Planned Obsolescence by Duopolists:

Consumers' Tolerance and its Welfare Implications

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Abstract

We give a very simple dynamic game in which sooner or later duopolists start to renew their products in the same time length, which generates turnover cycle of the quality levels of their products as an equilibrium outcome, regardless of the initial difference. Further, when consumers of higher-quality product can punish the firm whose product renewal is delayed, the more tolerant of the delay consumers are, the earlier each firm makes its product renewal in time length best for them. If consumers are fussy about product renewal, only one firm renews its product.

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1 Introduction

Planned obsolescence has been studied as games played by a monopolist and the consumers of its product (See, e.g., Utaka (2006), Waldman (1996, 2003), and references therein). This paper investigates a new aspect different from the traditional literature considering a dynamic duopoly, which can be extended to the case of n firms.

We give a very simple dynamic game in which sooner or later duopolists start to renew their products in the same time length, which generates turnover cycle of quality levels of goods produced by the duopolists as an equilibrium outcome, regardless of the initial difference, although there is neither asymmetric information nor uncertainty.[1]

In the literature on planned obsolescence, a market for the used goods plays an important role that affects the timing of product renewal. For simplicity, however, we take into account such effect as depreciation rate: consumers depreciate the products of firms as time passes by, since they move to the underlying market for the used goods.

In real practices in industries, firms can decide prices and the timing of product renewal (quality levels). Then, even in our simple model, we may generate much more complicated turnover cycle with "price-quality dynamics" However, this paper aims to show turnover cycle in a simple model. Hence, we exclude any possibility of generating price-quality cycle in the following way: if some time length does not yet passes by after a firm takes up the high position, no consumers of its higher-quality product move to market for used goods (due to their brand loyalty to the good of higher quality), although they actually depreciate the product in mind. as soon as that time length passes by, they punish the firm for delay of product renewal. The firm suffers some damage at each time (for, e.g., better maintenance service).

[1] Applying repeated game with imperfect monitoring, Rob-Sekiguchi(2004) showed that "a sophisticated system generates turnover cycle of quality levels". This paper aims to show that "turnover cycle can be generated by a much simpler system" similar to Rob-Sekiguchi and its simplicity facilitates comparative statics.

Welfare implications are given, in the case where consumers of higher-quality product can punish the firm whose product renewal is delayed. (A) As far as consumers are sufficiently "tolerant" of the delay in product renewal, sooner or later the duopolists start renewing their products in the same time length efficient for consumers. If consumers are fussy about renewal, then only one firm renews its product in time length efficient for them. (B) The more tolerant of delay in product renewal consumers are, the earlier the time comes at which the duopolists start renewing their products in time length best for consumers (if some intuitive conditions are met).

The remaining part of this paper is organized as follows. In order to clarify how turnover cycle is generated, section 2 gives the basic model and its results, where consumers' tolerance is not concerned. Section 3 begins with an example given for better understanding why and how consumers' tolerance is introduced to the basic model extended with time-variant instantaneous profit function of firms. The proofs of results in this section are all shown in Appendix. Finally, some brief remarks are given in section 4.

2 The Basic Model

2.1 time invariance, no tolerance

To show the key that generates turnover cycle, we begin our analysis with the time-invariant instantaneous profit function of firms, not installing consumers' tolerance of the delay of product renewal.

In this industry, there are two firms, α and β, living in discrete time horizon $\{0, 1, 2, \dots\}$. At each time t, each firm $i \in \{\alpha, \beta\}$ decides whether to make its "product renewal" ($s_i(t) = 1$) paying cost $C(> 0)$ or not ($s_i(t) = 0$) with no cost. In this paper, we do not analyze investment decision made by firms on product development. Interpret this situation as sequential "model changes" of products like automobiles.

This game starts at $t = 1$. Let $s_i = \{s_i(t)\}_{t=1}^{\infty}$ be a sequence of firm i's actions. Let $t_k(s_i)$ denote the time at which the k-th renewal is made in s_i.

Denote by $x_i(t)$ the quality level of firm i's product at time t. The initial condition is given as $(x_i(0))_{i=\alpha,\beta}$, where $x_i(0)$ is the quality level of firm i's product at $t = 0$.

As in the literature, we might have explicitly incorporated into our model a market for the used goods that affects the timing of product renewal. For simplicity, however, we take into account such effect as depreciation rate: consumers depreciate the products of firms as time passes by, since they move to the underlying market for the used goods.

