

What should the social planner do for keeping off the procrastination? *

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Abstract

We often see people procrastinate what should be done as soon as possible. Procrastination is often occurred in collective decision making. The stakeholders in collective decision making tend to waste time while trying to avoid responsibility and shifting blame to others. What should the social planner do for preventing the procrastination? In the nonperforming loans (NPLs) problem of Japan, the financial authorities did various policies. We focus on the three different types of policies: change the cost of having NPLs, change the cost of eliminating NPLs and set the deadline of final disposal of NPLs. We show that the procrastination is repressed by the taxation of having NPLs and the subsidy of eliminating NPLs. But, we reveal that the setup of deadline is insignificant whether the deadline has the punishment or not.

1 Introduction

Many persons have the experience to procrastinate what should be done as soon as possible. Procrastination generates the loss and the loss depresses the efficiency of society. In ‘lost decade’ of Japan, many reforms in politics, economy and society were delayed.

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Why did (or do) we procrastinate? Nowadays, some psychologists and behavioral economists find that human beings have the innate propensity to procrastinate even if they do not have the over-optimistic expectation. It is considered that the real identity of their propensity is hyperbolic discounting.¹ The persons with hyperbolic discounting plays down the gain and the loss in the future and plays up the gain and the loss in the present. Therefore, they prefer larger loss to smaller loss when the smaller loss comes sooner in time than the larger. Needless to say, this pattern of discounting is dynamically inconsistent.

But, most of decision-making is taken not by one person but by several persons. Are the stakeholders in the collective decision-making hyperbolic discounters? The previous researches show that the procrastination happens if the stakeholders are not hyperbolic discounting. It is reason why they tend to waste time trying to avoid responsibility and shifting blame to others.

Maynard Smith (1974) writes the situation as the game in the biological context. Two animals face each other across the dish. If they deter, they share the profit fifty-fifty but rack up a large cost in the battle. If one compromises and the other deters, deterring animal gets all of the dish. If they compromise, they share the profit fifty-fifty amicably. We call the game of this type as war of attrition.

Alesina and Drazen(1991) applies the war of attrition to fiscal policy.² In the timing game, they assume that the game continues till any player compromises and that the value of dish is depreciated. In terms of game theory, corresponding payoff structure is known as the game of chicken. This game has pure strategy efficient equilibria and mixed strategy inefficient equilibrium. It is popular that mixed strategy inefficient equilibrium is only stable under best-response dynamics. In this game, it is social optimal that any player compromises at $t = 0$. However, the probability that the game does not end at $t = 0$ is positive in a stable equilibrium. Therefore, the expected time of stabilization is 'rational' delay.

In Japan, the bad-debt disposal was delayed. Many economists think that the economy in the last decade in Japan could have been recovered earlier if the problem could have been tackled earlier.³ What is behind the delay is that the bank with an impaired balance sheet might attempt to "gamble for resurrection" and increase risky lending to "zombie" firms.

Fukuda et al(2005) cited the five factors behind this banking corporation's action as follows:

- (1) over-optimistic expectation
- (2) the regulatory of capital ratio (discretionary zone in the accounting rule)
- (3) moral hazard in the banking corporations
- (4) reconciliation of interests (war of attrition)

¹Laibson(1997), O'Donoghue and Rabin(2006) and Benabou and Tirole(2002) give the explanation to this discounting.

²Fudenberg and Tirole(1991) investigated the theory of this game.

³Many high income OECD countries have faced this problem. The term most of them settle this problem was three or five years. So, the term of Japan is exceptionally long. [Caprio and Klingebiel(2003)]

(5) too-big-to-fail

In this paper, we view the base of NPL problem as the reconciliation of interests. We focus on the three different types of policies: change the cost of having NPLs, change the cost of eliminating NPLs and set the deadline of final disposal of NPLs. We categorize the policies which was done and we reveal what should be done in the lost decade and estimate the policies used by the war of attrition.

The model in this paper is in the literature of finance. But, the model can be applied in the other literature. We can apply the model to the problem of greenhouse warming. The existence of greenhouse gas causes the damage as global warming. The reduction of the gas causes the damage in the world economy. Even if any government makes an effort for reducing greenhouse gas emission in the country, the problem can not be solved. The above problem has the following properties:

- (1) The existence of the problems itself produces the cost.
- (2) The final disposal of the problems produces the cost.
- (3) These costs increase as time goes on.
- (4) Collective decision-making.

