Capital Commitment, a Labour-managed Duopoly, and a Mixed Duopoly: A Labour-managed Firm's Reaction Functions Revisited

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November 19, 2006

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< Vanek (1970, pp. 114-116) states the following for a labour-managed (LM) firm's reaction functions. >:
(1) "in the short run the reaction functions should generally be positively sloped";
(2) in the long-run setting, "As a 'central' tendency, it can (thus) be expected that they will be just about perpendicular to the axes."
Miyamoto (1982) examines the first issue (1). → See Figure 1.

< Defect in Miyamoto (1982) >: capital is fixed in the short run. If a level of physical capital is optimally determined, could another result follow? → A two-stage game model in which capital commitment is a strategic variable.

< There seems to be no literature on the shape (existence) of an LM firm's 'long-run' reaction function. > ⇔ < This is closely related with the problem of the nonexistence of a unique interior optimum for an LM monopoly (duopoly). >: Pestieau and Thisse (1979), and Landsberger and Subotnik (1982) show that if a production function is homogeneous and its degree is equal to or less than unity, then the interior optimum does not exist for the LM monopoly.

< Solution >: (1) Ireland and Law (1982, Chapter 6) use homothetic production technology to ensure the existence of such an interior solution; (2) a great number of papers take an entry cost into account to deal with the problem. → Stewart (1991), Cremer and Crémer (1992), Futagami and Okamura (1996), Haruna (2001).
< Exception >: Lambertini and Rossini (1998) do not include a fixed entry cost. →

They use the assumption of linearly homogeneous technology. $\rightarrow$ This assumption produces the nonexistence of the interior optimum for the LM duopolists.

< Purpose of this paper >: without taking the fixed entry cost into account, (1) first this paper uses the assumption of homothetic production technology to define the shape of the LM firm's 'short-run' and 'long-run' reaction functions; (2) then, by using a two-stage game model of strategic interaction, in an LM Cournot duopoly I compare strategic equilibria where both LM firms' 'short-run' reaction functions meet and nonstrategic equilibria at which their 'long-run' reaction functions intersect. Furthermore, the similar problem is considered in a mixed duopoly.

An LM duopoly is composed of two LM firms.
A mixed duopoly is made up of an LM and a profit-maximizing (PM) firm.

< Assumptions >: (1) the homothetic production function $q = g(f(K, L))$, where $q = g(z)$ and $z = f(K, L)$. z is an index of input levels. $g'(z)$ is positive. $f(K, L)$ is homogeneous of degree one; (2) $g''(z)$ is at first positive and then negative (see Figure 4); (3) use of the labour cost function $L = l(q, K)$ rather than the production function (variable: $L \rightarrow q$); (4) l_{qq} is at first negative and then positive; (5) first $R_{zz} > 0$, and then $R_{zz} < 0$ (see Figure 5).

< Maximand of an LM firm >

$$\text{income per worker:} \quad y^i = \frac{R^i(q_1, q_2) - rK_i}{L_i}.$$

< Objective function of a PM firm >

$$\text{profit:} \quad \pi^j = R^j(q_1, q_2) - rK_j - wL_j.$$

< LM firm i's 'short-run' reaction function >: output is the only variable.

Proposition 1 $\omega^i \lesseqgtr 1$ *if and only if* $-p'(\hat{Q})\hat{q}_i^2 \gtreqless F_i$.

Suppose that the market demand is linear. $\rightarrow H^i_j = p'(Q)(1 - \omega^i) \rightarrow$ See Figure 1.
$\omega^i \equiv \partial \log L_i / \partial \log q_i = (q_i/L_i)(\partial L_i/\partial q_i)$: output elasticity of labour, which plays an important role in deciding on the slope of the LM firm's short-run reaction function.

< LM firm i's 'long-run' reaction function >: capital and output are simultaneously determined.

Proposition 2 *If the market demand is linear* $p''(Q) = 0$, *then an* LM *firm's long-run reaction function is negatively sloped.*

Proposition 3 *If each* PM *firm earns nonnegative profits at the* PM *Cournot equilibrium, then under symmetry the output levels of each* LM *firm at the* LM *Cournot equilibrium are equal to or lower than those of each* PM *firm at the* PM *Cournot equilibrium.*

< On the behaviour of LM firms in an LM duopoly with capital strategic interaction >: we are concerned with a two-stage game; (1) in the first stage of the game, each LM firm sets a level of capital stock; (2) in the second stage, it decides on its output levels. → Propositions 4 & 5 (See Tables 1 & 2) and Corollaries 2 & 3 → See Figures 2-a, b & c.