Given $x_i(0)$ and s_i, the quality level of firm i's product at time t is determined as follows:

$$x_i(t) = \begin{cases} \lambda^t x_i(0) & \text{if } 0 < t < t_1(s_i) \\ \lambda^{t-t_{k-1}(s_i)} x_i(t_{k-1}(s_i)) + a(t - t^i_{k-1}(s_i))s_i(t) & \text{if } t_{k-1}(s_i) \leq t < t_k(s_i), \end{cases}$$

where $k \geq 2$, $\lambda \in (0, 1)$ represents the rate of consumers' depreciation of product quality, and $a(\tau)$ stands for the magnitude of acceleration in quality which increases in consecutive time length $\tau(\geq 0)$ spent with no product introduction after the last renewal. We require that $a(\tau) > 0$ if $\tau > 0$ and $a(\tau) = 0$ if $\tau = 0$, where $\tau = t - t^i_{k-1}(s_i)$ if $t^i_{k-1}(s_i) \leq t < t^i_k(s_i)$ with $k \geq 2$, and it is t if $0 < t < t_1(s_i)$. We assume $\lim_{\tau\to\infty} a(\tau)/\tau = 0$.

Consider any time length $\gamma \in (1, \infty)$. For each γ, divide it into L shorter ones, and denote them by $\tau_1, \ldots, \tau_L$, i.e. $\tau_1 + \cdots + \tau_L = \gamma$, where $1 \leq L \leq \gamma$. Assume that there exists a unique time length $\gamma_{\max}$ in which they appreciate product renewal made only once more than that made some times, i.e., the depreciation rate λ satisfies that for any $L \in (1, \gamma_{\max}]$,

$$a(\gamma_{\max}) > \sum_{l=1}^{L-1} \lambda^{\tau_{l+1}+\cdots+\tau_L} a(\tau_l) + a(\tau_L). \tag{1}$$

It would not be unnatural to assume that for any $k \geq 2$,

$$\lambda^{\gamma_{\max}} x_i(t_{k-1}(s_i)) + a(\gamma_{\max}) > x_i(t_{k-1}(s_i)).$$

The history induced by $s = (s_\alpha, s_\beta)$ up to time t is described by

$$h(s, t) = ((s_\alpha(1), s_\beta(1)); \ldots ; (s_\alpha(t-1), s_\beta(t-1))).$$

Let σ_i denote a function that assigns an action $s_i(t)$ to $h(s,t)$, which is called firm i's (pure) strategy. Let Σ_i be the set of all strategy of player i.

Given $h(s,t)$ and $s(t) = (s_\alpha(t), s_\beta(t))$, firm i earns at time t its (gross) instantaneous profit (incl. production cost)

$$\pi_i^t(s(t) : h(s,t)) = \begin{cases} \pi_H & \text{if } x_i(t) > x_j(t) \\ \pi_L & \text{if } x_i(t) < x_j(t), \end{cases} \tag{2}$$

where $j \neq i$. In case of tie ($x_i(t) = x_j(t)$), firm i obtains π_L when $x_i(t-1) < x_j(t-1)$. We say that firm i is in a high (low) position at time t if it obtains π_H (π_L) at time t. Note that $\pi_i^t(\cdot : \cdot)$ is time invariant in this basic model. We say that "turnover" takes place when the positions of firms are reversed.

Define the long-run net profit of firm i by

$$\Pi_i(\sigma_\alpha, \sigma_\beta) = \lim_{n\to\infty} \inf \frac{\sum_{t=1}^n \{\pi_i^t(s(t) : h(s,t)) - s_i(t)C\}}{n}, \tag{3}$$

where, for a given sequence $\{y_n\}_{n\geq 1}$, $\lim_{n\to\infty} \inf y_n = \sup_{n\geq 1} \inf_{k\geq n} y_k$. $\sigma^* = (\sigma_\alpha^*, \sigma_\beta^*)$ is an equilibrium (in pure strategies) if

$$\begin{aligned} \Pi_\alpha(\sigma_\alpha^*, \sigma_\beta^*) &\geq \Pi_\alpha(\sigma_\alpha, \sigma_\beta^*) \quad \forall \sigma_\alpha \\ \Pi_\beta(\sigma_\alpha^*, \sigma_\beta^*) &\geq \Pi_\beta(\sigma_\alpha^*, \sigma_\beta) \quad \forall \sigma_\beta \end{aligned}$$

2.2 the results

The mixed strategies and equilibria in those strategies are defined in a usual manner. We here just mention the existence[2].

Proposition 1 *The basic model has equilibria at least in mixed strategies.*

Proof : See Appendix.

We hereafter confine attention to equilibria in pure strategies.

Assumption (a) $\pi_H - \pi_L \geq C$.

