Optimal Financial Contract and Financial Structure When Verifiability of Cash Flows Is Endogenous *

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Abstract

We analyze the optimal financial structure of a firm within a optimal contracting framework. Dispersing the financial structure makes the renegotiation of contracts more complicated on one hand and it makes a coordination of verification activity more difficult on the other hand. The optimal financial structure balances these two effects. We show that decentralized financial structure is optimal when a verification technology is efficient and projects are profitable in the short run, and main-banking structure is optimal otherwise. We also show that the optimum number of creditors is inmonotonous and incontinuous with the efficiency of verification technology and the profitability in the short run.

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1 Introduction

Why do entrepreneurs choose various financial structure? Some entrepreneurs finance the required fund from one or two investors and others finance it from very large number of investors. Moreover, even when the entrepreneurs finance it from the same number of investors, some finance it from them evenly and others finance a large portion of it from one or two of them. A possible reason of the variety of financial structure is that the financial structure affects the design of securities and the behavior of related parties. We expose an analysis on the design of securities and the choice of financial structure in this article.

The results show that, when the contracts satisfy a proportionality among creditors, it is optimal for entrepreneurs to offer a debt-like contract, which specify fixed payments, a liquidation rule (liquidation rights and dividends from liquidation), and enforced transfers contingent on the payments to each creditor.[1]

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[1]The construction of the debt-like contract is similar to Bolton and Scharfstein (1996). However, we explicitly consider the relationships between the entrepreneur and *each* creditor although they treat the creditors considerably collectively.

The financial structure affects renegotiation outcomes and decisions on verification activity following strategic defaults and changes the terms in the debt-like contracts as a result.[2] Dispersing the financial structure makes the renegotiation more complicated and reduces the entrepreneur's gain from it. On the other hand, the dispersion reduces each creditor's share of lending and discourages the creditors' incentive of a costly verification activity.[3]

These two effects changes terms of an optimal contract inversely. An increase in the degree of renegotiation-complexity discourages the entrepreneur's incentive of the misbehavior, which reduces the necessity of strict liquidation right and lowers the fixed payments. In contrast, the discouragement of verification activity crowds out the opportunity of verification and heighten the incentive of the misbehavior, which increases the necessity of strict liquidation right and heightens the fixed payments.

The optimal financial structure must balance the two effects. There are three candidates on optimal financial structure; active main-banking structure with maximum number (AMBSM), passive main-banking structure with maximum number (PMBSM), and most dispersed decentralized financial structure (MDDFS). AMBSM is a financial structure under which one creditor (main bank) holds a sufficient share of lending for undertaking the verification activity even when the entrepreneur makes some payments in the renegotiation, and as many as possible creditors divide the rest. In contrast, PMBSM is a financial structure under which one creditor (main bank) holds a sufficient share of lending for undertaking the verification activity only when the entrepreneur makes no payment and as many as possible creditors divide the rest. Last, MDDFS is a financial structure under which each creditor holds a minimum unit of lending and no creditor does not undertake the verification activity.

Which types of financial structure are the optimum is dependent of not only the profitability of project but also the efficiency of verification technology. When the expected cash flow in the distant future is insufficient for financing the project, MDDFS is never feasible. On the other hand, when it is sufficient for financing, AMBSM tends to be optimal as the efficiency of verification technology and the profitability in the short run are heightened, and MDDFS tends to be optimal as they are lowered. When the creditors other than the main-bank can be considered as bondholders, this result implies that (i) entrepreneurs who have a project producing sufficient return in the near future but low return in the distant future call for a main bank (ii) those who have a project with insufficient return in the near future but sufficient return in the distant future choose bond finance, and (iii) those who have a project with sufficient return both in the near and distant future have a relationship with a main bank although they can finance only from the bond market.

The optimum number of creditors is inmonotonous and incontinuous with the efficiency of verification technology and the profitability in the short run. For a lower level of them, MDDFS is optimal and consequently the optimum number is maximum feasible number (con-

[2]When the creditors cannot enforce any repayments, entrepreneurs have an incentive to default the repayments and renegotiate with the creditors in good states. This type of default is called strategic default.

[3]The verification activity in our model is not equivalent to that in costly-state-verification (CSV) models (Townsend (1979), Gale and Hellwig (1985), Williamson (1986), and so on). In CSV models, it is a series of activities for creditors to observe the cash flows and to enforce certain amount of transfer. In contrast, the verification activity in our model is a series of actions for creditors to make the information public and the enforcement by the court feasible.

stant). In contrast, for a middle-low and middle-high level of them, PMBSM and AMBSM are optimal,respectively, and the optimum number increases with them for each level, respectively. It follows that the optimum number jumps down when the a type of financial structure converts to another type of financial structure. Moreover, when the efficiency of verification technology and the profitability in the short run are sufficiently high, there exist several optimal numbers.

There are a large number of studies investigating the optimal debt structure. Several studies analyze the choice between bank loan and bond (Diamond (1991), Rajan (1992), Chemmanur and Fulghieri (1994)). Furthermore, several studies investigate the problem of designing the debt, for example, the terms of repayment, maturity, seniority, and control right (Berglöf and Von Thadden (1994), Rajan and Winton (1995), Welch (1997), Repullo and Suarez (1998), Park (2000)). Our study is distinguished from these studies since we focus on an analysis in the case that each creditor have a similarly designed security although their studies analyze the choices among various securities and/or permit an asymmetry among securities. It is true that the asymmetry among securities is a significant aspect in financial structure problem. However, several creditors share a similarly designed debt in reality, especially, in the case of short-term and unsecured financing. We access why one similarly designed debt is shared among several creditors and how it is shared in this study.

Several possible answers for the two questions are proposed in studies on optimal number of creditors (Bolton and Scharfstein (1996), Detragiache, Garella, and Guiso (2000), Diamond (2004), Bris and Welch (2005)). Bolton and Scharfstein (1996) is the base of our study. They show that an increase in the number of creditors heightens the complexity of bargaining among possible related parties and changes the expected payoff of an entrepreneur in the two manners; (i) it makes renegotiation more complicated, lowers an incentive of misbehavior, and consequently heightens the expected payoff, and (ii) it lowers the expected liquidation value due to more complicated bargaining between creditors and a possible buyer, heightens the necessity of liquidation, and consequently lowers the expected payoff.[4] However, their study, similar to the others, does not take the possibility of sharing the debt asymmetricly into account.[5] One of the most significant contribution of our study is that we investigate not only the number of creditors but also the shares of lending explicitly.

More theoretically, we reconsider the verifiability of cash flows. In an incomplete contracting framework, the cash flows are supposed to be observable but unverifiable for creditors due to their unobservability for outsiders (Hart and Moore (1998) and Bolton and Scharfstein (1996), for example). However, whether the cash flows are verifiable or not could be dependent of creditors' actions. We introduce the verification activity (a series of costly actions) and endogenously analyze the effect of financial structure on the decisions of it.[6]

This consideration on verifiability, as a result, provides some predictions on the relationship between strength of creditor right (efficiency of legal enforcement) and optimal financial

[4]we keep the former effect in this study and takes another aspect into our model instead of the latter.

[5]Bolton and Von Thadden (1998) analyze the optimal ownership sturucture endogenously. In their model, optimal ownership structure is a structure under which a blockholder owns a sufficiently large share for maintaining her incentive of monitoring and as many as possible investors holds the rest for heightening the liquidity in the market. Our results are parallel to their results although deriving forces and objective securities are different.

[6]The verification activity in our study is not equivalent to that in CSV models.

structure. Several studies have already investigated the relationship. Some of these studies show that strong creditor rights could distort the entrepreneurs' decisions and consequently change the value of projects (Bebchuk and Fried (1997) and Berkovitch, Israel, and Zender (1997), for example). Others show that strong creditor rights could reduce the incentive of screening activity for the creditors (Manove, Padilla, and Pagano (2001), Bianco, Jappelli, and Pagano (2005), and Zazzaro (2005)). Although these studies produce more precise analysis on the effect of legal enforcement, they do not access the optimal financial structure.

In contrast to theoretical literature, several studies investige the shares of financing precisely, not only the number of creditors.[7] Most related study is Esty and Megginson (2003).[8] They show that debt concentration is positively related to the strength of creditor right and the reliability of legal enforcement controlling the loan size and the other risks.[9]

Our results are partially inconsitent with the empirical results in the studies above. A possible reason for this inconsistency is that these studies suppose a monotonous relationship between concentrationa and the determinant in the background. Further empirical studies based on more precise and concrete theoretical prediction would be required.

The rest of this study is organized as follows. In Section 2, we describe the technologies and time structure in the model. We solve the model by a backward induction in Section 3 and Section 4. In Section 3, given a financial structure, we first analyze the behaviors of agents given a contract and next solve the problem on designing an optimal contract. Next, we analyze the optimal financial structures in Section 4. Finally, we expose our concluding remarks in Section 5.

2 Basic Model

2.1 Project

A risk-neutral entrepreneur without any consumption/investment good (capital) has a production technology (project) requiring $K \geq 2$ units of capital.[10] On the other hand, there are $N \geq K$ investors having sufficient amount of capital but no project. They are also risk-neutral.