Corollary 2 *In an* LM *Cournot duopoly model in which capital commitment is a strategic variable, each* LM *firm sets a level of physical capital corresponding to the slope of its 'short-run' reaction function that is negative, perpendicular to its own axis, or positive, according as the output elasticity of labour is smaller than, equal to, or larger than unity in output space.*

Corollary 3 *If the output elasticity of labour for each* LM *firm is equal to or larger than unity, then the levels of both its output and the industry output at the strategic equilibria of the* LM *duopoly are equal to or smaller than those at the strategic equilibrium of the* PM *duopoly.*

< Mixed duopoly and capital commitment >:

Propositions 6 & 7 (See Tables 3 & 4) → See Figures 3-a, b & c.

< Welfare analysis >:

Propositions 10 & 11 (See Tables 5 & 6) → The results of welfare analysis in an LM and a mixed duopoly.

≪ Conclusion ≫

(1) Under the linear demand, an LM firm's short-run reaction function is negatively sloped, horizontal, or positively sloped depending on the magnitude of the output elasticity of labour, and its long-run reaction function is negatively sloped.
(2) In an LM duopoly, whether each LM firm employs more capital, produces greater output and obtains greater income per worker at strategic equilibria than at non-strategic Cournot equilibria depends on the magnitude of its own output elasticity of labour.
(3) In a mixed duopoly, the LM firm tends to have more capital at the strategic equilibria than at the nonstrategic equilibria, whereas whether the PM firm under-invests depends on the magnitude of the output elasticity of labour for its LM competitor. In contrast, at the strategic equilibria the LM firm is likely to earn greater income per worker than, and the PM firm tends to earn less profit than at the nonstrategic equilibria, regardless of the magnitude of the output elasticity of labour for the LM firm.
(4) In both duopolies, whether net welfare rises or falls depends on the magnitude of the output elasticity of labour for the LM firm(s).

Table 1: Proposition 4

LM Duopoly		Output Elasticity of Labour		
	Capital Stock	(a) $\omega^i < 1$	(b) $\omega^i = 1$	(c) $\omega^i > 1$
Linear Demand	(i) $\Delta K_1 + \Delta K_2$	> 0	$= 0$	< 0
	(ii) ΔK_i (symmetry)	> 0	$= 0$	< 0

The market demand function is linear $p''(Q) = 0$.

Δ refers to any difference between a strategic and a corresponding nonstrategic equilibrium. For example, $\Delta K_i = K_i^S - K_i^C$.

Table 2: Proposition 5

LM Duopoly	Output & Income per Worker	Output Elasticity of Labour		
		(a) $\omega^i < 1$	(b) $\omega^i = 1$	(c) $\omega^i > 1$
Linear Demand	(i) Δq_i (symmetry)	> 0	$= 0$	< 0
	(ii) Δy^i (symmetry)	< 0	$= 0$	> 0

Table 3: Proposition 6

Mixed Duopoly	Capital Stock	Output Elasticity of Labour		
		(a) $\omega^i < 1$	(b) $\omega^i = 1$	(c) $\omega^i > 1$
Linear Demand	(i) $\Delta K_i + \Delta K_j$	> 0	> 0	$\gtreqless 0$
	(ii-i) ΔK_i	> 0	> 0	> 0
	(ii-j) ΔK_j	> 0	$\gtreqless 0^{(*)}$	< 0

(*): K_j is at the cost-minimizing level.

Table 4: Proposition 7

Mixed Duopoly Output, Income per Worker & Profits		Output Elasticity of Labour (a) $\omega^i < 1$	 (b) $\omega^i = 1$	 (c) $\omega^i > 1$
Linear Demand	(i-i) Δq_i	> 0 $(\Delta Q > 0)^{(*)}$	> 0	$\gtreqless 0$
	(i-j) Δq_j	> 0 $(\Delta Q > 0)^{(*)}$	$\gtreqless 0$	< 0
	(ii-i) Δy^i	> 0	$\Delta K_j > 0 \to$ > 0 $\Delta K_j = 0 \to$ > 0 $\Delta K_j < 0 \to$ > 0	> 0
	(ii-j) $\Delta \pi^j$	< 0	$\Delta K_j > 0 \to$ < 0 $\Delta K_j = 0 \to$ < 0 $\Delta K_j < 0 \to$ < 0	< 0

(*): Δq_i or Δq_j is positive.

