Capital Commitment, a Labour-managed Duopoly, and a Mixed Duopoly: A Labour-managed Firm's Reaction Functions Revisited

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Abstract

Under the linear demand, the slope of a labour-managed (LM) firm's short-run reaction function changes in sign depending on the magnitude of the output elasticity of labour, and its long-run reaction function is negatively sloped. I use a Cournot two-stage game model with capital strategic interaction. In an LM duopoly, whether LM firms employ more capital and produce greater output at strategic equilibria than at nonstrategic equilibria depends on the magnitude of the output elasticity of labour. In a mixed duopoly, the LM firm tends to have more capital at the strategic equilibria than at the nonstrategic equilibria, whereas whether the profit-maximizing firm under-invests depends on the magnitude of the output elasticity of labour for the LM firm. In both duopolies, whether the move between the two equilibria improves welfare also depends on the magnitude of the output elasticity of labour for the LM firm(s).

Keywords: Short-run and long-run reaction functions; Labour-managed and mixed duopolies; Capital commitment

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1 Introduction

There was a noticeable trend in economic theory to consider Japanese large enterprises as 'one type of labour-managed firm' and analyze their behaviour before the Japanese economy plunged into 'the lost decade'. In real world markets there also exist labour-managed oligopolistic enterprises as represented by 'Mondragon' in Spain. This paper focuses on an analysis of the behaviour of a labour-managed (LM) firm in an oligopoly, particularly a Cournot duopoly. Vanek (1970, pp. 114-116) develops a Cournot duopoly model of LM firms. He states the following for an LM firm's reaction functions: (1) although "there is a possibility that either or both of the short-run reaction curves would be negatively sloped", "in the short run the reaction functions should generally be positively sloped"; (2) "in the long-run setting they can be positively or negatively sloped and cannot be expected to have a significant elasticity, whether positive or negative. As a 'central' tendency, it can thus be expected that they will be just about perpendicular to the axes."

To examine the first issue, Miyamoto (1982) deals with a short-run duopolistic situation where labour is the only variable input and fixed costs of production are given. A production function is assumed to be at first convex and then concave in labour. The paper shows that if demand is linear, then the LM firm's short-run reaction function is negatively sloped, perpendicular to its own axis, or positively sloped depending on the amount of fixed costs. Capital is fixed in the short run. Since a given level of physical capital is not always optimally predetermined, it might be said that the above result emerges from a dynamic inefficient choice of fixed costs. If the level of physical capital is optimally determined, could another result follow? The question of deciding on the level of physical capital can be discussed in two cases: one is the case where capital and output are simultaneously determined; the other is that in which in the first stage of a two-stage game, each duopolist sets the level of physical capital, and in the second stage it decides on its output level. The purpose of this paper is to consider the question of what levels of capital and output an LM firm decides on if the simultaneous determination of capital and output is cast in a different context of the two-stage game in which capital commitment is a strategic variable. In other words, this paper compares strategic equilibria where both LM firms' 'short-run' reaction functions meet and nonstrategic ones at which their 'long-run' reaction functions intersect.

It seems that there is no literature on the shape of an LM firm's 'long-run' reaction function as pointed out by Vanek (1970). This situation seems to be closely related to the one referred to as a 'serious problem' by Haruna (2001, p. 61), that is, the problem that a unique interior optimum does not always exist in an analysis of the behaviour of LM firms. Pestieau and Thisse (1979), and Landsberger and Subotnik (1982) prove that if a production function is homogeneous and its degree is equal to or less than unity, then the interior optimum does not exist for the LM monopoly. Ireland and Law (1982, Chapter 6), and Haruna (2001, Chapter 2) discuss the problem of the nonexistence of the unique interior optimum for the LM monopoly. Ireland and Law use homothetic production technology to ensure the existence of such an interior solution. Haruna takes an entry cost into account to deal with the problem in a roundabout way.

It is quite common for game theory to be used to analyze the behaviour of an LM monopolist or oligopolist. Furthermore, in addressing the question of whether it enters an industry, an entry cost that can be viewed as fixed costs is very often taken into account. Stewart (1991), who follows these lines, considers strategic entry interactions both in an LM oligopoly made up of LM firms and in the mixed setting where LM and profit-maximizing (PM) firms may be present. In Cremer and Crémer (1992), firms play a two-stage game: in the first stage they simultaneously decide whether or not to enter, and if a firm enters, it incurs a fixed entry cost; in the second stage the firms that have entered play a Cournot-Nash oligopoly, and capital and labour are simultaneously determined. They make a comparison between equilibrium in the mixed duopoly made up of an LM and a PM firm and that in a PM duopoly composed of two PM firms. Futagami and Okamura (1996) use a three-stage game model of duopoly between an LM and a PM firm to compare the LM firm's capital and output levels and the PM firm's ones in the mixed duopoly. Their framework follows Brander and Spencer (1983), but in their model, if a firm decides to enter the market in the first stage, it incurs a fixed entry cost. Law and Stewart (1983) differ from the literature above in that they consider the question of whether Stackelberg equilibrium occurs in the mixed duopoly. Moreover, Haruna (2001, Chapter 9) makes a comparison between Cournot- and Bertrand-equilibria in an LM duopoly composed of two LM firms and those in a PM duopoly.

Lambertini and Rossini (1998) develop a two-stage game model to analyze the behaviour of LM and PM firms in an LM and a mixed Cournot duopoly with capital strategic interaction. Their model does not include a fixed entry cost, but they assume a Cobb-Douglas technology with constant returns to scale. In the first stage two firms set the level of physical capital, and in the second stage they decide on their output levels. This setting leads them to the result that in the second stage of the game, an LM firm's reaction function is positively sloped and in the mixed duopoly, the LM firm always over-invests, while the PM firm always under-invests. They, however, do not refer to the problem of the nonexistence of a nonstrategic equilibrium, that is, an interior solution at which capital and output is simultaneously determined. The assumption of linearly homogeneous production technology that they use does not ensure the existence of the interior optimum for the LM duopolist. Moreover, taking the result of Miyamoto (1982) into account, it could be said that Lambertini and Rossini (1998) take no account of the fact that in the second stage, the slope of the LM firm's reaction function changes in sign depending on the level of physical capital determined in the first stage.

As stated above, Vanek (1970, pp. 114-116) refers to the shape of an LM firm's 'long-run' reaction function in the sense that capital and output are simultaneously determined. But there seems to be no literature that, by using an LM Cournot duopoly model that does not include a fixed entry cost, discusses the shape of the LM firm's 'long-run' reaction function. Taking no account of the entry cost and assuming linearly homogeneous technology produces the nonexistence of an interior solution for the LM duopolist. Paying close attention to the results of Pestieau and Thisse (1979), and Landsberger and Subotnik (1982), Ireland and Law (1982, Chapter 6) is the first paper to show that a production function is required to be homothetic to ensure the existence of a unique interior optimum for the LM firms. But under homothetic production technology they restrict their analysis to the LM monopoly. Without taking the fixed entry cost into account, first this paper uses the assumption of homothetic technology to define the shape of the LM firm's 'short-run' and 'long-run' reaction functions clearly. Subsequently, by using a two-stage game model of strategic interaction, I compare equilibria generated by the LM firm's 'short-run' and 'long-run' reaction functions in an LM Cournot duopoly, respectively, that is, its capital and output levels at strategic equilibria and those at corresponding nonstrategic Cournot equilibria. Furthermore, the similar problem is dealt with in a mixed duopoly where the LM and the PM firm coexist.

The remainder of this paper is arranged as follows. In Section 2 we describe the model and assumptions. In Section 3, first we investigate the shape of an LM firm's 'short-run' and 'long-run' reaction functions, and then compare a nonstrategic Cournot-Nash equilibrium in an LM duopoly and that in a PM duopoly. Section 4 draws a comparison between strategic equilibria where both LM firms' 'short-run' reaction functions meet and non-strategic ones at which their 'long-run' reaction functions intersect. Section 5 is concerned with the question of whether in a mixed duopoly the LM and the PM firm have a tendency to over- or under-invest. In Section 6 we conduct welfare analysis in an LM and a mixed duopoly, respectively. We conclude in Section 7.

2 The Model

This paper develops a Cournot duopoly model that does not include a fixed entry cost. Assume that two firms produce homogeneous products and face the inverse market demand function $p(Q)$ where the total output Q is the sum of the firms' outputs, q_1 and q_2, and $p(Q)$ is downward-sloping, i.e., $Q = q_1 + q_2$ and $p'(Q) < 0$. The level of capital stock (plant and equipment) and the number of worker-members or employed workers are denoted by K and L, respectively. In the short run, the number of workers is variable and capital is fixed. But the firms have to incur fixed costs $F(= r\bar{K})$ where r is the rental price of one unit of capital and $\bar{K}$ is its predetermined level. In the long run, capital is also variable.[1]

Each firm produces a single product q according to the same homothetic production function $q = g(f(K, L))$, where $q = g(z)$ and $z = f(K, L)$. $g(z)$ is a monotonic increasing function of z that can be thought of as an index of input levels. The function $f(K, L)$ is homogeneous of degree one. In addition, we assume that at first $g''(z) > 0$ and then $g''(z) < 0$ as z increases. The function $f(K, L)$ is assumed to have the following properties:

$$f_K > 0, \quad f_L > 0, \quad f_{KK} < 0, \quad f_{LL} < 0, \quad f_{KL} > 0,$$

where we use subscripts to denote derivatives; f_K stands for $\partial f / \partial K$, for example. For tractability, however, we make use of the labour cost function[2] $L = l(q, K)$ rather than the production function. We place the following assumptions on this labour cost function:[3]

$$l_q > 0, \quad l_K < 0, \quad l_{qq} \gtreqless 0, \quad l_{KK} < 0, \quad l_{qK} < 0. \tag{1}$$

[1]In this paper, 'short-run' refers to the situation where the number of workers adjusts for a given level of capital stock to an equilibrium level, while 'long-run' means that not only the number of workers but also the capital stock is permitted to adjust to market forces. In addition, since we are also concerned with a mixed duopoly where an LM and a PM firm coexist, both firms face the same rental price of capital.

[2]This is termed a labour cost function by Meade (1974).

[3]If K is normal in production, then we have $f_{KL} f_L - f_K f_{LL} > 0$, from which it follows that $l_{qK} < 0$. In addition, since $l_{qq} = -[g''(z) f_L + f_{LL} g'(z)] / \{[g'(z)]^3 (f_L)^3\}$, we obtain $l_{qq} \gtreqless 0 \Leftrightarrow -f_{LL} g'(z) \gtreqless g''(z) f_L$.

Moreover, we assume that l_{qq} is at first negative and then positive as output q increases. This manipulation enables us to treat the level of output instead of the number of workers as a variable for firms.

Let $R^i(q_1, q_2) = p(Q)q_i$ stand for firm i's gross revenue. To ensure the existence of a unique interior optimum, we assume that this revenue function has the property that at first $\partial^2 R^i(z_i)/\partial z_i^2 > 0$ and then $\partial^2 R^i(z_i)/\partial z_i^2 < 0$ as the index of input levels z_i increases.[4]

Following Ward (1958), the appropriate maximand of an LM firm is assumed to be the dividend or income per worker:

$$y^i = \frac{R^i(q_1,\, q_2) - rK_i}{L_i}. \tag{2}$$

On the other hand, the objective function of a PM firm is given by

$$\pi^j = R^j(q_1,\, q_2) - rK_j - wL_j, \tag{3}$$

where w is the market rate of wage. Perfect competition prevails in factor markets.

