V_2(z)\}$$

where *R* is the set of all real numbers. From (A.10),

$$Z_0 \subset Z_1 \subset Z_{-1} \tag{A.14}$$

Now,

$$V_{10}(z,x) - V_{00}(z) = \max\{|x+\varepsilon-\tau|_+, |x+\varepsilon|_+ - \varepsilon + V_2(z) - V_3(z) + c\} \text{ for } z \in Z_0$$

$$= |x+\varepsilon|_+ \text{ for } z \notin Z_0$$

$$V_{11}(z,x) - V_{01}(z) = \max\{|x+\varepsilon-\tau|_+, |x+\varepsilon|_+ - \varepsilon + V_2(z) - V_1(z)\} \text{ for } z \in Z_1$$

$$= |x+\varepsilon|_+ \text{ for } z \notin Z_1$$

$$V_{1-1}(z,x) - V_{0-1}(z) = \max\{|x+\varepsilon-\tau|_+, |x+\varepsilon|_+ - \varepsilon + V_2(z) - V_3(z)\} \text{ for } z \in Z_{-1}$$

$$= |x+\varepsilon|_+ \text{ for } z \notin Z_{-1}$$

From (A.10) and (A.14), we can derive the second result:

$$V_{10}(z,x) - V_{00}(z) \ge V_{11}(z,x) - V_{01}(z) \ge V_{1-1}(z,x) - V_{0-1}(z)$$

where the inequalities are strict for some sets of (*z,x*) with positive measure given that *f* is continuous and *f*(*x*) > 0 for all *x*. ■

Proof of Proposition 1

Suppose $c = 0$. By definition, $V_{00}(z) = V_{0-1}(z)$ and $V_{10}(z,x) = V_{1-1}(z,x)$. Then, from the definition of V_i's, $V_1(z) = V_3(z)$. This leads to $V_1(z) = V_2(z) = V_3(z)$ by (A.13).

Now, from definition, $V_{0j}(z) = \max\{0;\ -\varepsilon\} + V_1(z)$ for all (0, *j*) firms and the firm should buy a technology only if $\varepsilon < 0$. At the same time, $V_{1j}(z,x) = \max\{|x+\varepsilon-\tau|_+;\ |x+\varepsilon|_+ - \varepsilon\} + V_1(z)$. Thus, the firm will sell its technology if $x > \tau - \varepsilon$ and $\varepsilon > \tau \geq 0$. Therefore, there exists no market premium for which some firms benefit from buying or selling technology in the market. Hence $f_t = 0$ for all *t*.

This result simplifies the value functions as follows:

$$V_{0j}(z) = \max\{\delta E(V_{00}\,|\,z);\ -d + \delta E(V_{10}\,|\,z)\},$$

$$V_{1j}(z,x) = \max\{\delta E(V_{00}\,|\,z);\ -d + \delta E(V_{10}\,|\,z);\ x + \delta E(V_{00}\,|\,z);\ -d + x + \delta E(V_{10}\,|\,z)\}$$

$$= |x|_+ + V_{0j}(z)$$

They mean that the project in stage II should be commercialized if and only if $x > 0$ and the project in stage I should be advanced to stage II only if

$$\delta E(V_{00}\,|\,z) \leq -d + \delta E(V_{10}\,|\,z) \text{ for all types}.$$

Furthermore, the right-hand side of this inequality is increasing in *z* while the left-hand side does not depend on *z* thus there is a critical threshold $\hat{z}$ such that the product should be developed if and only if $z > \hat{z}$. $\hat{z}$ is determined by $d = \delta E(V_{10} - V_{00}\,|\,z) = \delta \int_0^{+\infty} x f(x\,|\,z)dx$. This concludes the proof. ■

To prove Propositions 2 - 3.1, we use the following notation:

Define $\underline{V_{0j}}$, $\overline{V_{0j}}(z)$, $\underline{V_{1j}}(x)$ and $\overline{V_{1j}}(x,z)$ by

$$\underline{V_{01}} = \max\{\delta EV_{00}; -\varepsilon + \delta EV_{01}\},$$

$$\overline{V_{01}}(z) = \max\{-d + \delta E(V_{10}\,|\,z); -d - \varepsilon + \delta E(V_{11}\,|\,z)\}.$$

$$\underline{V_{00}} = \max\{-c + \delta EV_{0-1};\ -\varepsilon + \delta EV_{01}\},$$

$$\overline{V_{00}}(z) = \max\{-d - c + \delta E(V_{1-1}\,|\,z); -d - \varepsilon + \delta E(V_{11}\,|\,z)\},$$

$$\underline{V_{0-1}} = \max\{\delta EV_{0-1};\ \ -\varepsilon + \delta EV_{01}\},$$

$$\overline{V_{0-1}}(z) = \max\{-d + \delta E(V_{1-1}\,|\,z); -d - \varepsilon + \delta E(V_{11}\,|\,z)\}.$$

$$\underline{V_{11}}(x) = \max\{|x+\varepsilon-\tau|_+ + \delta EV_{00};\ |x+\varepsilon|_+ - \varepsilon + \delta EV_{01}\},$$

$$\overline{V_{11}}(z,x) = \max\{-d + |x+\varepsilon-\tau|_+ + \delta E(V_{10}\,|\,z);\ -d + |x+\varepsilon|_+ - \varepsilon + \delta E(V_{11}\,|\,z)\},$$

$$\underline{V_{10}}(x) = \max\{-c + |x+\varepsilon-\tau|_+ + \delta EV_{0-1};\ \ |x+\varepsilon|_+ - \varepsilon + \delta EV_{01}\},$$

$$\overline{V_{10}}(z,x) = \max\{-d - c + |x+\varepsilon-\tau|_+ + \delta E(V_{1-1}\,|\,z);\ -d + |x+\varepsilon|_+ - \varepsilon + \delta E(V_{11}\,|\,z)\},$$

$$\underline{V_{1-1}}(x) = \max\{|x+\varepsilon-\tau|_+ + \delta EV_{0-1};\ \ |x+\varepsilon|_+ - \varepsilon + \delta EV_{01}\},$$

$$\overline{V_{1-1}}(z,x) = \max\{-d + |x+\varepsilon-\tau|_+ + \delta E(V_{1-1}\,|\,z);\ -d + |x+\varepsilon|_+ - \varepsilon + \delta E(V_{11}\,|\,z)\}$$