The project lives for two periods. At date 1, it produces a random cash flow (short-term cash flow) y_1; $x > 0$ with probability θ or zero with probability $1 - \theta$. It also produces a random cash flow (long-term cash flow) y_2 at date 2, which is independent of the short-term cash flow, if it is continued. Let y be the expected value of the long-term cash flow. The distributions of both y_1 and y_2 are publicly known. However, the realized cash flows are

[7] Empirical studies on the number of creditors are, for example, Petersen and Rajan (1995), Ongena and Smith (2000), Detragiache, Garella, and Guiso (2000), and Farinha and Santos (2002).

[8] Sufi (2007) is another study on shares of lending in recent years. The theoretical background of his study is the shares' effect on the monitoring. He show the positive relationship between the concentration and the requirement of intense monitoring.

[9] Measurement (and the effects) of the strength of creditor right and efficiency of legal enforcement and is studied, for example, by La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1998) and Berkowitz, Pistor, and Richard (2003).

[10] We assume K is a natural number. This assumption is just for explanatory simplicity.

observed only by the entrepreneur and creditors who are financing the project. The cash flows of this project satisfy the following.

Assumption 1 (production technology)**.** (i) The short-term cash flow in the good state is not less than the expected value of the long-term cash flow, $x \geq y$. (ii) The short-term cash flow is sufficient for financing the project, $\theta x \geq K$.

This project can be liquidated at date 1. Each creditor can put the whole project on sale by herself at the end of date 1.[11] Let $\tilde{a}_i^L \in \{0, 1\}$ be creditor i's action concerning the liquidation; '1' represents 'liquidation' and the other does 'continuation'. This action is publicly observed. When the project is liquidated, it produces a liquidation value $L \in (0, y)$.[12] Define $\Delta \equiv y - L$. Δ represents the amount of liquidation loss. When the project is managed by a manager other than the original, it generates certain value less than y. We can consider the liquidation value as the maximum value produced by alternative uses. We suppose that the value is independent of the financial structure, especially, the number of creditors. In Bolton and Scharfstein (1996), the liquidation value depends on the number of creditors.[13] This assumption is imposed in order to focus the other effects of the financial structure analyzed below.

2.2 Verification

Since the cash flows are not observable for outsiders, especially for the court, the creditors cannot enforce the entrepreneur to make any payments if they do not make the court know the realized cash flows. In reality, it is possible that, although creditors notice their borrower's diversion, the court does not recognize it if the creditors do not undertake a series of actions such as seizing the objectives diverted to, collecting several pieces of evidence indicating the diversion, and reporting the evidence to the court, which ordinarily require certain amount of cost. We call the series of costly actions *verification activity*. Literature with the incomplete contracting model (Hart and Moore (1998) and Bolton and Scharfstein (1996), for example) assumes that cash flows are not observable for outsiders and is *unverifiable*. However, unobservability is not always equivalent to unverifiability. In our model, the verifiability is an endogenous outcome.

Note the verification activity in our model is not equivalent to that in costly-state-verification (CSV) models:[14] In CSV models, cash flows are private information for the entrepreneur. Therefore, the verification activity is a series of actions for creditors to observe the cash flows and to enforce certain amount of transfer. In contrast, the verification activity in our model

[11] We suppose that the capital is indivisible after the project has set up.

[12] The liquidation value may be a stochastic variable. When it is the case, we consider L as an expected liquidation value. Whether the liquidation value is stochastic or not does not affect the outcomes of our analysis when it is independent of the short-term cash flow.

[13] This is because each creditor is secured by one asset which is complementary to the other asset when the entrepreneur borrows from two creditors. In contrast, when the project consists of a set of assets which are indivisible from each other and every creditor takes the set of assets in the pledge and can liquidate it by her own choice, the number of creditors does not affect the liquidation value.

[14] CSV models originate in Townsend (1979), Gale and Hellwig (1985), and Williamson (1986).

is a series of actions for creditors to make the information public and the enforcement by the court feasible.

The verification technology is as follows: At the end of date 1, each creditor can undertake the verification activity at a cost of c. When more than one creditors do, the realized value of y_1 is verified (known by the court) with probability v. Let $\tilde{a}_i^V \in \{0, 1\}$ be creditor i's action concerning the verification activity; '1' represents 'undertaking' and the other does 'not undertaking'. Note that, under this technology, it is redundant for more than two creditors to choose '1' since the cost is duplicated but the probability is not changed. We suppose that the creditor i's action $\tilde{a}_i^V$ cannot be observed by the others and is unverifiable. This assumption may seem to be inexplainable since choosing '1' means going to court. However, it is probable for a creditor to go to court without the costly actions. Therefore, going to court does not reveal the creditor's action. Given the verification technology and the assumption on the action $\tilde{a}_i^V$, certain type of coordination problem takes place. We analyze the coordination problem later in the next section.[15]

2.3 Contract and Renegotiation

Now, we present the time structure explicitly. At the beginning of initial date (date 0), the entrepreneur chooses how much amount of capital raising from each investor. Note that the entrepreneur has no incentive to raise more than K units of capital. Therefore, we can consider that the entrepreneur chooses a share profile $(s_i)_{i=1}^N$, which implies raising $s_i K$ units of capital from investor i. By definition, $\sum_{i=1}^N s_i = 1$. We call this profile $(s_i)_{i=1}^N$ *financial structure*. Without loss of generality, we suppose $s_1 \geq \cdots \geq s_N \geq 0$. We call investors with $s_i > 0$ creditors. Let n be the maximum number such that $s_i > 0$. Then n represents the number of creditors and $\{i \mid s_i > 0\} \equiv \mathbb{C}$ is the set of creditors. Further, we assume the following:

Assumption 2 (minimum units of capital)**.** The amount of capital raised from one creditor is not less than one, that is, $s_n \geq \frac{1}{K}$.[16]

Next, the entrepreneur offers a contract in the manner of take-it-or-leave-it offer. We explain the contract later in this sub-section. If the contract is rejected by more than one creditor, the project is not financed and every agent gains nothing.

When the contract is accepted by every creditor, the project is financed and goes to next stage. At the beginning of date 1, the short-term cash flow y_1 is realized. Following the realization, the entrepreneur makes voluntary payments to each creditor. Let $\tilde{\mathbf{a}}^R \in \Re_+^n$ be the entrepreneur's action concerning the repayment to creditors. The entrepreneur pays $\tilde{a}_i^R \in \Re_+$ to creditor i. This action $\tilde{\mathbf{a}}^R$ is supposed to be publicly observed. For later use, define the total voluntary payment $\tilde{R} \equiv \sum_{i \in \mathbb{C}} \tilde{a}_i^R$.

[15]When the court can estimate the true state more accurately as pieces of evidence are accumulated, the probability of verification v increases as the number of creditors who choose '1' increases. When it is the case, not only the coordination problem among creditors but also the free-riding problem takes place. We suppose the verification technology above for the sake of focusing on the coordination problem. We would like to analyze a model with both problems in another paper.

[16]In other words, the domain of the feasible amount of capital from an investor is $\{0\} \cup [1, \infty)$. Behind this condition, we suppose an transaction cost between the entrepreneur and the investors, implicitly.

Observing the action, each creditor chooses the action concerning the liquidation $\tilde{a}_i^L$. As mentioned before, these actions are also observed publicly. Let $\tilde{\mathbf{a}}^L$ be a vector whose i-th element is $\tilde{a}_i^L$.

Following the actions concerning the liquidation, the entrepreneur can request a renegotiation. The outcome of renegotiation cannot be observed by outsiders, especially by the court.

Last, following the renegotiation, each creditor chooses the action concerning the verification activity $\tilde{a}_i^V \in \{0, 1\}$. When more than one creditors chooses '1', the cash flow y_1 is verified with probability v at the end of date 1.

This time structure at date 1 is rationalized as follows. Since the entrepreneur has no capital other than the capital invested in this project, the repayment action follows after the realization of y_1. Moreover, since both the liquidation and the verification are costly, the creditors chooses the actions $\tilde{a}_i^L$ and $\tilde{a}_i^V$ following $\tilde{\mathbf{a}}^R$. In addition, since we can consider that the verification technology requires some degree of time-interval although the liquidation does not, the liquidation actions precedes the verification actions in order not to lose the chance of liquidation. As presented later, the renegotiation is held to change the liquidation actions. Therefore, the renegotiation must immediately follow the decisions on liquidation.

Given the time structure above, the contract can be generally specified as

$$\left(\mathbf{a}^R, \sigma^L(\tilde{\mathbf{a}}^R; \tilde{a}_i^L), \mathbf{d}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L), \mathbf{p}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)\right).$$

First, the contract specifies a fixed payment vector $\mathbf{a}^R$. Note that the payments are not state contingent since the short-term cash flows are not verifiable at that time. Second, it specifies each creditor's liquidation right contingent on the entrepreneur's voluntary payments. In general, the liquidation-right term can be defined as a distribution on the space of liquidation actions $\{0, 1\}^n$ contingent on the voluntary payment vector; $\sigma^L(\tilde{\mathbf{a}}^R; \tilde{a}_i^L)$. Third, it specifies a division rule of the liquidation value contingent on the voluntary payments and the liquidation rights. $\mathbf{d}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \in \Re_+^n$ represents the dividend vector whose i-th component is the dividend creditor i receives from the liquidation. Last, it specifies enforced transfers to each creditor in the good state contingent on the voluntary payments and the liquidation rights; $p_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$. $\mathbf{p}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \in \Re_+^n$ is the enforced payment vector in the good state, whose i-th component is $p_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$. Note that the terms concerning the liquidation rights and the division rule are not contingent on the short-term cash flow. This is because the short-term cash flows are not verified at the time of the liquidation. We also note that the contract cannot specify terms concerning enforced transfers in the bad state and the verification activity since the former is not feasible and the latter is not enforceable.

