THE RETURNS TO SCALE EFFECT IN LABOUR PRODUCTIVITY GROWTH

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Abstract

Labour productivity is defined as output per unit of labour input. Economists acknowledge that technical progress as well as growth in capital inputs increases labour productivity. However, little attention has been paid to the fact that changes in labour input alone could also impact labour productivity. Since this effect disappears for the constant returns to scale short-run production frontier, we call it the *returns to scale effect*. We decompose the growth in labour productivity into two components: 1) the joint effect of technical progress and capital input growth, and 2) the returns to scale effect. We propose theoretical measures for these two components and show that they coincide with the index number formulae consisting of prices and quantities of inputs and outputs. We then apply the results of our decomposition to U.S. industry data for 1970–2007. It is acknowledged that labour productivity in the service sector grows much more slowly than in the goods-producing sector. We conclude that the returns to scale effect can explain a large part of the gap in labour productivity growth between the two sectors.

Key Words

Labour productivity, index numbers, Malmquist index, Törnqvist index, output distance function, input distance function

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1. Introduction

Economists broadly think of productivity as measuring the current state of the technology used in producing the firm's goods and services. The production frontier, consisting of inputs and the maximum output attainable from them, characterizes the prevailing state of technology. Productivity growth is often identified by the shift in the production frontier, reflecting changes in production technology.[1,2] However, productivity growth can also be driven by movement along the production frontier.

Even in the absence of changes in the production frontier, changes in the inputs used for production can lead to productivity growth, moving along the production frontier and making use of its curvature. Productivity growth that is induced by the movement along the production frontier is called the *returns to scale effect*. This effect does not reflect changes in the production frontier. Thus, in order to properly evaluate improvements in the underlying production technology reflecting the shift in the production frontier, we must disentangle the returns to scale effect from labour productivity.

Productivity measures can be classified into two types: total factor productivity (TFP) and partial factor productivity. The former index relates a bundle of total inputs to outputs, whereas the latter index relates a portion of total inputs to outputs. The present paper deals with labour productivity (LP) among several measures of partial factor productivity. LP is defined as output per labour input in the simple one-output one-labour-input case. Economy-wide LP is the critical determinant of a country's standard of living in the long-run. For example, U.S. history reveals that increases in LP have translated to nearly one-for-one increases in per capita income as well as real wages over a long period of time. The importance of LP as a source for the progress of economic well-being prompts many researchers to investigate what determines LP growth.[3] Technical progress and capital input growth have been identified as the main determinants of a country's enormous LP growth over long periods (Jorgenson and Stiroh 2000, Jones 2002) as well as the wide differences in LP across countries (Hall and Jones 1999). The present paper adds one more explanatory factor to LP growth.

LP relates labour inputs to outputs, holding technology and capital services fixed. The short-run production frontier, which consists of labour inputs and the maximum output attainable from them, represents the capacity of current technology to translate labour inputs to outputs. Both technical progress and capital input growth, which have been identified as the sources of LP growth, induce LP growth throughout the shift in the short-run production frontier. However, the returns to scale effect, which is the extent of LP growth induced by movement along the short-run production frontier, has never been exposed.

[1] See Griliches (1987). The same interpretation is also found in Chambers (1988).

[2] In principle, productivity improvement can occur through technological progress and gains in technical efficiency. Technical efficiency is the distance between the production plan and the production frontier. The present paper assumes a firm's profit maximizing behaviour, and in our model the current production plan is always on the current production frontier. The assumption of profit maximization is common in economic approaches to index numbers. See Caves, Christensen and Diewert (1982) and Diewert and Morrison (1986).

[3] The LP growth and the capital input growth are the abbreviations for the growth rates of LP and capital input. In this paper, the growth rate of LP between the current and previous periods is the ratio of LP in the current period to LP in the previous period.

We decompose LP growth into two components: 1) the joint effect of technical progress and capital input growth, 2) the returns to scale effect. First, we propose theoretical measures representing the two effects by using distance functions. Second, we derive the index number formulae consisting of prices and quantities and show that they coincide with theoretical measures, assuming a translog functional form for the underlying technology and the firm's profit maximizing behaviour.

Our approach to implementing theoretical measures is drawn from Caves, Christensen and Diewert (1982) (CCD). Using the distance function, CCD formulate the (theoretical) Malmquist productivity index, which measures the shift in the production frontier, and show that the Malmquist productivity index and the Törnqvist productivity index coincide, assuming the translog functional form for the underlying technology and the firm's profit maximizing behaviour.

The Törnqvist productivity index is a measure for the TFP growth calculated by the Törnqvist quantity indexes. It is an index number formula consisting of price and quantity observations. Equivalence between the two indexes breaks down if the underlying technology does not exhibit constant returns to scale. Its difference represents the returns to scale effect, which is the growth in TFP induced by movement along the production frontier. Thus, like Diewert and Nakamura (2007) and Diewert and Fox (2010), we can interpret that CCD decompose the TFP growth that is calculated by the Törnqvist quantity indexes into Malmquist productivity index and the returns to scale effect. The former component captures TFP growth induced by the shift in the short-run production frontier.

Many researchers have been concerned with the growth in TFP induced by movement along the underlying production frontier. CCD (1982) call it the *returns to scale factor*, Diewert and Nakamura (2007) call it the *returns to scale component*, and Lovell (2003) calls it the *scale effect*. In the literature of Data Envelopment Analysis (Balk, 2001 and Coelli et al. 2003), the product of *scale efficiency change*, *input mix effect* and *output mix effect* summarizes the TFP growth induced by movement along the production frontier, and it is interpreted as the returns to scale effect.

Although scholars have recognized the significance of the returns to scale effect for TFP growth, its effect on LP growth has never been addressed even though it is more important in explaining LP growth than in explaining TFP growth. When the underlying technology exhibits constant returns to scale, the scale economies effect disappears from TFP growth. However, it still plays a role for LP growth. This is because even if the underlying technology exhibits constant returns to scale, the short-run production frontier is likely not to exhibit constant returns to scale.

Triplett and Bosworth (2004, 2006) and Bosworth and Triplett (2007) observed that LP growth in the service sector was much less than in the service sector in U.S. economy since the early 1970s. As we discussed above, there are two underlying factors to LP growth; therefore, two explanations are possible for the low LP growth in the services sector: 1) the joint effect of technical progress and increases in capital inputs is modest, or 2) an increase in labour inputs induces negative returns to scale effects. We apply our decomposition result to U.S. industry data to compare the relative contributions of the two effects.

Section 2 illustrates the two effects underlying LP growth graphically. Section 3 discusses the measure of the joint effect of technical progress and capital input growth in the multiple-inputs multiple-outputs case. Section 4 discusses the returns to scale effect in the multiple-inputs multiple-outputs case. We show that the product of the joint effect of technical progress and capital input growth coincides with LP growth. Section 6 includes the application to the U.S. industry data. Section 5 presents the conclusions.

2. Two Sources of Labour Productivity Growth

We display graphically what derives LP growth using a simple model of one output y and two inputs: labour input x_L and capital input x_K. Suppose that a firm produces outputs y^0 and y^1 using inputs $(\boldsymbol{x}_K{}^0, x_L{}^0)$ and $(\boldsymbol{x}_K{}^1, x_L{}^1)$. Period t production technology is described by the period t production function $y = f^t(x_K, x_L)$ for $t = 0$ and 1. Let us begin by considering how this joint effect of technical progress and capital input growth raises LP. Figure 1 illustrates the case when the joint effects of technical progress and increases in capital inputs positively affect the employment of labour in production. The lower curve represents the period 0 short-run production frontier. It indicates how much output can be produced in period 0 by using a specified quantity of labour given the capital and technology available in period 0. Similarly, the higher curve represents the period 1 short-run production frontier. It indicates how much output can be produced from a given labour input in period 1 given the capital and technology available in period 1.

Since the short-run production frontier shifts upward, the output attainable from a given labour input x_L increases between the two periods such that $f^1(x_K{}^1, x_L) > f^0(x_K{}^0, x_L)$. The corresponding LP also grows such that $f^1(x_K{}^1, x_L)/x_L > f^0(x_K{}^0, x_L)/x_L$. Thus, the ratio $f^1(x_K{}^1, x_L)/f^0(x_K{}^0, x_L) = (f^1(x_K{}^1, x_L)/x_L)/(f^0(x_K{}^0, x_L)/x_L)$ captures the joint effect on LP growth of technical progress and capital input growth. Note that the ratio is also a measure of the distance between the short-run production frontiers of periods 0 and 1 in the direction of the y axis, evaluated at x_L. The ratio increases as the distance between the period 0 and the period 1 short-run production frontiers increases. Therefore, the joint effect of technical progress and increase in capital inputs can be captured throughout by measuring the shift in the short-run production frontier.

[Place Figure 1 appropriately here.]

Any quantity of labour input can produce more output in period 1 than in period 0, reflecting the positive joint effect of technical progress and increase in capital inputs. The firm increases its demand for labour input from $x_L{}^0$ to $x_L{}^1$, exploiting the increased productive capacity of labour input. Production takes place at A for period 0 and at B for period 1. The slope of the ray from the origin to A and B indicates the LP of each period. Since $y^1/x_L{}^1$ is smaller than $y^0/x_L{}^0$, LP declines between the two periods. The fact that LP can decline despite the outward shift in the short-run production frontier suggests that another factor contributes to LP growth.