3 An LM Firm's 'Short-run' and 'Long-run' Reaction Functions, and a Nonstrategic Cournot Equilibrium

This section first investigates the shape of an LM firm's 'short-run' and 'long-run' reaction functions, and then compares the capital and output levels and the capital-labour ratio at a nonstrategic equilibrium in an LM duopoly and those in a PM duopoly. Initially we are concerned with a short-run duopolistic situation where capital K is fixed at $\bar{K}$, so firms incur fixed costs $F(= r\bar{K})$. Suppose that in the short run the LM firm is free to choose its number of workers and hence adjust output q so as to maximize (2). Using subscripts to denote derivatives and differentiating (2) with respect to q_i, we obtain the first- and second-order conditions for maximization:

$$y_i^i \equiv \partial y^i/\partial q_i = [\partial R^i/\partial q_i - y^i(dl^i/dq_i)]/L_i$$

[4]Since output q is a function of z, revenue R is also a function of z. Differentiating revenue R with respect to q yields marginal revenue $R'(q) > 0$. On the other hand, differentiating the revenue function $R(z)$ with respect to z produces $R'(z) = R'(q)g'(z)$. Since $R'(q) > 0$ and $g'(z) > 0$, we obtain $R'(z) > 0$, which means that the revenue function $R(z)$ is also homothetic. $R''(q)$ would typically be expected to be negative. In addition, since $R''(z) = R''(q)[g'(z)]^2 + R'(q)g''(z)$, if at first $g''(z)(> 0)$ is large enough to ensure that $R''(z) > 0$ as z rises, and then $g''(z) < 0$, then the assumption about the curvature of $R(z)$ in the text is satisfied.

$$= (R^i_i - y^i l^i_q)\,/\,L_i = 0, \tag{4}$$

$$y^i_{ii} \equiv \partial^2 y^i\,/\,\partial q_i^2 = (R^i_{ii} - y^i l^i_{qq})\,/\,L_i < 0. \tag{5}$$

Differentiating the first-order condition (4) with respect to q_i and q_j and collecting terms yields

$$dq_i\,/\,dq_j = -(R^i_{ij} - y^i_j l^i_q)\,/\,(R^i_{ii} - y^i l^i_{qq}), \tag{6}$$

which indicates the slope of LM firm i's reaction function. Since the denominator of (6) is negative based on the second-order condition (5), the sign of the slope of the reaction function depends only on the sign of the numerator of (6). Define the elasticity of labour with respect to output (the output elasticity of labour) to be $\omega^i \equiv \partial \log L_i\,/\,\partial \log q_i = (q_i\,/\,L_i)(\partial L_i\,/\,\partial q_i)$. This can be thought of as a function of q_i, i.e., $\omega^i = \omega^i(q_i)$. Letting $H^i_j \equiv R^i_{ij} - y^i_j l^i_q$ in (6) leads to

$$H^i_j = p''(Q) q_i + p'(Q)(1 - \omega^i). \tag{7}$$

Consider the question of whether the slope of an LM firm's short-run reaction function depends on the amount of fixed costs. First we show that the LM firm's output level q_i is an increasing function of fixed costs F_i. Note that $F_i = rK_i$. An increase (or a decrease) in F_i results from an increase (or a decrease) in the rental price of capital r, the capital stock K_i, or both of them. For simplicity, we analyze the effects that a change in r and K_i each has on the output level of LM firm i. Given the other firm's output level q_j, differentiating the first-order condition (4) with respect to q_i and r, and collecting terms produces

$$dq_i\,/\,dr = -K_i l^i_q\,/\,[L_i(R^i_{ii} - y^i l^i_{qq})] > 0. \tag{8}$$

On the other hand, differentiating the first-order condition (4) with respect to q_i and K_i, and collecting terms yields

$$dq_i\,/\,dK_i = [l^i_q(dy^i\,/\,dK_i) + y^i l^i_{qK}]\,/\,(R^i_{ii} - y^i l^i_{qq}) > 0, \tag{9}$$

where $H^i_K \equiv -[l^i_q(dy^i\,/\,dK_i) + y^i l^i_{qK}] = -l^i_q(y^i_L l^i_K + y^i_K) - y^i l^i_{qK}$. This expression can be rewritten as $H^i_K = l^i_q(dF_i\,/\,dK_i)\,/\,L_i + y^i(l^i_q l^i_K\,/\,L_i - l^i_{qK})$. Define the elasticity of substitution σ for the function $f(K, L)$ to be $\sigma \equiv (f_K f_L)\,/\,(z f_{KL})$. If $\sigma \leq 1$, then $H^i_K > 0$ holds true (see Futagami and Okamura, 1996, Lemma 1). It follows from $H^i_K > 0$ and the second-order condition (5) that $dq_i\,/\,dK_i > 0$ holds true.

Thus, it turns out that given the other firm's output level q_j, the LM firm's output level q_i is a monotonic increasing function of fixed costs F_i. We therefore obtain:

Lemma 1 *Given the other firm's output level q_j, there exists some level of output q_i that uniquely corresponds to given F_i.*

Let $\hat{F}_i$ stand for the amount of fixed costs that uniquely corresponds to the output level $\hat{q}_i$ at which $\omega^i = 1$. Substituting $\hat{F}_i$ and $\hat{q}_i$ into the first-order condition, rearranging terms and making use of $\omega^i(\hat{q}_i) = 1$ results in $-p'(\hat{Q})\hat{q}_i^2 = \hat{F}_i$, where $p'(\hat{Q})$ represents the slope of the demand function evaluated at $\hat{q}_i$, given q_j. Then, let $p(\hat{Q})$ be the value of the demand function at $\hat{q}_i$, given q_j.

We therefore establish:

Proposition 1 $\omega^i \lesseqgtr 1$ *if and only if* $-p'(\hat{Q})\hat{q}_i^2 \gtreqless F_i$.

Proof. See Miyamoto (1982).

This proposition tells us on which portion of the labour cost function LM firm i is operating in the short run, given the other firm's output level q_j. Since the situation is rather complicated, we focus on the case in which the market demand function is linear, i.e., $p''(Q) = 0$, following Vanek (1970, p. 115). (7) is then reduced to $H_j^i = p'(Q)(1 - \omega^i)$. In addition, we have $-p'(\hat{Q})\hat{q}_i^2 = s\hat{R}^i/\varepsilon$, where ε is the price elasticity of demand evaluated at $\hat{q}_i$, and $s = \hat{q}_i/\hat{Q}$ and $\hat{R}^i$ are firm i's market share and revenue at $\hat{q}_i$, respectively.

The following are what Proposition 1 shows: when LM firm i's market share is larger and the price elasticity of demand is smaller at its output level where the output elasticity of labour is equal to unity, $s\hat{R}^i/\varepsilon$ tends to be larger than fixed costs F_i. The corresponding slope of the LM firm's short-run reaction function is negative. It is producing on the portion of the labour cost function where the output elasticity of labour is smaller than unity. Conversely, when LM firm i's market share is smaller and the price elasticity of demand is larger at the output level above, $s\hat{R}^i/\varepsilon$ tends to be smaller than fixed costs F_i. The corresponding slope of the LM firm's short-run reaction function is positive. It is producing on the portion of the labour cost function where the output elasticity of labour is larger than unity. When $s\hat{R}^i/\varepsilon$ is equal to fixed costs F_i, the slope of the LM firm's short-run reaction function is perpendicular to its own axis. The output elasticity of labour is equal to unity.[5]

[5] Differentiating (2) with respect to at first q_i and then q_j yields

$$\partial^2 y^i/\partial q_i \partial q_j = (1/L_i)[p''(Q)q_i + p'(Q)(1 - \omega^i)].$$

Thus, if $\omega^i \lesseqgtr 1$ and $p''(Q) = 0$, then $\partial^2 y^i/\partial q_i \partial q_j \lesseqgtr 0$. When the output elasticity of labour is smaller than unity, the other firm's output is a strategic substitute for LM firm i's output, whereas when the output elasticity of labour is larger than unity, the other firm's output is a strategic complement for LM firm i's output. For

An example of the homothetic production function is given by $q = A \log z$ and $z = f(K, L) = \sqrt{KL}$, where $K = \bar{K}$ in the short run. If the linear demand function is given by $p(Q) = a - Q$, then LM firm 1's short-run reaction function is

$$q_1 = \frac{(a - q_2) + A - \sqrt{(a - q_2)^2 + (A^2 - 4F_1)}}{2}, \tag{10}$$

where $a > 0$. In addition, we have $\hat{q}_1 = A/2$. When $A^2 - 4F_1 > 0$, LM firm 1's short-run reaction function is negatively sloped. When $A^2 - 4F_1 = 0$, its reaction function is perpendicular to its own axis. When $A^2 - 4F_1 < 0$, it is positively sloped. These reaction functions are depicted in Figure 1. < Insert Figure 1. > In this example a PM firm's short-run reaction function is negatively sloped.

Consider next the shape of LM firm i's 'long-run' reaction function. Partially differentiating (2) with respect to q_i and K_i, respectively, yields the first-order conditions for maximization:

$$y_i^i = (R_i^i - y^i l_q^i)/L_i = 0, \tag{11}$$

$$y_K^i = (-r - y^i l_K^i)/L_i = 0. \tag{12}$$

(11) and (12) are reduced to (see Ireland and Law, 1982, Chapter 6)

$$Z^i \equiv R_i^i g'(z_i) z_i - R^i = 0. \tag{13}$$

The second-order condition is given by

$$Z_i^i = z_i \{R_{ii}^i [g'(z_i)]^2 + R_i^i g''(z_i)\} < 0. \tag{14}$$

Differentiating (13) with respect to q_i and q_j and collecting terms leads to

$$dq_i/dq_j = -g'(z_i)[R_{ij}^i g'(z_i) z_i - R_j^i] / z_i \{R_{ii}^i [g'(z_i)]^2 + R_i^i g''(z_i)\}. \tag{15}$$

The denominator of (15) is negative based on the second-order condition. Let its numerator be $\gamma \equiv g'(z_i)[R_{ij}^i g'(z_i) z_i - R_j^i]$, which can be rewritten as

$$\gamma = p''(Q) q_i g'(z_i) z_i + p'(Q) q_i (\eta_z^i - 1), \tag{16}$$

where $\eta_z^i \equiv d \log q_i / d \log z_i$. Since $\eta_z^i > 1$,[6] if $p''(Q) = 0$, then we obtain $\gamma < 0$.

We therefore establish:

the concept of strategic substitutability/complementarity concerning the PM and the LM firm, see Bulow, Geanakoplos and Klemperer (1985), and Haruna (2001, pp. 249-250), respectively.

[6]To keep things simple, consider an LM monopoly. In this case, the first-order condition for maximization

Proposition 2 *If the market demand is linear $p''(Q) = 0$, then an* LM *firm's long-run reaction function is negatively sloped.*

Making use of the example above of the homothetic production function yields the following long-run reaction function for LM firm 1:

$$q_1 = \frac{(a - q_2) + 2A - \sqrt{(a - q_2)^2 + 4A^2}}{2}. \quad (17)$$

Compare the output levels at a nonstrategic Cournot equilibrium in an LM duopoly and those in a PM duopoly. Letting $i = 1$, then $i = 2$ in (13) yields a two-equation simultaneous system in unknowns z_1 and z_2 which can be solved to yield $\mathbf{z}^{lm} = (z_1^{lm}, z_2^{lm})$ and corresponding LM Cournot equilibrium outputs $\mathbf{q}^{lm} = (q_1^{lm}, q_2^{lm})$. On the other hand, the first- and second-order conditions for profit maximization are given by, respectively,[7]

$$Z^j \equiv R_j^j g'(z_j) - c = 0, \quad (19)$$

$$Z_j^j = R_{jj}^j [g'(z_j)]^2 + R_j^j g''(z_j) < 0. \quad (20)$$

Letting $j = 1$, then $j = 2$ in (19) first yields a two-equation simultaneous system in unknowns z_1 and z_2, and then produces $\mathbf{z}^{pm} = (z_1^{pm}, z_2^{pm})$ and corresponding PM Cournot equilibrium outputs $\mathbf{q}^{pm} = (q_1^{pm}, q_2^{pm})$. Suppose that each PM duopolist earns nonnegative profits at the PM Cournot equilibrium. Under symmetry $\pi^j(\mathbf{z}^{pm}) = R^j(\mathbf{z}^{pm}) - cz_j^{pm} \geq 0$. Evaluating Z^i for each LM duopolist at this PM equilibrium leads to $R_i^i(\mathbf{q}^{pm})g'(z_j^{pm})z_j^{pm} - R^i(\mathbf{z}^{pm}) = cz_j^{pm} - R^i(\mathbf{z}^{pm}) \leq 0$, from which it follows that $\mathbf{z}^{lm} \leq \mathbf{z}^{pm}$ and $\mathbf{q}^{lm} \leq \mathbf{q}^{pm}$.

We therefore obtain:

Proposition 3 *If each* PM *duopolist earns nonnegative profits at the* PM *Cournot equilibrium, then under symmetry the output levels of each* LM *duopolist at the* LM *Cournot equilibrium are equal to or lower than those of each* PM *duopolist at the* PM *Cournot equilibrium.*

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is given by $R(q) = R'(q)g'(z)z$. Moreover, this equation can be rewritten as $R(q)(1 - \eta_z) = p'(q)qg'(z)z$, the right side of which is negative. This implies that $\eta_z > 1$ holds true. Although it is said that the LM monopoly is operating on an increasing returns portion of the production function in the long run, under the homothetic technology it produces on a portion of the production function where marginal productivity exceeds average productivity in terms of an index of input levels.

[7] Partially differentiate profit (3) with respect to q_j and K_j, and we obtain the first-order conditions,

$$R_j^j - wl_q^j = 0, \quad \text{and} \quad -r - wl_K^j = 0. \quad (18)$$

Letting $c = w / f_L^j = r / f_K^j$ leads to (19) in the text (see Ireland and Law, 1982, Chapter 6).