Now, we present the feasibility constraints on these terms as follows:

$$R \equiv \sum_{i \in \mathbb{C}} a_i^R \leq x \tag{1}$$

$$\forall \tilde{\mathbf{a}}^R \, \forall \tilde{\mathbf{a}}^L \quad \sigma^L(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L) \in [0, 1] \tag{2}$$

$$\forall \tilde{\mathbf{a}}^R \quad \sum_{\tilde{\mathbf{a}}^L} \sigma^L(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L) = 1 \tag{3}$$

$$\forall \tilde{\mathbf{a}}^R \, \forall \tilde{\mathbf{a}}^L \quad D(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \equiv \sum_{i \in \mathbb{C}} d_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \leq L \tag{4}$$

$$\forall \tilde{\mathbf{a}}^R \, \forall \tilde{\mathbf{a}}^L \quad P(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \equiv \sum_{i \in \mathbb{C}} p_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \leq x - \tilde{R} \tag{5}$$

Since the entrepreneur has no other capital than the input to this project, the first and the last constraints must be satisfied. On the other hand, since $D(\cdot, \cdot)$ represents the total dividend from the liquidation, it is never larger than the liquidation value. The second and the third must be satisfied since $\sigma^L(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L)$ is a distribution function.

In addition, in order to focus on a financial structure in which every creditor holds a similarly designed security, we impose on the contract a proportionality defined as follows:

Definition 1 (proportionality)**.** The contract is proportional among creditors if the followings hold:

$$\begin{gathered} \forall i \quad a_i^R = s_i R, \\ \forall i \, \forall \tilde{\mathbf{a}}^R \, \forall \tilde{\mathbf{a}}^L \quad d_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = s_i D(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \quad \text{if } \tilde{a}_i^R = s_i \tilde{R} \text{ for all } i \\ \forall i \, \forall \tilde{\mathbf{a}}^L \quad p_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = s_i P(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \quad \text{if } \tilde{a}_i^R = s_i \tilde{R} \text{ for all } i \end{gathered}$$

The first implies that the fixed payments prescribed in the contract are proportional to the share of lending. The others implies that, when the voluntary payments are proportional to the share, the contract treats the creditors proportionally. Note this constraints requires $d_i(\mathbf{0}, \tilde{\mathbf{a}}^L) = s_i D(\mathbf{0}, \tilde{\mathbf{a}}^L)$ and $p_i(x, \mathbf{0}, \tilde{\mathbf{a}}^L) = s_i P(x, \mathbf{0}, \tilde{\mathbf{a}}^L)$ for all $\tilde{\mathbf{a}}^L$ for all i, especially, which are the terms applied in the case of complete default.

Next, we explain the renegotiation, explicitly. The entrepreneur renegotiates (or recontracts) with the creditors to change the terms concerning the repayments and liquidation (liquidation rights and division rule). Let $\left(\mathbf{a}^{R\prime}, \mathbf{a}^{L\prime}, \mathbf{d}'\right)$ be the renegotiated terms.[17] We suppose that the outcome of the renegotiation follows the Shapley value wherever possible. Note that the entrepreneur cannot change the term concerning the enforced transfers. This is because the outcome of renegotiation is not observable for outsiders and is unverifiable.

The feasibility constraints on the renegotiated terms are as follows.

$$R' \equiv \sum_{i \in \mathbb{C}} a_i^{R\prime} \leq y_1 \tag{6}$$

$$D' \equiv \sum_{i \in \mathbb{C}} d_i' \leq L \tag{7}$$

[17] In general, the renegotiated term concerning the liquidation rights can be stochastic. However, it is obvious from the analysis in the next section that a deterministic term dominates every stochastic term.

Note that $a_i^{R'} \in \Re_+$ is the amount of repayment altering the amount voluntarily paid $\tilde{a}_i^R$, that is, the entrepreneur additionally transfers $a_i^{R'} - \tilde{a}_i^R$ to creditor i through the renegotiation.

3 Optimal Contract

3.1 Verification

We solve the model by a backward induction. First, we analyze the creditors' decisions on verification activity $\tilde{\mathbf{a}}^V$ following a realization of $\left(y_1, \tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L\right)$. Note that the verification activity is never undertaken in the bad state since the court cannot enforce any transfers in the state. Therefore, we consider the decisions in the good state from now on.

Given the verification technology, whether or not a creditor chooses to undertake the verification activity depends on an effective enforced payment vector $\mathbf{t}$, whose i-th component is t_i. One might think that $\mathbf{t}$ is the vector prescribed in the contract, $\mathbf{p}\left(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L\right)$. However, the court cannot enforce the entrepreneur to pay any amount more than $x - R'$, which is the realized cash flow subtracted by the total *renegotiated payment* R'. Given the proportionality constraint, the effective enforced payment for creditor i can be written as

$$t_i = s_i \min\left(P\left(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L\right), x - R'\right). \tag{8}$$

We now analyze the decisions on the verification activity more precisely. Since this stage of decisions on $\tilde{a}_i^V$ is a n-player-simultaneous game, there may be multiple equilibria. Possible equilibria are such that only one creditor chooses '1' (undertaking) with probability one (*single verifier equilibrium*), more than two creditors chooses '1' with a positive probability less than one (*multiple verifier equilibrium*), and every creditor chooses '0' (*no verifier equilibrium*). Suppose $t_i \geq \frac{c}{v}$ for a creditor i. Then she will certainly undertake the verification activity if the others never undertake it. Suppose $n \geq 2$ and $t_k \geq \frac{c}{v} > t_{k+1}$ holds for $k \in \{1, \ldots, n-1\}$, then there are k single verifier equilibria.[18] Note that each single verifier equilibrium produces the same verification probability v although it does various payoff distribution among creditors.

In the case of $k \geq 2$, there also exist some multiple verifier equilibria. Although the verification provability of a multiple verifier equilibrium is probably various, it is always less than v since every creditor chooses '0' with a positive probability in the equilibrium.[19] Therefore, when there exists i such that $t_i \geq \frac{c}{t}$, we use the single verifier equilibrium outcome, that is, we consider that the verification probability is v, for further analysis.[20] On the other hand, the verification probability is 0 when $t_1 < \frac{c}{v}$ (no verifier equilibrium).

[18] t_i is decreasing in i following from the construction of $(s_i)_{i \in \mathbb{C}}$ and the proportionality.

[19] Suppose, for example, $n = 3$ and $t_2 > \frac{c}{v} > t_3$, there are three equilibria in the class of mixed-strategy equilibrium; $(1, 0, 0)$, $(0, 1, 0)$, and $(\alpha_1, \alpha_2, 0)$, where the components represent the probability choosing '1' and $(\alpha_1, \alpha_2) = \left(1 - \frac{c}{vt_2}, 1 - \frac{c}{vt_1}\right)$. The verification probability is v in the former two equilibria, but it is $\{1 - (1-\alpha_1)(1-\alpha_2)\}\, v < v$ in the last equilibrium.

[20] Our equilibrium selection can be rationalized as follows: Suppose the creditors can negotiate each other in order to coordinate $\tilde{\mathbf{a}}^V$ before they simultaneously choose their own action. Then any agreements consistent with an equilibrium are sustainable. Since the outcome of single verifier equilibrium is efficient for the creditors, they agree to delegate the verification activity to one creditor for whom $t_i \geq \frac{c}{v}$ and share the cost.

Lemma 1 (equilibrium verification probability)**.** *Given an effective enforced transfer vector* $\mathbf{t}$*, the equilibrium verification probability* $v^*(\mathbf{t})$ *is* v *if and only if* $t_1 \geq \frac{c}{v}$*.*

Proof. Obvious following from the explanation above. □

In the last of this sub-section, we note that who bears the verification cost and/or how the cost is shared does not affects the analysis in the following sub-section. It does not affect the outcome of renegotiation since it is unobservable and unverifiable. In addition, it does not affect the original contract either since the sub-game of verification activity does not take place in the equilibrium path.[21]

3.2 Renegotiation

Now, we examine the renegotiation following a profile $\left(y_1, \tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L\right)$. First, we note that the entrepreneur is willing to change the terms only when the change can produce some additional value to the coalition containing the entrepreneur and creditors, which is the case that nature chooses $\tilde{\mathbf{a}}^L \neq \mathbf{0}$ following the prescribed liquidation-right term $\sigma^L(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L)$. Furthermore, the additional value is maximized by changing $\tilde{\mathbf{a}}^L$ to $\mathbf{0}$ and the maximum is $\Delta = y - L$.

When nature chooses $\tilde{\mathbf{a}}^L = \mathbf{0}$ instead, the entrepreneur has no incentive to change the terms since the renegotiation cannot produce any additional value. On the other hand, the entrepreneur cannot deter the liquidation if he has sufficient amount of cash flow to compensate the creditors for the deterrence of liquidation since he cannot commit any transfer at date 2. We could consider that the negotiation does not take place in these cases. However, we do consider that the terms are not changed although the renegotiation takes place and call the terms unchanged through the renegotiation *renegotiated terms* even in these cases.