Note that $V_{0j}(z) = \max\{\underline{V_{0j}}, \overline{V_{0j}}(z)\}$ and $V_{1j}(x,z) = \max\{\underline{V_{1j}}(x), \overline{V_{1j}}(x,z)\}$. Let $\hat{z}_{0,j}(\varepsilon,\tau)$ be the number such that $\underline{V_{0j}} = \overline{V_{0j}}(z)$ and $\hat{z}_{1,j}(x,\varepsilon,\tau)$ be the number such that $\underline{V_{1j}}(x) = \overline{V_{1j}}(x,z)$. $\hat{z}_{i,j}$ is uniquely determined because $\underline{V_{ij}}$ is constant with respect to *z* while $\overline{V_{ij}}$ is increasing in *z*. Then, by definition, $\underline{V_{0j}} < \overline{V_{0j}}(z)$ ($\underline{V_{1j}}(x) < \overline{V_{1j}}(x,z)$) for $z > \hat{z}_{0,j}(\varepsilon,\tau)$ ($z > \hat{z}_{1,j}(x,\varepsilon,\tau)$) implying that the first-stage project advances to the second stage and $\underline{V_{0j}} > \overline{V_{0j}}(z)$ ($\underline{V_{1j}}(x) > \overline{V_{1j}}(x,z)$) for $z < \hat{z}_{0,j}(\varepsilon,\tau)$ ($z < \hat{z}_{1,j}(x,\varepsilon,\tau)$) meaning that the project is terminated.

Proof of Proposition 2

Substitute in $\tau = 0$ into (A.5), (A.7) and (A.9), and obtain

$$V_{11}(z,x) = |x+\varepsilon|_+ + V_{01}(z), \tag{A.15}$$

$$V_{10}(z,x) = |x+\varepsilon|_+ + V_{00}(z), \tag{A.16}$$

and

$$V_{1-1}(z,x) = |x+\varepsilon|_+ + V_{0-1}(z). \tag{A.17}$$

Hence, all firms with a project in the second stage (*i.e.,* the (1,1), (1,0) and (1,-1) firms) should launch the product if and only if $x > -\varepsilon$. Furthermore, since

$$V_{11}(z,x) - V_{01}(z) = V_{10}(z,x) - V_{00}(z) = V_{1-1}(z,x) - V_{0-1}(z), \tag{A.18}$$

$\hat{z}_{0,1}(\varepsilon,\tau) = \hat{z}_{0,0}(\varepsilon,\tau) = \hat{z}_{0,-1}(\varepsilon,\tau) = \hat{z}_{1,1}(x,\varepsilon,\tau) = \hat{z}_{1,0}(x,\varepsilon,\tau) = \hat{z}_{1,-1}(x,\varepsilon,\tau) \equiv \hat{z}(\varepsilon)$ by definition where $\hat{z}(\varepsilon)$ is defined by $d = \delta E(V_{1j} - V_{0j} \mid \hat{z}) = \delta \int_{-\varepsilon}^{+\infty} x f(x \mid \hat{z}) dx$. This concludes the proof. ■

Proof of Proposition 3.1

The first half of the proposition (i.e. the existence of the thresholds) is trivial from the definition of $\hat{z}_{i,j}$ introduced above. For the second half, we will first characterize $\hat{z}_{0,j}(\varepsilon,\tau)$ to prove $\hat{z}_{0,1}(\varepsilon,\tau) \le \hat{z}_{0,0}(\varepsilon,\tau) \le \hat{z}_{0,-1}(\varepsilon,\tau)$. We do so by identifying the thresholds for three different ranges of ε: low, intermediate, and high. Then, we will characterize $\hat{z}_{1,j}(x,\varepsilon,\tau)$ to prove the rest of the proposition.

z-thresholds for firms (0,1), (0,0), and (0,-1)

When ε is low enough

The (0, *j*) firm will buy new technology in the market in the neighborhood of $z = \hat{z}_{0,j}(\varepsilon,\tau)$. Thus,

$$V_{01}(z) = V_{00}(z) = V_{0-1}(z) = \max\{\delta E V_{01};\ -d + \delta E(V_{11} \mid z)\} - \varepsilon$$
$$\begin{cases} > \max\{\delta E V_{00};\ -d + \delta E(V_{10} \mid z)\} & \text{for } j = 1 \\ > \max\{\delta E V_{0-1};\ -d + \delta E(V_{1-1} \mid z)\} - c & \text{for } j = 0 \\ > \max\{\delta E V_{0-1};\ -d + \delta E(V_{1-1} \mid z)\} & \text{for } j = -1 \end{cases} \tag{A.19}$$

at $z = \hat{z}_{0,j}(\varepsilon,\tau)$. Let $\tilde{z}_1(\varepsilon,\tau)$ be the solution for

$$\delta E V_{01} = -d + \delta E(V_{11} \mid z). \tag{A.20}$$

Then, $\hat{z}_{01}(\varepsilon,\tau)=\hat{z}_{00}(\varepsilon,\tau)=\hat{z}_{0-1}(\varepsilon,\tau)=\tilde{z}_1(\varepsilon,\tau)$ by definition.