The path from A to B can be divided into two parts: the vertical jump from A to A' and the movement along the period 1 short-run production frontier from A' to B. Along the vertical jump from A to A', the LP changes from $y^0/x_L{}^0$ to $f^1(x_K{}^1, x_L{}^0)/x_L$.[1] Its ratio $(y^1/x_L{}^1)/(f^1(x_K{}^1, x_L{}^0)/x_L{}^0)$ is considered to be the growth in LP induced by the shift in the short-run production frontier, which is the joint effect of technical progress and an increase in capital inputs. LP growth is offset by the change in labour input from $x_L{}^0$ to $x_L{}^1$. The movement along the period 1 short-run production frontier from A' to B reduces LP from $f^1(x_K{}^1, x_L{}^0)/x_L{}^0$ to $(y^1/x_L{}^1)$. We call the LP growth induced by movement along the short-run production frontier $(y^1/x_L^1)/(f^1(x_K^1, x_L^0)/x_L^0)$ the returns to scale effect.

In this section, we illustrate two sources of LP growth using the simple one-output two-inputs model. However, the division of the path from *A* to *B* into two steps from *A* to *A'* and from *A'* to *B* is merely an example. It is also possible to decompose the change from *A* to *B* into the movement along the period 0 short-run production function from *A* to *B'* and the vertical jump from *B'* to *B*. In this case, the former movement reflects the returns to scale effect, and the latter movement reflects the joint effect of technical progress and capital input growth.

For measuring the joint effect of technical progress and capital input growth, the important consideration is the quantity of labour input at which the distance between two short-run production frontiers is evaluated. For measuring the returns to scale effect, whether we consider the movement along the period 0 or 1 short-run production frontier matters. Hereafter, we generalize our discussion to the more general multiple-inputs multiple-outputs case and propose measures for the two effects that are immune from choosing the arbitrary benchmark.

3. Joint Effect of Technical Progress and Capital Input Growth

A firm is considered as a productive entity transforming inputs into outputs. We assume that there are *M* (net) outputs, $\boldsymbol{y} = [y_1,\ldots,y_M]^T$ and $P + Q$ inputs consisting of *P* types of capital inputs $\boldsymbol{x}_K = [x_{K,1},\ldots,x_{K,P}]^T$ and *Q* types of labour inputs $\boldsymbol{x}_L = [x_{L,1},\ldots, x_{L,Q}]^T$.[4] The period *t* production possibility set S^t consists of all the feasible combinations of inputs and outputs and it is defined as follows:

(1) $S^t \equiv \{(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L) : (\boldsymbol{x}_K, \boldsymbol{x}_L) \text{ can produce } \boldsymbol{y}\}$

We assume S^t is a closed set that exhibits a free disposability property and includes the origin.[5] The period *t* production frontier, which is the boundary of S^t, is represented by the *period t input requirement function* G^t. It is defined as follows:

(2) $F^t(\boldsymbol{y}, \boldsymbol{x}_{K,-1}, \boldsymbol{x}_L) \equiv \min_{x_{K,1}} \{x_{K,1} : (\boldsymbol{y}, x_{K,1}, \boldsymbol{x}_{K,-1}, \boldsymbol{x}_L) \in S^t\}$

It represents the minimum amount of the first capital input that a firm can use at period *t*, producing output quantities $\boldsymbol{y}$, holding other capital inputs $\boldsymbol{x}_{K-1} = [x_{K,2},\ldots, x_{K,P}]^T$ and labour inputs $\boldsymbol{x}_L$ fixed. This function, which is originally formulated for the period *t* production frontier, can be used for characterizing the period *t* short-run production frontier. Given period *t* capital input $\boldsymbol{x}_K^t$, the set of outputs and labour inputs satisfying $x_{K,1}{}^t = F^t(y, x_{K,-1}{}^t, x_L)$ forms the period *t* short-run production frontier.

CCD (1982) measures the shift in the production frontier by using the output distance function. Adjusting their approach, we also use the output distance function to measure the shift in the short-run production frontier. Using the input requirement function, the *period t output distance function* for $t = 0$ and 1 is defined as follows:

[4] Outputs include intermediate inputs. If output *m* is an intermediate input, then $y_m < 0$. Hence, the nominal value of total (net) outputs $\boldsymbol{p}\cdot\boldsymbol{y}$ is the value-added that a firm generates.

[5] Free disposability property means that given $(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L) \in S^t$, if $(\boldsymbol{y}^*, \boldsymbol{x}_K^*, \boldsymbol{x}_L^*) \geq (\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$, $(\boldsymbol{y}^*, \boldsymbol{x}_K^*, \boldsymbol{x}_L^*) \in S^t$.

$$(3)\ D_O^t(\boldsymbol{y},\boldsymbol{x}_K,\boldsymbol{x}_L)\equiv \min_{\delta}\left\{\delta : F^t\left(\frac{\boldsymbol{y}}{\delta},\boldsymbol{x}_{K,-1},\boldsymbol{x}_L\right)\le x_{K,1}\right\}.$$

Given capital inputs $\boldsymbol{x}_K$ and labour inputs $\boldsymbol{x}_L$, $D_O{}^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ is the minimum contraction of outputs $\boldsymbol{y}$ so that the contracted outputs $\boldsymbol{y}/D_O{}^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$, capital inputs $\boldsymbol{x}_K$, and labour inputs $\boldsymbol{x}_L$ are on the period t production frontier. If $(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ is on the period t production frontier, $D_O{}^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ equals one. Note that $D_L{}^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ is linearly homogeneous in $\boldsymbol{y}$.

We can also relate it to the short-run production frontier. Given labour inputs $\boldsymbol{x}_L$, $D_O{}^t(\boldsymbol{y}, \boldsymbol{x}_K{}^t, \boldsymbol{x}_L)$ is the minimum contraction of outputs so that the contracted outputs $\boldsymbol{y}/D_O{}^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ and labour input $\boldsymbol{x}_L$ are on the period t short-run production frontier. Thus, $D_O{}^t(\boldsymbol{y}, \boldsymbol{x}_K{}^t, \boldsymbol{x}_L)$ provides a radial measure of the distance of $\boldsymbol{y}$ to the period t short-run production frontier. We measure the shift in the short-run production frontier by comparing the distance of $\boldsymbol{y}$ to the short-run production frontiers of the period 0 and 1. It is defined as follows:[6]

$$(4)\ SHIFT(\boldsymbol{y},\boldsymbol{x}_L)\equiv\frac{D_O^0(\boldsymbol{y},\boldsymbol{x}_K^0,\boldsymbol{x}_L)}{D_O^1(\boldsymbol{y},\boldsymbol{x}_K^1,\boldsymbol{x}_L)}.$$

If technical progress and capital input growth have a positive effect on the productive capacity of labour between period 0 and 1, the short-run production frontier shifts outward. Given labour inputs $\boldsymbol{x}_L$, more outputs can be produced. Thus, the minimum contraction factor for given outputs $\boldsymbol{y}$ declines such that $D_O{}^1(\boldsymbol{y}, \boldsymbol{x}_K{}^1, \boldsymbol{x}_L) \le D_O{}^0(\boldsymbol{y}, \boldsymbol{x}_K{}^0, \boldsymbol{x}_L)$, leading to $SHIFT(\boldsymbol{y}, \boldsymbol{x}_L) \ge 1$. Similarly, the negative joint effect of technical progress and capital input growth leads to $SHIFT(\boldsymbol{y}, \boldsymbol{x}_L) \le 1$.

Each choice of reference vectors $(\boldsymbol{y}, \boldsymbol{x}_L)$ might generate a different measure of the shift in the short-run production frontier going from period 0 to period 1. We calculate two measures by using different reference vectors $(\boldsymbol{y}^0, \boldsymbol{x}_L{}^0)$ and $(\boldsymbol{y}^1, \boldsymbol{x}_L{}^1)$. Since these reference outputs and labour inputs are actually chosen in each period, they are equally reasonable. Following Fisher (1922) and CCD (1982), we use the geometric mean of these measures as a theoretical measure of the joint effect of technical progress and capital input growth, *SHIFT*, as follows:[7]

$$(5)\ SHIFT\equiv\sqrt{SHIFT(\boldsymbol{y}^0,\boldsymbol{x}_L^0)\cdot SHIFT(\boldsymbol{y}^1,\boldsymbol{x}_L^1)}.$$

The case of one output and one labour input offers us a graphical interpretation of *SHIFT*. In Figure 1, it reduces to the following formula.

$$(6)\ SHIFT=\sqrt{(f^1(\boldsymbol{x}_L^1,\boldsymbol{x}_L^0)/\boldsymbol{y}^0)(\boldsymbol{y}^1/f^0(\boldsymbol{x}_L^0,\boldsymbol{x}_L^1))}.$$

Given a quantity of labour input, the ratio of the output attainable from such a labour input at period 1 to the output attainable at period 0 represents the extent to which the

[6] CCD (1982) and Färe et al (1994) introduce a measure of the shift in the production frontier by using the ratio of the output distance function. Given $(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$, Färe et al (1994) measure the shift in the production frontier by $D_O{}^0(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)/D_O{}^1(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$.

[7] Since the firm's profit maximization is assumed, it is possible to adopt a different formulation for the measure of the shift in the short-run production frontier: $SHIFT = (D_O{}^0(\boldsymbol{y}^1, \boldsymbol{x}_K{}^0, \boldsymbol{x}_L{}^1)/D_O{}^0(\boldsymbol{y}^0, \boldsymbol{x}_K{}^0, \boldsymbol{x}_L{}^0))^{1/2}(D_O{}^1(\boldsymbol{y}^1, \boldsymbol{x}_K{}^1, \boldsymbol{x}_L{}^1)/D_O{}^1(\boldsymbol{y}^0, \boldsymbol{x}_K{}^1, \boldsymbol{x}_L{}^0))^{1/2}$. This formulation is closer to the original Malmquist productivity (TFP growth) index, which was introduced by CCD (1982).

short-run production function expands. *SHIFT* is the geometric mean of those ratios conditional on $\boldsymbol{x}_L^0$ and $\boldsymbol{x}_L^1$.