Table 5: Welfare Analysis: Proposition 10

<table>
<tr><td colspan="2">LM Duopoly

Welfare</td><td colspan="3">Output Elasticity of Labour</td></tr>
<tr><td colspan="2"></td><td>(a) $\omega^i < 1$</td><td>(b) $\omega^i = 1$</td><td>(c) $\omega^i > 1$</td></tr>
<tr><td rowspan="3">Linear Demand</td><td>(i) $W_i^{(*)}$</td><td colspan="3">> 0</td></tr>
<tr><td>(ii) $\Delta W^{(\dagger)}$</td><td>$> 0^{(\dagger\dagger)}$</td><td>$= 0$</td><td>$< 0^{(\dagger\dagger)}$</td></tr>
<tr><td>(iii) $W_i^{(\ddagger)}$</td><td colspan="3">> 0</td></tr>
</table>

(*): $W_i > 0$ at the nonstrategic equilibrium under condition A$^{(\S)}$.

(†): The signs of ΔW under symmetry.

(††): $W_i > 0$ at $\mathbf{K}^*$ under condition A.

(‡): $W_i > 0$ at the strategic equilibrium under condition A.

(§): Condition A is that $|(y^i - w)l_K^i|$ be sufficiently small.

Table 6: Welfare Analysis: Proposition 11

Mixed Duopoly	Welfare	Output Elasticity of Labour		
		(a) $\omega^i < 1$	(b) $\omega^i = 1$	(c) $\omega^i > 1$
Linear Demand	(i-i) $W_i^{(*)}$	> 0		
	(i-j) $W_j^{(**)}$	> 0		
	(ii) $\Delta W^{(\dagger)}$	$> 0^{(\#)}$	> 0	$\gtreqless 0 \quad (< 0)^{(\#\#)}$
	(iii-i) $W_i^{(\S)}$	> 0		
	(iii-j) $W_j^{(\S\S)}$	> 0		

(*): $W_i > 0$ at the nonstrategic equilibrium under conditions $q_j < 2q_i$ and A.

(**): If $\frac{1-\omega^i}{2} q_i < q_j$, then $W_j > 0$ at the nonstrategic equilibrium.

(†): $W_i > 0$ at $\mathbf{K}^*$ under conditions A and B$^{(\P)}$.

(#): If $\frac{1-\omega^i}{1+\omega^i} q_i \leq q_j$, then $W_j > 0$ at $\mathbf{K}^*$.

(##): The condition is that W^j should largely dominate W^i.

(§): $W_i > 0$ at the strategic equilibrium under conditions A and B.

(§§): If $\frac{1-\omega^i}{1+\omega^i} q_i \leq q_j$, then $W_j > 0$ at the strategic equilibrium.

(¶): Condition B is that $q_j \leq q_i$.