Moreover, we derive $f^i_L / f^i_K = y^i / r$ from (11) and (12), and $f^j_L / f^j_K = w / r$ from (18). Evaluating (11) and (12) at the PM Cournot equilibrium yields, respectively,

$$R^i_i(\mathbf{q}^{pm})g'(z^{pm}_j)f^i_L - y^i(\mathbf{z}^{pm}) = w - y^i(\mathbf{z}^{pm}) = [cz^{pm}_j - R^i(\mathbf{z}^{pm})] / L^{pm}_j \leq 0, \tag{21}$$

$$-r - y^i(\mathbf{z}^{pm})l^i_K = [y^i(\mathbf{z}^{pm}) - w](r / w) \geq 0. \tag{22}$$

(21) and (22) imply that at the LM Cournot equilibrium each LM duopolist's levels of use of L and K are lower than or equal to, and higher than or equal to those at the PM Cournot equilibrium, given a level of the other input, respectively. Thus, we have $(K_j / L_j)^{pm} \leq (K_i / L_i)^{lm}$.

We therefore have:[8]

Corollary 1 *The capital-labour ratio of each* LM *duopolist at the* LM *Cournot equilibrium is higher than or equal to that of each* PM *duopolist at the* PM *Cournot equilibrium.*

4 An LM Duopoly and Strategic Equilibria

In this section, by using a two-stage game model that does not include a fixed entry cost, we analyze the behaviour of LM firms in an LM duopoly with capital strategic interaction. In the first stage of the game, each LM duopolist sets a level of physical capital, and in the second stage it decides on its output levels. The solution concept is a subgame-perfect Nash equilibrium.[9]

First, focusing on the second stage, we find an optimal level of output for LM firm i, given the capital stock levels, K_1 and K_2, determined in the first stage. The first- and second-order conditions for income per worker maximization are given by (4) and (5), which can be rewritten as

$$H^i(q_i, q_j, K_i) = R^i_i - y^i l^i_q = 0, \tag{23}$$

[8]Given homothetic technology, Ireland and Law (1982) show that a (profitable) PM monopoly produces greater output than an LM monopoly. Gal-Or, Landsberger and Subotnik (1980) obtain the same result on condition that labour is a non-inferior input. Futagami and Okamura (1996) show that in a mixed duopoly the LM duopolist employs a higher capital-labour ratio than the PM duopolist.

[9]Lambertini and Rossini (1998) show that LM firms choose their capital commitments according to the level of the interest rate, while PM firms over-invest regardless of the cost of capital in a PM duopoly. But they do not compare nonstrategic and strategic equilibria in an LM duopoly. This paper uses the assumption of homothetic technology to generate an interior optimum for the LM firms, thereby making a comparison of the equilibria above. This drastically differentiates their analysis from mine.

$$H^i_i(q_i,\, q_j, K_i) = R^i_{ii} - y^i l^i_{qq} < 0, \tag{24}$$

where $H^i_i \equiv \partial H^i / \partial q_i$. To ensure the uniqueness of equilibrium, we use the following condition:

$$\Lambda \equiv H^1_1 H^2_2 - H^1_2 H^2_1 > 0. \tag{25}$$

Differentiating (23) with respect to q_i and q_j, we have the slope of a reaction function for LM firm i,

$$dq_i \,/\, dq_j = -H^i_j \,/\, H^i_i \lesseqgtr 0 \quad \text{if} \quad \omega^i \lesseqgtr 1 \quad \text{and} \quad p''(Q) = 0, \tag{26}$$

where $H^i_j = p''(Q)q_i + p'(Q)(1-\omega^i)$. Let first $i = 1$, then $i = 2$ in (23) and we can write q_1 and q_2 as functions of K_1 and K_2,

$$q_1 = \varphi^1(K_1, K_2); \quad q_2 = \varphi^2(K_1, K_2). \tag{27}$$

Totally differentiating (23) with respect to q_1, q_2, and K_1 yields the following 2×2 simultaneous system:

$$H^1_1 dq_1 + H^1_2 dq_2 = -H^1_K dK_1, \tag{28}$$

$$H^2_1 dq_1 + H^2_2 dq_2 = 0, \tag{29}$$

where $H^1_K \equiv \partial H^1 / \partial K_1 = -l^1_q(y^1_L l^1_K + y^1_K) - y^1 l^1_{qK}$. It turns out that if the elasticity of substitution for the function $f(K, L)$ satisfies $\sigma \leq 1$, then $H^1_K > 0$ holds true.

We therefore obtain:

Lemma 2 $\varphi^i_i \equiv \partial q_i / \partial K_i = -H^i_K H^j_j / \Lambda > 0$; $\varphi^j_i = H^i_K H^j_i / \Lambda \lesseqgtr 0$, if $\omega^j \lesseqgtr 1$ and $p''(Q) = 0$.

If LM firm j produces on the portion of the labour cost function where $l^j_{qq} > 0$ is satisfied,[10] then the sum of output effects with respect to K_i is

$$\varphi^i_i + \varphi^j_i = H^i_K(-H^j_j + H^j_i) \,/\, \Lambda > 0, \tag{30}$$

where $-H^j_j + H^j_i = -p'(Q)[1 + (q_j / L_j) l^j_q] + y^j l^j_{qq}$.

Find next an optimal level of capital stock for LM firm i in the first stage. Substitute (27) into income per worker (2) and we can write y^i as a function of K_1 and K_2,

$$y^i = \Phi^i(\varphi^i(K_1,\, K_2), \varphi^j(K_1,\, K_2), K_i). \tag{31}$$

[10] It is sufficiently probable for the LM firms to produce on the portion above of the labour cost function. See footnote 3.

The Nash equilibrium of this two-stage game occurs when each LM firm maximizes its income per worker with respect to capital, given the level of capital stock chosen by its LM competitor. Partially differentiating (31) with respect to K_i and making use of (23) yields the first-order condition for income per worker maximization for LM firm i,

$$\begin{aligned}\Phi_i^i &= [(R_i^i - y^i l_q^i)\varphi_i^i + R_j^i \varphi_i^j - (r + y^i l_K^i)]\,/\,L_i \\ &= [R_j^i \varphi_i^j - (r + y^i l_K^i)]\,/\,L_i = 0.\end{aligned} \tag{32}$$

The second-order condition is

$$\Phi_{ii}^i = [R_j^i \varphi_{ii}^j + \varphi_i^j dR_j^i\,/\,dK_i - y^i(l_{KK}^i + l_{qK}^i \varphi_i^i)]\,/\,L_i < 0. \tag{33}$$

We use the following condition:

$$|\Phi_{ii}^i| > |\Phi_{ij}^i|, \tag{34}$$

where[11]

$$\Phi_{ij}^i = [R_j^i \varphi_{ij}^j + \varphi_i^j dR_j^i\,/\,dK_j - (\partial y^i\,/\,\partial K_j) l_K^i - y^i l_{qK}^i \varphi_j^i]\,/\,L_i. \tag{35}$$

(34) implies

$$\Sigma \equiv \Phi_{11}^1 \Phi_{22}^2 - \Phi_{12}^1 \Phi_{21}^2 > 0, \tag{36}$$

which is the Routh-Hurwicz condition for reaction function stability in capital stock space.

On the other hand, letting $i = 1$, then $i = 2$ in the first-order conditions (11) and (12) produces a four-equation simultaneous system in unknowns q_i and K_i. Solving this simultaneous system generates a nonstrategic Cournot equilibrium. Let $\mathbf{q}^C = (q_1^C, q_2^C)$ and $\mathbf{K}^C = (K_1^C, K_2^C)$ denote the optimal levels of output and the corresponding optimal levels of capital at the nonstrategic equilibrium, respectively. Evaluating Φ_i^i at this equilibrium yields

$$\Phi_i^i(\mathbf{K}^C) = R_j^i \varphi_i^j\,/\,L_i. \tag{37}$$

It follows from Lemma 2 that if $\omega^j \lesseqgtr 1$ and $p''(Q) = 0$, then $\varphi_i^j \lesseqgtr 0$.

We therefore establish the following proposition concerning the determination of capital stock level (see the Mathematical Appendix for the proof):

[11] Find out the sign of each term on the right side of (35). We have $dR_j^i\,/\,dK_j = R_{ji}^i \varphi_j^i + R_{jj}^i \varphi_j^j$, but if the linear demand and symmetry are assumed, then $\varphi_i^j dR_j^i\,/\,dK_j = (\varphi_j^i)^2 R_{ji}^i < 0$. In addition, $\partial y^i\,/\,\partial K_j = R_j^i \varphi_j^j\,/\,L_i < 0$. The sign of φ_{ij}^j of $R_j^i \varphi_{ij}^j$ is ambiguous. But under symmetry, using the examples above of the homothetic production function and the linear demand produces $\varphi_{ij}^j = 8r^2\,/\,(4a^2 - 4aA + 9A^2 - 32F_j)^{3/2} > 0$. In addition, we have $\varphi_j^i \gtreqless 0 \Rightarrow y^i l_{qK}^i \varphi_j^i \lesseqgtr 0$. If $\varphi_j^i < 0$, then $\Phi_{ij}^i < 0$. Using the examples above leads to the result that $\Phi_{ij}^i < 0$. It is likely that we will obtain $dK_i\,/\,dK_j = -\Phi_{ij}^i\,/\,\Phi_{ii}^i < 0$.

Proposition 4 *Suppose that the market demand function is linear, i.e.,* $p''(Q) = 0$.

(i) (a) *If the output elasticity of labour for each* LM *duopolist is smaller than unity, then at the strategic equilibrium the total capital stock of both* LM *duopolists is greater than at the corresponding nonstrategic Cournot equilibrium.* (b) *If the output elasticity of labour is equal to unity, then the total capital stock is on the same level at both the strategic and the nonstrategic equilibrium.* (c) *If the output elasticity of labour is larger than unity, then the converse is true.*

(ii) *Suppose perfect symmetry.* (a) *If the output elasticity of labour for each* LM *duopolist is smaller than unity, then at the strategic equilibrium its level of capital stock is higher than at the corresponding nonstrategic equilibrium.* (b) *If the output elasticity of labour is equal to unity, then each* LM *duopolist sets the identical level of capital stock at both the strategic and the nonstrategic equilibrium.* (c) *If the output elasticity of labour is larger than unity, then the converse is true.*

Moreover, we obtain the following proposition for the output level and income per worker of each LM firm:

Proposition 5 *Suppose the linear demand, i.e.,* $p''(Q) = 0$, *and perfect symmetry.*

(i) (a) *If the output elasticity of labour for each* LM *duopolist is smaller than unity, then at the nonstrategic equilibrium it chooses greater output than at the nonstrategic equilibrium.* (b) *If the output elasticity of labour is equal to unity, then it produces the identical level of output at both the strategic and the nonstrategic equilibrium.* (c) *If the output elasticity of labour is larger than unity, then at the strategic equilibrium it produces smaller output than at the nonstrategic equilibrium.*

(ii) (a) *If the output elasticity of labour is smaller than unity, then at the strategic equilibrium the income per worker of each* LM *duopolist is smaller than at the nonstrategic equilibrium.* (b) *If the output elasticity of labour is equal to unity, then it is the same at both the strategic and the nonstrategic equilibrium.* (c) *If the output elasticity of labour is larger than unity, then the converse is true.*

Propositions 4 and 5 lead to the corollary for the shape of an LM firm's short-run reaction function:

Corollary 2 *In an* LM *Cournot duopoly model in which capital commitment is a strategic variable, each* LM *firm sets a level of physical capital corresponding to the slope of its 'short-run' reaction function that is negative, perpendicular to its own axis, or positive, according as the output elasticity of labour is smaller than, equal to, or larger than unity in output space.*

In addition, making use of Proposition 3 yields:

Corollary 3 *If the output elasticity of labour for each* LM *firm is equal to or larger than unity, then the levels of both its output and the industry output at the strategic equilibria of the* LM *duopoly are equal to or smaller than those at the strategic equilibrium of the* PM *duopoly.*

In the LM duopoly the outputs of the two LM firms are strategic substitutes if $\omega^i < 1$ and strategic complements if $\omega^i > 1$. Thus, when $\omega^i < 1$, they have a tendency to increase the total amount of capital stock, increase total output, and diminish the income per worker, whereas when $\omega^i > 1$, they have an opposite tendency. The sharp contrast between the PM and the LM duopoly is that in the former the outputs of the two PM firms are strategic substitutes and so these firms have a tendency to increase the total amount of capital stock and total output, while in the latter, when $\omega^i > 1$, unlike the PM firms, the LM firms have a tendency to reduce them.

Consider first the case of $\omega^i < 1$ to give an intuitive explanation of these results. Since each LM firm's short-run reaction function is negatively sloped, the same remarks as made for the PM duopoly apply to this case. Suppose that one LM firm raises its own level of capital stock in the first stage while the other LM firm ignores the possibility of its strategic use. Then, the former increases its output level and income per worker in the second stage, but the latter reduces its output level and income per worker. Thus, it also increases its own level of capital stock in the first stage.

Turn next to the case of $\omega^i > 1$ in which the slope of each LM firm's short-run reaction function is positive. Suppose that one LM firm reduces its own level of capital stock in the first stage while the other LM firm's level of the capital stock is held constant. Then, the former decreases the output level and increases its income per worker in the second stage, whereas the latter increases its output level and reduces income per worker. Hence, it reduces its own level of capital stock in the first stage, too.