[2]Under an assumption weaker than Assumption (a), Lemma 1 can be extended to the case of mixed strategies.

Lemma 1 *Suppose that the basic model has equilibria in pure strategies. Let* $\{\{x_i^*(t)\}_{t=1}^\infty, \{x_j^*(t)\}_{t=1}^\infty\}$ *be a pair of quality ladders induced by an arbitrary equilibrium* σ^* *in pure strategies. If Assumption (a) is met, then for any equilibrium in pure strategies,*

$$\mu_i(\sigma^*) := \lim_{n\to\infty} \inf \frac{|\{t \le n : x_i^*(t) > x_j^*(t)\}|}{n} > 0, \quad i = \alpha, \beta.$$

Proof : Suppose that there exists an equilibrium σ^* with $\mu_\alpha(\sigma^*) = 0$, w.l.o.g. Then, in that equilibrium,

$$\begin{aligned} \Pi_\alpha(\sigma_\alpha^*, \sigma_\beta^*) &= \lim_{n\to\infty} \inf \frac{\sum_{t=1}^n \{\pi_L - s_\alpha^*(t)C\}}{n}, \\ \Pi_\beta(\sigma_\alpha^*, \sigma_\beta^*) &= \lim_{n\to\infty} \inf \frac{\sum_{t=1}^n \{\pi_H - s_\beta^*(t)C\}}{n}. \end{aligned} \tag{4}$$

Clearly, $s_\alpha^*(t) = 0$ and $s_\beta^*(t) = 0$ at almost every time t. Then, firm α obtains π_L in the limit. Suppose that α deviates to a strategy σ'_α such that $s'_\alpha(t) = 1$ whenever $t - t_{k-1}(s'_\alpha) = \gamma_{\max}$ for $k \ge 2$. Since firm β takes $s_\beta^*(t) = 0$ at almost every time t, there is a time n' at which α overtakes β's position and is in the high position forever after n', and so α obtains $\pi_H - C/\gamma_{\max}$ in the limit. Since Assumption (a) implies $\pi_H - C/\gamma_{\max} > \pi_L$, firm α has incentive to take σ'_α.

Let us confirm that there is no equilibrium σ^* with $\mu_\alpha(\sigma^*) = 0$. If exists, a possible case that leads to (4) is described as follows: take any positive integer $z(> 1)$. on the path of equilibrium, each firm takes actions such that

$$\begin{aligned} &x_\beta(t_1) > x_\alpha(t_1) \quad \text{at } t_1 := t_1(s_\alpha^*) = t_1(s_\beta^*). \\ &t_k := t_k(s_\alpha^*) = t_k(s_\beta^*) = z^k \quad \forall k \ge 2. \end{aligned}$$

off the path of equilibrium, as soon as β observes α's deviation from the above on-the-equilibrium-path actions, it takes retaliatory actions by which β takes back the high position within finite time length. Since $\lim_{\tau\to\infty} a(\tau)/\tau = \lim_{t\to\infty} \lambda^t = 0$, for any z, we can find such a large integer $\hat{k}$ that

$$x_\beta(t_{\hat{k}}(s_\beta^*)) - x_\alpha(t_{\hat{k}}(s_\alpha^*)) < a(\gamma_{\max}),$$

Because of (1), if firm α deviates to σ'_α forever after the time $t_{\hat{k}}$, α can take up the high position at least one time length in every time length $\gamma_{\max}$, even if β takes the same strategy after observing α's deviation to σ'_α, contradicting the existence of σ^* with $\mu_\alpha(\sigma^*) = 0$.

The same argument applies to the other cases where α and β have different integers z_α and z_β, when we take different times $\hat{k}$ and $\hat{k}'$ so large that $x_\beta(t) - x_\alpha(t) < a(\gamma_{\max})$, where $t = t_{\hat{k}}(s^*_\alpha)$ is such that $t_{\hat{k}'-1}(s^*_\beta) < t \leq t_{\hat{k}'}(s^*_\beta)$ and $x_\beta(t) > x_\alpha(t)$. □