SHIFT is a theoretical measure defined by the unknown distance functions and there are alternative ways of implementing it. We show that the theoretical measure coincides with a formula of price and quantity observations under the assumption of a firm's profit maximizing behaviour and a translog functional form for the output distance function.[8] Our approach is drawn from CCD (1982) that implement the Malmquist productivity index, which is the theoretical measure of the shift in the production frontier. They show that the Malmquist productivity index coincides with a different index number formula of price and quantity observations, which is called Törnqvist productivity index, under some assumption.[9]

CCD (1982) also show that the first-order derivatives of the distance function D^t with respect to quantities at the period t actual production plan ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^t$, $\boldsymbol{x}_L^t$) are computable from price and quantity observations. Their equivalence result between the Malmquist and Törnqvist productivity indexes relies on these relationships. Utilizing the same relationships, we also show that *SHIFT* coincides with an index number formula of price and quantity observations. Although all the following equations (7)-(16) have been already derived by CCD (1982), we outline how to compute the first-order derivatives of the distance functions from price and quantity observations below for completeness of discussion. The implicit function theorem is applied to the input requirement function $F^t(\boldsymbol{y}/\delta, \boldsymbol{x}_{K,-1}, \boldsymbol{x}_L) = x_{K,1}$ to solve for $\delta = D_O^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ around ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^t$, $\boldsymbol{x}_L^t$).[10] Its derivatives are represented by the derivatives of $F^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$. We have the following equations for t = 0 and 1:

$$(7)\, \nabla_y D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \frac{1}{\boldsymbol{y}^t \cdot \nabla_y F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)} \nabla_y F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t).$$

$$(8)\, \nabla_{x_K} D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \frac{1}{\boldsymbol{y}^t \cdot \nabla_y F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)} \begin{bmatrix} -1 \\ \nabla_{x_{K,-1}} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t) \end{bmatrix}.$$

$$(9)\, \nabla_{x_L} D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \frac{1}{\boldsymbol{y}^t \cdot \nabla_y F^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t)} \nabla_{x_L} F^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t).$$

We assume the firm's profit maximizing behaviour. Thus, ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^t$, $\boldsymbol{x}_L^t$) >> 0_{N+P+Q} is a solution to the following period t profit maximization problem for t = 0 and 1:

$$(10)\, \max\{\boldsymbol{p}^t \cdot \boldsymbol{y} - r_1^t F^t(\boldsymbol{y}, \boldsymbol{x}_{K,-1}, \boldsymbol{x}_L) - \boldsymbol{r}_{-1}^t \cdot \boldsymbol{x}_{K,-1} - \boldsymbol{w}^t \cdot \boldsymbol{x}_L\}$$

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[8] Alternative approaches are to estimate the underlying distance function by econometric and linear programming approaches. Either of them requires sufficient empirical observations. Our approach originated by CCD (1982) is applicable as long as price and quantity observations are available at the current and the reference periods. See Nishimizu and Page (1982) for the application of the econometric technique and see Färe et al. (1994) for the application of the linear programming technique.

[9] CCD (1982) justify the formula that is Törqvist output quantity index divided by Törqvist quantity index. In addition to the translog functional form for the output distance function, they assume the firm's cost minimizing and revenue maximizing. Their approach is called the index number approach, finding a formula of price and quantity observations that approximate a theoretical measure.

[10] We assume the following three conditions are satisfied for t = 0 and 1: 1) F^t is differentiable at the point ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^t$, $\boldsymbol{x}_L^t$),: 2) $\boldsymbol{y}^t >> 0_M$ and 3) $\boldsymbol{y}^t \cdot \nabla_y F(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) > 0$.

Outputs are sold at the positive producer prices $\boldsymbol{p} = [p_1,\ldots, p_M]^T >> 0$, capital inputs are purchased at the positive rental prices $\boldsymbol{r} = [r_1,\ldots, r_P]^T >> 0$, and labour inputs are purchased at the positive wages $\boldsymbol{w} = [w_1,\ldots, w_Q]^T >> 0$. Note that $\boldsymbol{r}_{-1} = [r_{-1},\ldots, r_P]^T$. The period t profit maximization problem yields the following first order conditions for $t = 0$ and 1:

(11) $\boldsymbol{p}^t = r_1^{\ t}\nabla_{\boldsymbol{y}} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)$.

(12) $\boldsymbol{r}_{-1}^{\ t} = -r_1^{\ t}\nabla_{\boldsymbol{x}_{K,-1}} F^s(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)$.

(13) $\boldsymbol{w}^t = -r_1^{\ t}\nabla_{\boldsymbol{x}_L} F^s(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)$.

By substituting (11)-(13) into (7)-(9), we obtain the following equations for $t = 0$ and 1:

(14) $\nabla_{\boldsymbol{y}} D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \boldsymbol{p}^t / \boldsymbol{p}^t \cdot \boldsymbol{y}^t$.

(15) $\nabla_{\boldsymbol{x}_K} D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = [r_1^{\ t} / \boldsymbol{p}^t \cdot \boldsymbol{y}^t]\begin{bmatrix} -1 \\ \nabla_{\boldsymbol{x}_{K,-1}} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t) \end{bmatrix} = [1/ \boldsymbol{p}^t \cdot \boldsymbol{y}^t]\begin{bmatrix} -r_1 \\ -\boldsymbol{r}_{-1} \end{bmatrix}$.

(16) $\nabla_{\boldsymbol{x}_L} D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = -\boldsymbol{w}^t / \boldsymbol{p}^t \cdot \boldsymbol{y}$.

The above equations (14)-(16) allow us to compute the derivatives of the distance function, without knowing the output distance function itself. The information of the derivatives is useful for calculating the values of the output distance functions. However, one disadvantage is that the derivatives of the period t output distance function need to be evaluated at the period t actual production plan ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^{\ t}$, $\boldsymbol{x}_L^{\ t}$) in equations (14)-(16) for $t = 0$ and 1. The distance functions evaluated at the hypothetical production plan such as ($\boldsymbol{y}^1$, $\boldsymbol{x}_K^{\ 0}$, $\boldsymbol{x}_L^{\ 1}$) and ($\boldsymbol{y}^0$, $\boldsymbol{x}_K^{\ 1}$, $\boldsymbol{x}_L^{\ 0}$) also constitute *SHIFT*. Hence, the above equations are not enough for implementing *SHIFT*. In addition to a firm's profit maximization, we further assume a following translog functional form for the period t output distance function for $t = 0$ and 1. It is defined as follows;

$$
\begin{aligned}
\ln D_O^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L) \equiv{}& \alpha_0^{\ t} + \sum_{m=1}^{M} \alpha_m^{\ t} \ln y_m + (1/2)\sum_{i=1}^{M}\sum_{j=1}^{M} \alpha_{i,j} \ln y_i \ln y_j \\
&+ \sum_{p=1}^{P} \beta_p^{\ t} \ln x_{K,p} + (1/2)\sum_{i=1}^{P}\sum_{j=1}^{P} \beta_{i,j} \ln x_{K,i} \ln x_{K,j} \\
&+ \sum_{q=1}^{Q} \chi_q^{\ t} \ln x_{L,q} + (1/2)\sum_{i=1}^{Q}\sum_{j=1}^{Q} \chi_{i,j} \ln x_{L,i} \ln x_{L,j} \\
&+ \sum_{m=1}^{M}\sum_{p=1}^{P} \delta_{m,p} \ln y_m \ln x_{K,p} + \sum_{m=1}^{M}\sum_{q=1}^{Q} \varepsilon_{m,q} \ln y_m \ln x_{L,q} \\
&+ \sum_{p=1}^{P}\sum_{q=1}^{Q} \phi_{p,q} \ln x_{K,p} \ln x_{L,q}
\end{aligned} \tag{17}
$$

where the parameters satisfy the following restrictions:

(18) $\alpha_{i,j} = \alpha_{j,i}$ for all i and j such as $1 \le i < j \le M$;

(19) $\beta_{i,j} = \beta_{j,i}$ for all i and j such as $1 \le i < j \le P$;

(20) $\chi_{i,j} = \chi_{j,i}$ for i and j such as $1 \le i < j \le Q$;

(21) $\sum_{n=1}^{N} \alpha_n^t = 1$;

(22) $\sum_{i=1}^{M} \alpha_{i,m} = 0$ for $m = 1,\ldots,M$;

(23) $\sum_{m=1}^{M} \delta_{m,p} = 0$ for $p = 1,\ldots,P$;

(24) $\sum_{i=1}^{M} \varepsilon_{m,q} = 0$ for $q = 1,\ldots,Q$.

Restrictions (21)-(24) guarantee the linear homogeneity in *y*. The translog functional form characterized (17)-(24) is a flexible functional form so that it can approximate an arbitrary output distance function to the second order at an arbitrary point. Thus, assuming this functional form does not harm any generality of the output distance function. Note that the coefficients for the linear terms and the constant term are allowed to vary across periods. Thus, technical progress under the translog distance function is by no means limited to Hicks neutral and is able to represent a vast variety of technical progress is possible. Under the assumption of the profit maximizing behaviour and the translog functional form, a theoretical measure *SHIFT* is computed from price and quantity observations.