We use the examples above of the homothetic production function and the linear demand to illustrate these propositions and corollaries with figures (see Figure 2).

Figure 2–(a) corresponds to the case in which the output elasticity of labour for each LM duopolist is smaller than unity. In this case the same result emerges in both the LM and the PM duopoly (see Brander and Spencer, 1983, Propositions 2 and 3). Each LM firm's short-run reaction function is negatively sloped.[12] Figure 2–(b) is concerned with

[12]Figure 2–(a) shows that each PM firm's output at the nonstrategic equilibrium of the PM duopoly is greater than each LM firm's one at the strategic equilibrium of the LM duopoly. Corollary 3 does not refer to this situation. But the numerical calculation using the examples above of homothetic technology and linear

the case where the output elasticity of labour is equal to unity and thus both the strategic and the nonstrategic equilibrium coincide. Each LM firm's short-run reaction function is perpendicular to its own axis. In Figure 2–(c) the output elasticity of labour is larger than unity, and so each LM firm's short-run reaction function is positively sloped.

A PM duopoly does not have cases corresponding to Figures 2–(b) and (c). The clear contrast between the LM and the PM duopoly is that the levels of both capital and output that each LM duopolist chooses and the slope of its short-run reaction function change according as the output elasticity of labour for the LM firm is smaller than, equal to, or larger than unity.

5 A Mixed Duopoly and Capital Commitment

Let us compare strategic and nonstrategic Cournot equilibria in a mixed duopoly made up of an LM and a PM firm. In this section, i and j that are used as superscripts or subscripts refer to the LM and the PM firm, respectively.

We first deal with a Cournot two-stage game in which capital commitment is a strategic variable. In the second stage, the first- and second-order conditions for income per worker maximization are given by (23) and (24). The corresponding first- and second-order conditions for profit maximization are

$$G^j(q_i, q_j, K_j) = R^j_j - wl^j_q = 0, \tag{38}$$

$$G^j_j(q_i, q_j, K_j) = R^j_{jj} - wl^j_{qq} < 0. \tag{39}$$

In addition, $G^j_i = R^j_{ji} < 0$. Differentiating the first-order condition (38) with respect to q_j and q_i produces the slope of the PM firm's short-run reaction function,

$$dq_j\,/\,dq_i = -G^j_i\,/\,G^j_j < 0. \tag{40}$$

Differentiating (23) and (38) with respect to q_i, q_j, K_i, and K_j, respectively, we obtain the following two-equation simultaneous system:

$$H^i_i dq_i + H^i_j dq_j = -H^i_K dK_i, \tag{41}$$

$$G^j_i dq_i + G^j_j dq_j = -G^j_K dK_j, \tag{42}$$

where $G^j_K = -wl^j_{qK} > 0$. To ensure the uniqueness of equilibrium, we assume

$$\Gamma \equiv H^i_i G^j_j - H^i_j G^j_i > 0. \tag{43}$$

demand yields the result as illustrated in the Figure.

q_i and q_j can be rewritten as functions of K_i and K_j,

$$q_i = \phi^i(K_i, K_j); \quad q_j = \phi^j(K_i, K_j). \tag{44}$$

Differentiating (44) with respect to K_i and K_j yields

$$\phi_i^i = -H_K^i G_j^j / \Gamma > 0 \quad \text{if} \quad \sigma \leq 1, \quad \text{and} \quad \phi_j^i = H_j^i G_K^j / \Gamma \lesseqgtr 0 \quad \text{if} \quad \omega^i \lesseqgtr 1, \tag{45}$$

$$\phi_i^j = H_K^i G_i^j / \Gamma < 0, \quad \text{and} \quad \phi_j^j = -H_i^i G_K^j / \Gamma > 0, \tag{46}$$

where ϕ_j^i stands for $\partial q_i / \partial K_j$, for example. If each firm produces on the portion of the labour cost function where $l_{qq}^{i(j)} > 0$, then the sums of output effects with respect to K_i and K_j are given by, respectively,

$$\phi_i^i + \phi_i^j = H_K^i(-G_j^j + G_i^j) / \Gamma > 0 \quad \text{and} \quad \phi_j^i + \phi_j^j = G_K^j(H_j^i - H_i^i) / \Gamma > 0, \tag{47}$$

where $-G_j^j + G_i^j = -p'(Q) + wl_{qq}^j > 0$.

In the first stage, the income per worker and profits are represented by, respectively,

$$y^i = \Phi^i(\phi^i(K_i, K_j), \phi^j(K_i, K_j), K_i), \tag{48}$$

$$\pi^j = \Pi^j(\phi^i(K_i, K_j), \phi^j(K_i, K_j), K_j). \tag{49}$$

The first- and second-order conditions for maximization of (48) with respect to K_i are given by (32) and (33) in which 'φ' is replaced by 'ϕ'. The same remark applies to Φ_{ij}^i. On the other hand, partially differentiating (49) with respect to K_j leads to the following first- and second-order conditions for profit maximization:

$$\begin{aligned} \Pi_j^j &= R_j^j \phi_j^j + R_i^j \phi_j^i - wl_q^j \phi_j^j - r - wl_K^j \\ &= R_i^j \phi_j^i - r - wl_K^j = 0, \end{aligned} \tag{50}$$

$$\Pi_{jj}^j = (R_{ii}^j \phi_j^i + R_{ij}^j \phi_j^j)\phi_j^i + R_i^j \phi_{jj}^i - w(l_{qK}^j \phi_j^j + l_{KK}^j) < 0. \tag{51}$$

Moreover, Π_{ji}^j is given by

$$\Pi_{ji}^j = (R_{ii}^j \phi_i^i + R_{ij}^j \phi_i^j)\phi_j^i + R_i^j \phi_{ji}^i - wl_{qK}^j \phi_i^j. \tag{52}$$

In this section we assume the condition (34). The following condition is also used:

$$|\Pi_{jj}^j| > |\Pi_{ji}^j|. \tag{53}$$

(34) and (53) imply, together with (33) and (51),

$$\Omega \equiv \Phi_{ii}^i \Pi_{jj}^j - \Phi_{ij}^i \Pi_{ji}^j > 0. \tag{54}$$

We therefore establish (see the Mathematical Appendix for the proof):

Proposition 6 *Suppose that the market demand is linear, i.e., $p''(Q) = 0$.*

(a) *If the output elasticity of labour for an* LM *firm is smaller than unity, then at the strategic equilibrium the total capital stock of both* LM *and* PM *firms is greater than at the corresponding nonstrategic Cournot equilibrium. Furthermore, if the own effects of each firm's capital stock on marginal profit (marginal income per worker) dominate its cross effects on marginal income per worker (marginal profit), then at the strategic equilibrium each firm's level of capital stock is higher than at the nonstrategic equilibrium.*

(b) *If the output elasticity of labour for the* LM *firm is equal to unity, then at the strategic equilibrium the total capital stock of both* LM *and* PM *firms is greater than at the nonstrategic equilibrium. Furthermore, at the strategic equilibrium, the* LM *firm's capital stock is greater than at the nonstrategic equilibrium, while the* PM *firm chooses the efficient level of capital that is defined as that minimizing total production costs. But it is ambiguous whether at the strategic equilibrium the* PM *firm sets a lower or higher level of capital stock than at the nonstrategic equilibrium.*

(c) *When the output elasticity of labour for the* LM *firm is larger than unity, if the own effects of each firm's capital stock on marginal profit (marginal income per worker) dominate its cross effects on marginal income per worker (marginal profit), then at the strategic equilibrium, the* LM *firm's capital stock is greater than, while the* PM *firm sets a lower level of capital stock than, at the nonstrategic equilibrium.*

A mixed duopoly shows a sharp contrast with an LM or a PM duopoly. In the LM duopoly a level of physical capital that each LM firm chooses changes according as the output elasticity of labour is smaller than, equal to, or larger than unity, while in the mixed duopoly the LM firm always employs more capital at strategic equilibria than at nonstrategic equilibria. In contrast, in the mixed duopoly, whether the PM firm over- or under-invests depends on the magnitude of the output elasticity of labour for its LM competitor, that is, if $\omega^i < 1$, then at the strategic equilibrium the PM firm chooses more capital than at the nonstrategic equilibrium, while if $\omega^i > 1$, then the converse is true. This contrast results from the fact that the LM firm's output is a strategic substitute for that of the PM firm, whereas the PM firm's output is a strategic substitute if $\omega^i < 1$ and a strategic complement if $\omega^i > 1$, for the LM firm's output.

Moreover, we establish:

Proposition 7 *Suppose the linear demand, i.e., $p''(Q) = 0$.*

(i) (a) *If the output elasticity of labour for the* LM *firm is smaller than unity, then at the strategic*

equilibrium the industry output is greater than at the nonstrategic equilibrium. At the former at least one of the two firms' outputs is greater than at the latter. (b) *If the output elasticity of labour for the* LM *firm is equal to unity, then the* PM *firm's 'long-run' and 'short-run' reaction functions intersect at the strategic equilibrium. At the strategic equilibrium the* LM *firm produces greater output than at the nonstrategic equilibrium.* (c) *If the output elasticity of labour for the* LM *firm is larger than unity, then at the strategic equilibrium the* PM *firm produces smaller output than at the nonstrategic equilibrium. But it is ambiguous whether at the former the* LM *firm produces greater or smaller output than at the latter.*
(ii) (a) *When the output elasticity of labour for the* LM *firm is smaller than unity, at the strategic equilibrium the* LM *firm is likely to obtain larger income per worker than, and the* PM *firm is likely to earn less profit than at the nonstrategic equilibrium.* (b) *Suppose that the output elasticity of labour for the* LM *firm is equal to unity.* [*i*] *If at the strategic equilibrium the* PM *firm sets a higher level of capital stock than at the nonstrategic equilibrium, then at the strategic equilibrium the* LM *firm's income per worker is likely to be larger than at the nonstrategic equilibrium. At the former the* PM *firm's profits are smaller than at the latter.* [*ii*] *If at the strategic equilibrium the* PM *firm's capital stock is at the same level as, or smaller than at the nonstrategic equilibrium, then at the former the* LM *firm's income per worker is larger than, and the* PM *firm's profits are smaller than at the latter.* (c) *If the output elasticity of labour for the* LM *firm is larger than unity, then at the strategic equilibrium the* LM *firm's income per worker is greater than at the nonstrategic equilibrium, while the* PM *firm's profits are likely to be smaller.*

Even if an LM duopolist's competitor is PM, its short-run reaction function is negatively sloped, perpendicular to its own axis, or positively sloped according to $\omega^i \lesseqgtr 1$. Proposition 7 does not always give a definite answer to the question of whether at a strategic equilibrium each firm produces greater or smaller output than at a nonstrategic equilibrium. Let us take an example to compare strategic and nonstrategic equilibria in output space. It turns out that an LM duopolist always produces on the portion of the labour cost function where marginal productivity is greater than average productivity in terms of an index of input levels. Thus, we can approximate the homothetic production function by $q = z - 1$ and $z = \sqrt{KL}$. Let the market demand function be $p = a - bQ$, where $a > 0$ and $b > 0$.

In Figure 3 the shape of the reaction functions is illustrated according to $\omega^i \lesseqgtr 1$. <Insert Figure 3.> Figure 3 shows that however large ω^i may be, at the strategic equilibrium the LM firm produces greater output than at the nonstrategic equilibrium, whereas the PM firm chooses smaller output. Furthermore, when $\omega^i = 1$, the LM firm chooses the corresponding level of output $\hat{q}_i = 1$. In this case, note that the LM firm's short-run reaction function that is perpendicular to its own axis intersects with the PM firm's short-run reaction function at the point where the latter long-run and short-run reaction functions meet.

Let us give an intuitive explanation of these results. Since it is ambiguous whether the move from a nonstrategic equilibrium to a strategic equilibrium leads to an increase or a decrease in the LM firm's output levels in the cases of $\omega^i \lesseqgtr 1$ and the PM firm's output level in the case of $\omega^i > 1$, we focus on the signs of a change in the levels of output as indicated in Figure 3 and thus when $\omega^i < 1$, $\Delta q_i > 0$, $\Delta q_j < 0$ and also $\Delta Q > 0$. Suppose that the LM firm raises its own level of capital stock while the PM firm ignores the possibility of its strategic use. Then, the former increases its output level and income per worker in the second stage, while the latter reduces its output level and profits in the second stage. Thus, it also increases its own level of capital stock to prevent its market share and profits from falling drastically. On the other hand, suppose that the PM firm raises its own level of capital stock, whereas the LM firm ignores the possibility of its strategic use. Then, the PM firm increases its output level and profits in the second stage, while the LM firm reduces its output level and income per worker in the second stage. Hence, it also raises its own level of capital stock.