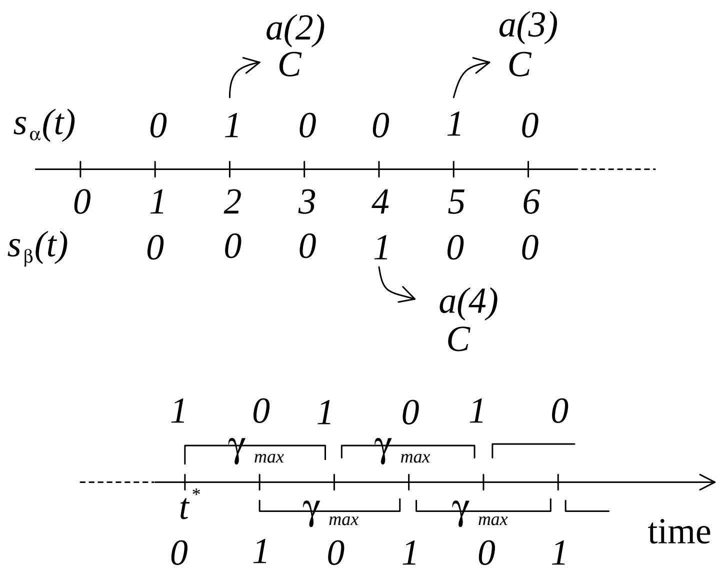

Figure 1: product renewal and turnover cycle

Proposition 2 *Let Assumption (a) is met. Then, we have the following.* (i) *The basic model has equilibria in pure strategies.* (ii) *In any equilibrium σ^* in pure strategies, there is time $t^*(< \infty)$ after which both firms renew their products in every time length $\gamma_{\max}$.* (iii) *Turnover takes place whenever each firm renews its product after the time t^* (i.e., together with* (ii)*, turnover cycle is generated).*

Proof : We begin with (ii). The proof of (i) is shown last.

(ii) Suppose that in an equilibrium σ^* in pure strategies, if any, only firm β does not renew its product in time length $\gamma_{\max}$ infinitely many times, w.l.o.g. Take the smallest integer $\hat{n}$ such that

$$\hat{n}(\frac{a(\gamma_{\max})}{\gamma_{\max}} - \sup\{\frac{a(\tau)}{\tau} : \tau \in \mathbb{N} \setminus \{\gamma_{\max}\}\}) > |x_\alpha(0) - x_\beta(0)|.$$

Let $\hat{t}$ be the earliest time by which time firm β has not renewed $\hat{n}$ times its product in time length $\gamma_{\max}$. Clearly, $\hat{t} < \infty$ for any initial difference $x_\beta(0) - x_\alpha(0)$ in quality levels of their products.

(case 1) Suppose $x_\alpha(0) < x_\beta(0)$. If firm α renews its product in every time length $\gamma_{\max}$ until $\hat{t}$, then it overtakes β's position by time $\hat{t}$. Since it is at most finite times that firm α does not renew its product in time length $\gamma_{\max}$ according to $\{s^*_\alpha(t)\}^\infty_{t=\hat{t}+1}$, firm α is in the high position always in the limit after time $\hat{t}$. Since $\hat{t} < \infty$, the costs (at most $\hat{t}C$) that α must pay until it overtakes β disappear in the limit (converges to zero as n tends to infinity). Hence, $\mu_\beta(\sigma^*) = 0$, contradicting Lemma 1.

(case 2) Suppose $x_\alpha(0) > x_\beta(0)$. The same argument as in the latter part (after $\hat{t}$) of case 1 applies.

Further, by an argument similar to case 1, it is shown that there is no equilibrium in which infinitely many times both firms do not renew their products in time length $\gamma_{\max}$. Hence, if an equilibrium σ^* in pure strategies exists, then at most finitely many times in σ^* each firm renews its product not in time length $\gamma_{\max}$.

(iii) Suppose not. Since, by Proposition 2 (ii), both firms renew their products in time length $\gamma_{\max}$ after time t^*, either firm i has $\mu_i(\sigma^*) = 0$, contradicting Lemma 1.

(i) Assumption (a) is equivalent to

$$\frac{1}{\gamma_{\max}}\pi_H + \frac{\gamma_{\max}-1}{\gamma_{\max}}\pi_L - \frac{C}{\gamma_{\max}} \geq \pi_L.$$

If an equilibrium in pure strategies exists, each firm obtains π_H at least one time length in every time length $\gamma_{\max}$ by (ii), paying $C/\gamma_{\max}$ in the limit. When a firm deviates to any other strategies, it obtains at most π_L in limit. Hence, pure strategies described in (ii) constitute an equilibrium σ^*. □

We can interpret t^* as the time after which "stationary" decision process will begin. I would be interesting that we might have described what happens before that stationarity is attained. Unfortunately, however, "anything goes" before t^* in any equilibrium, since (the sum of) instantaneous profits of firms gained until t^* converges to zero as n tends to infinity. To obtain clearer results prior to the time t^*, we need a stronger equilibrium notion, which is examined in the next section.