<u>When ε is in the intermediate range</u>

As ε increases, the following equation would hold first at $z=\hat{z}_{0,j}(\varepsilon,\tau)$:

$$\max\{\delta EV_{01};\ -d+\delta E(V_{11}\mid z)\}-\varepsilon\begin{cases}=\max\{\delta EV_{00};\ -d+\delta E(V_{10}\mid z)\} & \text{for } j=1\\ =\max\{\delta EV_{0-1};\ -d+\delta E(V_{1-1}\mid z)\}-c & \text{for } j=0\\ =\max\{\delta EV_{0-1};\ -d+\delta E(V_{1-1}\mid z)\} & \text{for } j=-1\end{cases}\quad \text{(A.21)}$$

For j = 1, when the above equation holds, $\hat{z}_{0,1}(\varepsilon,\tau)$ should satisfy

$$\delta EV_{01}-\varepsilon=-d+\delta E(V_{10}\mid z), \qquad \text{(A.22)}$$

namely, the firm will either buy new technology without advancing its own project to the second stage or pursue developing its own technology without buying any from the market. To show this, Suppose not. Then, $\delta EV_{01}<-d+\delta E(V_{11}\mid z)$ or $\delta EV_{00}>-d+\delta E(V_{10}\mid z)$ at $z=\hat{z}_{0,1}(\varepsilon,\tau)$. When the former is true, $\delta E(V_{10}\mid z)-\delta EV_{00}>\delta E(V_{11}\mid z)-\delta EV_{01}>d$ by Lemma 1, thus $\delta EV_{00}<-d+\delta E(V_{10}\mid z)$. But, these inequalities imply that $\underline{V_{01}}<\overline{V_{01}}(z)$ at $z=\hat{z}_{0,1}(\varepsilon,\tau)$ contradicting the definition of $\hat{z}_{0,1}(\varepsilon,\tau)$. Similarly, when $\delta EV_{00}>-d+\delta E(V_{10}\mid z)$ at $z=\hat{z}_{0,1}(\varepsilon,\tau)$, $\delta EV_{01}>-d+\delta E(V_{11}\mid z)$ by Lemma 1. Again, the result leads to $\underline{V_{01}}>\overline{V_{01}}(z)$ at $z=\hat{z}_{0,1}(\varepsilon,\tau)$ contradicting the definition of $\hat{z}_{0,1}(\varepsilon,\tau)$. Hence, (A.22) should determine the solution $\hat{z}_{0,1}(\varepsilon,\tau)$ for (A.21). Note that z is decreasing in ε.

Similarly, for the intermediate level of ε, we can show that $\hat{z}_{0,0}(\varepsilon,\tau)$ is the solution for

$$-d+\delta E(V_{11}\mid z)-\varepsilon=\delta EV_{0-1}-c \qquad \text{(A.23)}$$

and $\hat{z}_{0,-1}(\varepsilon)$ is the one for

$$-d+\delta E(V_{11}\mid z)-\varepsilon=\delta EV_{0-1}. \qquad \text{(A.24)}$$

Note that both $\hat{z}_{0,0}(\varepsilon,\tau)$ and $\hat{z}_{0,-1}(\varepsilon,\tau)$ are increasing in ε.

<u>When ε is sufficiently high</u>

The firm will no longer buy technology in the market in the neighborhood of $z=\hat{z}_{0,j}(\varepsilon,\tau)$. Thus,

$$V_{01}(z)=\max\{\delta EV_{00};\ -d+\delta E(V_{10}\mid z)\}>\max\{\delta EV_{01};\ -d+\delta E(V_{11}\mid z)\}-\varepsilon, \quad \text{(A.25)}$$

$$\begin{aligned}V_{00}(z)&=\max\{\delta EV_{0-1};\ -d+\delta E(V_{1-1}\mid z)\}-c\\ &>\max\{\delta EV_{01};\ -d+\delta E(V_{11}\mid z)\}-\varepsilon\end{aligned} \qquad \text{(A.26)}$$

$$V_{0-1}(z)=\max\{\delta EV_{0-1};\ -d+\delta E(V_{1-1}\mid z)\}>\max\{\delta EV_{01};\ -d+\delta E(V_{11}\mid z)\}-\varepsilon \quad \text{(A.27)}$$

at $z=\hat{z}_{0,j}(\varepsilon,\tau)$. Let $\tilde{z}_0(\varepsilon,\tau)$ and $\tilde{z}_{-1}(\varepsilon,\tau)$ be the solutions for

$$\delta EV_{00}=-d+\delta E(V_{10}\mid z) \qquad \text{(A.28)}$$

and

$$\delta EV_{0-1}=-d+\delta E(V_{1-1}\mid z), \qquad \text{(A.29)}$$

respectively. By definition, $\hat{z}_{0,1}(\varepsilon,\tau)=\tilde{z}_0(\varepsilon,\tau)$ and $\tilde{z}_{-1}(\varepsilon,\tau)=\hat{z}_{0,0}(\varepsilon,\tau)=\hat{z}_{0,-1}(\varepsilon,\tau)$. Furthermore, by Lemma 1, $E(V_{10}\mid z)-EV_{00}>E(V_{11}\mid z)-EV_{01}>E(V_{1-1}\mid z)-EV_{0-1}$. These inequalities imply

$$\hat{z}_{0,1}(\varepsilon,\tau)=\tilde{z}_0(\varepsilon,\tau)<\tilde{z}_1(\varepsilon,\tau)<\tilde{z}_{-1}(\varepsilon,\tau)=\hat{z}_{0,0}(\varepsilon,\tau)=\hat{z}_{0,-1}(\varepsilon,\tau) \qquad (30)$$

when ε is high enough to lead to (A.25) and (A.26) at $z = \hat{z}_{0,j}(\varepsilon,\tau)$.