If $\omega^i > 1$, then $\Delta q_i > 0$ and $\Delta q_j < 0$. Suppose that the LM firm raises its own level of capital stock while the PM firm's level of capital stock is kept constant. Then, the former increases its output level and income per worker in the second stage, while the latter reduces its profits and output level in the second stage. Thus, it reduces its own level of capital stock to earn more profits. On the other hand, suppose that the PM firm reduces its own level of capital stock to gain while the LM firm's level of capital stock is kept constant. Although the latter increases its output level and income per worker in the second stage, it raises its own level of capital stock to increase its market share and income per worker greatly.

Based on Propositions 6 and 7, we cannot make a definite statement about the question of whether at a strategic equilibrium of a mixed duopoly the capital and output levels of an LM firm are higher or lower than those of a PM firm.[13] However, we can obtain the following result concerning the capital-output ratio at the strategic equilibria of the mixed

[13]It is because in comparing strategic and nonstrategic equilibria, an absolute value of an increment or deduction in each firm's capital and output is ambiguous.

duopoly; replacing φ_i^j with ϕ_i^j in (32) and rewriting it yields

$$r + wl_K^i = R_j^i \phi_i^j - (y^i - w) l_K^i. \tag{55}$$

Making use of $\phi_i^j < 0$, "$\phi_j^i \lesseqgtr 0$ if $\omega^i \lesseqgtr 1$ and $p''(Q) = 0$", and (50) produces:

Proposition 8 *Suppose the linear demand, i.e., $p''(Q) = 0$, and $y^i \geq w$.*
(a) *If the output elasticity of labour for an* LM *firm is smaller than unity, then the capital-labour ratios of both the* LM *and the* PM *firm are higher than that of the cost-minimizing firm, i.e., $f_L^i / f_K^i > y^i / r \geq w / r$ and $f_L^j / f_K^j > w / r$.*
(b) *If the output elasticity of labour for the* LM *firm is equal to or larger than unity, then the capital-labour ratio of the LM firm is higher than, whereas the capital-labour ratio of the* PM *firm is equal to or lower than, that of the cost-minimizing firm, i.e., $f_L^i / f_K^i > w / r \geq f_L^j / f_K^j$.*

Compare the output levels of both an LM and a PM firm at a nonstrategic Cournot equilibrium of a mixed duopoly. The first-order conditions for the LM and the PM duopolist are given by Z_i in (13) and Z_j in (19), respectively.[14] Suppose that $y^i = (R^i - rK_i) / L_i \geq w$ at this equilibrium. It follows from this inequality that $R^i - cz_i \geq 0$. Using the first-order conditions (13) and (19) leads to $R_i^i g'(z_i) - c \geq R_j^j g'(z_j) - c = 0$, which means that $z_i \leq z_j$ and so $q_i \leq q_j$.

We therefore establish:

Proposition 9 *At a nonstrategic mixed Cournot equilibrium, if the income per worker of an* LM *firm is greater than or equal to the market rate of wage, then the output level of the* LM *duopolist is equal to or lower than that of the* PM *duopolist.*

Moreover, $f_L^i / f_K^i = y^i / r$ is derived from (11) and (12), and $f_L^j / f_K^j = w / r$ from (18). At the nonstrategic Cournot equilibrium, the supposition of $y^i \geq w$ yields $f_L^i / f_K^i \geq w / r = f_L^j / f_K^j$, from which it follows that $K_i / L_i \geq K_j / L_j$.

We therefore establish:

Corollary 4 *At the nonstrategic mixed Cournot equilibrium the capital-labour ratio of the* LM *duopolist is higher than or equal to that of the* PM *duopolist.*

[14] To ensure the uniqueness of equilibrium, we assume that $\Theta \equiv Z_i^i Z_j^j - Z_j^i Z_i^j > 0$, where $Z_j^i = [R_{ij}^i g'(z_i) g'(z_j) - (R_j^i / z_j) g'(z_j)] z_j$ and $Z_i^j = R_{ji}^j g'(z_i) g'(z_j)$.

6 Welfare Analysis

This section is concerned with an analysis of the welfare effects of changes in capital stock invested by each firm at first in an LM duopoly and then in a mixed duopoly. To measure welfare changes in the LM duopoly, let us define social welfare as the sum of consumer and producer surplus. This welfare is given by

$$W(K_i, K_j) = \int_0^{q_i+q_j} p(\chi)d\chi - [(rK_i + wL_i) + (rK_j + wL_j)], \tag{56}$$

where $q_i = \varphi^i(K_i, K_j)$. Differentiating (56) with respect to K_i yields

$$W_i \equiv \frac{\partial W}{\partial K_i} = [p(Q) - wl_q^i]\varphi_i^i + [p(Q) - wl_q^j]\varphi_i^j - (r + wl_K^i). \tag{57}$$

From (4) $p(Q) - y^i l_q^i = -p'(Q)q_i$. Under perfect symmetry, (57) can be rewritten as

$$\begin{aligned} W_i &= [p(Q) - wl_q^i](\varphi_i^i + \varphi_i^j) - (r + wl_K^i) \\ &= -p'(Q)q_i(\varphi_i^i + \varphi_i^j) - (r + y^i l_K^i) + (y^i - w)l_q^i(\varphi_i^i + \varphi_i^j) + (y^i - w)l_K^i. \end{aligned} \tag{58}$$

We have $-p'(Q)q_i > 0$ and $\varphi_i^i + \varphi_i^j > 0$. Furthermore, it should be noted that it suffices for an absolute value of $(y^i - w)l_K^i$ to be sufficiently small to obtain a higher probability of having $W_i > 0$. If the difference between the income per worker of each LM firm y^i and the market rate of wage w is small, each LM firm employs more capital-intensive technology, or both conditions are met simultaneously, $|(y^i - w)l_K^i|$ is sufficiently small (hereafter condition A).[15]

The results about the welfare effects of changes in capital stock invested by each LM duopolist are summarized in the following proposition (see *Welfare Analysis* (i), (ii) and (iii) in the Mathematical Appendix):

Proposition 10 *Suppose that the difference between the income per worker of each* LM *firm and the market rate of wage is small, it employs more capital-intensive technology, or both conditions are met simultaneously.*

(i) *At the nonstrategic equilibrium, a small increase in capital stock by either* LM *firm is welfare improving;*

(ii) *Assuming symmetry, the move from the nonstrategic equilibrium to the strategic equilibrium improves welfare, keeps it unchanged, or deteriorates it according as the output elasticity of labour is smaller than, equal to, or greater than unity.*

(iii) *At the strategic equilibrium, the same remark as* (i) *applies here.*

[15] Since worker-members must have an incentive to work for the LM firm, I assume that $y^i \geq w$.

Let W^{pm} and W_j^{pm} denote social welfare in a PM duopoly and differentiation of the social welfare function with respect to PM firm j's capital stock, respectively.[16] In the theoretical framework of this paper, assuming perfect symmetry, we have $W_j^{pm} > 0$ at both strategic and nonstrategic equilibria, whereas under condition A, $W_i^{lm} > 0$ holds true.

Another remarkable contrast between an LM and a PM duopoly is that while $\Delta W^{pm} > 0$, whether the move from a nonstrategic equilibrium to a strategic equilibrium is welfare improving in an LM duopoly depends on the magnitude of the output elasticity of labour, i.e., $\Delta W^{lm} \gtreqless 0$ according as $\omega^i \lesseqgtr 1$. This contrast results from the fact that the outputs of the two firms are strategic substitutes in the PM duopoly, whereas in the LM duopoly those of the two firms are strategic substitutes if $\omega^i < 1$ and strategic complements if $\omega^i > 1$. Furthermore, since we have already obtained the result that the move from the nonstrategic equilibria to the strategic equilibria causes the LM industry output to increase, remain unchanged, or decrease according as the output elasticity of labour is smaller than, equal to, or larger than unity, the contrast above is not counterintuitive.

Consider next the welfare effects of changes in capital stock by an LM and a PM firm at nonstrategic and strategic equilibria, and the question of whether the move from the former to the latter is welfare improving, in a mixed duopoly. In addition, let i and j used as superscripts or subscripts refer to the LM and the PM firm, respectively.

Differentiation of the social welfare function (56) with respect to K_i and K_j yields

$$\text{(LM)}: \quad W_i = -p'(Q)(q_i\phi_i^i + q_j\phi_i^j) - (r + y^i l_K^i) + (y^i - w)l_q^i\phi_i^i + (y^i - w)l_K^i, \tag{59}$$

$$\text{(PM)}: \quad W_j = -p'(Q)(q_i\phi_j^i + q_j\phi_j^j) - (r + wl_K^j) + (y^i - w)l_q^i\phi_j^i, \tag{60}$$

where $q_i = \phi^i(K_i, K_j)$ and $q_j = \phi^j(K_i, K_j)$.

We summarize the results of a mixed duopoly in the following proposition (see *Welfare Analysis* (iv), (v) and (vi) in the Mathematical Appendix):

Proposition 11 *Suppose that the difference between the income per worker of each* LM *firm and the market rate of wage is small, it employs more capital-intensive technology, or both conditions are met simultaneously.*
(iv) *If the* PM *firm's output level is smaller than double the* LM *firm's one at the nonstrategic equilibrium, then a small increase in capital stock by the* LM *is welfare improving. On the other hand, if the former is greater than* $(1 - \omega^i)/2$ *times the latter at the nonstrategic equilibrium, then*

[16] Superscripts pm and lm refer to a PM and an LM duopoly, respectively.

that in capital stock by the PM *firm is welfare improving.*

(v) *Suppose that the* LM *firm's output level is greater than the* PM *firm's one at some point between the nonstrategic and the strategic equilibria. Then, the move from the former to the latter is welfare improving when the output elasticity of labour is smaller than or equal to unity. On the other hand, when the output elasticity of labour is larger than unity, it is ambiguous whether the move above is welfare improving. However, if an increment in welfare due to a change in capital stock by the* PM *firm largely dominates that by the* LM *firm, then the move above leads to deterioration in welfare when the output elasticity of labour is greater than unity.*

(vi) *At the strategic equilibrium, if the* LM *firm's output level is greater than the* PM *firm's one, then a small increase in capital stock by the* LM *is welfare improving. On the other hand, if the* PM *firm' output level is greater than* $\frac{1-\omega^i}{1+\omega^i}$ *times the* LM *firm's one, then it is highly likely that a small increase in capital stock by the* PM *firm will lead to improvement in welfare.*

Since both LM and PM firms' long-run reaction functions are negatively sloped, the result of part (iv) of this proposition is the same as that obtained in the PM duopoly.

It suffices to have the condition (hereafter condition B) that the LM firm should produce at least as much output as the PM firm, to obtain the results of parts (v) and (vi). Under condition B a small increase in capital stock by the LM firm increases welfare. It is because imposing not only condition A but B on the LM firm implies that it should produce as efficiently as the PM firm in terms of output expansion effect.

Turn to the result of part (v). Since the PM firm's output is a strategic complement for the LM firm's one while the latter is a strategic substitute for the former, there is a high probability that an increment in welfare due to a change in capital stock by the PM firm largely dominates that by the LM firm. Then, since the PM firm tends to reduce its own level of capital stock when $\omega^i > 1$, the move above may be welfare deteriorating.

In general, a small increase in capital stock by the PM firm at the strategic equilibrium leads to the result that $W_j > 0$, whereas that by the LM firm does not always yield $W_i > 0$, depending on the demand and cost conditions. When $\omega^i < 1$, it is likely that we will have $W_i > 0$ more easily, because the price elasticity of demand is small and the LM firm's market share is large. However, when the price elasticity of demand is larger and the LM firm's market share is not so large, we obtain $\omega^i > 1$. Consequently, the probability of being $W_i < 0$ is higher. If the difference between the LM firm's income per worker and the market rate of wage is significant, or the LM firm does not employ so capital-intensive technology, then we have $W_i < 0$ easily. This means that at the strategic equilibrium the

LM firm might over-invest beyond a welfare maximum in the mixed duopoly.

7 Conclusion

First this paper showed that under the linear demand, a labour-managed firm's 'short-run' reaction function is negatively sloped, perpendicular to its own axis, or positively sloped depending on the magnitude of the output elasticity of labour, and that its 'long-run' reaction function is negatively sloped.

Then, using a Cournot two-stage game model with capital strategic interaction, I compared strategic and nonstrategic Cournot equilibria. In an LM duopoly, when the output elasticity of labour for each LM firm is smaller than unity, at the strategic equilibrium it employs more capital, produces greater output and earns smaller income per worker than at the nonstrategic equilibrium. When the output elasticity of labour is equal to unity, the strategic and the nonstrategic equilibrium coincide where the capital stock, output and income per worker of the LM firms are at the identical levels. When it is larger than unity, at the strategic equilibrium each LM firm has less capital and smaller output, and obtains more income per worker than at the nonstrategic equilibrium. It is the output elasticity of labour for each LM firm that has a decisive influence in determining the level of physical capital corresponding to the slope of the LM firm's short-run reaction function which is negatively sloped, perpendicular to its own axis, or positively sloped.