Proposition 1: Assume that the output distance functions D_O^0 and D_O^1 have the translog functional form defined by (17)-(24) and a firm follows competitive profit maximizing behaviour in period t = 0 and 1. Then, the joint effect of technical progress and capital input growth, *SHIFT*, can be computed from observed prices and quantities as follows:

$$(25)\ \ln SHIFT = \sum_{m=1}^{M} s_m \ln\left(\frac{y_m^1}{y_m^0}\right) - \sum_{q=1}^{Q} s_{L,q} \ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$

where s_m and $s_{L,q}$ are the average value-added shares of output *m* and labour input *q* between periods 0 and 1 such that;

$$s_m = \frac{1}{2}\left(\frac{p_m^0 y_m^0}{\boldsymbol{p}^0 \cdot \boldsymbol{y}^0} + \frac{p_m^1 y_m^1}{\boldsymbol{p}^1 \cdot \boldsymbol{y}^1}\right) \text{ and } s_{L,q} = \frac{1}{2}\left(\frac{w_q^0 x_{L,q}^0}{\boldsymbol{p}^0 \cdot \boldsymbol{y}^0} + \frac{w_q^1 x_{L,q}^1}{\boldsymbol{p}^1 \cdot \boldsymbol{y}^1}\right)$$

The index number formula in (25) can be interpreted as the ratio of a quantity index of outputs to a quantity index of labour input. Note that no data of price and quantity of capital inputs appear in this formula. Even though the shift in the short-run production frontier reflects technical progress as well as the change in capital input, we can measure its shift without resort to capital input data explicitly.

4. Returns to Scale Effect

As is shown in Figure 1, the shift in the short-run production frontier is not the only factor contributing to the growth in LP. Even when there is no change in the short-run production frontier, the movement along the frontier could raise LP, exploiting the concavity of the short-run production frontier. We call the LP growth induced by the movement along the short-run production frontier the *returns to scale effect*. In the simple model consisting of one output and one labour input, LP is defined as output per one unit of labour input. Therefore, LP growth, which is the growth rate of LP between the current period and the previous period, coincides with the growth rate of output divided by the growth rate of labour input. Since the returns to scale effect is the LP growth induced by the movement along the short-run production frontier, it is computed by the growth rates of output and labour input between the two end points of the movement. Figure 2 shows how the movement along the period t short-run production frontier from point C to D affects LP. Comparing two points C and D, the growth rate of output is $f^t(\boldsymbol{x}_K^t, \boldsymbol{x}_L^1)/f^t(\boldsymbol{x}_K^t, \boldsymbol{x}_L^0)$ and the growth ratio of labour input $\boldsymbol{x}_L^1/\boldsymbol{x}_L^0$. The growth rate of LP between two points coincides with the growth rate of output divided by that of labour input so that $(f^t(\boldsymbol{x}_K^t, \boldsymbol{x}_L^1)/f^t(\boldsymbol{x}_K^t, \boldsymbol{x}_L^0))/(\boldsymbol{x}_L^1/\boldsymbol{x}_L^0) = (f^t(\boldsymbol{x}_K^t, \boldsymbol{x}_L^1)/\boldsymbol{x}_L^1)/(f^t(\boldsymbol{x}_K^t, \boldsymbol{x}_L^0)/\boldsymbol{x}_L^0)$.

[Place Figure 2 appropriately here.]

We generalize the growth rates of labour input and output between two points on the period t short-run production frontier in order to measure the returns to scale effect in the multiple-inputs multiple-outputs case. First, we investigate how to measure the counterpart of the growth rate of labour inputs in the multiple-inputs multiple-outputs case. CCD (1982) define the input quantity index, which is the counterpart of the growth rate of total inputs, by comparing the radial distance between two input vectors and the period t production frontier. The input distance function is used for the radial scaling of total inputs. Adapting the input distance function used by CCD (1982), we introduce the labour input distance function that measure the radial distance of labour inputs $\boldsymbol{x}_L$ to the period t production frontier. The *period t labour input distance function* for t = 0 and 1 is defined as follows:

$$(26)\ D_L^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L) \equiv \max_\delta \left\{ \delta : F^t\left(\boldsymbol{y}, \boldsymbol{x}_{K,-1}, \frac{\boldsymbol{x}_L}{\delta} \right) \le x_{K,1} \right\}.$$

Given outputs $\boldsymbol{y}$ and capital inputs $\boldsymbol{x}_K$, $D_L^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ is the maximum contraction of labour inputs $\boldsymbol{x}_L$ so that the contracted labour inputs $\boldsymbol{x}_L/D_L^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ and capital inputs $\boldsymbol{x}_K$ with outputs $\boldsymbol{y}$ are on the period t production frontier. If $(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ is on the period t production frontier, $D_L^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ equals one. Note that $D_L^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ is linearly homogeneous in $\boldsymbol{x}_K$.

We can also relate it to the short-run production frontier. Given outputs $\boldsymbol{y}$, $D_L^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L)$ is the maximum contraction of labour inputs so that the contracted labour inputs $\boldsymbol{x}_L/D_L^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L)$ and outputs $\boldsymbol{y}$ are on the period t short-run production frontier. Thus, $D_L^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L)$ provides a radial measure of the distance of $\boldsymbol{x}_L$ to the period t short-run production frontier. We construct the counterpart of the growth rate of labour input by comparing two labour inputs $\boldsymbol{x}_L^0$ and $\boldsymbol{x}_L^1$ to the period t short-run production frontier. It is defined as follows;

$$(27)\ LABOUR(t, \boldsymbol{y}) \equiv D_L^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^1) / D_L^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^0)$$

If labour inputs grow between two periods, $\boldsymbol{x}_L^1$ moves further away from the origin than $\boldsymbol{x}_L^0$, meaning that the labour input vector $\boldsymbol{x}_L^1$ is larger than the labour input vector $\boldsymbol{x}_L^0$. The maximum contraction factor for producing outputs $\boldsymbol{y}$ with the period t capital inputs $\boldsymbol{x}_K^t$ using the period t technology increases such that $D_L^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^0) \le D_L^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^1)$, leading to $LABOUR(t,\boldsymbol{y}) \ge 1$. Similarly, if labour inputs shrinks between two periods, $\boldsymbol{x}_L^1$ moves closer to the origin than $\boldsymbol{x}_L^0$, leadings to $LABOUR(t,\boldsymbol{y}) \le 1$.

Second, we investigate how to measure the growth rate of outputs on the period t short-run production frontier. In the multiple-inputs multiple-outputs case, there is the set of outputs attainable from given labour inputs $\boldsymbol{x}_L$ with period t technology and capital inputs. It is a part of the period t short-run production frontier. Since $D_O^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L)$ provides a radial measure of the distance of $\boldsymbol{y}$ to the period t short-run production frontier, $D_O^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L)$ provides a radial measure of the distance of $\boldsymbol{y}$ to the set of outputs attainable from labour inputs $\boldsymbol{x}_L^t$ with period t technology and capital input. We construct the counterpart of the growth rate of outputs on the period t short-run production frontier by comparing outputs $\boldsymbol{y}$ to the sets of outputs attainable from $\boldsymbol{x}_L^0$ and $\boldsymbol{x}_L^1$ with period t capital input $\boldsymbol{x}_K^t$. It is defined as follows:

$$(28)\ OUTPUT(t, \boldsymbol{y}) \equiv D_O^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^0) / D_O^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^1)$$

If the labour input growth makes it possible to produce more outputs with given capital input using technology, the set of outputs attainable from $\boldsymbol{x}_L^1$ shifts outward to that of outputs attainable from $\boldsymbol{x}_L^0$. Thus, the minimum contraction factor for given outputs $\boldsymbol{y}$ declines such that $D_O^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^1) \leq D_O^t(\boldsymbol{y}, \boldsymbol{x}_K^t, \boldsymbol{x}_L^0)$, leading to $OUTPUT(t, \boldsymbol{y}) \geq 1$. Similarly, if the change in labour inputs reduces outputs attainable from given capital and labour inputs leads to $OUTPUT(t, \boldsymbol{y}) \leq 1$.

Using the counterparts of the growth rate of outputs and labour inputs between two points, we can propose the measure of LP growth between the two points on the period t short-run production frontier. When we use the period t short-run production and outputs as reference technology and outputs, the returns to scale effect is defined as follows:

$$(29)\quad \begin{aligned} SCALE(t, \boldsymbol{y}^t) &\equiv OUTPUT(t, \boldsymbol{y}^t) / LABOUR(t, \boldsymbol{y}^t) \\ &= \left(\frac{D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^0)}{D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^1)} \right) \Big/ \left(\frac{D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^1)}{D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^0)} \right). \end{aligned}$$

Each choice of the reference technology might generate a different measure of the returns to scale effect going from period 0 to period 1. We calculate two measures by using two different reference technologies: the period 0 and 1 short-run production frontiers. Since these reference technology are equally reasonable, we use the geometric mean of these measures as a theoretical index of the returns to scale effect, *SCALE*, as follows;

$$(30)\ SCALE \equiv \sqrt{SCALE(0, \boldsymbol{y}^0) \cdot SCALE(1, \boldsymbol{y}^1)}\,.$$

The case of one output and one labour input offers us a graphical interpretation of *SHIFT*. In Figure 1, it reduces to the following formula.

$$(31)\ SCALE = \sqrt{\left(\frac{f^0(x_K^0, x_L^1)}{y^0} \Big/ \frac{x_L^1}{x_L^0} \right) \left(\frac{y^1}{f^1(x_K^1, x_L^0)} \Big/ \frac{x_L^1}{x_L^0} \right)}\,.$$

Given the period t short-run production frontier, the ratio of the LP associated with x_L^0 to the LP associated with x_L^1 represents the LP growth induced by the change in labour input. *SCALE* is the geometric mean of those ratios conditional on the period 0 and 1 short-run production frontiers.