In order to prove the inequalities $\hat{z}_{0,1}(\varepsilon,\tau) \le \hat{z}_{0,0}(\varepsilon,\tau) \le \hat{z}_{0,-1}(\varepsilon,\tau)$ for any value of ε, remember our findings earlier that $\hat{z}_{0,1}(\varepsilon,\tau)$ decreases from $\tilde{z}_1(\varepsilon,\tau)$ to $\tilde{z}_0(\varepsilon,\tau)$ while $\hat{z}_{0,0}(\varepsilon,\tau)$ and $\hat{z}_{0,-1}(\varepsilon,\tau)$ increase from $\tilde{z}_1(\varepsilon,\tau)$ to $\tilde{z}_{-1}(\varepsilon,\tau)$. Then, it is obvious that the first inequality holds. To prove the second inequality, it is enough to show that, for any ε, $\hat{z}_{0,0}(\varepsilon,\tau) \le \hat{z}_{0,-1}(\varepsilon,\tau)$ when $\hat{z}_{0,-1}(\varepsilon,\tau)$ satisfies (A.24). Suppose $\hat{z}_{0,-1}(\varepsilon,\tau)$ is the solution of (A.24). From (A.21) and (A.24),
$\overline{V_{00}}(\hat{z}_{0,-1}(\varepsilon,\tau)) = -d - \varepsilon + \delta E(V_{11} \mid \hat{z}_{0,-1}(\varepsilon,\tau)) \ge \underline{V_{00}} = \max\{-c + \delta EV_{0-1};\quad -\varepsilon + \delta EV_{01}\}$. By definition, $\hat{z}_{0,0}(\varepsilon,\tau) \le \hat{z}_{0,-1}(\varepsilon,\tau)$.

z-thresholds for firms (1,1), (1,0), and (1,-1)

We will characterize $\hat{z}_{1,j}(\varepsilon, x)$ for three different ranges of x: (1) $x < -\varepsilon$, (2) $x > -\varepsilon + \tau$, and (3) $-\varepsilon \le x \le -\varepsilon + \tau$.

Case (1): $x < -\varepsilon$.

In this case, the project in the second stage will be terminated and thus $V_{1j}(z,x) = V_{0j}(z)$. Hence,

$$\hat{z}_{1,j}(x,\varepsilon,\tau) = \hat{z}_{0,j}(\varepsilon,\tau). \qquad \text{(A.31)}$$

Case (2): $x > -\varepsilon + \tau$.

In this case, the firm will either commercialize its own technology or sell it in the market. If ε is low enough, the firm will launch its own product in the neighborhood of $z = \hat{z}_{1,j}(x,\varepsilon,\tau)$. Thus,

$$\begin{aligned} &V_{11}(z,x) = V_{10}(z,x) = V_{1-1}(z,x) = \max\{\delta EV_{01};\ -d + \delta E(V_{11} \mid z)\} + x \\ &\begin{cases} > \max\{\delta EV_{00};\ -d + \delta E(V_{10} \mid z)\} + x + \varepsilon - \tau & \text{for } j = 1 \\ > \max\{\delta EV_{0-1};\ -d + \delta E(V_{1-1} \mid z)\} + x + \varepsilon - \tau - c & \text{for } j = 0 \\ > \max\{\delta EV_{0-1};\ -d + \delta E(V_{1-1} \mid z)\} + x + \varepsilon - \tau & \text{for } j = -1 \end{cases} \end{aligned} \qquad \text{(A.32)}$$

Then, $\hat{z}_{11}(x,\varepsilon,\tau) = \hat{z}_{10}(x,\varepsilon,\tau) = \hat{z}_{1-1}(x,\varepsilon,\tau) = \tilde{z}_1(\varepsilon,\tau)$ where $\tilde{z}_1(\varepsilon,\tau)$ was earlier defined as the solution for $\delta EV_{01} = -d + \delta E(V_{11} \mid z)$. Therefore, note that $\hat{z}_{1,j}(x,\varepsilon,\tau)$ does not depend on x and

$$\hat{z}_{1,j}(x,\varepsilon,\tau) = \hat{z}_{0,j}(\varepsilon,\tau) = \tilde{z}_1(\varepsilon,\tau) \qquad \text{(A.33)}$$

for all j if ε is low enough.

For the intermediate level of ε, from the same argument that led to (A.22), Lemma 1 suggests that the following equations hold at $z = \hat{z}_{1,j}(x,\varepsilon,\tau)$:

$$\begin{aligned} &\delta EV_{01} = -d + \delta E(V_{10} \mid z) + \varepsilon - \tau && \text{for } j = 1 \\ &-d + \delta E(V_{11} \mid z) = \delta EV_{0-1} + \varepsilon - \tau - c && \text{for } j = 0 \\ &-d + \delta E(V_{11} \mid z) = \delta EV_{0-1} + \varepsilon - \tau && \text{for } j = -1 \end{aligned} \qquad \text{(A.34)}$$

Note that $\hat{z}_{1,1}(x,\varepsilon,\tau)$ is decreasing in ε and $\hat{z}_{1,0}(x,\varepsilon,\tau)$ and $\hat{z}_{1,-1}(x,\varepsilon,\tau)$ are increasing in ε by (A.34).

By comparing with (A.22)-(A.24),

$$\delta EV_{01} > -d + \delta E(V_{10} \mid \hat{z}_{0,1}(\varepsilon,\tau)) + \varepsilon - \tau$$
$$-d + \delta E(V_{11} \mid \hat{z}_{0,0}(\varepsilon,\tau)) > \delta EV_{0-1} + \varepsilon - \tau - c$$
$$-d + \delta E(V_{11} \mid \hat{z}_{0,-1}(\varepsilon,\tau)) > \delta EV_{0-1} + \varepsilon - \tau$$

Therefore,

$$\hat{z}_{1,1}(x,\varepsilon,\tau) > \hat{z}_{0,1}(\varepsilon,\tau),\ \hat{z}_{1,0}(x,\varepsilon,\tau) < \hat{z}_{0,0}(\varepsilon,\tau) \text{ and } \hat{z}_{1,-1}(x,\varepsilon,\tau) < \hat{z}_{0,-1}(\varepsilon,\tau) \quad \text{(A.35)}$$

for the intermediate level of ε where $\hat{z}_{1,j}(x,\varepsilon,\tau)$ satisfies (A.34).