In a mixed duopoly, the LM firm tends to have the higher level of capital at the strategic equilibria than at the nonstrategic equilibria, whereas whether the PM firm under-invests depends on the magnitude of the output elasticity of labour for its LM competitor. In contrast, at the strategic equilibria the LM firm is likely to earn greater income per worker than, and the PM firm tends to earn less profit than at the nonstrategic equilibria, regardless of the magnitude of the output elasticity of labour for the LM firm. Furthermore, at the nonstrategic mixed Cournot equilibrium the LM firm produces the same output as or smaller output than the PM firm.

In an LM duopoly, if the difference between the income per worker of each LM firm and the market rate of wage is small, each LM firm employs more capital-intensive technology, or both conditions are met simultaneously, a small increase in capital stock by either LM firm is welfare improving at the strategic and nonstrategic equilibria. Whether the move from nonstrategic equilibria to strategic equilibria increases welfare depends on the magnitude of the output elasticity of labour and the condition above.

In a mixed duopoly, at the nonstrategic equilibria a small increase in capital stock by the LM firm increases welfare under the condition above. That by the PM firm is also welfare improving. In general, at the strategic equilibria the small increase in capital stock by the PM firm leads to the improvement in welfare, whereas for that by the LM firm to increase welfare, in addition to the above condition, it is necessary that the LM firm's output should be greater than or equal to the PM firm's one. Whether the move above improves welfare depends on the magnitude of the output elasticity of labour for the LM firm and the two conditions above.

The arresting feature of this paper is that I used the homothetic production technology to ensure the existence of an interior optimum at which capital and output are simultaneously determined. The assumption of homotheticity generates a nonstrategic Nash equilibrium for an LM Cournot duopoly and thus enables us to reach the different results from those obtained so far for an LM and a mixed duopoly.

Mathematical Appendix[17]

Proof of Proposition 4

(i) Let $\mathbf{K}^C = (K_1^C, K_2^C)$ and $\mathbf{K}^S = (K_1^S, K_2^S)$ denote the optimal levels of capital at a nonstrategic Cournot and a strategic equilibrium, respectively. Δ refers to any difference between the two equilibria. For example, $\Delta K_i = K_i^S - K_i^C$, $\Delta\Phi_i^i = \Phi_i^i(\mathbf{K}^S) - \Phi_i^i(\mathbf{K}^C)$, etc. Applying 'the mean value theorem' to Φ_i^i yields

$$\Delta\Phi_i^i = \Phi_{i1}^i(\mathbf{K}^*)\Delta K_1 + \Phi_{i2}^i(\mathbf{K}^*)\Delta K_2, \tag{61}$$

where $\mathbf{K}^*$ is some point between $\mathbf{K}^C$ and $\mathbf{K}^S$. Letting first $i = 1$, then $i = 2$ in (61) yields a two-equation simultaneous system in unknowns ΔK_1 and ΔK_2 which can be solved to produce

$$\Delta K_1 = (\Phi_{22}^2\Delta\Phi_1^1 - \Phi_{12}^1\Delta\Phi_2^2)\,/\,\Sigma, \tag{62}$$

$$\Delta K_2 = (\Phi_{11}^1\Delta\Phi_2^2 - \Phi_{21}^2\Delta\Phi_1^1)\,/\,\Sigma. \tag{63}$$

Adding (62) and (63) leads to

$$\Delta K_1 + \Delta K_2 = [(\Phi_{22}^2 - \Phi_{21}^2)\Delta\Phi_1^1 + (\Phi_{11}^1 - \Phi_{12}^1)\Delta\Phi_2^2]\,/\,\Sigma. \tag{64}$$

Since $\Phi_i^i(\mathbf{K}^S) = 0$, it follows from (37) that we have

$$\Delta\Phi_i^i = \Phi_i^i(\mathbf{K}^S) - \Phi_i^i(\mathbf{K}^C) \lesseqgtr 0 \quad \text{if} \quad \omega^j \lesseqgtr 1 \quad \text{and} \quad p''(Q) = 0. \tag{65}$$

Since $\Phi_{ii}^i - \Phi_{ij}^i < 0$ by the assumption (34), we obtain

$$\Delta K_1 + \Delta K_2 \gtreqless 0 \quad \text{if} \quad \omega^1 \lesseqgtr 1,\ \omega^2 \lesseqgtr 1,\ \text{and } p''(Q) = 0. \tag{66}$$

(ii) Since $\Delta K_1 = \Delta K_2 = \Delta K$, we can set $\omega^1 = \omega^2 = \omega$. Thus, we have

$$\Delta K \gtreqless 0 \quad \text{if} \quad \omega \lesseqgtr 1 \quad \text{and} \quad p''(Q) = 0. \tag{67}$$

Proof of Proposition 5

(i) Applying 'the mean value theorem' to $q_i = \varphi^i(K_1, K_2)$ yields

$$\Delta q_i = \varphi_1^i(\mathbf{K}^*)\Delta K_1 + \varphi_2^i(\mathbf{K}^*)\Delta K_2. \tag{68}$$

[17] I applied 'the mean value theorem' referred to by Brander and Spencer (1983) to the proofs in this Appendix.

Under perfect symmetry, $\varphi^i_j = \varphi^j_i$. In addition, using (30) and (67) produces

$$\Delta q_i = (\varphi^i_1 + \varphi^i_2)\Delta K \gtreqless 0 \quad \text{if} \quad \omega \lesseqgtr 1 \quad \text{and} \quad p''(Q) = 0. \tag{69}$$

(ii) Under perfect symmetry, applying 'the mean value theorem' to the income per worker (31) yields

$$\begin{aligned}\Delta\Phi^i &= \Phi^i(\mathbf{K}^S) - \Phi^i(\mathbf{K}^C) \\ &= \Phi^i_1 \Delta K_1 + \Phi^i_2 \Delta K_2 = (\Phi^i_1 + \Phi^i_2)\Delta K.\end{aligned} \tag{70}$$

Moreover, we obtain

$$\Phi^i_1 + \Phi^i_2 = [(\varphi^i_1 + \varphi^i_2)R^i_j - (r + y^i l^i_K)] \,/\, L_i. \tag{71}$$

We have $(\varphi^i_1 + \varphi^i_2)R^i_j < 0$. But, the sign of $r + y^i l^i_K$ evaluated at some point between $\mathbf{K}^C$ and $\mathbf{K}^S$ changes depending on the sign of ΔK_i in (67). (a) When $\omega < 1$, $\Delta K_i > 0$. Since $r + y^i l^i_K > 0$, $\Phi^i_1 + \Phi^i_2 < 0$, so $\Delta\Phi^i < 0$. The income per worker is smaller. (b) When $\omega = 1$, $\Delta K_i = 0$, which means $r + y^i l^i_K = 0$. Since $\Delta\Phi^i = 0$, the income per worker remains constant. In this case the strategic and the nonstrategic equilibrium coincide. (c) When $\omega > 1$, $\Delta K_i < 0$, so $r + y^i l^i_K < 0$. If $(\varphi^i_1 + \varphi^i_2)R^i_j < r + y^i l^i_K < 0$, then $\Phi^i_1 + \Phi^i_2 < 0$ and thus $\Delta\Phi^i > 0$.[18] The income per worker is greater.

Proof of Proposition 6

Applying 'the mean value theorem' to Φ^i_i and Π^j_j yields

$$\Delta\Phi^i_i = \Phi^i_{ii}(\mathbf{K}^*)\Delta K_i + \Phi^i_{ij}(\mathbf{K}^*)\Delta K_j, \tag{72}$$

$$\Delta\Pi^j_j = \Pi^j_{ji}(\mathbf{K}^*)\Delta K_i + \Pi^j_{jj}(\mathbf{K}^*)\Delta K_j. \tag{73}$$

Solving a two-equation simultaneous system composed of (72) and (73) in ΔK_i and ΔK_j yields

$$\Delta K_i = (\Pi^j_{jj}\Delta\Phi^i_i - \Phi^i_{ij}\Delta\Pi^j_j) \,/\, \Omega, \tag{74}$$

$$\Delta K_j = (\Phi^i_{ii}\Delta\Pi^j_j - \Pi^j_{ji}\Delta\Phi^i_i) \,/\, \Omega. \tag{75}$$

Adding (74) and (75) leads to

$$\Delta K_i + \Delta K_j = [(\Pi^j_{jj} - \Pi^j_{ji})\Delta\Phi^i_i + (\Phi^i_{ii} - \Phi^i_{ij})\Delta\Pi^j_j] \,/\, \Omega. \tag{76}$$

[18] The numerical calculation using the examples above in the text supports $\Delta\Phi^i > 0$.

Since $\Phi^i_i(\mathbf{K}^C) = R^i_j\phi^j_i/L_i > 0$, we have $\Delta\Phi^i_i < 0$. Note that "$\Pi^j_j(\mathbf{K}^C) = R^j_i\phi^i_j \gtreqless 0$ if $\omega^i \lesseqgtr 1$ and $p''(Q) = 0$". By assumption $\Pi^j_{jj} - \Pi^j_{ji} < 0$ and $\Phi^i_{ii} - \Phi^i_{ij} < 0$.

(a) Suppose $\omega^i < 1$. Since $\Pi^j_j(\mathbf{K}^C) > 0$, $\Delta\Pi^j_j < 0$, so $\Delta K_i + \Delta K_j > 0$. In addition, if we assume that $|\Pi^j_{jj}| > |\Phi^i_{ij}|$ $|\Phi^i_{ii}| > |\Pi^j_{ji}|$, then $\Delta K_i > 0$ and $\Delta K_j > 0$.

(b) Suppose $\omega^i = 1$. We obtain $\phi^i_j = 0$ and thus $\Pi^j_j(\mathbf{K}^C) = 0$, from which it follows that $\Delta\Pi^j_j = 0$. Thus, $\Delta K_i + \Delta K_j > 0$. Moreover, $\Delta K_i > 0$. On the other hand, $\Delta K_j \gtreqless 0$. But, since at the strategic equilibrium $\Pi^j_j = -r - wl^j_K = 0$, the PM firm chooses the same level of physical capital as the cost-minimizing firm.

(c) Suppose $\omega^i > 1$. Since $\Pi^j_j(\mathbf{K}^C) < 0$, $\Delta\Pi^j_j > 0$. If we assume hat $|\Pi^j_{jj}| > |\Phi^i_{ij}|$ and $|\Phi^i_{ii}| > |\Pi^j_{ji}|$, then $\Delta K_i > 0$ and $\Delta K_j < 0$. But $\Delta K_i + \Delta K_j \gtreqless 0$. The numerical calculation using the examples above in the text supports $\Delta K_i + \Delta K_j > 0$.

Proof of Proposition 7

(i) Applying 'the mean value theorem' to $q_i = \phi^i(K_i, K_j)$ and $q_j = \phi^j(K_i, K_j)$ yields

$$\Delta q_i = \phi^i_i(\mathbf{K}^*)\Delta K_i + \phi^i_j(\mathbf{K}^*)\Delta K_j, \text{ and } \Delta q_j = \phi^j_i(\mathbf{K}^*)\Delta K_i + \phi^j_j(\mathbf{K}^*)\Delta K_j. \tag{77}$$

Note that it follows from (47) that $\phi^i_i + \phi^j_i > 0$ and $\phi^i_j + \phi^j_j > 0$.

(a) Suppose that $\omega^i < 1$. Since $\Delta K_i > 0$ and $\Delta K_j > 0$, $\Delta Q = \Delta q_i + \Delta q_j = (\phi^i_i + \phi^j_i)\Delta K_i + (\phi^i_j + \phi^j_j)\Delta K_j > 0$. $\Delta q_i \gtreqless 0$ and $\Delta q_j \gtreqless 0$, but $\Delta Q > 0$ means that at least one of Δq_i and Δq_j is positive.

(b) When $\omega^i = 1$, $\phi^i_j = 0$. Thus, the LM firm chooses the level of output $\hat{q}_i$ corresponding to $\omega^i = 1$. The PM firm produces the corresponding level of output q^S_j. The PM firm's long-run reaction function passes through the strategic equilibrium point. We have $\Delta q_i = \phi^i_i\Delta K_i > 0$, while since $\Delta K_j \gtreqless 0$, $\Delta q_j \gtreqless 0$.

(c) When $\omega^i > 1$, $\phi^i_j > 0$. We have $\Delta K_i > 0$ and $\Delta K_j < 0$, so $\Delta q_i \gtreqless 0$ and $\Delta q_j < 0$.

In all the three cases, the numerical calculations using the examples above in the text show that $\Delta Q = \Delta q_i + \Delta q_j > 0$.