As CCD (1982) apply the implicit function theorem to the input requirement function with the output distance function such as $F^t(\boldsymbol{y}^t/\delta, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t) = x_{K,1}^t$ where $\delta = D_O^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t)$, we apply the implicit function theorem to the input requirement function with the labour input distance function such as $F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t/\delta) = x_{K,1}^t$ where $\delta = D_L^t(\boldsymbol{y}^t$,

$\boldsymbol{x}_K^t, \boldsymbol{x}_L^t$). In this case, $D_L^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$ is differentiable around the point ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^t$, $\boldsymbol{x}_L^t$).[11] Its derivatives are represented by the derivatives of $F^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L)$. We have the following equations for t = 0 and 1:

$$(32)\ \nabla_y D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \frac{1}{\boldsymbol{x}_L^t \cdot \nabla_{x_L} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)} \nabla_y F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t) .$$

$$(33)\ \nabla_{x_K} D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \frac{1}{\boldsymbol{x}_L^t \cdot \nabla_{x_L} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)} \begin{bmatrix} -1 \\ \nabla_{x_{K,-1}} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t) \end{bmatrix} .$$

$$(34)\ \nabla_{x_L} D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \frac{1}{\boldsymbol{x}_L^t \cdot \nabla_{x_L} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t)} \nabla_{x_L} F^t(\boldsymbol{y}^t, \boldsymbol{x}_{K,-1}^t, \boldsymbol{x}_L^t) .$$

We assume that $(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) >> 0_{N+P+Q}$ is a solution to the period t profit maximization problem (10) for t = 0 and 1. By substituting (11)-(13) obtained from the profit maximization into (32)-(34), we obtain the following equations for t = 0 and 1:

$$(35)\ \nabla_y D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = -\boldsymbol{p}^t / \boldsymbol{w}^t \cdot \boldsymbol{x}_L^t .$$

$$(36)\ \nabla_{x_K} D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = [1/\boldsymbol{w}^t \cdot \boldsymbol{x}_L^t] \begin{bmatrix} r_1 \\ \boldsymbol{r}_{-1} \end{bmatrix} .$$

$$(37)\ \nabla_{x_L} D_L^t(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) = \boldsymbol{w}^t / \boldsymbol{w}^t \cdot \boldsymbol{x}_L^t .$$

The above equations (35)-(37) allow us to compute the derivatives of the labour input distance function, without knowing the labour input distance function itself. The information of the derivatives is useful for calculating the values of *SCALE*, which is defined by the distance functions. However, one disadvantage is that the derivatives of the period t distance function need to be evaluated at the period t actual production plan ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^t$, $\boldsymbol{x}_L^t$) in equations (35)-(37) for t = 0 and 1. The distance functions evaluated at the hypothetical production plan such as ($\boldsymbol{y}^1, \boldsymbol{x}_K^0, \boldsymbol{x}_L^1$) and ($\boldsymbol{y}^0, \boldsymbol{x}_K^1, \boldsymbol{x}_L^0$) also constitute *SCALE*. Hence, the above equations are not enough for obtaining *SCALE*. In addition to the firm's profit maximization, we further assume the translog functional form for the period t labour input distance function for t = 0 and 1. It is defined as follows;

$$(38)\quad \begin{aligned} \ln D_L^t(\boldsymbol{y}, \boldsymbol{x}_K, \boldsymbol{x}_L) \equiv\ & \tau_0{}^t + \sum_{m=1}^M \tau_m{}^t \ln y_m + (1/2)\sum_{i=1}^M \sum_{j=1}^M \tau_{i,j} \ln y_i \ln y_j \\ & + \sum_{p=1}^P \upsilon_p{}^t \ln x_{K,p} + (1/2)\sum_{i=1}^P \sum_{j=1}^P \upsilon_{i,j} \ln x_{K,i} \ln x_{K,j} \\ & + \sum_{q=1}^Q \omega_q{}^t \ln x_{L,q} + (1/2)\sum_{i=1}^Q \sum_{j=1}^Q \omega_{i,j} \ln x_{L,i} \ln x_{L,j} \\ & + \sum_{m=1}^M \sum_{p=1}^P \xi_{m,p} \ln y_m \ln x_{K,p} + \sum_{m=1}^M \sum_{q=1}^Q \psi_{m,q} \ln y_m \ln x_{L,q} \\ & + \sum_{p=1}^P \sum_{q=1}^Q \zeta_{p,q} \ln x_{K,p} \ln x_{L,q} \end{aligned}$$

where the parameters satisfy the following restrictions:

(39) $\tau_{i,j} = \tau_{j,i}$ for all i and j such as $1 \le i < j \le M$;

[11] We also assume the following three conditions are satisfied for t = 0 and 1: 1) F^t is differentiable at the point ($\boldsymbol{y}^t$, $\boldsymbol{x}_K^t$, $\boldsymbol{x}_L^t$),: 2) $\boldsymbol{y}^t >> 0_M$ and 3) $\boldsymbol{y}^t \cdot \nabla_y F(\boldsymbol{y}^t, \boldsymbol{x}_K^t, \boldsymbol{x}_L^t) > 0$.

(40) $\upsilon_{i,j} = \upsilon_{j,i}$ for all i and j such as $1 \le i < j \le P$;

(41) $\omega_{i,j} = \omega_{j,i}$ for i and j such as $1 \le i < j \le Q$;

(42) $\sum_{q=1}^{Q} \omega_q^t = 1$;

(43) $\sum_{i=1}^{Q} \omega_{i,q} = 0$ for q = 1,..., Q;

(44) $\sum_{q=1}^{Q} \psi_{m,q} = 0$ for m = 1,..., M ;

(45) $\sum_{q=1}^{Q} \zeta_{p,q} = 0$ for p = 1,..., P.

The equation (38) is the same functional form defined by (17) that we assume for the output distance function in the discussion of *SHIFT*. However, the restrictions on parameters on the labour input distance function differ from those on the output distance function. We replace the restrictions (21)-(24) by (42)-(45). While the restrictions (21)-(24) guarantee the linear homogeneity in outputs $\boldsymbol{y}$ for the output distance function, the restrictions (42)-(45) guarantees the linear homogeneity in labour inputs $\boldsymbol{x}_L$ for the labour input distance function. The translog functional form characterized by (38)-(45) is a flexible functional form so that it can approximate an arbitrary labour input distance function to the second order at an arbitrary point. Thus, assuming this functional form does not harm any generality of the labour input distance function. Note that the coefficients for the linear terms and the constant term are allowed to vary across periods. Thus, technical progress under the translog distance function is by no means limited to Hicks neutral and a vast variety of technical progress is possible. Under the assumption of the profit maximizing behaviour and the translog functional form, a theoretical index of the returns to scale, *SCALE* is computable from price and quantity observations.

Proposition 2: Assume that the output distance functions D_O^0 and D_O^1 have the translog functional form defined by (17)-(24), the labour input distance functions D_L^0 and D_L^1 have the translog functional form defined by (38)-(45), and a firm follows competitive profit maximizing behaviour in period t = 0 and 1. Then, the returns to scale effect, *SCALE*, can be computed from observable prices and quantities as follows:

$$(46)\quad \ln SCALE = \sum_{q=1}^{Q} s_{L,q} \ln\left(\frac{x_{L.q}^1}{x_{L,q}^0}\right) - \sum_{q=1}^{Q} \bar{s}_{L,q} \ln\left(\frac{x_{L.q}^1}{x_{L,q}^0}\right)$$

where $s_{L,q}$ is the average value-added shares of labour input q and $\bar{s}_{L,q}$ is the average labour-compensation share of labour input q between periods 0 and 1 such that;

$$s_{L,q} = \frac{1}{2}\left(\frac{w_q^0 x_{L,q}^0}{\boldsymbol{p}^0 \cdot \boldsymbol{y}^0} + \frac{w_q^1 x_{L,q}^1}{\boldsymbol{p}^1 \cdot \boldsymbol{y}^1}\right) \text{ and } \bar{s}_{L,q} = \frac{1}{2}\left(\frac{w_q^0 x_{L,q}^0}{\boldsymbol{w}^0 \cdot \boldsymbol{x}_L^0} + \frac{w_q^1 x_{L,q}^1}{\boldsymbol{w}^1 \cdot \boldsymbol{x}_L^1}\right).$$

The index number formula in the right-hand side of (46) can be interpreted as the ratio of the quantity indexes of labour inputs. Both terms in (46) are the weighted geometric average of the growth rates for labour inputs. The first term uses the value added share and the second term uses the labour compensation share. Thus, if capital share, which is the nominal share of capital inputs to the value added, is large, the difference between two terms (46) also becomes large, making the value of *SCALE* larger. Conversely, in the case when no capital inputs are utilized for production, the returns to scale effect disappears.