3 Consumers' Tolerance

3.1 an example

price competition under vertical product differentiation

Section 2 dealt with the time-invariant instantaneous profit function of firms. We hereafter incorporate the time-variant one into our model, considering the following example as an underlying market structure.

Consider a continuum of consumers uniformly distributed on $(0,1)$, each indexed by λ with demand for one unit of either firm α's product or β's. A consumer λ has her utility function $u(t) = \lambda x(t-1) - p(t)$, where λ is her depreciation rate to quality level $x(t-1) \in \{x_\alpha(t-1), x_\beta(t-1)\}$ of a commodity, and $p(t) \in \{p_\alpha(t), p_\beta(t)\}$ is the price she actually pays to the commodity of quality $\lambda x(t-1)$. Each firm produces its commodity at the cost c per unit of output. Let $\triangle x(t-1) := x_\alpha(t-1) - x_\beta(t-1) > 0$. Assume that $0 < p_\alpha(t) - p_\beta(t) < \triangle x(t-1)$. Firms compete in prices.

The consumer who is indifferent to whether to buy α's good or β's is at $\theta_0 = (p_\alpha(t) - p_\beta(t))/(x_\alpha(t-1) - x_\beta(t-1))$. The demand for α's good is $1-\theta_0$ and that for β's is θ_0, and so the instantaneous profit of firm α at time t is

$(p_\alpha(t) - c)(1 - \theta_0)$ and that of firm β is $(p_\beta(t) - c)\theta_0$. The equilibrium price is hence $p^*_\alpha(t) = c + (2/3)\triangle x(t-1)$ and $p^*_\beta(t) = c + (1/3)\triangle x(t-1)$. Thus, in the equilibrium of price competition at time t, firm α obtains $\pi_H(t) = (5/6)^2\triangle x(t-1)$ and β obtains $\pi_L(t) = (1/6)^2\triangle x(t-1)$.

As $\triangle x(t-1)$ increases, the product of firm β is evaluated lower, which might decrease the demand for its products. Under such larger product differentiation, however, firm α increases its price. Hence, β also benefits from larger $\triangle x(t-1)$.

In the dynamic consideration of this example, firms decide both prices and the timing of product renewal (quality levels). Then, we may generate "price-quality cycle" as well as turnover cycle.[3] However, this paper aims to show turnover cycle in a simple model. Hence, we here exclude any possibility of generating price-quality cycle in the following way: if consecutive time length m does not yet passes by after a firm takes up the high position, no consumers of its higher-quality product move to market for used goods (due to their brand loyalty to the good of higher quality), although they actually depreciate the product in mind. as soon as that time length passes by, they punish the firm for delay of product renewal.

3.2 the extended model

Considering the above example as an underlying market structure, modify (2) and its related parts in the basic model as follows. Let firm i be in the high position at time t, and let $y_i(t)$ denote the quality level of its product at which consumers trade in the market at that time. Consumers are tolerant of the delay of product renewal in this way: if consecutive time length m does not passes by at time t after firm i takes up the high position, then

$$y_i(t) = x_i(t_{k-1}(s_i)) \text{ for any } t \text{ such that } t_{k-1}(s_i) \le t < t_k(s_i),$$

[3]Gale and Rosenthal (1994) studied the price-quality cycle introducing consumers' cognitive delay of product quality. Their analysis was also confined to monopoly.

where $k \geq 2$, although they actually appreciate firm i's product as $x_i(t)$ in mind. Let $y_i(t) = x_i(0)$ for any t such that $0 \leq t < t_1(s_i)$. if that time length passes by, $y_i(t) = x_i(t)$ and consumers punish the firm. Firm i suffers the damage $d(t)$ from the punishment as additional costs (e.g. for better maintenance service), as long as it is in the high position.

Hence, given a history $h(s,t)$ and a pair $s(t)$ of actions, firm $i \in \{\alpha, \beta\}$ earns at time t its (gross) instantaneous profit

$$\pi_i^t(s(t) : h(t)) = \begin{cases} \tilde{\pi}_H(t) - d(t) \cdot I(\mathcal{A}) & \text{if } x_i(t) > x_j(t) \\ \tilde{\pi}_L(t) & \text{if } x_i(t) \leq x_j(t), \end{cases}$$

where $j \neq i$, $I(\mathcal{A})$ is an identification function that assigns 1 if a condition $\mathcal{A}$ is met or 0 otherwise, and $d(t)$ represents damage firm i suffers if $\mathcal{A}$ is met and satisfies $\tilde{\pi}_H(t) - \tilde{\pi}_L(t) > d(t) > 0$ at any time t. The condition $\mathcal{A}$ is that consecutive time length m passes by at time t after firm i takes up the high position. Let $\tilde{\pi}_H(t) = \tilde{\pi}_H(\triangle x(t))$ and $\tilde{\pi}_L(t) = \tilde{\pi}_L(\triangle x(t))$, where $\triangle x(t) := |y_i(t) - x_j(t)|$. Both are increasing in $\triangle x(t)$, as is in the example. Assume that