Now, we consider the renegotiated terms closely. First, we provide the Shapley values for the creditors as benchmarks of the required compensations for the deterrence of liquidation. The Shapley value for a creditor i is given by

$$\begin{cases} d_i + \frac{1}{m+1}\Delta & \text{for } i \in \{i' \mid \tilde{a}^L_{i'} = 1\} \\ 0 & \text{for } i \notin \{i' \mid \tilde{a}^L_{i'} = 1\}, \end{cases} \tag{9}$$

where $d_i = d_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$ and m represents the number of creditors with liquidation right, that is, $m = \sum_{i \in \mathbb{C}} \tilde{a}^L_i$. This is calculated as follows. Creditor i gains d_i when the renegotiation is not held. Since every creditor with liquidation right can liquidate the project by herself, a creditor with liquidation right produces the marginal value Δ only when joining a coalition with the entrepreneur and all the other creditors with liquidation right, which comes about with probability $\frac{1}{m+1}$.

When $y_1 = x$, the entrepreneur can deter the liquidation if and only if the residual cash flow is not less than the total amount of dividend of the creditors who have got the liquidation right, that is,

$$x - \tilde{R} \geq \sum_{i \in \{i' \mid \tilde{a}^L_{i'} = 1\}} d_i. \tag{10}$$

[21] See the proof of Lemma 2 in Appendix A for more precise explanation.

Note that even when the residual cash flow is less than the sum of creditors' Shapley value $D + \frac{m}{m+1}\Delta$, both the entrepreneur and the creditors have an incentive to deter the liquidation.

Whether the inequality holds or not depends on both $\tilde{\mathbf{a}}^R$ and $\tilde{\mathbf{a}}^L$). Given a $\tilde{\mathbf{a}}^R$, the set of $\tilde{\mathbf{a}}^L$ with which the original contract changed through the renegotiation can be written as

$$\mathbb{L}(\tilde{\mathbf{a}}^R) \equiv \left\{ \mathbf{a}^{L\prime} \mid x - \tilde{R} \geq \sum_{i \in \{i' \mid a_{i'}^{L\prime}=1\}} d_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) \right\}. \tag{11}$$

We can express the renegotiated terms as follows;

$$a_i^{R\prime}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = \begin{cases} \tilde{a}_i^R + d_i + \min\left(\frac{x - \tilde{R} - \sum_{i \in \{i' \mid \tilde{a}_{i'}^L = 1\}} d_i}{m},\ \frac{1}{m+1}\Delta \right) & \text{for } i \in \{i' \mid \tilde{a}_{i'}^L = 1\} \\ \tilde{a}_i^R & \text{for } i \notin \{i' \mid \tilde{a}_{i'}^L = 1\} \end{cases} \tag{12}$$

$$a_i^{L\prime}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = 0 \quad \text{for all } i$$
$$d_i{}'(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = d_i \quad \text{for all } i$$

if $\tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R)$, and

$$a_i^{R\prime}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = \tilde{a}_i^R \quad \text{for all } i$$
$$a_i^{L\prime}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = \tilde{a}_i^L \quad \text{for all } i$$
$$d_i{}'(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = d_i \quad \text{for all } i$$

otherwise. Moreover, the total renegotiated payment is written as

$$R'(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = \begin{cases} \min\left(x,\ \tilde{R} + \sum_{i \in \{i' \mid \tilde{a}_{i'}^L = 1\}} d_i + \frac{1}{m+1}\Delta\right) & \text{if } \tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R) \\ \tilde{R} & \text{otherwise.} \end{cases} \tag{13}$$

In the last of this sub-section, we reconsider the decisions on verification activity, which is the sub-games following the decisions on the renegotiation. The effective enforced transfer is affected by the renegotiated payment following from (8). Since the renegotiated payment is a function of $(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$ as expressed above, the effective enforced transfer can be written as $t_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$, the vector as $\mathbf{t}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$, and the total as $T(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$.

3.3 Optimal Contract

Now, we explain the optimal contract.

Definition 2 (debt-like contract)**.** A contract is *debt-like contract* if and only if it prescribes an amount of constant payment, a liquidation right, a secured value, and a legal penalty, as follows;

- fixed-payment term: $\mathbf{a}^R$

- liquidation-right: $\sigma^L(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L) = \begin{cases} 0 \quad \text{for all } \tilde{\mathbf{a}}^L & \text{if } \tilde{\mathbf{a}}^R \geq \mathbf{a}^R \\ \begin{cases} \beta \in [0,1] & \text{for } \tilde{a}^L = \mathbf{1} \\ 1-\beta & \text{for } \tilde{a}^L = \mathbf{0} \\ 0 & \text{for else} \end{cases} & \text{otherwise} \end{cases}$

- dividend term: $\mathbf{d}(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L)$ such that $d_i(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L) = s_i L$ for all i

- enforced-transfer term: $\mathbf{p}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L)$ such that $p_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = \begin{cases} 0 & \text{if } \tilde{\mathbf{a}}^R \geq \mathbf{a}^R \\ s_i x - \tilde{a}_i^R & \text{otherwise} \end{cases}$

Lemma 2. *It is optimal for the entrepreneur to offer the debt-like contract with appropriate value of β to his creditors.*

Proof. See Appendix A. □

Since the liquidation is inefficient, the project should not be liquidated wherever possible. However, the entrepreneur has no incentive to pay anything if he incurs no penalty in the case of default. There are two devices for the penalty; the liquidation and legal penalty. Since any cash-flow-contingent liquidation right is not feasible, offering a stricter liquidation right decreases the expected value in the bad state. In contrast, the legal penalty plays just a deterrent against the strategic default and generates no inefficiency in equilibrium. Therefore, the legal penalty is the maximum feasible value, that is, $P(\tilde{\mathbf{a}}^R, \cdot) = x - \tilde{R}$. In addition, the dividend from the liquidation is also the maximum feasible value, that is, $D(\cdot, \cdot) = L$ independent of the voluntary payments and the realized liquidation rights, in order to deter the default as efficiently as possible. Moreover, the liquidation-right term is the harshest prescription since it means that when the entrepreneur defaults the repayment to a creditor, not only the defaulted creditor but also the other creditors get the liquidation right with probability β.

Given the debt-like contract, partial defaults are dominated by complete default, that is, the entrepreneur prefers paying nothing rather than paying some amount but being default of some creditors' repayments.[22] Therefore, we can focus on the incentive of the complete default in the good state.

The renegotiated payment to creditor i following the complete default is rewritten as

$$a_i^{R\prime}(\mathbf{0}, \tilde{\mathbf{a}}^L) = \begin{cases} s_i L + \frac{1}{n+1}\Delta \equiv a_{cd,i}^{R}{}' & \text{if } \tilde{\mathbf{a}}^L = \mathbf{1} \text{ with probability } \beta \\ 0 & \text{if } \tilde{\mathbf{a}}^L = \mathbf{0} \text{ with probability } 1-\beta \end{cases} \tag{14}$$

and the total renegotiated payment is

$$R'(\mathbf{0}, \tilde{\mathbf{a}}^L) = \begin{cases} L + \frac{n}{n+1}\Delta \equiv R_{cd}{}' & \text{if } \tilde{\mathbf{a}}^L = \mathbf{1} \text{ with probability } \beta \\ 0 & \text{if } \tilde{\mathbf{a}}^L = \mathbf{0} \text{ with probability } 1-\beta. \end{cases} \tag{15}$$

Note the first line in (14) holds since $\frac{x-L}{n} > \frac{1}{n+1}\Delta$ following from $\Delta = y - L$ and $x \geq y$.

[22] See Appendix A for more explanation.

Given the renegotiated payments, the effective enforced transfer to creditor i following the complete default is

$$t_i(\mathbf{0}, \tilde{\mathbf{a}}^L) = \begin{cases} s_i(x - R') \equiv t_{cd,i} & \text{if } \tilde{\mathbf{a}}^L = \mathbf{1} \text{ with probability } \beta \\ s_i x & \text{if } \tilde{\mathbf{a}}^L = \mathbf{0} \text{ with probability } 1-\beta. \end{cases} \tag{16}$$

Now, define the equilibrium verification probabilities following the complete default as $v^{*\prime} \equiv v^*(\mathbf{t}_{cd})$ and $v^* \equiv v^*(\mathbf{p})$. The former is the equilibrium verification probability when nature gives the creditors the liquidation right following the complete default and the latter is the equilibrium verification probability when nature does not give the liquidation right following the complete default.

Therefore, given a financial structure and the debt-like contract described above, the entrepreneur's maximization problem is written as

$$\max_{R,\beta} \quad \theta(x - R + y) + (1-\theta)(1-\beta)y \tag{OF}$$

$$\text{s.t.} \quad \theta R + (1-\theta)\beta L - K \geq 0 \tag{PC}$$

$$x - R + y \geq x + \beta(-R_{cd}{}' + y - v^{*\prime}(x - R_{cd}{}')) + (1-\beta)(y - v^* x) \tag{IC}$$

$$0 \leq R \leq x \tag{FC-R}$$

$$0 \leq \beta \leq 1. \tag{FC-β}$$

The objective function represents the entrepreneur's expected payoff. In the good state, the entrepreneur gains x and pays R in total at date 1and he gains y in expectation at date 2. In the bad state, on the other hand, he gains y in expectation only when the project is not liquidated.