Starting from understanding that the two contribution factors exists for the LP growth, we reached the index number formula for these factors, independently. Our result, however, does not deny the possibility of the existence of other explanatory factors to the LP growth. Fortunately, two factors of *SHIFT* and *SCALE* can fully explain the LP growth. The product of *SHIFT* and *SCALE* coincides with the index of the LP growth, as follows:

Corollary 1: Assume that the output distance functions $D_O{}^0$ and $D_O{}^1$ have the translog functional form defined by (17)-(24), the labour input distance functions $D_L{}^0$ and $D_L{}^1$ have the translog functional form defined by (38)-(45), and a firm follows competitive profit maximizing behaviour in period t = 0 and 1. Then, the product of *SHIFT* and *SCALE* can be computed from observed prices and quantities as follows:

$$\text{(47)}\ \ln SHIFT + \ln SCALE = \sum_{m=1}^{M} s_m \ln\left(\frac{y_m^1}{y_m^0}\right) - \sum_{q=1}^{Q} \bar{s}_{L,q} \ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right),$$

where s_m is the average value-added shares of output m and $\bar{s}_{L,q}$ is the average labour-compensation share of labour input q between periods 0 and 1 such that;

$$s_m = \frac{1}{2}\left(\frac{p_m^0 y_m^0}{\boldsymbol{p}^0 \cdot \boldsymbol{y}^0} + \frac{p_m^1 y_m^1}{\boldsymbol{p}^1 \cdot \boldsymbol{y}^1}\right) \text{ and } \bar{s}_{L,q} = \frac{1}{2}\left(\frac{w_q^0 x_{L,q}^0}{\boldsymbol{w}^0 \cdot \boldsymbol{x}_L^0} + \frac{w_q^1 x_{L,q}^1}{\boldsymbol{w}^1 \cdot \boldsymbol{x}_L^1}\right).$$

The right side of the equation (47) represents the logarithm of LP growth. The first term of the right hand side coincides with the Törnqvist quantity index of outputs and the second term is the Törnqvist quantity index of labour inputs. The equation (47) allows us to fully decompose LP growth into two components such as *SHIFT* and *SCALE* in the case of multiple inputs and outputs. It is a generalization of the simple explanation in the case of one input and one output that the LP grows is induced by the shift in the production frontier and the movement along the production frontier.

5. An Application to U.S. Industry Data

We apply the decomposition result to U.S. industry data. Dataset covering the period 1970-2005 is taken from the comprehensive industry dataset called the EU KLEMS Growth and Productivity Accounts.[12] Industry data drawn from the database consist of gross outputs and intermediate inputs at current and constant prices, and hours worked of employment by 31 industries. These industry data are organized according to the System of Industry Classification (SIC) adopted by the U.S. official statistics.[13]

[12] Data are downloaded from the EU KLEMS website (http://www.euklems.net/). The detailed explanation about this comprehensive international database is found in O'Mahony and Timmer (2009). The U.S. industry data of EU KLEMS is constructed by Dale Jorgenson and his research group. See Jorgenson, Ho and Stiroh (2008).

[13] For each industry, there exist one type of gross output and one type of intermediate input. Their deflator varies across industries. Labour input is hours worked by total employment. Total employment in each industry includes employees and the self-employed engaged in the production of the industry.

They are categorized either as the goods producing sector (goods sector, thereafter) or the services providing sector (services sector, thereafter).

[Place Table 1 appropriately here.]

Table 1 compares LP growth and its components across the whole economy, the goods sector and the services sector. For the entire sample period 1970-2005, the returns to scale effect has a negative impact on LP growth, reflecting the average growth rate of hours worked of 1.43 percent. While the average rate of the joint effect of technical progress and capital input growth is 2.04 percent, it is largely offset by the returns to scale effect of -0.52 percent. There is a hypothesis so called "Baumol's disease" stating that LP in the services sector is likely to be stagnated and lower than that of the goods sector, which has been widely advocated by Triplett and Bosworth (2004, 2006) and Bosworth and Triplett (2007). Such difference in LP growth between two sectors is also documented in our dataset. The average growth rate of the goods sector LP of 2.33 percent is almost twice as large as that of the goods sector LP of 1.27 percent. While the returns to scale effect is subtle and negligible in the goods sector, the significant negative returns to scale effect is found in the services sector. Removing the returns to scale effect largely raises the services sector LP growth. The more than half of the difference in LP growth between two sectors can be explained by the difference in the returns to scale effect. The joint effects of technical progress and capital input growth in both sectors are very close on average over the period 1970-2005: 2.33 percent for the goods sector and 2.01 percent for the services sector. It reflects that the increase in hours worked is taking place mostly in the services sector. While hours worked for the goods sector grows at the average growth rate of 0.05 percent only, hours worked for the services sector grows at the average growth rate of 1.97 percent.

It is convenient to divide the entire sample period 1970-2005 into two periods: the "productivity slowdown" period 1970-1995 and the "productivity resurgence" period 1995-2005.[14] Productivity slowdown in U.S. economy started since the early 1970s with the average growth rate of 1.2 percent during the period 1970-1995. Productivity growth surged after 1995, with the average growth rate of 2.31 percent during the period 1995-2005. As Triplett and Bosworth (2004, 2006) and Bosworth and Triplett (2007) pointed out, the services sector LP grows slowly especially during the productivity slowdown period, with the average growth rate of 1 percent during the period 1970-1995. At the same period, the goods sector grows with the average growth rate of 1.57 percent. However, once we control the returns to scale effect and consider the joint effect of technical progress and capital input growth only, the services sector with the average rate of 1.82 percent forereaches the goods sector with the average rate of 1.62 percent. Thus, even though LP growth is smaller in the services sector than in the goods sector, the productive capacity of labour, which is the output attainable from given labour inputs, increases more in the services sector than in the goods sector. It reflects that the large increase in hours worked in the services sector restraints LP from growing largely. While the hours worked in the goods sector grow slightly during the period 1970-1995, it even declines during the period 1995-2005 leading to the positive returns to scale effect. On the other hand, hours worked in the services sector increases throughout the sample period, leading to

[14] It is known that the stagnation of the U.S. economy started since 1973. Since the dataset is available since 1970 and the period 1970-1973 is too enough to identify the underlying trend, we deal with the period 1970-1995.

the negative returns to scale effect. Thus, the part of the gap in LP growth explained by the gap in the returns to scale effect between two sectors become even larger in the period 1995-2005.

[Place Table 2 appropriately here.]

The returns to scale effect, growth in hours worked and capital share by industry are shown in Table 2.[15] The patter found in the aggregate study based on the sector data in Table 1 is also documented in the detailed industries. Most of the industries in services sector show the negative returns to scale effect. It is especially significant from 1970-1995. On the other hand, many industries in the goods sector show modest returns to scale effect from 1970-1995 and the positive returns to scale effect from 1995-2005. *Textiles, textile products, leather and footwear* (SIC17-19) and *coke, refined petroleum products and nuclear fuel* (SIC 23) are exceptional industries in the goods sector. Reflecting largely decreasing hours worked, the positive and large returns to scale effects are observed in both industries within the goods sector in the period 1970-1995.

The returns to scale effect depends on capital share as well as increases in hours worked. The detailed industry study reveals cases when the returns to scale effect gets larger for different reasons. On average over the whole sample period, the most significant returns to scale effect is found in *Real estate activities* (SIC 70), averaging -2.35 percent. *Renting of equipment and other business activities* (SIC 71-74) shows the second largest returns to scale effect in magnitude. However, its average growth rate is -0.97 percent, which is much smaller than the value of the *real estate activities.* The reason why the returns to scale effect becomes large differs between two industries. *Renting of equipment and other business activities* has the highest growth of hours worked, averaging 4.68 percent, leading to the second large returns to scale effect. However, hours worked in *real estate activities* does not grow significantly with the average growth rate of 2.61 percent. For example, although hours worked in *health and social work* (SIC N) grows larger with the average growth rate of 3.31 percent, its rate of returns effect is -0.56 on average, which is smaller than even one quarter of *real estate activities*. The reason of the largest returns to scale effect is because the *real estate activities* has the large capital share 90.52 percent. The impact of the growth in hours worked on the returns to scale effect is largely amplified by the large capital share.

6. Conclusion

In this paper, we distinguished two effects on LP growth by examining the short-run production frontier. The joint effect of technical progress and capital input growth appears as the growth in LP that is induced by the shift in the short-run production frontier. The returns to scale effect appears as the LP growth induced by the movement along the short-run production frontier. The LP growth calculated by Törnqvist quantity indexes is fully decomposed into the product of these two effects. We applied this decomposition result to U.S. industry data for the period 1970-2005. It is shown that a large part of the difference in LP growth between the goods sector and the services sector can be explained by the returns to scale effect.

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[15] Since *private households with employed persons* is not endowed with capital inputs and all the production has been conducted by using labour inputs only, there is no returns to scale effect for this industry. Thus, this industry is excluded from Table 2.

In this paper, we assumed the firm's profit maximizing behaviour and ruled out inefficient production processes. If we relax the firm's profit maximizing behaviour, another factor—*technical efficiency change*—appears in the decomposition of LP growth. Even with no change in the short-run production frontier and no change in labour input, a firm can approach closer to the short-run production frontier by improving technical efficiency. For example, a firm improves technical efficiency by increasing output up to the maximum level attainable from given labour inputs under current technology. For the implementation of the decomposition of the LP growth without assuming a firm's profit maximizing behaviour, we can estimate the distance function by using econometric techniques or Data Envelopment Analysis's linear programming technique. However, we leave this exercise to the future research.