(ii) Applying 'the mean value theorem' to Φ^i and Π^j yields

$$\Delta\Phi^i = \Phi^i_i(\mathbf{K}^*)\Delta K_i + \Phi^i_j(\mathbf{K}^*)\Delta K_j, \tag{78}$$

$$\Delta\Pi^j = \Pi^j_i(\mathbf{K}^*)\Delta K_i + \Pi^j_j(\mathbf{K}^*)\Delta K_j. \tag{79}$$

(a) When $\omega^i < 1$, $\phi^i_j < 0$, from which it follows that $\Delta K_i > 0$ and $\Delta K_j > 0$. Thus, $\Phi^i_j = R^i_j\phi^j_j/L_i < 0$ and $\Phi^i_i = (R^i_j\phi^j_i - r - y^i l^i_K)/L_i > 0$. Note that $\phi^j_i = H^i_K G^j_i/\Gamma < 0$ and

$\phi^j_j = -H^i_i G^j_K/\Gamma > 0$, where $G^j_i = p'(Q) < 0$. $\omega^i < 1$ means the small price elasticity of demand, so it is probable for $|p'(Q)|$ to be large enough to have $\phi^j_j + \phi^j_i < 0$. If $\Delta K_i \geq \Delta K_j$, then we obtain $\Delta\Phi^i > 0$.

On the other hand, $\Pi^j_i = R^j_i \phi^i_i < 0$ and $\Pi^j_j = R^j_i \phi^i_j - r - w l^j_K > 0$. Note that if $\sigma \leq 1$, $\phi^i_i = -H^i_K G^j_j/\Gamma > 0$, and if $\omega^i < 1$, $\phi^i_j = H^i_j G^j_K/\Gamma < 0$. Since $H^i_j = p'(Q)(1-\omega^i) < 0$, $\phi^i_i + \phi^i_j > 0$ is probable, so if $\Delta K_i \geq \Delta K_j$, then we have $\Delta\Pi^j < 0$.

(b) When $\omega^i = 1$, $\phi^i_j = 0$. $\Delta K_i > 0$, while $\Delta K_j \gtreqless 0$. [*i*] Suppose that $\Delta K_j > 0$. As shown in the case above, we can have $\phi^j_j + \phi^j_i < 0$. If $\Delta K_i \geq \Delta K_j$, then $\Delta\Phi^i > 0$. Since $\Pi^j_j < 0$, $\Delta\Pi^j < 0$. [*ii-a*] If $\Delta K_j = 0$, then $\Delta\Phi^i > 0$ and $\Delta\Pi^j < 0$. [*ii-b*] Suppose that $\Delta K_j < 0$. $\Phi^i_i > 0$ and $\Phi^i_j < 0$ yield $\Delta\Phi^i > 0$. Since $\Pi^j_j > 0$ and $\Pi^j_i < 0$, $\Delta\Pi^j < 0$.

(c) When $\omega^i > 1$, $\phi^i_j > 0$. We have $\Delta K_i > 0$ and $\Delta K_j < 0$, so $\Delta\Phi^i > 0$. On the other hand, since $\Pi^j_j < 0$ and $\Pi^j_i < 0$, $\Pi^j_j \Delta K_j > 0$ and $\Pi^j_i \Delta K_i < 0$. Supposing that $\Delta K_i > |\Delta K_j|$, we obtain $\Delta\Pi^j < 0$. The numerical calculation using the examples above in the text supports this result.

Welfare Analysis

(i) Evaluate (58) at the nonstrategic equilibrium where $-(r + y^i l^i_K) = 0$ and condition A implies $W_i > 0$.

(ii) Consider the welfare effect of an increase in capital stock by each LM firm at some point $\mathbf{K}^*$ between the strategic and nonstrategic equilibria. Applying 'the mean value theorem' to the social welfare function W yields

$$\Delta W = W_i \Delta K_i + W_j \Delta K_j, \tag{80}$$

where W_i and W_j are evaluated at $\mathbf{K}^*$. Under symmetry, $\Delta K_i \gtreqless 0$ according as $\omega^i \lesseqgtr 1$.

Case (a): $\omega^i < 1$.

Since $\Delta K_i > 0$ and $\Phi^i_{ii} < 0$, $R^i_j \varphi^j_i > r + y^i l^i_K$ at $\mathbf{K}^*$. We then have

$$W_i > 0 \quad \text{if} \quad -p'(Q)q_i(\varphi^i_i + 2\varphi^j_i) + (y^i - w)l^i_q(\varphi^i_i + \varphi^j_i) + (y^i - w)l^i_K > 0. \tag{81}$$

Verify first $\varphi^i_i + 2\varphi^j_i > 0$. If the elasticity of substitution for the function $f(K, L)$ satisfies $\sigma \leq 1$, then $H^i_K > 0$ holds true. It is sufficiently probable for the LM firms to produce on the portion of the labour cost function where $l^j_{qq} > 0$ (see footnote 3). It follows from $\Lambda > 0$ that the following expression holds true:

$$\varphi^i_i + 2\varphi^j_i = \frac{H^i_K}{\Lambda}(-H^j_j + 2H^j_i) = \frac{H^i_K}{\Lambda}(y^j l^j_{qq} - 2p'(Q)\omega^j) > 0.$$

Hence, condition A implies $W_i > 0$, from which it follows that under symmetry $\Delta W > 0$.

Case (b): $\omega^i = 1$.

Since $\Delta K_i = 0$, under symmetry $\Delta W = 0$.

Case (c): $\omega^i > 1$.

Since $\Delta K_i < 0$ and $\Phi^i_{ii} < 0$, $R^i_j \varphi^j_i < r + y^i l^i_K$ at $\mathbf{K}^*$. Differentiation of the social welfare function W with respect to K_i yields (58). Since $K^S_i < K^C_i$, $-(r + y^i l^i_K) > 0$ at $\mathbf{K}^*$. If condition A is satisfied, then we have $W_i > 0$ with a higher probability than when (58) is evaluated at the nonstrategic equilibrium. Thus, assuming symmetry, we obtain $\Delta W < 0$.[19]

(iii) Evaluate W_i at the strategic equilibrium. Since $R^i_j \varphi^j_i = r + y^i l^i_K$, we obtain

$$W_i = -p'(Q) q_i (\varphi^i_i + 2\varphi^j_i) + (y^i - w) l^i_q (\varphi^i_i + \varphi^j_i) + (y^i - w) l^i_K. \tag{82}$$

As in Case (a), condition A implies $W_i > 0$.

(iv) Evaluate (59) and (60) at the nonstrategic equilibria where $-(r + y^i l^i_K) = 0$ and $-(r + w l^j_K) = 0$. Let us prove that if $q_j < 2q_i$, then $q_i \phi^i_i + q_j \phi^j_i > 0$. The left side of this inequality is rewritten as

$$\begin{aligned} q_i \phi^i_i + q_j \phi^j_i &= \frac{H^i_K}{\Gamma}(-q_i G^j_j + q_j G^j_i) \\ &= \frac{H^i_K}{\Gamma}[(-2q_i + q_j) p'(Q) + q_i w l^j_{qq}]. \end{aligned}$$

Since $\Gamma > 0$, supposing $q_j < 2q_i$ leads to the result that $q_i \phi^i_i + q_j \phi^j_i > 0$. Since $\phi^i_i > 0$, under conditions $q_j < 2q_i$ and A, $W_i > 0$.

Verify that if $\frac{1-\omega^i}{2} q_i < q_j$, then $-p'(Q)(q_i \phi^i_j + q_j \phi^j_j)$ of the right side of (60) would be positive and dominate the right side. $q_i \phi^i_j + q_j \phi^j_j$ is rewritten as

$$\begin{aligned} q_i \phi^i_j + q_j \phi^j_j &= \frac{H^i_K}{\Gamma}(q_i H^i_j - q_j H^i_i) \\ &= \frac{H^i_K}{\Gamma}\{p'(Q)[q_i(1-\omega^i) - 2q_j] + q_j y^i l^i_{qq}\}. \end{aligned}$$

If $\frac{1-\omega^i}{2} q_i < q_j$, then the expression above is positive. If $1 - \omega^i \leq 0$, then the condition above is satisfied. Even if $1 - \omega^i > 0$, it is likely that the condition will be satisfied, because $\frac{1-\omega^i}{2}$ is sufficiently small. In contrast, $(y^i - w) l^i_q \phi^i_j$ of (60) seems to have only a little effect on the determination of its sign. It is because the own effects of output on marginal income per worker dominate the cross effects, i.e., $|H^i_i| > |H^i_j|$, and thus $|\phi^i_j|$ is likely to be very small. This means that W_j would be positive.

[19] The numerical calculation using $q = z - 1, z = \sqrt{KL}$ and $p = a - bQ$ ($a > 0, b > 0$) supports the results derived in Cases (a), (b), and (c).

(v) Consider whether the move between the two equilibria leads to the improvement in welfare.

Case (a): $\omega^i < 1$.

Since $\Delta K_i > 0$ for the LM firm, $R^i_j \phi^j_i > r + y^i l^i_K$. Making use of this inequality produces

$$W_i > 0 \quad \text{if} \quad -p'(Q)[q_i \phi^i_i + (q_i + q_j)\phi^j_i] + (y^i - w) l^i_q \phi^i_i + (y^i - w) l^i_K > 0. \tag{83}$$

Let us verify that if $q_j \leq q_i$, then $q_i \phi^i_i + (q_i + q_j)\phi^j_i > 0$. Since this expression is rewritten as

$$q_i \phi^i_i + (q_i + q_j)\phi^j_i = \frac{H^i_K}{\Gamma}[(-q_i + q_j)p'(Q) + q_i w l^j_{qq}],$$

supposing $q_j \leq q_i$ (condition B) produces the result that the expression above is positive. When the LM firm's market share is larger and the price elasticity of demand is smaller, $\omega^i < 1$ holds true, so it is highly likely that $q_j \leq q_i$. Thus, under conditions A and B, $W_i > 0$.

Since $\Delta K_j > 0$ for the PM firm, $R^j_i \phi^i_j > r + w l^j_K$, so we have

$$W_j > 0 \quad \text{if} \quad -p'(Q)[(q_i + q_j)\phi^i_j + q_j \phi^j_j] + (y^i - w) l^i_q \phi^i_j > 0. \tag{84}$$

Let us verify that if $\frac{1-\omega^i}{1+\omega^i} q_i \leq q_j$, then $(q_i + q_j)\phi^i_j + q_j \phi^j_j > 0$. The left side of this inequality is rewritten as

$$(q_i + q_j)\phi^i_j + q_j \phi^j_j = \frac{G^j_K}{\Gamma}\{p'(Q)[q_i(1-\omega^i) - q_j(1+\omega^i)] + q_j y^i l^i_{qq}\}.$$

Since $G^j_K > 0$, supposing $\frac{1-\omega^i}{1+\omega^i} q_i \leq q_j$ yields the result that $(q_i + q_j)\phi^i_j + q_j \phi^j_j > 0$.

Since $|\phi^i_j|$ would be small, the first term of the conditional expression in (84) seems to dominate the second term, so the probability of having $W_j > 0$ is much higher than that of being $W_i > 0$.

Therefore, if $\frac{1-\omega^i}{1+\omega^i} q_i \leq q_j$ is satisfied, then conditions A and B imply $\Delta W > 0$.

Case (b): $\omega^i = 1$.

Proposition 1 states that $\omega^i = 1$ is equivalent to $-p'(\hat{Q})\hat{q}_i^2 = rK_i$. Under the given demand and cost conditions, given the other firm's output q_j, we obtain LM firm i's unique output level $\hat{q}_i$ and corresponding level of capital stock $\hat{K}_i$ at which $\omega^i = 1$. Correspondingly, $\hat{q}_j$ and $\hat{K}_j$ are uniquely determined. Since evaluating W_i and W_j at $\mathbf{K}^*$ invalidates $\omega^i = 1$, let us evaluate them in the neighbourhood of the strategic equilibrium. This implies $\Delta K_j = 0$. On the other hand, under conditions A and B, $\Delta K_i > 0$ leads to $W_i > 0$ as in Case (a). We then have $\Delta W > 0$.

Case (c): $\omega^i > 1$.

Since $\Delta K_i > 0$, $W_i > 0$ under conditions A and B as in Case (a). In contrast, $\Delta K_j < 0$ produces $-(r + wl_K^j) > 0$ at $\mathbf{K}^*$. $\omega^i > 1$ implies $\phi_j^i > 0$. Moreover, $\phi_j^j > 0$. From (60) $W_j > 0$. Consequently, $\Delta W \gtreqless 0$.

However, it should be noted that $\phi_j^i > 0$ in (60), whereas $\phi_i^j < 0$ in (83). Even if $\Delta K_i > |\Delta K_j|$, it is highly likely that W_j will largely dominate W_i and so $W_j|\Delta K_j|$ is larger than $W_i \Delta K_i$. This implies $\Delta W < 0$.