The first constraint is the participation constraint of the creditors. Note that this constraint is common for every creditor following from the proportionality. Each creditor inputs $s_i K$ and gains $s_i R$ in the good state and is distributed $s_i L$ when the project is liquidated in the bad state.

The second constraint is the incentive compatibility constraint. It requires that the entrepreneur has no incentive to default completely under the optimal contract. The right-hand side represents the expected payoff when the entrepreneur chooses the entire strategic default; the entrepreneur voluntarily pays $R_{cd}{}'$ in total through the renegotiation and is enforced to pay $x - R_{cd}{}'$ with probability $v^{*\prime}$ when the creditors get the liquidation right (with probability β), and pays nothing and is enforced to pay x with probability v^* otherwise. Note that this is the maximum deviation payoff as mentioned before. This inequality is rewritten as

$$R \leq v^* x + \beta \left[R_{cd}{}' + v^{*\prime}(x - R_{cd}{}') - v^* x\right]. \tag{17}$$

The remaining two constraints are feasibility constraints.

We now solve the maximization problem. Note that this problem may have no solution, which implies the project is not financiable under the financial structure.

Lemma 3 (financiability)**.** *Given a financial structure, (i) the project is financiable if $K \leq \theta v^* x$, and (ii) if $K > \theta v^* x$, the project is financiable if and only if the following holds;*

$$\theta \left[R_{cd}{}' + v^{*\prime}(x - R_{cd}{}')\right] + (1-\theta)L \geq K. \tag{18}$$

Proof. (i) If setting $(R, \beta) = (\frac{K}{\theta}, 0)$, every constraints are satisfied. (ii) When the project is financiable given the financial structure, (PC) holds with equality. If it was not the case, reducing R could make the entrepreneur better off while the constraints are still satisfied. In addition, the feasibility constraint on the total repayment is not binding as confirmed in Appendix A. Therefore, there exists a set (R, β) satisfying every constraints if and only if

$$0 \leq \frac{K - \theta v^* x}{\theta \left[{R_{cd}}' + v^{*\prime}(x - {R_{cd}}') - v^* x \right] + (1-\theta) L} \leq 1.$$

If $K > \theta v^* x$, the inequality holds if and only if (18) holds. $\square$

We can intuitively explain this lemma as follows. Even when the creditors do not have the liquidation right, the entrepreneur will voluntarily pay the expected value of the legal penalty $v^* x$ in the good state. Therefore, the voluntary payment is sufficient for the creditors to recover the lending if $K \leq \theta v^* x$. When the creditors use the liquidation as the additional threat-device, the entrepreneur will voluntarily pay the sum of the renegotiated payments and the expected value of the legal penalty, ${R_{cd}}' + v^{*\prime}(x - {R_{cd}}')$, in the good state, and then the maximum value the creditors gains is the left-hand side of (18).[23] Therefore, the creditors accept the contract if (18) holds.

As confirmed in the proof of Lemma 3, (PC) holds with equality in optimum. Substituting (PC) with equality, the objective function is rewritten as

$$\theta x + y - K - (1-\theta)\beta\Delta. \tag{19}$$

The first three terms in (19) represent the net present value of the project in the first-best case, where the project is not liquidated. The last term is the expected loss derived from the inefficient liquidation. This inefficiency is following from the incompleteness of contracts.[24] The degree of inefficiency increases as the liquidation right becomes stricter since liquidating the project with higher probability increases the expected loss.

Now, we present the optimal liquidation-right term.

Lemma 4 (optimal liquidation right)**.** *Suppose the project is financiable. Then the optimal liquidation-right term is (i) zero if $K \leq \theta v^* x$, and (ii) a positive value*

$$\bar{\beta} = \frac{K - \theta v^* x}{\theta \left[{R_{cd}}' + v^{*\prime}(x - {R_{cd}}') - v^* x \right] + (1-\theta) L} \tag{20}$$

if $K > \theta v^* x$.

Proof. (i) Following from (19), the objective is maximized when $\beta = 0$. Since $(R, \beta) = (\frac{K}{\theta}, 0)$ satisfies every constraint when $K \leq \theta v^* x$, the result holds. (ii) When $K > \theta v^* x$ and (18) hold, (IC) also holds with equality. If it held with strict inequality, then the entrepreneur could be better off by increasing R by some infinitely small amount ϵ and reducing β by

[23] ${R_{cd}}' + v^{*\prime}$ may be less than $v^* x$ when $v^* = v$ and $v^{*\prime} = 0$. When it is the case, the creditors have an incentive to reduce the renegotiated payments contrary to the entrepreneur. We suppose that the renegotiated payment vector is ${\mathbf{a}_{cd}^{R}}'$ even when the creditors have the incentive to reduce the payments. This is just for simplicity.

[24] The contract is incomplete since it cannot prescribe any cash-flow contingent terms.

$\frac{\theta}{1-\theta}\epsilon$. Such a change would not change the expected payoff of the creditors, increase the expected payoff of the entrepreneur, and keep the incentive compatibility constraint still being satisfied. □

When $K \leq \theta v^* x$, the verification produces a sufficient incentive for the entrepreneur to pay a necessary amount and consequently the liquidation is not required. The optimal total payment is $\frac{K}{\theta}$ in this case.

On the contrary, when $K > \theta v^* x$, the stochastic liquidation is required in order to induce the prescribed repayment. The numerator represents the additionally required incentive. On the other hand, the first term of the denominator is the maximum additional incentive derived from the liquidation and the second term is the expected value from the liquidation. The optimal total payment is $\bar{R} = v^* x + \bar{\beta}\left[{R_{cd}}' + v^{*\prime}(x - {R_{cd}}') - v^* x\right]$ in this case.

4 Optimal Financial Structure

The financial structure affects both the renegotiation and verification outcome. We now analyze the effects and the optimal financial structure given the debt-like contract. Following from Lemma 1, if $vx < c$, $v^* = 0$ under any financial structure. On the other hand, if $vx \geq c$, there exists a number $\bar{n} \geq 1$ such that $\frac{K-(n-1)}{K} vx \geq c$ for $n \leq \bar{n}$ and $\frac{K-(n-1)}{K} vx < c$ for $n > \bar{n}$. Concentration on creditor 1 can heighten his incentive of verification and makes him single verifier. $\bar{n}$ is the maximum number with which creditor 1 undertakes the verification activity in the case without any payments. If borrowing from more than $\bar{n}$ creditors, the share of the creditor with maximum share (creditor 1) is not sufficient for providing the creditor the incentive of verification since each creditor must have the minimum share $\frac{1}{K}$.

We similarly consider $v^{*\prime}$. Let the total renegotiated payment given a number of creditors be ${R_{cd}}'(n) \equiv L + \frac{n}{n+1}\Delta$. Obviously, this is increasing in n. If $v(x - {R_{cd}}'(1)) < c$, $v^{*\prime} = 0$ under any financial structure. On the other hand, if $v(x - {R_{cd}}'(1)) \geq c$, there exists a number $\tilde{n} \geq 1$ such that $\frac{K-(n-1)}{K} v(x - {R_{cd}}'(1)) \geq c$ for $n \leq \tilde{n}$ and $\frac{K-(n-1)}{K} v(x - {R_{cd}}'(1)) < c$ for $n > \tilde{n}$. $\tilde{n}$ is the maximum number with which creditor 1 undertakes the verification activity in the case with renegotiated payments.

Summing up, we have the following.

Lemma 5 (maximum verification-producing number)**.** *(i) Suppose $vx \geq c$. Then the equilibrium verification probability in the case without any payments, v^*, can be v if and only if $n \leq \bar{n}$. (ii) Suppose $v(x - {R_{cd}}'(1)) \geq c$. Then the equilibrium verification probability in the case with renegotiated payments, $v^{*\prime}$, can be v if and only if $n \leq \tilde{n}$.*

Proof. Following from the explanation above. □

Now, we analyze the financiability and the optimal financial structure, explicitly. First of all, we define three types of financial structure as follows.

Definition 3 (three types of financial structure)**.** A financial structure is (i) *decentralized financial structure* if and only if every creditor occupies a fairly small share and then the creditors never undertake the verification activity, that is, $v^* = v^{*\prime} = 0$, (ii) *passive main-banking* *structure* if and only if one creditor undertakes the verification activity only in the case without any payment, that is, $v^* = v$ but $v^{*\prime} = 0$, and (iii) *active main-banking structure* if and only if one creditor undertakes the verification activity even in the case with a renegotiated payment, that is, $v^* = v^{*\prime} = v$.

When the financing is considerably dispersed and the shares of each creditor is fairly small, the creditors never undertake the verification activity. We call such a type of financial structure decentralized financial structure. Within the type, we call the most dispersed one *most dispersed decentralized financial structure (MDDFS)*, which is a financial structure such that

$$s_i = \begin{cases} \frac{1}{K} & \text{for } i \leq K \\ 0 & \text{for } i \geq K+1. \end{cases}$$

When one creditor holds a sufficient share, the creditor will play the role of single verifier. We call the creditor *main bank* and the type of financial structure under which one creditor undertakes the verification activity *main-banking structure*. This type of financial structure is divided to two types, passive main-banking structure and active main-banking structure. When the share of main bank is relatively small, main bank undertakes the verification activity when the entrepreneur makes no payment but does not when the entrepreneur repays renegotiated payments. In contrast, when the share of main bank is relatively large, main bank undertakes the verification activity even when the entrepreneur makes renegotiated payments. Since the attitude of main bank in the former case is passive comparing to that in the latter case, we call the former type of financial structure passive main-banking structure and the latter type active main-banking structure. Passive main-banking structure with n creditors is a financial structure such that

$$\begin{cases} s_i \geq \frac{c}{vx} & \text{for } i = 1 \\ s_i = 0 & \text{for } i \geq n+1 \\ s_i \geq \frac{1}{K} & \text{for else,} \end{cases}$$

where $n \leq \bar{n}$, and active main-banking structure with n creditors is a financial structure such that

$$\begin{cases} s_i \geq \frac{c}{v(x - {R_{cd}}'(n))} & \text{for } i = 1 \\ s_i = 0 & \text{for } i \geq n+1 \\ s_i \geq \frac{1}{K} & \text{for else,} \end{cases}$$

where $n \leq \tilde{n}$.