Appendix A

Proof of Proposition 1

$$\ln SHIFT = \left(\frac{1}{2}\right)\ln\left(\frac{D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{D_O^1(\boldsymbol{y}^0,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}\right)+\left(\frac{1}{2}\right)\ln\left(\frac{D_O^0(\boldsymbol{y}^1,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}\right)$$

$$=\left(\frac{1}{2}\right)\ln\left(\frac{D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{D_O^1(\boldsymbol{y}^0,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}\right)+\left(\frac{1}{2}\right)\ln\left(\frac{D_O^0(\boldsymbol{y}^1,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}\right)$$

Since the firm's profit maximization is assumed, the period t production plan is on the period t production frontier for t = 0 and 1.

$$=\left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln y_m}+\frac{\partial\ln D_O^1(\boldsymbol{y}^0,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln y_m}+\frac{\partial\ln D_O^0(\boldsymbol{y}^1,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln y_m}+\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln y_m}\right)\ln\left(\frac{y_m^1}{y_m^0}\right)$$

$$\left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}+\frac{\partial\ln D_O^1(\boldsymbol{y}^0,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln x_{L,q}}+\frac{\partial\ln D_O^0(\boldsymbol{y}^1,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln x_{L,q}}+\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$

using the translog identity in CCD (1982)

$$=\left(\frac{1}{2}\right)\sum_{m=1}^{M}\left(\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln y_m}+\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln y_m}\right)\ln\left(\frac{y_m^1}{y_m^0}\right)$$
$$+\left(\frac{1}{2}\right)\sum_{q=1}^{Q}\left(\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}+\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$
$$+\left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(-\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln y_m}+\frac{\partial\ln D_O^1(\boldsymbol{y}^0,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln y_m}+\frac{\partial\ln D_O^0(\boldsymbol{y}^1,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln y_m}-\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln y_m}\right)\ln\left(\frac{y_m^1}{y_m^0}\right)$$
$$+\left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(-\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}+\frac{\partial\ln D_O^1(\boldsymbol{y}^0,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln x_{L,q}}+\frac{\partial\ln D_O^0(\boldsymbol{y}^1,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln x_{L,q}}-\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$
$$=\left(\frac{1}{2}\right)\sum_{m=1}^{M}\left(\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln y_m}+\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln y_m}\right)\ln\left(\frac{y_m^1}{y_m^0}\right)$$
$$+\left(\frac{1}{2}\right)\sum_{q=1}^{Q}\left(\frac{\partial\ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}+\frac{\partial\ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$

from the equation (17).

$$=\left(\frac{1}{2}\right)\sum_{m=1}^{M}\left(\frac{p_m^0 y_m^0}{\boldsymbol{p}^0\cdot\boldsymbol{y}^0}+\frac{p_m^1 y_m^1}{\boldsymbol{p}^1\cdot\boldsymbol{y}^1}\right)\ln\left(\frac{y_m^1}{y_m^0}\right)-\left(\frac{1}{2}\right)\sum_{q=1}^{Q}\left(\frac{w^0 x_{L,q}^0}{\boldsymbol{p}^0\cdot\boldsymbol{y}^0}+\frac{w^1 x_{L,q}^1}{\boldsymbol{p}^1\cdot\boldsymbol{y}^1}\right)\ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$

substituting equations (14)-(16).

Proof of Proposition 2

$$\ln SCALE = \left(\frac{1}{2}\right)\ln\left(\left(\frac{D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}\right)\Big/\left(\frac{D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}\right)\right) + \left(\frac{1}{2}\right)\ln\left(\left(\frac{D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}\right)\Big/\left(\frac{D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}\right)\right)$$

$$= \left(\frac{1}{2}\right)\ln\left(\left(\frac{D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}\right)\left(\frac{D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}\right)\right) - \left(\frac{1}{2}\right)\ln\left(\left(\frac{D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}\right)\left(\frac{D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}\right)\right)$$

Since the firm's profit maximization is assumed, the period t production plan is on the period t production frontier for t = 0 and 1.

$$= \left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(\frac{\partial \ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln x_{L,q}} + \frac{\partial \ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}\right)\ln\left(\frac{y_m^1}{y_m^0}\right)$$

$$\left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(\frac{\partial \ln D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln x_{L,q}} + \frac{\partial \ln D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}} + \frac{\partial \ln D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$

using the translog identity in CCD (1982)

$$= \left(\frac{1}{2}\right)\sum_{m=1}^{M}\left(\frac{\partial \ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^0}{x_{L,q}^1}\right)$$

$$-\left(\frac{1}{2}\right)\sum_{q=1}^{Q}\left(\frac{\partial \ln D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^1}{x_{L,q}^0}\right)$$

$$+\left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(-\frac{\partial \ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_O^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln x_{L,q}} + \frac{\partial \ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln x_{L,q}} - \frac{\partial \ln D_O^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^0}{x_{L,q}^1}\right)$$

$$-\left(\frac{1}{4}\right)\sum_{m=1}^{M}\left(-\frac{\partial \ln D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^1)}{\ln x_{L,q}} + \frac{\partial \ln D_L^0(\boldsymbol{y}^0,\boldsymbol{x}_K^0,\boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^1)}{\ln x_{L,q}} - \frac{\partial \ln D_L^1(\boldsymbol{y}^1,\boldsymbol{x}_K^1,\boldsymbol{x}_L^0)}{\ln x_{L,q}}\right)\ln\left(\frac{x_{L,q}^0}{x_{L,q}^1}\right)$$

$$= \left(\frac{1}{2}\right) \sum_{m=1}^{M} \left(\frac{\partial \ln D_O^0(\boldsymbol{y}^0, \boldsymbol{x}_K^0, \boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_O^1(\boldsymbol{y}^1, \boldsymbol{x}_K^1, \boldsymbol{x}_L^1)}{\ln x_{L,q}} \right) \ln \left(\frac{x_{L,q}^0}{x_{L,q}^1} \right)$$

$$- \left(\frac{1}{2}\right) \sum_{q=1}^{Q} \left(\frac{\partial \ln D_L^0(\boldsymbol{y}^0, \boldsymbol{x}_K^0, \boldsymbol{x}_L^0)}{\ln x_{L,q}} + \frac{\partial \ln D_L^1(\boldsymbol{y}^1, \boldsymbol{x}_K^1, \boldsymbol{x}_L^1)}{\ln x_{L,q}} \right) \ln \left(\frac{x_{L,q}^1}{x_{L,q}^0} \right)$$

from the equation (17) and (38).

$$= \left(\frac{1}{2}\right) \sum_{q=1}^{Q} \left(\frac{w^0 x_{L,q}^0}{\boldsymbol{p}^0 \cdot \boldsymbol{y}^0} + \frac{w^1 x_{L,q}^1}{\boldsymbol{p}^1 \cdot \boldsymbol{y}^1} \right) \ln \left(\frac{x_{L,q}^1}{x_{L,q}^0} \right) - \left(\frac{1}{2}\right) \sum_{q=1}^{Q} \left(\frac{w^0 x_{L,q}^0}{\boldsymbol{w}^0 \cdot \boldsymbol{x}_L^0} + \frac{w^1 x_{L,q}^1}{\boldsymbol{w}^1 \cdot \boldsymbol{x}_L^1} \right) \ln \left(\frac{x_{L,q}^1}{x_{L,q}^0} \right)$$

substituting equations (14)-(16) and equations (35)-(37).

References

Balk, B.M. (2000), "Divisia Price and Quantity Indexes: 75 Years After", Department of Statistical Methods, Statistics Netherlands

Bosworth, B.P. and J.E. Triplett (2007), "The Early 21st Century U.S. Productivity Expansion is Still in Services", *International Productivity Monitor* 14, 3-19.

Caves, D.W., L. Christensen and W.E. Diewert (1982), "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity", *Econometrica* 50, 1393-1414.

Chambers, R.G. (1988), *Applied Production Analysis: A Dual Approach*, New York: Cambridge University Press.

Diewert, W.E. (1974), "Applications of Duality Theory,", in M.D. Intriligator and D.A. Kendrick (ed.), *Frontiers of Quantitative Economics,* Vol. II, Amsterdam: North-Holland, pp. 106-171.

Diewert, W.E. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 114-145.

Diewert, W.E. (1978), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900.

Diewert, W.E. and C.J. Morrison (1986), "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade", *Economic Journal* 96, 659-679.

Diewert, W.E. and A.O. Nakamura (2007), "The Measurement of Productivity for Nations", in J.J. Heckman and E.E. Leamer (ed.), *Handbook of Econometrics,* Vol. 6, Chapter 66, Amsterdam: Elsevier, pp. 4501-4586.

Fare, R., S. Grosskopf, M. Norris and Z. Zhang (1994), "Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries", *American Economic Review* 84, 66-83.

Griliches, Z. (1987), "Productivity: Measurement Problems", pp. 1010-1013 in J. Eatwell, M. Milgate and P. Newman (ed.), *The New Palgrave: A Dictionary of Economics.* New York: McMillan.

Hall, R.E., and C.I. Jones (1999). "Why do some countries produce so much more output per worker than others?", *Quarterly Journal of Economics* 114, 83-116.

Hotelling, H. (1932), "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions", *Journal of Political Economy* 40, 577-616.

Hulten, C.R. (1978), "Growth Accounting with Intermediate Inputs", *Review of Economic Studies* 45, 511-518.

Jones, C.I. (2002), "Sources of U.S. Economic Growth in a World of Idea", *American Economic Review* 92, 220-239

Jorgenson, D.W. and K.J. Stiroh (2000), "Raising the Speed Limit: U.S. Economic Growth in the Information Age", *Brookings Papers on Economic Activity* 2, 125-211.

Jorgenson, D.W., M.S. Ho and K.J. Stiroh (2008), "A Retrospective Look at the U.S. Productivity Growth Resurgence", *Journal of Economic Perspective* 22, 3-24.

Kohli, U. (2004b), “Real GDP, Real Domestic Income and Terms of Trade Changes”, *Journal of International Economics* 62, 83-106.