When ε is sufficiently high, the firm will sell its technology in the market in the neighborhood of $z = \hat{z}_{1,j}(x,\varepsilon,\tau)$.

$$\begin{aligned} V_{11}(z) &= \max\{\delta EV_{00};\ -d + \delta E(V_{10} \mid z)\} + x + \varepsilon - \tau \\ &> \max\{\delta EV_{01};\ -d + \delta E(V_{11} \mid z)\} + x \end{aligned} \quad \text{(A.36)}$$

and

$$\begin{aligned} V_{10}(z) + c = V_{1-1}(z) &= \max\{\delta EV_{0-1};\ -d + \delta E(V_{1-1} \mid z)\} + x + \varepsilon - \tau \\ &> \max\{\delta EV_{01};\ -d + \delta E(V_{11} \mid z)\} + x \end{aligned} \quad \text{(A.37)}$$

at $z = \hat{z}_{1,j}(x,\varepsilon,\tau)$. Remember $\tilde{z}_0(\varepsilon,\tau)$ and $\tilde{z}_{-1}(\varepsilon,\tau)$ are the solutions for $\delta EV_{00} = -d + \delta E(V_{10} \mid z)$ and $\delta EV_{0-1} = -d + \delta E(V_{1-1} \mid z)$, respectively. By definition, $\hat{z}_{1,1}(x,\varepsilon,\tau) = \tilde{z}_0(\varepsilon,\tau)$ and $\hat{z}_{1,0}(x,\varepsilon,\tau) = \hat{z}_{1,-1}(x,\varepsilon,\tau) = \tilde{z}_{-1}(\varepsilon,\tau)$. From (30),

$$\hat{z}_{1,1}(x,\varepsilon,\tau) = \tilde{z}_0(\varepsilon,\tau) < \tilde{z}_1(\varepsilon,\tau) < \tilde{z}_{-1}(\varepsilon,\tau) = \hat{z}_{1,0}(x,\varepsilon,\tau) = \hat{z}_{1,-1}(x,\varepsilon,\tau) \quad \text{(A.38)}$$

for a sufficiently high ε.

With (A.33), (A.35) and (A.38), $\tilde{z}_0(\varepsilon,\tau) \le \hat{z}_{0,1}(\varepsilon,\tau) \le \hat{z}_{1,1}(x,\varepsilon,\tau) \le \tilde{z}_1(\varepsilon,\tau)$, $\tilde{z}_1(\varepsilon,\tau) \le \hat{z}_{1,0}(x,\varepsilon,\tau) \le \hat{z}_{0,0}(\varepsilon,\tau) \le \tilde{z}_{-1}(\varepsilon,\tau)$ and $\tilde{z}_1(\varepsilon,\tau) \le \hat{z}_{1,-1}(x,\varepsilon,\tau) \le \hat{z}_{0,-1}(\varepsilon,\tau) \le \tilde{z}_{-1}(\varepsilon,\tau)$. We need to show $\hat{z}_{0,0}(\varepsilon,\tau) \le \hat{z}_{1,-1}(x,\varepsilon,\tau)$ for $\tau < c$. It suffices to show $\hat{z}_{0,0}(\varepsilon,\tau) \le \hat{z}_{1,-1}(x,\varepsilon,\tau)$ only in the range of ε where $\tilde{z}_1(\varepsilon,\tau) < \hat{z}_{1,-1}(x,\varepsilon,\tau) < \tilde{z}_{-1}(\varepsilon,\tau)$ because the result is trivial if $\hat{z}_{1,-1}(x,\varepsilon,\tau) = \tilde{z}_1(\varepsilon,\tau)$ or $\tilde{z}_{-1}(\varepsilon,\tau)$. When $\tilde{z}_1(\varepsilon,\tau) < \hat{z}_{1,-1}(x,\varepsilon,\tau) < \tilde{z}_{-1}(\varepsilon,\tau)$,

$$-d + \delta E(V_{11} \mid \hat{z}_{1,-1}(x,\varepsilon,\tau)) = \delta EV_{0-1} + \varepsilon - \tau > \max\{\delta EV_{01}; -d + \varepsilon - \tau + \delta E(V_{1-1} \mid \hat{z}_{1,-1}(x,\varepsilon,\tau))\}$$

from (A.34). Then, for the same ε,

$$\begin{aligned} \overline{V_{00}}(\hat{z}_{1,-1}(x,\varepsilon,\tau)) &= \max\{-d - c + \delta E(V_{1-1} \mid \hat{z}_{1,-1}(x,\varepsilon,\tau));\ -d - \varepsilon + \delta E(V_{11} \mid \hat{z}_{1,-1}(x,\varepsilon,\tau))\} \\ &= -d - \varepsilon + \delta E(V_{11} \mid \hat{z}_{1,-1}(x,\varepsilon,\tau)) \\ &= \delta EV_{0-1} - \tau > \max\{-c + \delta EV_{0-1};\ -\varepsilon + \delta EV_{01}\} = \underline{V_{00}} \end{aligned}$$

Therefore, $\hat{z}_{0,0}(\varepsilon,\tau) < \hat{z}_{1,-1}(x,\varepsilon,\tau)$ concluding the proof for the case.

Case (3): $-\varepsilon \le x \le -\varepsilon + \tau$.