$$y_i(t_k(s_i)) = \lambda^m x_i(t_{k-1}(s_i)) + a(m) \geq x_i(t_{k-1}(s_i)) = y_i(t_{k-1}(s_i))$$

for any $k \geq 2$ and that $\lambda^m x_i(0) + a(m) \geq x_i(0)$. Hence, time length m reflects consumers' "ratchet" on the quality level of product. Let $\gamma_{\max} < m$. Since $\gamma_{\max}$ is the most desirable time length for product renewal from consumers' viewpoint, we call $m - \gamma_{\max}$ consumers' "tolerance" level to the delay of product renewal. Let $d := \lim_{n\to\infty} \inf \sum_{t=1}^n d(t)/n$.

Assumption (a') $\tilde{\pi}_H(\epsilon) - \tilde{\pi}_L(a(\gamma_{\max})) \geq C$ for any $\gamma_{\max} \in (1, \infty)$ and any $\epsilon \in (0, \gamma_{\max})$. (b) $C < d \cdot m$.

Note that the difference $\triangle x(t^*)$ in quality levels is determined by an equilibrium σ^*, but t^* is not completely determined by any equilibrium σ^*. Hence, a stronger equilibrium notion is given here. Define an equilibrium $\sigma^{**} := (\sigma_\alpha^{**}, \sigma_\beta^{**})$ with pre-t^* preference by (1) it is an equilibrium that leads to the earliest t^* and (2) for each $i \in \{\alpha, \beta\}$,

$$\begin{aligned}&\textstyle\sum_{t=1}^{t^*}\{\pi_i^t(s^{**}(t) : h(s^{**}, t)) - s_i^{**}(t)C\}\\ &\quad \geq \textstyle\sum_{t=1}^{t^*}\{\pi_i^t((s_i^*(t), s_j^{**}(t),) : h((s_i^*, s_j^{**}, t)) - s_i^*(t)C\}\end{aligned}$$

for any $s_i^* \in \Sigma_i$.

The proof of the following proposition is given in Appendix, since it is quite similar to Lemma 1 and Proposition 2.

Proposition 3 *Under Assumptions* (a') *and* (b)*, we have the following.* (i) *The extended model has equilibria in pure strategies.* (ii) *In any equilibrium σ^* in pure strategies of the extended model, there is time $t^*(< \infty)$ after which each firm $i \in \{\alpha, \beta\}$ renews its product in every time length $\gamma_{\max}$ and so turnover cycle is generated if and only if, for any $i, j \in \{\alpha, \beta\}$ with $i \neq j$, m is so large that $\lambda^m(x_i(0) - x_j(0)) < a(\gamma_{\max})$.* (iii) *When $m < t^*$, in any equilibrium σ^{**} with pre-t^* preference of the extended model, t^* does not become larger as consumers' tolerance level m becomes larger.*

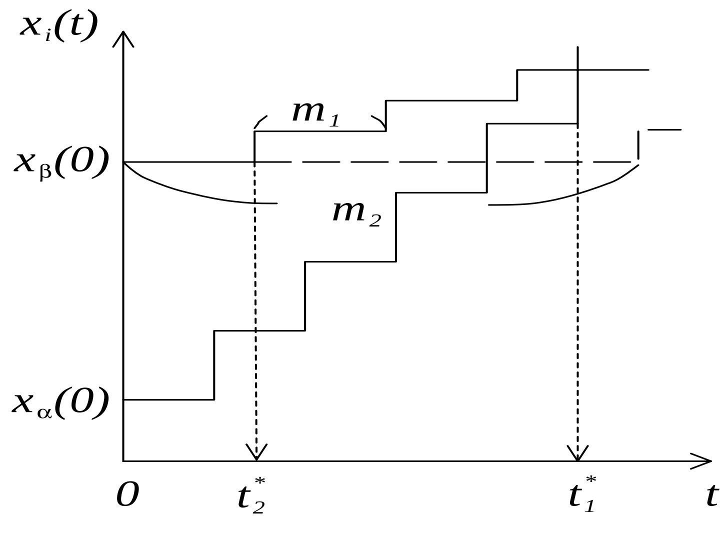

Figure 2: comparative statics

Proposition 3 (ii) implies that only one firm who is in the high position renews its product in every time length $m(> \gamma_{\max})$ if consumers are so fussy about product renewal that $\lambda^m(x_i(0)-x_j(0)) \geq a(\gamma_{\max})$. As far as consumers are sufficiently "tolerant" of the delay in product renewal, sooner or later the duopolists start renewing their products in the same time length $\gamma_{\max}$. The next corollary is a direct implication of Proposition 3 (ii).