(vi) Evaluate W_i and W_j at the strategic equilibrium where $R_j^i \phi_i^j = r + y^i l_K^i$ for the LM firm and $R_i^j \phi_j^i = r + wl_K^j$ for the PM firm. We therefore obtain

$$\text{(LM)}: \quad W_i = -p'(Q)[q_i \phi_i^i + (q_i + q_j)\phi_i^j] + (y^i - w) l_q^i \phi_i^i + (y^i - w) l_K^i, \tag{85}$$

$$\text{(PM)}: \quad W_j = -p'(Q)[(q_i + q_j)\phi_j^i + q_j \phi_j^j] + (y^i - w) l_q^i \phi_j^i. \tag{86}$$

(85) and (86) are identical with the conditional expressions in (83) and (84), respectively. Thus, under conditions A and B as in Case (a), $W_i > 0$. On the other hand, when $\omega^i \geq 1$, $\phi_j^i \geq 0$ and so $W_j > 0$. When $\omega^i < 1$, if $\frac{1-\omega^i}{1+\omega^i} q_i \leq q_j$, then it is highly likely that $W_j > 0$.

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Figure 1: An example of a labour-managed firm's short-run reaction functions

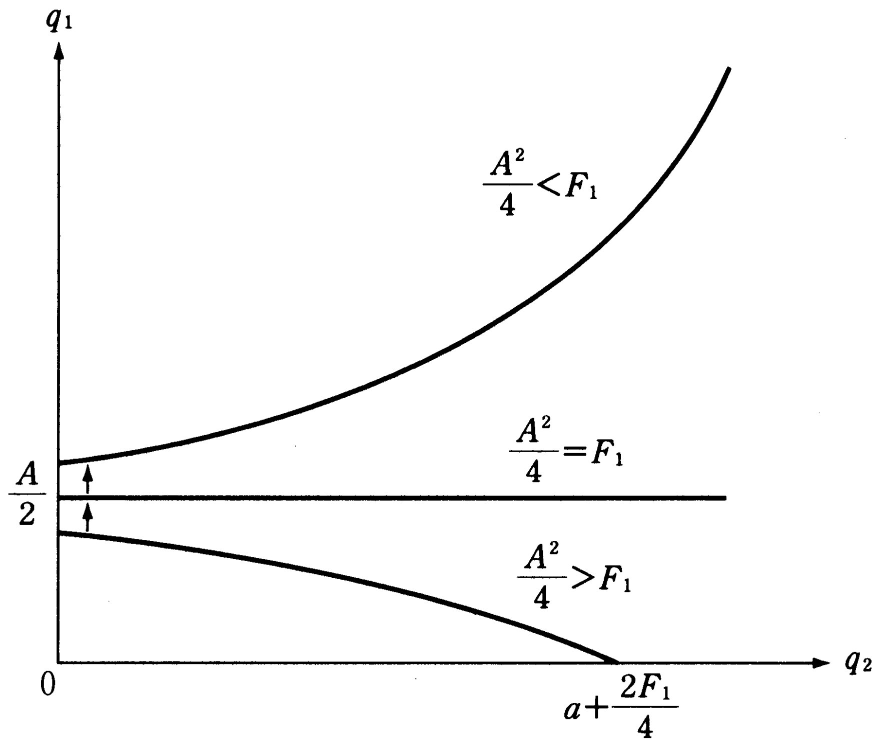
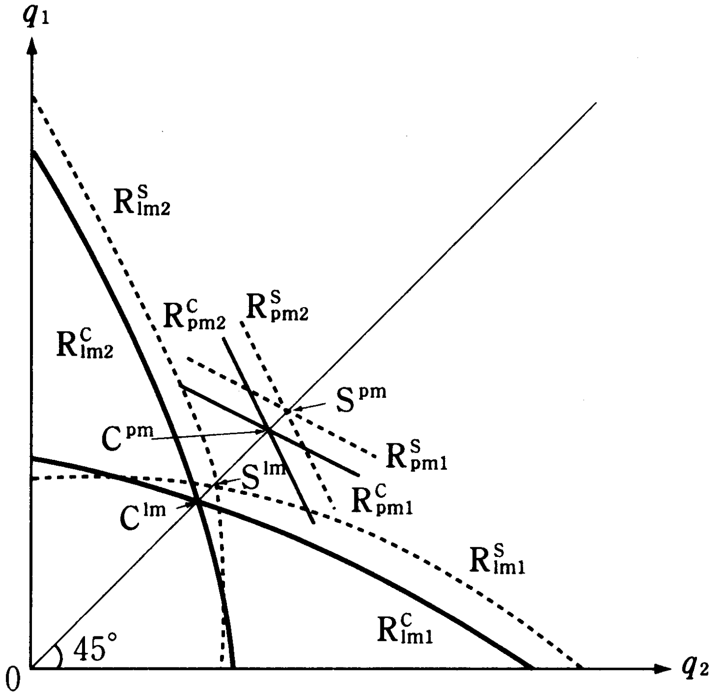
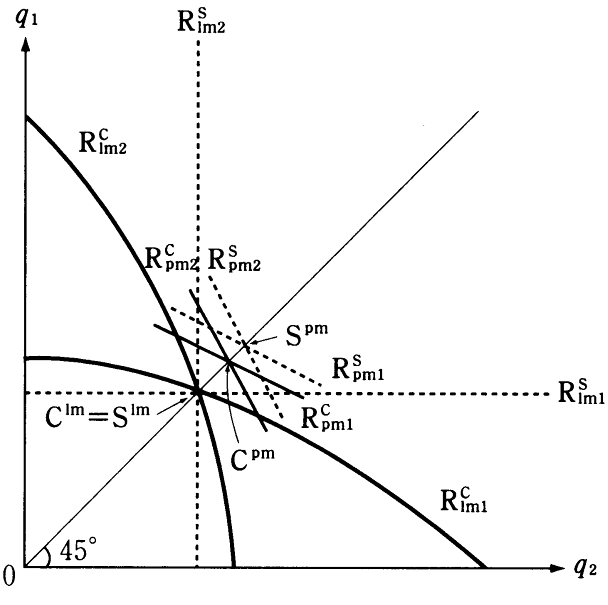


Figure 2: An LM firm's and a PM firm's short-run and long-run reaction functions

Case (a): $\omega^i < 1$



Case (b): $\omega^i = 1$

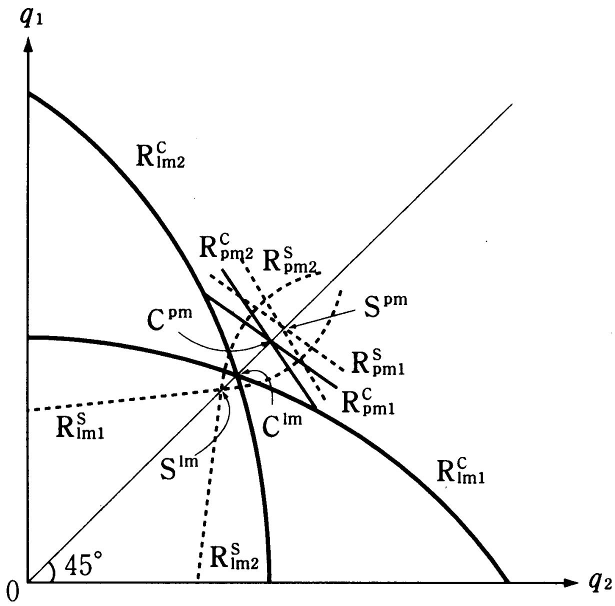


C^{lm}, C^{pm}: A nonstrategic Cournot equilibrium in an LM (PM) duopoly

S^{lm}, S^{pm}: A strategic equilibrium in an LM (PM) duopoly

$R^{S(C)}_{lm(pm)i}$: LM (PM) firm *i*'s short-run (long-run) reaction function

Case (c): $\omega^i > 1$



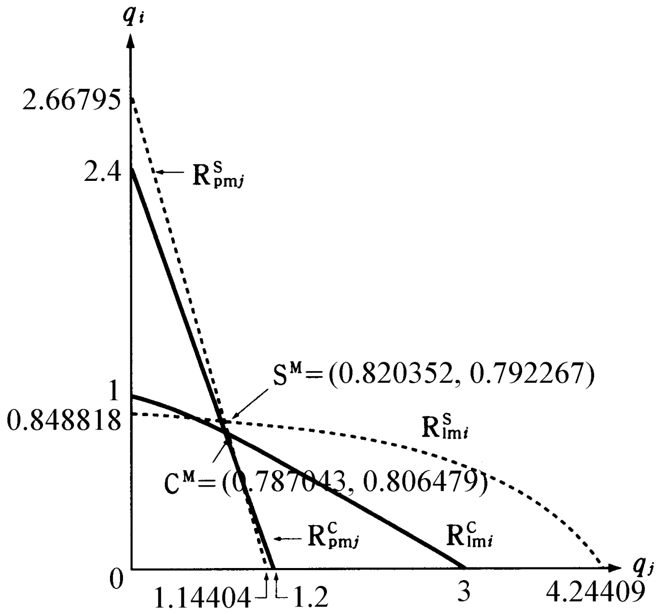
C^{lm}, C^{pm}: A nonstrategic Cournot equilibrium in an LM (PM) duopoly

S^{lm}, S^{pm}: A strategic equilibrium in an LM (PM) duopoly

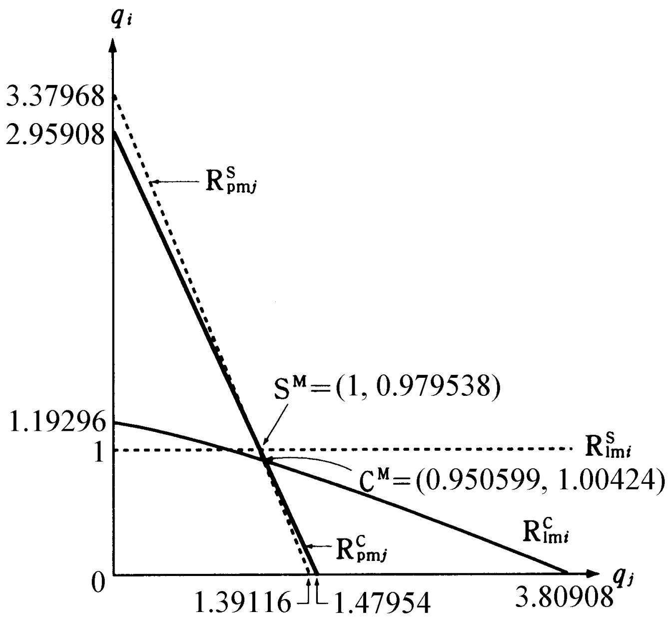
$R^{S(C)}_{lm(pm)i}$: LM (PM) firm *i*'s short-run (long-run) reaction function

Figure 3: An LM firm's and a PM firm's short-run and long-run reaction functions in a mixed duopoly

Case (a): $a = 6, b = 2, c = 1.2, w = 3.6, r = 0.1$; $\omega^i < 1$



Case (b): $a = 7.61815, b = 2, c = 1.7, w = 7.225, r = 0.1$; $\omega^i = 1$

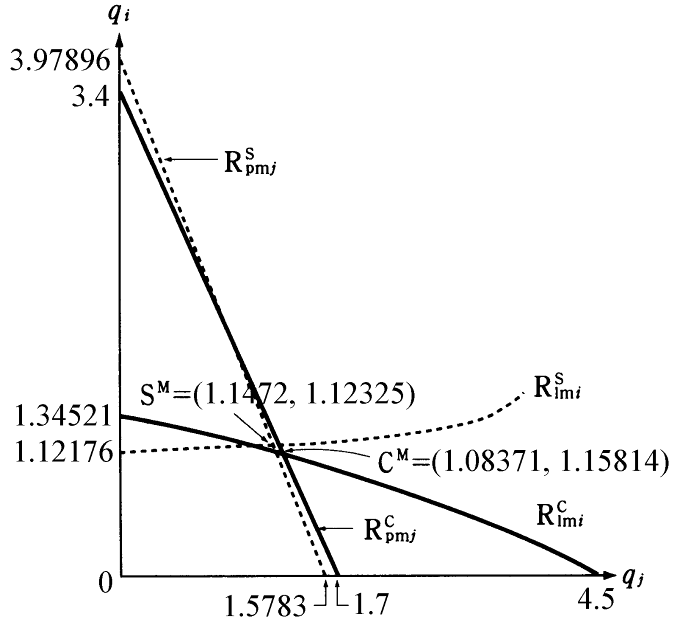


C^M: A nonstrategic Cournot equilibrium in a mixed duopoly

S^M: A strategic equilibrium in a mixed duopoly

$R^{S(C)}_{lm(pm)i(j)}$: LM (PM) firm $i(j)$'s short-run (long-run) reaction function

Case (c): $a = 9, b = 2, c = 2.2, w = 12.1, r = 0.1$; $\omega^i > 1$



C^M: A nonstrategic Cournot equilibrium in a mixed duopoly

S^M: A strategic equilibrium in a mixed duopoly

$R^{S(C)}_{lm(pm)i(j)}$: LM (PM) firm $i(j)$'s short-run (long-run) reaction function