In this case, firms never buy or sell technology and the value functions are simplified to

$$V_{11}(z,x) = \max\{\delta E(V_{00} \mid z); -d + \delta E(V_{10} \mid z); x + \delta E(V_{01} \mid z); -d + x + \delta E(V_{11} \mid z)\}, \quad \text{(A.39)}$$

$$V_{10}(z,x)=\max\{-c+\delta E(V_{0-1}\mid z);-c-d+\delta E(V_{1-1}\mid z);x+\delta E(V_{01}\mid z);-d+x+\delta E(V_{11}\mid z)\},$$

(A.40)

and

$$V_{1-1}(z,x)=\max\{\delta E(V_{0-1}\mid z);-d+\delta E(V_{1-1}\mid z);x+\delta E(V_{01}\mid z);-d+x+\delta E(V_{11}\mid z)\}. \quad (A.41)$$

Define $\tilde{z}_{11}(x,\varepsilon,\tau)$, $\tilde{z}_{10}(x,\varepsilon,\tau)$ and $\tilde{z}_{1-1}(x,\varepsilon,\tau)$ as follows:

$$-d+\delta E(V_{10}\mid \tilde{z}_{11})=x+\delta E(V_{01}\mid \tilde{z}_{11}) \quad (A.42)$$

$$-c+\delta E(V_{0-1}\mid \tilde{z}_{10})=-d+x+\delta E(V_{11}\mid \tilde{z}_{10}) \quad (A.43)$$

and

$$\delta E(V_{0-1}\mid \tilde{z}_{1-1})=-d+x+\delta E(V_{11}\mid \tilde{z}_{1-1}). \quad (A.44)$$

Then, by definition, $\tilde{z}_{11}(x,\varepsilon,\tau)$ is increasing in *x*, and $\tilde{z}_{10}(x,\varepsilon,\tau)$ and $\tilde{z}_{1-1}(x,\varepsilon,\tau)$ are decreasing in *x*. Define thresholds $\hat{z}_{1,j}(x,\varepsilon,\tau)$ for $-\varepsilon\le x\le -\varepsilon+t$ as follows:

$$\hat{z}_{1,1}(x,\varepsilon,\tau)=\max\{\tilde{z}_0(\varepsilon,\tau),\min\{\tilde{z}_1(\varepsilon,\tau),\tilde{z}_{11}(x,\varepsilon,\tau)\}\}, \quad (A.45)$$

$$\hat{z}_{1,0}(x,\varepsilon,\tau)=\max\{\tilde{z}_1(\varepsilon,\tau),\min\{\tilde{z}_{-1}(\varepsilon,\tau),\tilde{z}_{10}(x,\varepsilon,\tau)\}\}, \quad (A.46)$$

and

$$\hat{z}_{1,-1}(x,\varepsilon,\tau)=\max\{\tilde{z}_1(\varepsilon,\tau),\min\{\tilde{z}_{-1}(\varepsilon,\tau),\tilde{z}_{1-1}(x,\varepsilon,\tau)\}\}. \quad (A.47)$$

where $\tilde{z}_1(\varepsilon,\tau)$, $\tilde{z}_0(\varepsilon,\tau)$ and $\tilde{z}_{-1}(\varepsilon,\tau)$ are defined earlier in this proof and satisfy $\delta EV_{01}=-d+\delta E(V_{11}\mid z)$, $\delta EV_{00}=-d+\delta E(V_{10}\mid z)$ and $\delta EV_{0-1}=-d+\delta E(V_{1-1}\mid z)$, respectively.

Now, suppose $z>\hat{z}_{11}(x,\varepsilon,\tau)$. By definition (A.45), one of the following two inequalities holds: $z>\tilde{z}_1(\varepsilon,\tau)$, which is equivalent to $\delta E(V_{01}\mid z)<-d+\delta E(V_{11}\mid z)$, or $z>\tilde{z}_{11}(x,\varepsilon,\tau)$, which is equivalent to $-d+\delta E(V_{10}\mid z)>x+\delta E(V_{01}\mid z)$. (A.45) also implies $z>\tilde{z}_0(\varepsilon,\tau)$, which is equivalent to $\delta E(V_{00}\mid z)<-d+\delta E(V_{10}\mid z)$. Then, $V_{11}(z,x)=-d+\max\{\delta E(V_{10}\mid z);\ x+\delta E(V_{11}\mid z)\}$ and the firm should advance its project in stage I to stage II. Similarly, if $z<\hat{z}_{11}(x,\varepsilon,\tau)$, from (A.45), one of the following two inequalities holds: $z<\tilde{z}_{11}(x,\varepsilon,\tau)$, which is equivalent to $-d+\delta E(V_{10}\mid z)<x+\delta E(V_{01}\mid z)$, or $z<\tilde{z}_0(\varepsilon,\tau)$, which is equivalent to $\delta E(V_{00}\mid z)>-d+\delta E(V_{10}\mid z)$. The definition also implies $z<\tilde{z}_1(\varepsilon,\tau)$, which is equivalent to $\delta E(V_{01}\mid z)>-d+\delta E(V_{11}\mid z)$. Then, $V_{11}(z,x)=\max\{\delta E(V_{00}\mid z);x+\delta E(V_{01}\mid z)\}$ implying that the firm should not advance its stage I project. Similarly, we can show that $e_1^*=1$ if $z>\hat{z}_{1,j}(x,\varepsilon,\tau)$ and $e_1^*=0$ if $z<\hat{z}_{1,j}(x,\varepsilon,\tau)$, for $j=0$ and -1, too.

You can easily verify that $\hat{z}_{1,j}(x,\varepsilon,\tau)$ is continuous in the entire region.