Corollary 1 *In Proposition 3* (iii)*, the larger m becomes, the smaller t^* becomes, if the increment in m is sufficiently large, if $\lambda^m x_i(t_{k-1}(s_i))+a(m) > x_i(t_{k-1}(s_i))$ for any $k \geq 2$.*

4 Remarks

This paper did not consider investment decision for "new product" that is to be made by firms. However, we can obtain all the same results, if the investment is decided by 0-1 (no investment or do investment). It suffices to modify the acceleration $a(\cdot)$ in such a way that consumers can observe the investment decision and $a(\tau)$ is determined by the cumulative number τ of investments. Planned obsolescence with more general R&D investment was studied by Fishman and Rob (2000), although their consideration was limited to monopoly. We have to solve a stochastic game in order to extend their model to duopoly.

We can apply our model to the case of n firms, and obtain the same results if $\gamma_{\max} \geq n$. When $\gamma_{\max} < n$, some firms are to make their product renewal at the same time. In this case, we have to consider a possibility of firms' cartel in prices. The stability of that cartel is also to be studied in our model as a future research.

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Appendix

Proof of Proposition 1: We can describe any strategy of any player $i \in \{\alpha, \beta\}$ as a real number between 0.00 and $1.11\cdots$. Consider a strategy $\sigma' \in \Sigma_i$ corresponding to $1.0\cdots010\cdots$ where 1's appear after $\gamma_{\max}(> 1)$ consecutive 0's. We can find a real number $r := 1.1\cdots10\cdots$ with finitely many consecutive 1's such that strategies corresponding to any numbers greater than r are dominated by σ'. That is because it induces the fastest growth of $x_i(t)$ by (1) and its disadvantage in early times disappears in the limit by the definition (3) of the long-run net profit. Hence, it suffices to consider $[0, r]$. Let $\triangle_i$ be a set of firm i's mixed strategies defined on $[0, r]$. For any $i \in \{\alpha, \beta\}$, $\triangle_i$ is compact and convex and Π_i is continuous on $\triangle_i$. Hence, the existence of equilibria in mixed strategies is guaranteed. $\square$

Proof of Proposition 3 (ii): Confirm first that Lemma 1 hold true under Assumptions (a') also in the extended model if $\lambda^m(x_\beta(0)-x_\alpha(0)) < a(\gamma_{\max})$. The remaining part is completely the same as the proof of Proposition 2 (ii). Hence, we only show that Lemma 1 holds true.

Suppose that there exists an equilibrium σ^* with $\mu_\alpha(\cdot,\cdot) = 0$, w.l.o.g. Then, in that equilibrium,

$$\begin{aligned} \Pi_\alpha(\sigma^*_\alpha, \sigma^*_\beta) &= \lim_{n\to\infty} \inf \frac{\sum_{t=1}^n \tilde{\pi}_L(t) - s^*_\alpha(t)C}{n}, \\ \Pi_\beta(\sigma^*_\alpha, \sigma^*_\beta) &= \lim_{n\to\infty} \inf \frac{\sum_{t=1}^n \{\tilde{\pi}_H(t) - d \cdot I(\mathcal{A}) - s^*_\beta(t)C\}}{n}. \end{aligned} \tag{5}$$

For any $t \in [t_{k-1}(s_\beta), t_k(s_\beta))$, $\triangle x(t) = x_\beta(t_{k-1}(s_\beta)) - x_\alpha(t)$, since $y_\beta(t) = x_\beta(t_{k-1}(s_\beta))$. If firm α does not renew its product, $x_\alpha(t+1)$ decreases due to consumers' depreciation. Since $\tilde{\pi}_L(t)$ is increasing in $\triangle x(t)$, it is best for firm α to choose $s^*_\alpha(t) = 0$ at almost any time t. Then, firm β chooses $s^*_\beta(t) = 0$ at almost any time t such that $t - t_{k-1}(s_\beta) < m$ for any $k \geq 2$. (Otherwise, β must spent more costs for more times of product renewal.)