Since the objective of the entrepreneur is maximized by minimizing the liquidation probability, β, following from (19), the optimal financial structure is such a structure that minimizes β. The dispersion of financing has two effects on β. It increases the total renegotiated payment ${R_{cd}}'(n)$, which decreases the value of β (*complicated-renegotiation effect*). This is because it makes the renegotiation more complicated and induces the entrepreneur to pay larger amount in total.

On the other hand, it tends to reduce both the equilibrium verification probability in the case without any payment, v^*, and that in the case with a renegotiated payment, $v^{*\prime}$, to zero, which increases the value of β. The reason of this feature is explained as follows. Undertaking the verification activity in the case without any payment is collectively beneficial when $vx \geq c$ and that in the case with renegotiated payments is collectively beneficial when $v(x - R_{cd}'(n)) \geq c$. However, since the action concerning the verification is unobservable, the coordination of the verification activity which makes one creditor choose "undertaking" and the others choose "not undertaking" is not feasible if the activity is not privately beneficial for creditor 1. The dispersion makes the share of creditor 1 smaller and consequently causes the coordination-failure (*coordination-failure-in-delegated-verification effect*).

The optimal financial structure balances these two effects. When the verification activity is not beneficial, that is, $vx < c$, the latter effect is never effective. Therefore, MDDFS, under which every creditor lends the minimum unit of fund, is optimal. Under this structure, no creditor undertakes the verification activity.

Proposition 1 (optimal financial structure when the verification activity is not beneficial)**.** *Suppose the verification activity is not beneficial, that is, $vx < c$. Then the project is financiable if and only if*

$$\theta R_{cd}'(K) + (1 - \theta)L \geq K, \tag{21}$$

and the optimal financial structure is MDDFS.

The financiability condition follows from Lemma 3. Moreover, we have the following lemma concerning the feasibility of MDDFS.

Lemma 6 (feasibility of most dispersed financial structure)**.** *When the long-term cash flow is insufficient for financing the project, that is, $y < K$, the entrepreneur cannot finance the project with the most dispersed decentralized financial structure.*

Proof. The project is financed with MDDFS if and only if (21) holds. However, when $y < K$, the inequality does not hold since $R_{cd}'(K) = L + \frac{K}{K+1}\Delta < y$ and $L < y$. □

In contrast, when the verification activity is sufficiently beneficial, that is, $vx \geq c$ and $\theta vx \geq K$, the complicated-renegotiation effect does not take place since the creditors have no incentive to liquidate the project due to the sufficient expected gains from the verification activity. In optimum, one creditor holds a sufficient share of lending to pay the costly verification activity. Since the latter effect is not effective, there is a continuum of optimal financial structure. Financing from one investor is one optimum. On the other hand, financing from the maximum verification-sustainable number in the case without any payment, $\min(\bar{n}, K)$, (borrowing one unit from each $\min(\bar{n}, K) - 1$ investors and the rest from the main-bank) is another optimum.

Proposition 2 (optimal financial structure when the verification activity is sufficiently beneficial)**.** *Suppose the verification activity is beneficial, $vx \geq c$, and the expected gain from verification is sufficient for financing, $\theta vx \geq K$. Then the project is financiable and is never liquidated at the end of date 1, and the optimal financial structure is main-banking structure, whether passive one or active one.*

When the verification activity is beneficial but insufficient, that is, $vx \geq c$ but $\theta vx \geq K$, the trade-off between the two effects takes place. There are three candidates of optimal structure; active main-banking structure with maximum number $\min(\tilde{n}, K)$ (AMBSM), passive main-banking structure with maximum number $\min(\bar{n}, K)$ (PMBSM), and most dispersed decentralized financial structure (MDDFS). Under main-banking structure, whether passive or active one, the main bank plays the role of single verifier. She gives a discipline to the entrepreneur against the strategic default by the verification activity. The other creditors rely on the main-bank's activity but give another discipline to the entrepreneur by making the renegotiation more complicated. Therefore, AMBSM dominates any active main-banking structure with less number and PMBSM dominates any active main-banking structure with less number. On the other hand, under decentralized financial structure, only complicated-renegotiation effect takes place since no creditor undertakes the verification activity. Therefore, MDDFS dominates any decentralized financial structure with other number than K.

The benefit of verification depends on not only the verification technology, v and c, but also the production technology concerning the cash flow in the good state, x. The verification activity is more beneficial, the more efficient the verification technology is (higher v and lower c) and the more profitable the project in the short run is (higher x). The degree of the benefit affects the optimal financial structure as follows: An increase in v and/or x enhances the significance of coordination concerning the verification activity. In addition, it also heightens the complexity of renegotiation under AMBSM, and PMBSM since it increases $\tilde{n}$ and $\bar{n}$. An increase in c also heightens the complexity of renegotiation under AMBSM and PMBSM. Comparing MDDFS to PMBSM, MDDFS is preferable when the complicated-renegotiation effect outweighs the coordination-failure-on-verification-activity effect since the dispersion under MDDFS is not less than that under PMBSM. Similarly, comparing PMBSM to AMBSM, PMBSM is preferable when the complicated-renegotiation effect outweighs the coordination-failure-on-verification-activity effect since the dispersion under PMBSM is not less than that under AMBSM. Therefore, the optimal financial structure tends to convert from MDDFS to PMBSM, and further to AMBSM as the benefit of verification activity increases.

Proposition 3 (optimal financial structure when the verification activity is not sufficiently beneficial)**.** *Suppose the verification activity is beneficial, $vx \geq c$, but the expected gain from verification is insufficient for financing, $\theta vx \leq K$. Then the optimal financial structure is (i) AMBSM when the verification activity is fairly beneficial, (ii) PMBSM when the activity is not so beneficial, and (iii) MDDFS when the activity is less beneficial.*

When the long-term cash flow is insufficient, MDDFS is not feasible following from Lemma 6. Therefore, the project is financiable only when main-banking structure is feasible. This is the case that the the verification activity is beneficial. On the other hand, main-banking structure is optimal when the verification activity is sufficiently beneficial and when the verification activity is relatively beneficial following from Proposition 2 and 3. In contrast, decentralized financial structure is optimal when the verification activity is not beneficial or relatively inbeneficial following from Proposition 1 and 3. Therefore, we hold the following.

Proposition 4 (main-banking structure or decentralized financial structure)**.** *(i) When the long-term cash flow is insufficient, the entrepreneur can finance the project with main-banking* *structure when the verification technology is efficient (high v and low c) and the project is profitable in the short run (high x), and cannot finance the project with any financial structure otherwise. (ii) When the long-term cash flow is sufficient, the entrepreneur chooses main-banking structure when the verification technology is efficient (high v and low c) and the project is profitable in the short run (high x), and chooses (most dispersed) decentralized financial structure otherwise.*

When the necessary units of fund K is large, most dispersed decentralized financial structure is considered as a structure financing by bond only. In contrast, main-banking structure with high number of creditors is considered as a structure financing both by bank loan and bond. Therefore, our study is related to those in the existing corporate finance literature on a firm's choice between bank loans and bond finance. A great degree of this literature, for example Rajan (1992), supposes the existence of informational difference between banks and bondholders exogenously. In contrast, we do not suppose any informational and technological asymmetry among creditors. Each creditor observes the information concerning the real state of the project and has same opportunity and technology to verify the state. However, each creditor's incentive to undertake the verification activity depends on the financial structure; one creditor has the incentive under main-banking structure but no creditor has the incentive under decentralized financial structure (bond finance).

Proposition 4 implies that (i) entrepreneurs who have a project producing sufficient return in the near future but low return in the distant future call for a main bank (ii) those who have a project with insufficient return in the near future but sufficient return in the distant future choose bond finance, and (iii) those who have a project with sufficient return both in the near and distant future have a relationship with a main bank although they can finance only from the bond market.

Up to now, we have considered whether the optimal financial structure is main-banking structure or decentralized financial structure. Finally, we consider the optimal number of creditors. As mentioned above, the optimal financial structure tends to convert from MDDFS to PMBSM, and further to AMBSM as the benefit of verification activity increases. When MDDFS is optimal, the optimum number is the maximum feasible number K. When the optimal financial structure converts from MDDFS to PMBSM, the optimum number converts from K to $\bar{n}$, which is probable to be less than K. Moreover, when the optimal financial structure converts from PMBSM to AMBSM, the optimum number converts from $\bar{n}$ to $\tilde{n}$, where $\bar{n}$ is strictly larger than $\tilde{n}$ ordinarily. Note that $\bar{n}$ and $\tilde{n}$ are increasing in v and x and decreasing in c. Note also that the optimum number is not unique when the benefit of verification activity is fairly high, that is, both $vx \geq c$ and $\theta vx \geq K$. Therefore, we have the following.