$\hat{z}_{11}(x,\varepsilon,\tau)$ is non-decreasing in *x*, and

$$\hat{z}_{0,1}(\varepsilon,\tau)=\hat{z}_{1,1}(x',\varepsilon,\tau)|_{x'<-\varepsilon}\le \hat{z}_{1,1}(x,\varepsilon,\tau)\le \hat{z}_{1,1}(x',\varepsilon,\tau)|_{x'>\tau-\varepsilon}=\hat{z}_{1,1}(\tau-\varepsilon,\varepsilon,\tau).$$

$\hat{z}_{10}(x,\varepsilon,\tau)$ is non-increasing in *x* and

$$\hat{z}_{1,0}(t-\varepsilon,\varepsilon,\tau)=\hat{z}_{1,0}(x',\varepsilon,\tau)|_{x'>\tau-\varepsilon}\le \hat{z}_{1,0}(x,\varepsilon,\tau)\le \hat{z}_{1,0}(x',\varepsilon,\tau)|_{x'<-\varepsilon}=\hat{z}_{0,0}(\varepsilon,\tau)$$

$\hat{z}_{1-1}(x,\varepsilon,\tau)$ is non-increasing in *x* and

$$\hat{z}_{1,-1}(\tau-\varepsilon,\varepsilon,\tau)=\hat{z}_{1,-1}(x',\varepsilon,\tau)|_{x'>\tau-\varepsilon}\le \hat{z}_{1,-1}(x,\varepsilon,\tau)\le \hat{z}_{1,-1}(x',\varepsilon,\tau)|_{x'<-\varepsilon}=\hat{z}_{0,-1}(\varepsilon,\tau).$$

These inequalities imply that the following inequalities proven for $x > -\varepsilon + \tau$ and $x < -\varepsilon$ can be extended to $-\varepsilon \le x \le -\varepsilon + \tau$: $\hat{z}_{01}(\varepsilon) \le \hat{z}_{11}(\varepsilon, x) \le \hat{z}_{10}(\varepsilon, x) \le \hat{z}_{00}(\varepsilon)$, $\hat{z}_{10}(\varepsilon, x) \le \hat{z}_{1-1}(\varepsilon, x) \le \hat{z}_{0-1}(\varepsilon)$ and, when $\tau < c$, $\hat{z}_{01}(\varepsilon) \le \hat{z}_{11}(\varepsilon, x) \le \hat{z}_{10}(\varepsilon, x) \le \hat{z}_{00}(\varepsilon) \le \hat{z}_{1-1}(\varepsilon, x) \le \hat{z}_{0-1}(\varepsilon)$.

This concludes the proof of Proposition 3.1. ■

Proof of Proposition 3.2

When $x < -\varepsilon$, the stage II project is always terminated and the firm possibly buys a technology project from another firm. Thus, if the threshold $\hat{x}_{1,j}(z,\varepsilon,\tau)$ exists, $\hat{x}_{1,j}(z,\varepsilon,\tau) \ge -\varepsilon$. When $x > -\varepsilon + \tau$, the project in the final stage is either commercialized or sold in the technology market. Hence, if such a threshold exists, $\hat{x}_{1,j}(z,\varepsilon,\tau) \le -\varepsilon + \tau$.

Now, suppose $-\varepsilon \le x \le -\varepsilon + \tau$. The technology project in stage II will be either commercialized by the firm itself or terminated. Define $\hat{x}_j(z,\varepsilon,\tau)$ as follows:

$$\hat{x}_1(z,\varepsilon,\tau) = \begin{cases} \min\{\max\{\delta E(V_{00} - V_{01}), -\varepsilon\}, -\varepsilon + \tau\} & \text{for } z < \tilde{z}_0 \\ \min\{\max\{\delta E(V_{10} - V_{01} \mid z) - d, -\varepsilon\}, -\varepsilon + \tau\} & \text{for } \tilde{z}_0 \le z \le \tilde{z}_1 \\ \min\{\max\{\delta E(V_{10} - V_{11} \mid z), -\varepsilon\}, -\varepsilon + \tau\} & \text{for } z > \tilde{z}_1 \end{cases} \quad \text{(A.48)}$$

$$\hat{x}_0(z,\varepsilon,\tau) = \begin{cases} \min\{\max\{\delta E(V_{0-1} - V_{01}) - c, -\varepsilon\}, -\varepsilon + \tau\} & \text{for } z < \tilde{z}_1 \\ \min\{\max\{\delta E(V_{0-1} - V_{11} \mid z) + d - c, -\varepsilon\}, -\varepsilon + \tau\} & \text{for } \tilde{z}_1 \le z \le \tilde{z}_{-1} \\ \min\{\max\{\delta E(V_{1-1} - V_{11} \mid z) - c, -\varepsilon\}, -\varepsilon + \tau\} & \text{for } z > \tilde{z}_{-1} \end{cases} \quad \text{(A.49)}$$

$$\hat{x}_{-1}(z,\varepsilon,\tau) = \begin{cases} \min\{\max\{\delta E(V_{0-1} - V_{01}), -\varepsilon\}, -\varepsilon + \tau\} & \text{for } z < \tilde{z}_1 \\ \min\{\max\{\delta E(V_{0-1} - V_{11} \mid z) + d, -\varepsilon\}, -\varepsilon + \tau\} & \text{for } \tilde{z}_1 \le z \le \tilde{z}_{-1} \\ \min\{\max\{\delta E(V_{1-1} - V_{11} \mid z), -\varepsilon\}, -\varepsilon + \tau\} & \text{for } z > \tilde{z}_{-1} \end{cases} \quad \text{(A.50)}$$

We will show that the project in stage II will be commercialized only if $x > \hat{x}_j(z,\varepsilon,\tau)$. We will prove the claim only for $j = 1$. The proofs for $j = 0$ and -1 are similar and omitted.