Figure 1: An example of a labour-managed firm's short-run reaction functions

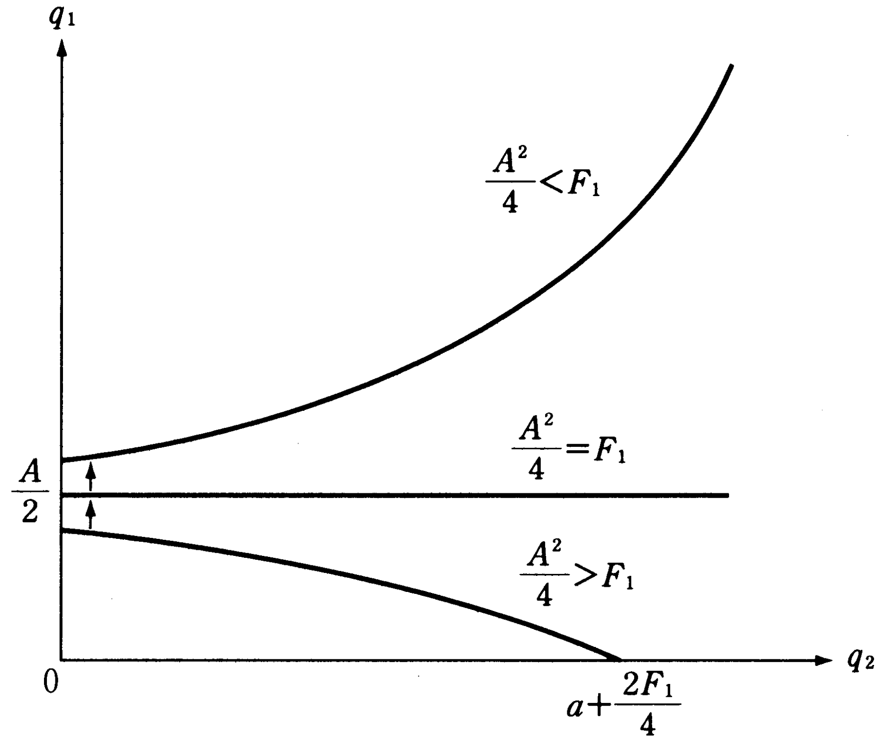
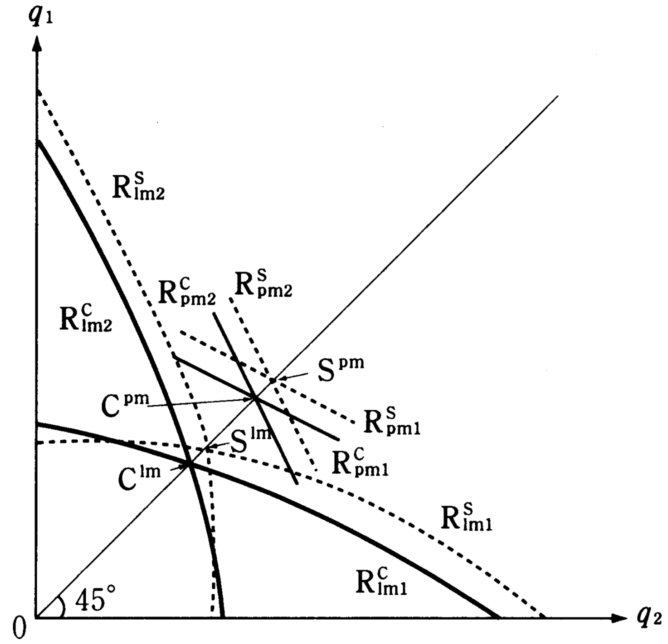
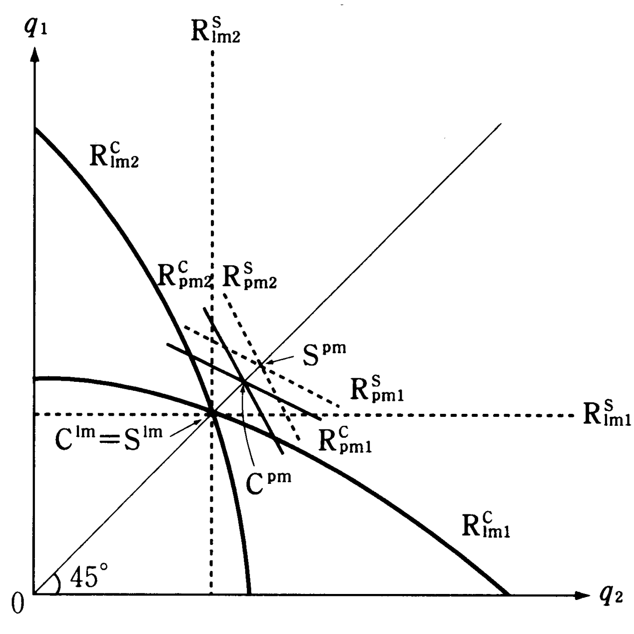