Proposition 5 (optimum number of creditors)**.** *The optimum number of creditors is inmonotonous and incontinuous with the efficiency of verification technology and the profitability in the short run. Furthermore, it is not unique when the efficiency of verification technology and the profitability in the short run is fairly high.*

Needless to say, several studies consider the debt concentration. However, lots of these studies only consider the number of creditors (theoretical studies; Bolton and Scharfstein

(1996), Detragiache, Garella, and Guiso (2000), Diamond (2004), and Bris and Welch (2005), and empirical studies; Petersen and Rajan (1995), Ongena and Smith (2000), Detragiache, Garella, and Guiso (2000), Farinha and Santos (2002)). One of the most significant contribution of this study is considering the shares among creditors endogenously.

In this respect, this study is most related to the empirical study by Esty and Megginson (2003). They measure the level of debt concentration not only with the number of creditors but also several measures of concentration, for example, Herfindahl-Hirschman Index and the largest single bank share. Their study based on several theoretical results; debt concentration (i) makes delegated monitoring easier and more intensive and (ii) makes re-organization easier and more efficient, but (iii) makes re-negotiation following strategic default easier and more attractive. (i) is based on large number of literature on the banks' monitoring, for example, Diamond (1984). (ii) and (iii) are based on the consideration of bargaining following defaults by Bolton and Scharfstein (1996). (i) and (ii) are the advantage of concentration but (iii) is the disadvantage of it. Since we incorporates (i), although ex-post verification activity instead of monitoring, and (iii) into our model, our results are considerably similar to their theoretical background.

They show that debt concentration is positively related to the strength of creditor right and the reliability of legal enforcement controlling the loan size and the other risks.[25] They conclude that this result is consistent with the idea that the effects in (i) and (ii) are significant when the strength of creditor right and the reliability of legal enforcement are high, and the effect in (iii) is significant otherwise. However, we consider that, if examining the effects more precisely as in our study, their empirical results are partially inconsistent with the theoretical results since the debt concentration should be not monotonous with the efficiency of enforcement.

There are several precise studies on the effects of the strength and efficiency of legal enforcement. Our study only accesses the effect through the ex-post enforcement. However, it also affects the entrepreneurs' decisions on management (Bebchuk and Fried (1997), Berkovitch, Israel, and Zender (1997)) and the investors' incentive of screening (Manove, Padilla, and Pagano (2001), Bianco, Jappelli, and Pagano (2005), Zazzaro (2005)). Although our results are restrictive since lacking these respects, we investigate the effect on the financial structure instead.

5 Conclusion

We first show that, when the contracts satisfy a proportionality among creditors, debt-like contract, which specify fixed payments, a liquidation rule (liquidation rights and dividends from liquidation), and enforced transfers contingent on the payments to each creditor, is optimal. We further consider the optimal financial structure and the optimum number of creditors.

The financial structure affects renegotiation outcomes and decisions on verification activity following strategic defaults and changes the terms in the debt-like contracts as a result.

[25]Measurement (and the effects) of the strength of creditor right and efficiency of legal enforcement and is studied, for example, by La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1998) and Berkowitz, Pistor, and Richard (2003).

Dispersing the financial structure makes the renegotiation more complicated and reduces the entrepreneur's gain from it. On the other hand, the dispersion reduces each creditor's share of lending and discourages the creditors' incentive of a costly verification activity.

These two effects changes terms of an optimal contract inversely. An increase in the degree of renegotiation-complexity discourages the entrepreneur's incentive of the misbehavior, which reduces the necessity of strict liquidation right and lowers the fixed payments. In contrast, the discouragement of verification activity crowds out the opportunity of verification and heighten the incentive of the misbehavior, which increases the necessity of strict liquidation right and heightens the fixed payments.

The optimal financial structure must balance the two effects. There are three candidates on optimal financial structure; active main-banking structure with maximum number (AMBSM), passive main-banking structure with maximum number (PMBSM), and most dispersed decentralized financial structure (MDDFS). AMBSM is a financial structure under which one creditor (main bank) holds a sufficient share of lending for undertaking the verification activity even when the entrepreneur makes some payments in the renegotiation, and as many as possible creditors divide the rest. In contrast, PMBSM is a financial structure under which one creditor (main bank) holds a sufficient share of lending for undertaking the verification activity only when the entrepreneur makes no payment and as many as possible creditors divide the rest. Last, MDDFS is a financial structure under which each creditor holds a minimum unit of lending and no creditor does not undertake the verification activity.

Which types of financial structure are the optimum is dependent of not only the profitability of project but also the efficiency of verification technology. When the expected cash flow in the distant future is insufficient for financing the project, MDDFS is never feasible. On the other hand, when it is sufficient for financing, AMBSM tends to be optimal as the efficiency of verification technology and the profitability in the short run are heightened, and MDDFS tends to be optimal as they are lowered. When the creditors other than the main-bank can be considered as bondholders, this result implies that (i) entrepreneurs who have a project producing sufficient return in the near future but low return in the distant future call for a main bank (ii) those who have a project with insufficient return in the near future but sufficient return in the distant future choose bond finance, and (iii) those who have a project with sufficient return both in the near and distant future have a relationship with a main bank although they can finance only from the bond market.

The optimum number of creditors is inmonotonous and incontinuous with the efficiency of verification technology and the profitability in the short run. For a lower level of them, MDDFS is optimal and consequently the optimum number is maximum feasible number (constant). In contrast, for a middle-low and middle-high level of them, PMBSM and AMBSM are optimal,respectively, and the optimum number increases with them for each level, respectively. It follows that the optimum number jumps down when the a type of financial structure converts to another type of financial structure. Moreover, when the efficiency of verification technology and the profitability in the short run are sufficiently high, there exist several optimal numbers.

One of the most significant contribution of this study is that we investigate not only the number of creditors but also the shares of lending explicitly. Although several number of studies investigate optimal concentration of debt, few of them consider the shares of lending explicitly. Our results are fruitful for further empirical studies.

Another contribution of this study is endogenous consideration on verifiability. Most of existing studies following an incomplete contracting framework are suppose that the cash flows are not verifiable due to the unobservability of them. However, whether the cash flows are verifiable or not could be dependent of creditors' actions. We introduce the verification activity (a series of costly actions) and endogenously analyze the effect of financial structure on the decisions of it. This consideration, as a result, provides some predictions on the relationship between strength of creditor right (efficiency of legal enforcement) and optimal financial structure. Although several studies have already investigated the relationship, our results could complement the results of them.

Appendix A

Given a contract $(\mathbf{a}^R, \sigma^L(\tilde{\mathbf{a}}^R; \tilde{\mathbf{a}}^L), \mathbf{d}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L), \mathbf{p}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L))$, the expected payoffs of the entrepreneur and the creditors are as follows:

- entrepreneur's expected payoff

$$\theta\Pi_0(x, \mathbf{a}^R) + (1-\theta)\Pi_0(0, \mathbf{0})$$

- creditor i's expected payoff ($i \in \mathbb{C}$)

$$\theta\Pi_i(x, \mathbf{a}^R) + (1-\theta)\Pi_i(0, \mathbf{0}),$$

where

$$\Pi_0(y_1, \tilde{\mathbf{a}}^R) = \begin{cases} x - ER(\tilde{\mathbf{a}}^R) + EL(\tilde{\mathbf{a}}^R) - ED(\tilde{\mathbf{a}}^R) - ET(\tilde{\mathbf{a}}^R) & \text{for } y_1 = x \\ EL(\mathbf{0}) - ED(\mathbf{0}) & \text{for } y_1 = 0 \end{cases}$$

and

$$\Pi_i(y_1, \tilde{\mathbf{a}}^R) = \begin{cases} s_i\left(ER(\tilde{\mathbf{a}}^R) + ED(\tilde{\mathbf{a}}^R) + ET(\tilde{\mathbf{a}}^R)\right) & \text{for } y_1 = x \text{ and } \tilde{\mathbf{a}}^R \neq \mathbf{0} \\ s_i\left(ER(\tilde{\mathbf{a}}^R) + ED(\tilde{\mathbf{a}}^R) + ET(\tilde{\mathbf{a}}^R)\right) - EC(\tilde{\mathbf{a}}^R) & \text{for } y_1 = x \text{ and } \tilde{\mathbf{a}}^R = \mathbf{0} \\ s_i ED(\mathbf{0}) & \text{for } y_1 = 0 \end{cases}$$

for a single-verifier creditor and

$$\Pi_i(y_1, \tilde{\mathbf{a}}^R) = \begin{cases} s_i\left(ER(\tilde{\mathbf{a}}^R) + ED(\tilde{\mathbf{a}}^R) + ET(\tilde{\mathbf{a}}^R)\right) & \text{for } y_1 = x \\ s_i ED(\mathbf{0}) & \text{for } y_1 = 0 \end{cases}$$