When $z < \tilde{z}_0$, $\delta EV_{00} > -d + \delta E(V_{10} \mid z)$ by definition. By applying Lemma 1, the inequality implies $\delta EV_{01} > -d + \delta E(V_{11} \mid z)$. Then, from (A.39), $V_{11}(z,x) = \max\{\delta E(V_{00} \mid z); x + \delta E(V_{01} \mid z)\}$. Therefore, the project in stage II should be commercialized only when $x > \delta E(V_{00} - V_{01}) = \hat{x}_1(\varepsilon, z)$ assuming $-\varepsilon \le x \le -\varepsilon + \tau$. When $\tilde{z}_0 \le z \le \tilde{z}_1$, $\delta EV_{01} \ge -d + \delta E(V_{11} \mid z)$ and $\delta EV_{00} \le -d + \delta E(V_{10} \mid z)$ by definition. Then, from (A.39) again, $V_{11}(z,x) = \max\{-d + \delta E(V_{10} \mid z); x + \delta E(V_{01} \mid z)\}$. Hence, the stage II product should be launched only when $x > \delta E(V_{10} - V_{01} \mid z) - d = \hat{x}_1(z,\varepsilon,\tau)$ assuming $-\varepsilon \le x \le -\varepsilon + \tau$. When $z > \tilde{z}_1$, $\delta EV_{01} < -d + \delta E(V_{11} \mid z)$. By Lemma 1 again, we get $\delta EV_{00} < -d + \delta E(V_{10} \mid z)$. $V_{11}(z,x) = \max\{-d + \delta E(V_{10} \mid z); -d + x + \delta E(V_{11} \mid z)\}$ and the technology project should be put forth to the market if $x > \delta E(V_{10} - V_{11} \mid z) = \hat{x}_1(z,\varepsilon,\tau)$ assuming $-\varepsilon \le x \le -\varepsilon + \tau$.

To show that $\hat{x}_1(z,\varepsilon,\tau)$ is non-decreasing in z, see its definition in (A.48). Clearly, $\hat{x}_1(z,\varepsilon,\tau)$ is non-decreasing in $z \le \tilde{z}_1(\varepsilon,\tau)$. When $z > \tilde{z}_1(\varepsilon,\tau)$, since firm (1,0) is more likely to commercialize or sell its own project, the value function V_{10} is more sharply increasing in the value of its project in stage II than V_{11}. Therefore, $E_{z_{t+1},x_t|z_t}(V_{10}(z_{t+1},x_t)-V_{11}(z_{t+1},x_t)\,|\,z_t)$ is increasing in z_t. Therefore, $\hat{x}_1(z,\varepsilon,\tau)$ is non-decreasing in z everywhere. Similarly, from (A.49) -(A.50), $\hat{x}_0(z,\varepsilon,\tau)$ and $\hat{x}_{-1}(z,\varepsilon,\tau)$ are non-increasing in z. This concludes the proof of (ii),(iii) in Proposition 3.2. ■

Proof of Proposition 3.3

We prove only i. of the proposition. The proof for the rest is similar. Suppose $\delta E(V_{00}-V_{01}\,|\,\tau,\varepsilon)>-\varepsilon$. Since $\hat{x}_1(z,\varepsilon,\tau)$ is non-decreasing in z by Proposition 3.2, $\hat{x}_1(z,\varepsilon,\tau)>-\varepsilon$ for all z. Then, by (A.48), $\delta E(V_{00}-V_{01})>-\varepsilon$ and $\delta E(V_{10}-V_{11}\,|\,z)>-\varepsilon$. They imply that $V_{01}(z)=\max\{\delta EV_{00};\ -d+\delta E(V_{10}\,|\,z)\}\begin{cases}>-\varepsilon+\delta EV_{01}\\ >-d-\varepsilon+\delta E(V_{11}\,|\,z)\end{cases}$ suggesting that firm (0,1) never buys a project. Similarly, firm (1,1) with $x < -\varepsilon$ never buys a project because $V_{11}(z,x)=V_{01}(z)$ in the case.

Now suppose $\delta E(V_{00}-V_{01}\,|\,\tau,\varepsilon)<-\varepsilon$ and consider firm (0,1) with $z<\hat{z}_{01}(\varepsilon,\tau)$. By the definition of $\hat{z}_{01}$,

$$\underline{V_{01}}=\max\{\delta EV_{00};\ -\varepsilon+\delta EV_{01}\}>\overline{V_{01}}(z)=\max\{-d+\delta E(V_{10}\,|\,z);\ -d-\varepsilon+\delta E(V_{11}\,|\,z)\}.$$

Furthermore, $\delta E(V_{00}-V_{01}\,|\,\tau,\varepsilon)<-\varepsilon$ leads to $\delta EV_{00}<-\varepsilon+\delta EV_{01}$. Hence, $V_{01}(z)=-\varepsilon+\delta EV_{01}$ suggesting that the firm will buy a technology project. The same argument holds for firm (1,1) with $x<-\varepsilon$.

When $\delta E(V_{10}-V_{11}\,|\,z,\tau,\varepsilon)<-\varepsilon$ holds for all z, by (A.48), $\hat{x}_1(z,\varepsilon,\tau)<-\varepsilon$ for all z, because $\hat{x}_1(z,\varepsilon,\tau)$ is non-decreasing in z. This result implies $\delta E(V_{00}-V_{01})<-\varepsilon$ and $\delta E(V_{10}-V_{11}\,|\,z)<-\varepsilon$. Then, given $x<-\varepsilon$, firm (0,1) and firm (1,1) have the same value function and

$$V_{11}(z,x)=V_{01}(z)=\max\{\delta EV_{00};\ -d+\delta E(V_{10}\,|\,z);\ -\varepsilon+\delta EV_{01};\ -d-\varepsilon+\delta E(V_{11}\,|\,z)\}$$
$$=\max\{-\varepsilon+\delta EV_{01};\ -d-\varepsilon+\delta E(V_{11}\,|\,z)\}$$

where the last equality implies that the firm always buy a project in the technology market. This concludes the proof. ■