Figure 2: An LM firm's and a PM firm's short-run and long-run reaction functions

Case (a): $\omega^i < 1$



Case (b): $\omega^i = 1$

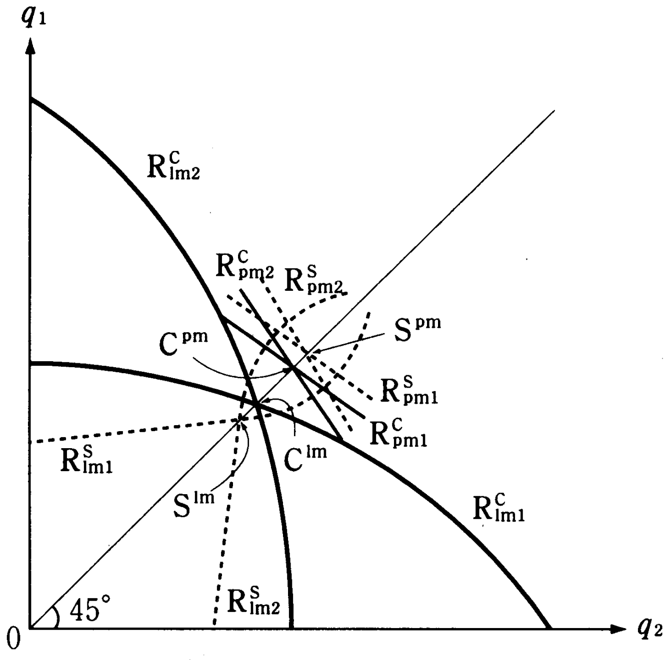


C^{lm}, C^{pm}: A nonstrategic Cournot equilibrium in an LM (PM) duopoly

S^{lm}, S^{pm}: A strategic equilibrium in an LM (PM) duopoly

$R^{S(C)}_{lm(pm)i}$: LM (PM) firm i's short-run (long-run) reaction function

Case (c): $\omega^i > 1$



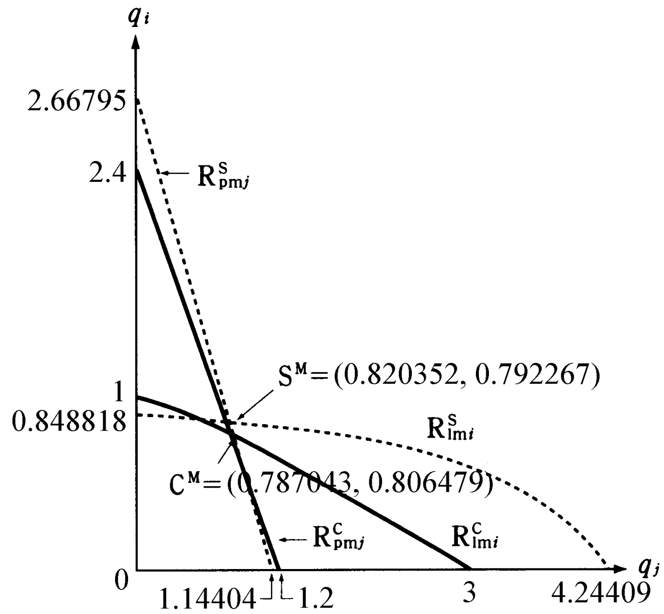
C^{lm}, C^{pm}: A nonstrategic Cournot equilibrium in an LM (PM) duopoly

S^{lm}, S^{pm}: A strategic equilibrium in an LM (PM) duopoly

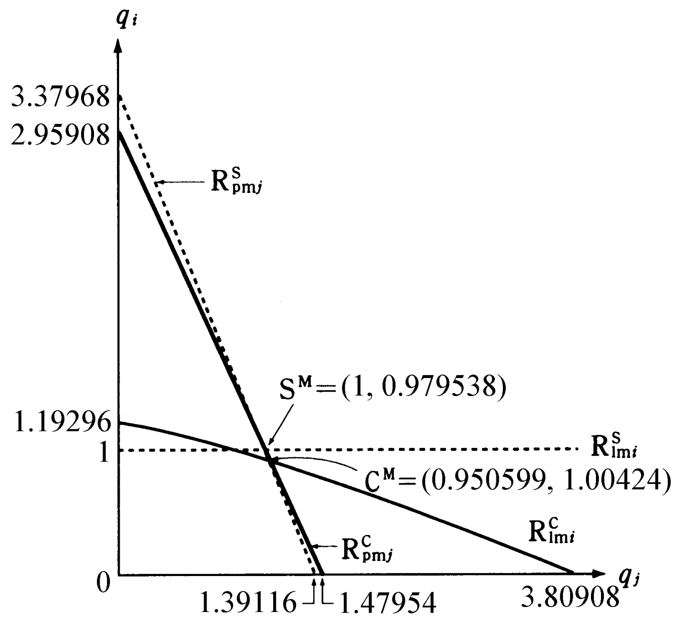
$R^{S(C)}_{lm(pm)i}$: LM (PM) firm i's short-run (long-run) reaction function

Figure 3: An LM firm's and a PM firm's short-run and long-run reaction functions in a mixed duopoly