Nishimizu, M., and J.M. Page (1982), “Total Factor Productivity Growth, Technical Progress and Technical Efficiency Change: Dimensions of Productivity Change in Yugoslavia, 1965-78”, *Economic Journal* 92, 920-936.

O’Mahony, M. and M.P. Timmer (2009), “Output, Input and Productivity Measures at the Industry Level: the EU KLEMS Database”, *Economic Journal* 119, F374-F403.

Samuelson, P.A. (1953), “Prices of Factors and Goods in General Equilibrium”, *Review of Economic Studies* 21, 1-20.

Triplett, J.E. and B.P. Bosworth (2004), *Services Productivity in the United States: New Sources of Economic Growth*, Washington, D.C.: Brookings Institution Press.

Triplett, J.E. and B.P. Bosworth (2006), “‘Baumol’s Disease’ Has Been Cured: IT and Multi-factor Productivity in U.S. Services Industries”, *The New Economy and Beyond: Past, Present, and Future*. Dennis W. Jansen, (eds.), Cheltenham: Edgar Elgar.

Table 1: Labour Productivity Growth and Its Components (%), 1970-1995, 1995-2005

	1970-2005	1970-1995	1995-2005
Whole Economy			
Labour Productivity Growth	1.51	1.20	2.31
Technical Progress and Capital Input Growth	2.04	1.78	2.67
Returns to Scale Effect	-0.52	-0.59	-0.36
Hours Worked	1.43	1.62	0.96
Capital Share	36.43	36.01	37.48
Goods Sector			
Labour Productivity Growth	2.33	1.52	4.34
Technical Progress and Capital Input Growth	2.33	1.62	4.13
Returns to Scale Effect	-0.01	-0.10	0.21
Hours Worked	0.05	0.30	-0.57
Capital Share	34.59	33.40	37.56
Services Sector			
Labour Productivity Growth	1.27	1.00	1.93
Technical Progress and Capital Input Growth	2.01	1.82	2.48
Returns to Scale Effect	-0.74	-0.82	-0.54
Hours Worked	1.97	2.19	1.44
Capital Share	37.42	37.40	37.47

Table 2: Industry Labour Productivity and Returns to Scale Effect (%), 1970-2005, 1970-1995, 1995-2005

	Labour Productivity			Returns to Scale Effect			Hours Worked			Capital Share		
	1970-2005	1970-1995	1995-2005	1970-2005	1970-1995	1995-2005	1970-2005	1970-1995	1995-2005	1970-2005	1970-1995	1995-2005
Goods Sector												
Agriculture, hunting, forestry and fishing	3.00	3.18	2.54	-0.38	-0.37	-0.39	0.75	0.68	0.90	53.33	55.15	48.77
Mining and quarrying	-0.09	0.32	-1.12	-0.06	-0.03	-0.15	0.02	0.05	-0.05	67.54	66.48	70.20
Food products, beverages and tobacco	2.78	2.86	2.59	0.20	0.14	0.37	-0.62	-0.54	-0.81	39.25	36.64	45.77
Textiles, textile products, leather and footwear	4.72	3.82	6.97	0.85	0.32	2.18	-3.58	-1.59	-8.55	22.31	21.01	25.57
Other manufacturing	2.59	1.56	5.18	-0.01	-0.14	0.33	-0.02	0.48	-1.25	25.22	24.22	27.71
Wood and products of wood and cork	0.96	0.99	0.90	-0.17	-0.23	-0.01	0.38	0.58	-0.10	31.74	32.53	29.77
Pulp, paper, paper products, printing and publishing	1.60	0.58	4.16	0.01	-0.32	0.82	0.02	1.09	-2.64	29.17	28.40	31.11
Coke, refined petroleum products and nuclear fuel	3.68	8.77	-9.04	0.94	0.64	1.69	-1.53	-1.11	-2.60	61.74	59.62	67.05
Chemicals and chemical products	2.13	1.62	3.42	0.20	-0.07	0.86	-0.34	0.12	-1.48	48.42	44.95	57.07
Rubber and plastics products	4.08	3.81	4.77	-0.14	-0.40	0.50	0.66	1.61	-1.71	23.37	21.26	28.65
Other non-metallic mineral products	2.24	1.62	3.78	0.07	0.05	0.13	-0.38	-0.39	-0.35	26.40	23.49	33.68
Basic metals and fabricated metal products	2.02	1.23	4.02	0.32	0.23	0.55	-1.18	-0.96	-1.73	28.86	26.21	35.49
Machinery, nec	2.10	0.99	4.87	0.19	0.01	0.64	-1.06	-0.18	-3.25	23.93	25.09	21.04
Electrical and optical equipment	11.05	9.18	15.71	0.10	-0.13	0.69	-0.22	0.60	-2.28	25.37	22.87	31.60
Transport equipment	2.62	1.27	5.99	0.03	-0.09	0.34	-0.26	0.14	-1.26	22.30	20.65	26.41
Electricity, gas and water supply	1.12	0.10	3.67	0.04	-0.42	1.20	-0.02	0.66	-1.70	67.82	66.80	70.37
Construction	-1.65	-1.55	-1.93	-0.31	-0.22	-0.53	2.10	1.61	3.32	13.66	12.15	17.43
Services Sector												
Sale and repair of motor vehicles and motorcycles	3.66	2.83	5.73	-0.61	-0.61	-0.61	1.18	1.11	1.35	43.77	42.07	48.02
Wholesale trade and commission trade	3.84	4.27	2.78	-0.34	-0.44	-0.08	1.26	1.67	0.24	26.49	25.60	28.73
Retail trade, repair of household goods	1.98	0.84	4.84	-0.20	-0.23	-0.12	1.37	1.65	0.68	15.24	13.07	20.66
Transport and storage	1.70	1.57	2.03	-0.35	-0.39	-0.25	1.26	1.46	0.77	29.09	27.76	32.41
Post and telecommunications	5.11	4.81	5.87	-0.25	-0.29	-0.14	0.51	0.60	0.30	54.39	52.45	59.26
Financial intermediation	2.80	2.60	3.30	-0.78	-0.80	-0.73	2.03	2.26	1.45	41.76	37.96	51.25
Renting of equipment and other business activities	-0.82	-1.55	1.02	-0.97	-1.14	-0.57	4.68	5.37	2.97	20.19	20.68	18.96
Hotels and restaurants	-1.54	-2.63	1.18	-0.62	-0.75	-0.28	3.04	3.84	1.05	21.24	18.99	26.86
Other community, social and personal services	1.02	1.30	0.32	-0.38	-0.37	-0.42	2.51	2.44	2.68	15.32	15.13	15.78
Public admin and defence; compulsory social security	1.03	1.15	0.72	-0.08	-0.10	-0.03	0.19	0.22	0.11	33.26	33.71	32.16
Education	0.25	0.30	0.11	-0.53	-0.55	-0.47	2.21	2.22	2.19	23.99	25.24	20.87
Health and social work	-0.28	-1.03	1.61	-0.56	-0.67	-0.30	3.31	3.95	1.70	16.69	16.29	17.68
Real estate activities	0.90	0.73	1.33	-2.35	-2.55	-1.84	2.61	2.82	2.08	89.76	90.52	87.88

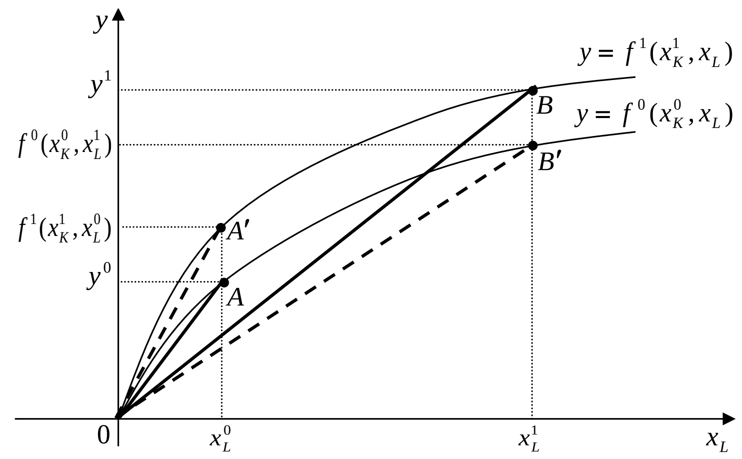

Figure 1: Labour Productivity Growth and Shift in the Short-run Production Frontier

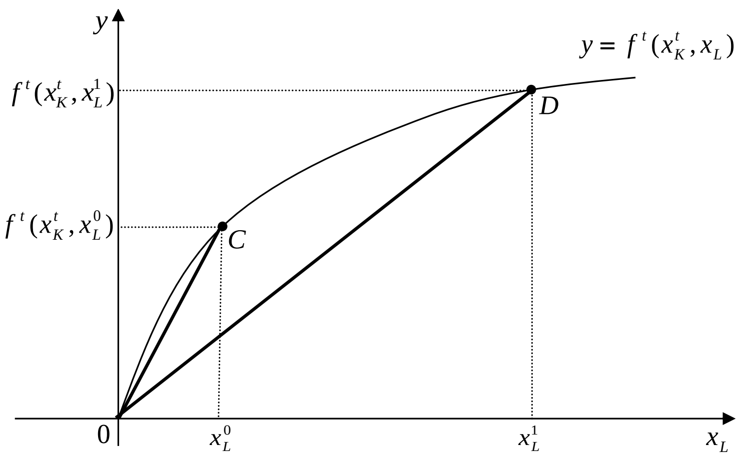

Figure 2: Returns to Scale Effect and Movement along the Short-run Production Frontier