By time $t'(< \infty)$, if firm β had not renewed its product for more than m consecutive time length, then it would be punished by consumers forever after t'. Consider a strategy σ'_β such that $s'_\beta(t) = 1$ when $t - t_{k-1}(s'_\beta) = m$ and $s'_\beta(t) = 0$ otherwise. Since $\tilde{\pi}_H(t) - d < \tilde{\pi}_H(t) - C/m$ by Assumption (b), σ'_β benefits firm β more. Hence, firm β renews its product at time t such that $t - t_{k-1}(s^*_\beta) = m$.

Suppose that firm α deviates to a strategy σ'_α such that $s_\alpha(t) = 1$ whenever $t - t_{k-1}(s_\alpha) = \gamma_{\max}$ for $k \geq 2$. Since firm β takes $s_\beta(t) = 1$ when $t - t_{k-1}(s^*_\beta) = m$ with $k \geq 2$, there is a time n' at which α overtakes β's position and is in a high position forever after n'. It is easy to see by Assumption (a') that $\lim_{n\to\infty} \inf(\tilde{\pi}_H(t) - \tilde{\pi}_L(t))/n > C/\gamma_{\max}$, and so firm α has incentive to deviate to σ'_α.

Let $x_\beta(0) > x_\alpha(0)$. If $\lambda^m(x_\beta(0) - x_\alpha(0)) < a(\gamma_{\max})$, then firm α deviates to σ'_α at $t = m + \gamma_{\max}$. Because of (1), α has at least one time length in every time length $\gamma_{\max}$ even if firm β takes the same strategy after observing α's deviation, contradicting the existence of σ^* with $\mu_\alpha(\sigma^*) = 0$.

When $\lambda^m(x_\beta(0) - x_\alpha(0)) \geq a(\gamma_{\max})$, firm α cannot overtake firm β even once, if firm β makes a retaliatory actions against α's deviation to σ'_α at $t = m + \gamma_{\max}$. □

Proof of Proposition 3 (i): Assumption (a') is equivalent to

$$\frac{1}{\gamma_{\max}}\tilde{\pi}_H(\epsilon) + \frac{\gamma_{\max} - 1}{\gamma_{\max}}\tilde{\pi}_L(\gamma_{\max}) - \frac{C}{\gamma_{\max}} \geq \tilde{\pi}_L(\gamma_{\max}),$$

for any $\gamma_{\max}$ and any $\epsilon \in (0, \gamma_{\max})$. If an equilibrium in pure strategies exists, then each firm obtains $\tilde{\pi}_H(t)$ at least one time length in every time length $\gamma_{\max}$ by (ii), paying $C/\gamma_{\max}$ in the limit, and (ii) further suggests that $0 < \triangle x(t) < \gamma_{\max}$. When a firm i deviates to any other strategies, the firm obtains at most $\tilde{\pi}_L(a(\gamma_{max}))$ in the limit, since $\lim_{t\to\infty} \lambda^t x_i(t) = 0$ if $s_i(t) = 0$ always, and since the other firm makes its product renewal every time length $\gamma_{\max}$ Hence, pure strategies described in (ii) constitute an equilibrium σ^*. □

Proof of Proposition 3 (iii): We begin with the following lemma.

Lemma 2 *Let $m < t^*$. In any equilibrium σ^{**} with pre-t^* preference of the extended model, the firm in the higher position renews its product in time length m until the time t^*, if it can keep the high position by doing so.*

Proof : Since delay of product renewal gives damage $d(t)$ at any time t, and so the firm i in the high position renews its product in every time length no more than m. Given an equilibrium σ^*, the difference $\triangle x(t^*)$ in quality levels is determined. Even if firm i in the high position enlarges $y_i(t) - x_j(t)$ by its product renewal at some time $t(< t^*)$, it induces firm j in the low position to recover the enlarged difference by its product renewal at time $t' \in [t, t^*]$. The instantaneous profits of firms depend only on the difference $y_i(t) - x_j(t)$. Hence, in the equilibrium σ^{**}, firm i in the high position renews its product in time length that makes it to pay the renewal cost C as few times as possible, i.e., in every time length m. The time of first turnover s t^*. The requirements (1) and (2) in the definition of σ^{**} are now met. □

By using Lemma 2, we can see that t^* does not become larger when m becomes $m'(> m)$, if firm j in a low position keeps initial σ_j^{**} intact. Assumption (a') is a sufficient condition for firm j not to delay its product renewal. Figure 2 illustrates the proof in the case of Corollary 1. In figure 2, firm β is in the high position at $t = 0$. If $\lambda^m x_i(t_{k-1}(s_i)) + a(m) = x_i(0)$ for any $k \geq 2$, $t_1^* = t_2^*$. If $m_2 - m_1$ is not sufficiently large, again $t_1^* = t_2^*$. $\square$