Case (a): $a = 6, b = 2, c = 1.2, w = 3.6, r = 0.1$; $\omega^i < 1$



Case (b): $a = 7.61815, b = 2, c = 1.7, w = 7.225, r = 0.1$; $\omega^i = 1$

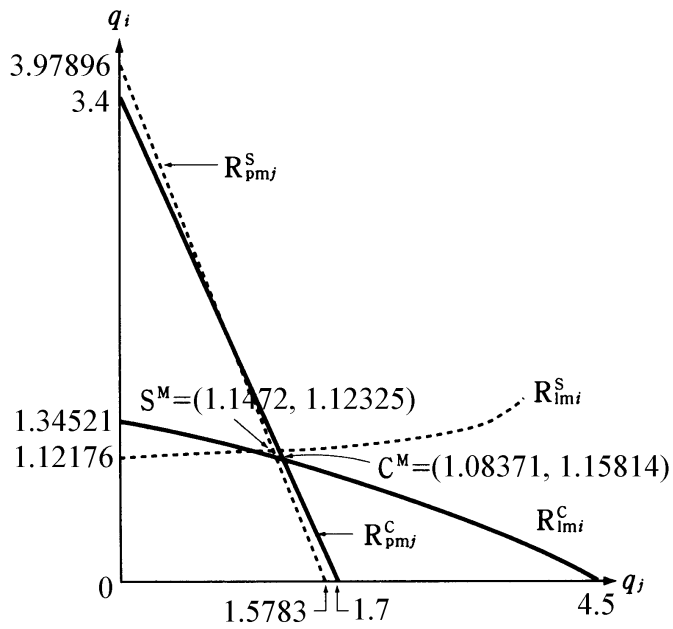


$\mathrm{C^M}$: A nonstrategic Cournot equilibrium in a mixed duopoly

$\mathrm{S^M}$: A strategic equilibrium in a mixed duopoly

$\mathrm{R^{S(C)}_{lm(pm)\mathit{i}(\mathit{j})}}$: LM (PM) firm $i(j)$'s short-run (long-run) reaction function

Case (c): $a = 9, b = 2, c = 2.2, w = 12.1, r = 0.1$; $\omega^i > 1$



C^M: A nonstrategic Cournot equilibrium in a mixed duopoly

S^M: A strategic equilibrium in a mixed duopoly

$R^{S(C)}_{lm(pm)i(j)}$: LM (PM) firm $i(j)$'s short-run (long-run) reaction function

Figure 4: Homothetic production function

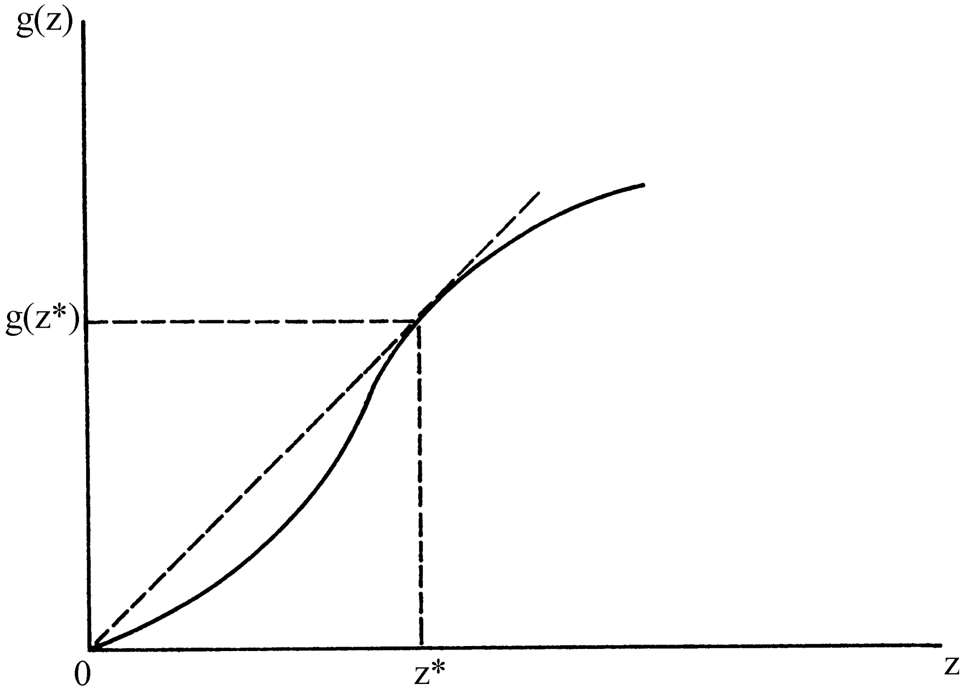


Figure 5: LM monopoly with homothetic technology