for the other creditors, and

$$
\begin{aligned}
ER(\tilde{\mathbf{a}}^R) &= \sum_{\tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) R'(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L) + \sum_{\tilde{\mathbf{a}}^L \notin \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L)\tilde{R},\\
EL(\tilde{\mathbf{a}}^R) &= \sum_{\tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R)\cap\{\mathbf{0}\}} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) y + \sum_{\tilde{\mathbf{a}}^L \notin \mathbb{L}(\tilde{\mathbf{a}}^R)\cap\{\mathbf{0}\}} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) L,\\
ED(\tilde{\mathbf{a}}^R) &= \sum_{\tilde{\mathbf{a}}^L \notin \mathbb{L}(\tilde{\mathbf{a}}^R)\cap\{\mathbf{0}\}} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) D(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L),\\
ET(\tilde{\mathbf{a}}^R) &= \begin{cases} \sum_{\tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) T(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L) + \sum_{\tilde{\mathbf{a}}^L \notin \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) P(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L) & \text{for } \tilde{\mathbf{a}}^R \neq \mathbf{0}\\ \sum_{\tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) v^*(\mathbf{t}(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L)) T(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L) & \\ + \sum_{\tilde{\mathbf{a}}^L \notin \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) v^*(\mathbf{p}(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L)) P(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L) & \text{for } \tilde{\mathbf{a}}^R = \mathbf{0}, \end{cases}\\
EC(\tilde{\mathbf{a}}^R) &= \sum_{\tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) c^*(\mathbf{t}(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L)) + \sum_{\tilde{\mathbf{a}}^L \notin \mathbb{L}(\tilde{\mathbf{a}}^R)} \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L) c^*(\mathbf{p}(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L)),\\
c^*(\mathbf{t}') &= \begin{cases} c & \text{if } v^*(\mathbf{t}') = v\\ 0 & \text{if } v^*(\mathbf{t}') = 0. \end{cases}
\end{aligned}
$$

$\Pi_0(y_1, \tilde{\mathbf{a}}^R)$ represents the entrepreneur's expected payoff given a state ($y_1 = x$ or $y_1 = 0$; good or bad) and a set of voluntary payments $\tilde{\mathbf{a}}^R$. It consists of the realized cash flow, expected total payment (voluntary or renegotiated one) $ER(\tilde{\mathbf{a}}^R)$, expected value from long-term cash flow $EL(\tilde{\mathbf{a}}^R)$, expected total dividend $ED(\tilde{\mathbf{a}}^R)$, and expected total enforced payment $ET(\tilde{\mathbf{a}}^R)$.

There are several points worth noting. First, these expression takes the renegotiation into account. As presented before, given the contract $(\mathbf{a}^R, \sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L), \mathbf{d}(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L), \mathbf{p}(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L))$ and voluntary payments $\tilde{\mathbf{a}}^R$, the renegotiation takes place when the short-term cash flow is x and nature chooses a liquidation-right allocation

$$
\tilde{\mathbf{a}}^L \in \mathbb{L}(\tilde{\mathbf{a}}^R) \equiv \left\{ \mathbf{a}^{L'} \mid x - \tilde{R} \geq \sum_{i \in \{i' \mid a_{i'}^{L'} = 1\}} d_i(\tilde{\mathbf{a}}^R,\tilde{\mathbf{a}}^L) \right\}
$$

following the liquidation-right term $\sigma^L(\tilde{\mathbf{a}}^R;\tilde{\mathbf{a}}^L)$. Consequently, the inefficient liquidation takes place only when

$$
\tilde{\mathbf{a}}^L \notin \mathbb{L}(\tilde{\mathbf{a}}^R) \cap \{\mathbf{0}\}.
$$

Second, the expected enforced payment are dependent of whether $\tilde{\mathbf{a}}^R \neq \mathbf{0}$ or not. Although the enforced payment is certainly enforced when $\tilde{\mathbf{a}}^R \neq \mathbf{0}$, it is stochastically enforced otherwise. Finally, only creditor 1 pays the verification cost, c, if she undertakes the verification activity.

The entrepreneur could deviate the contract if the deviation increased his payoff. The possible deviation is paying $\tilde{\mathbf{a}}^R \neq \mathbf{a}$ instead of $\mathbf{a}$ in the good state. Therefore, the contract must satisfy $\Pi_0(x, \mathbf{a}^R) \geq \Pi_0(x, \tilde{\mathbf{a}})$ for all $\tilde{\mathbf{a}}^R \neq \mathbf{a}$.

Note that the entrepreneur offers a contract in a take-it-or-leave-it manner. Therefore, the optimal contract is such a contract that maximize the entrepreneur's objective

$$\theta\Pi_0(x, \mathbf{a}^R) + (1-\theta)\Pi_0(0, \mathbf{0})$$

subject to participation constraints

$$\theta\Pi_i(x, \mathbf{a}^R) + (1-\theta)\Pi_i(0, \mathbf{0}) \geq s_i K \quad \text{for all } i \in \mathbb{C},$$

incentive compatible constraints

$$\Pi_0(x, \mathbf{a}^R) \geq \Pi_0(x, \tilde{\mathbf{a}}) \quad \text{for all } \tilde{\mathbf{a}}^R \neq \mathbf{a},$$

feasibility constraints ((1), (2), (3), (4), and (5)), and proportionality constraints defined in Definition 1.

Solving the maximization problem, we can show the result in Lemma 3. Note that the lemma just expresses that the debt-like contract is an solution of the maximization problem formulated above.

We provide a sketch of the solution. First, consider the problem without considering the feasibility constraint concerning the repayment (1) in the meanwhile. Then we can show

(i) $\sigma^L(\mathbf{a}^R; \mathbf{0}) = 1$ is optimal; since the liquidation reduces the surplus, it must not take place in the case of execution.

(ii) $d_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = s_i L$ for all i, $\tilde{\mathbf{a}}^R$, and $\tilde{\mathbf{a}}^L$ is optimal; given the result above, setting the dividend as high as possible mitigates the incentive of deviation.

(iii) $\mathbf{p}(\mathbf{a}^R, \tilde{\mathbf{a}}^L) = \mathbf{0}$ for all $\tilde{\mathbf{a}}^L$ is optimal; whether recovering via voluntary payments or enforced payments is indifferent so long as the voluntary payments are positive.

(iv) $\sigma^L(\tilde{\mathbf{a}}^R; \mathbf{0}) = 1$ for all $\tilde{\mathbf{a}}^R \geq \mathbf{a}^R$ is optimal; even when setting the liquidation-right term like this, the entrepreneur has no incentive to make larger repayments.

(v) $\mathbf{p}(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = \mathbf{0}$ for all $\tilde{\mathbf{a}}^R \geq \mathbf{a}^R$ is optimal; even when setting the enforced-payment term like this, the entrepreneur has no incentive to make larger repayments.

(vi) $p_i(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = s_i x - \tilde{a}_i^R$ for all $\tilde{\mathbf{a}}^R$ such that $\tilde{a}_i^R < a_i^R$ for some is for all $\tilde{\mathbf{a}}^L$ is optimal; since it does not produce any inefficiency in equilibrium, setting as high enforced payments in the case of diversion as possible is optimal.

(vii) $\sigma^L(\tilde{\mathbf{a}}^R, \tilde{\mathbf{a}}^L) = \begin{cases} \beta & \text{for } \tilde{\mathbf{a}}^L = \mathbf{1} \\ 1-\beta & \text{for } \tilde{\mathbf{a}}^L = \mathbf{0} \\ 0 & \text{for else} \end{cases}$ for all $\tilde{\mathbf{a}}^R$ such that $\tilde{a}_i^R < a_i^R$ for some is for all $\tilde{\mathbf{a}}^L$ is optimal; Set $\sum_{\tilde{\mathbf{a}}^L \neq \mathbf{0}} \sigma^L(\mathbf{0}; \tilde{\mathbf{a}}^L) = \beta \in [0, 1]$ without loss of generality. Given this, setting $\sigma^L(\mathbf{0}; \mathbf{1}) = \beta$ and $\sigma^L(\mathbf{0}; \tilde{\mathbf{a}}^L) = \mathbf{0}$ for $\tilde{\mathbf{a}}^L \neq \mathbf{0}, \mathbf{1}$ makes $ER(x, \mathbf{0})$ the largest and consequently minimizes $\Pi_0(x, \mathbf{0})$. When setting the liquidation-right term as proposed, $\Pi(x, \tilde{\mathbf{a}}^R) \leq \Pi(x, \mathbf{0})$ holds for all $\tilde{\mathbf{a}}^R$ such that $\tilde{a}_i^R < a_i^R$ for some is.

When setting the terms as proposed, the feasibility constraint concerning the fixed repayments (1) is never binding. Suppose $R > x$ holds in the optimum of the problem without the constraint (1). Then the creditors' expected payoffs are strictly larger than their inputs since

$$\begin{aligned}\theta\Pi_i(x,\mathbf{a}) + (1-\theta)\Pi_i(0,\mathbf{0}) &= \theta s_i \left(ER(\mathbf{a}^R) + ED(\mathbf{a}^R) + ET(\mathbf{a}^R)\right) + (1-\theta)s_i ED(\mathbf{0}) \\ &\geq \theta s_i ER(\mathbf{a}^R) \\ &\geq \theta s_i R \\ &> \theta s_i x \\ &\geq s_i K\end{aligned}$$

following from Assumption 1 (ii). Setting $R = x$ instead and unchanging the other terms increases the entrepreneur's payoff and still satisfies the other constraints; the participation constraints are obviously satisfied following from the above inequality and the incentive compatibility constraints are satisfied since it does not change the incentive of paying nothing.

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