We can see these properties in many cases of joint ventures and politics. This paper will also provide the solution of these problems other than finance.

The remainder of this paper is organized as follows. Section 2 presents a benchmark model and equilibrium. Section 3 investigates the change of having or eliminating NPLs. Section 4 introduces the deadline. Section 5 concludes the paper. Formal proofs are collected in the Appendix.

2 Benchmark

2.1 Model

At first, we present the model describing the reconciliation of interests. There are nonperforming loans at $t = 0$. If the elimination of the loans is postponed, the loans increase exponentially.

We assume that the amount of bad debt is $D(t)$ and that the cost of final disposal of bad debt is $bD(t)$. For simplicity, we assume $D(t)$ increases at the rate of $\gamma - 1 (\geq 0)$. So, we set $D(t) = D_0\gamma^t (D_0 > 0)$.

There is a ceiling on the total amount of loaned money defined by the net asset. If a collection of doubtful receivables is done, the bank can lend the other good company in the scope of bad debts write-off recovery. In fact, the existence of bad debts causes opportunity cost. We assume that the cost is $aD(t)$ at t .

We set discount factor δ . And we define $V(t)$ as the net present value from eliminating bad loans at t . So, this value is

$$\begin{aligned} V(t) &= -\delta^t bD(t) - a[D(0) + \delta D(1) + \cdots + \delta^{t-1} D(t-1)] \\ &= -(\gamma\delta)^t bD_0 - \frac{1 - (\gamma\delta)^t}{1 - \gamma\delta} aD_0. \end{aligned}$$

When should the final disposal of bad debt be done? We set the period when the disposal should be done T^* . T^* is the solution of $\max_{t \geq 0} V(t)$.

- i) $\gamma\delta \geq 1$ The cost of final disposal and social cost increases progressively. So, $V(t)$ is decreasing function. Therefore, $T^* = 0$.
- ii) $\gamma\delta < 1$ The present value of cost of final disposal decreases and social cost increases. T^* is decided by the balance of the both sides.

$$\begin{aligned} V(t+1) - V(t) &= -[(\gamma\delta)^{t+1} - (\gamma\delta)^t]bD_0 \\ &\quad - [(1 + \dots + (\gamma\delta)^{t-1} + (\gamma\delta)^t) - (1 + \dots + (\gamma\delta)^{t-1})]aD_0 \\ &= [b(1 - \gamma\delta)(\gamma\delta)^t - a(\gamma\delta)^t]D_0. \end{aligned}$$

$$\text{So, } V(t+1) - V(t) > 0 \Leftrightarrow (1 - \gamma\delta)b > a.$$

Therefore,

- (a) if $1 - \gamma\delta > \frac{a}{b}$, $T^* = \infty$.
- (b) if $1 - \gamma\delta < \frac{a}{b}$, $T^* = 0$.

We assume that the best timing of final disposal is 0 ($T^* = 0$). Hence,

Assumption .1 $1 - \gamma\delta < \frac{a}{b}$.

There are two players (A,B). They have a nonperforming loan. One player has the half of the loan and the other has the other half. The cost which they eliminate bad debt is $bD(t) = bD_0\gamma^t$. They negotiate how much they share the cost. We define the player i 's share of the cost as C_i . The actions which they can take are compromising(C) and deterring(D) at each period. If player A compromises and player B deters, $C_A = bD(t), C_B = 0$. If player A deters and player B compromises, $C_A = 0, C_B = bD(t)$. If player A and player B compromise, the final disposal is taken by act. So, $C_A = C_B = \frac{1}{2}bD(t)$.

If player A and player B deter, the final disposal is procrastinated. In this case, the existence of bad debts generates the opportunity cost, $aD(t)$. They split the cost between A and B (because each player has the same amount of the loan). And they will negotiate in the next term again.

Now, we permit mixed strategy. So, we defined $p_i(t)$ as the probability that player i deters at t . $p_i(t) = 0$ means Compromise and $p_i(t) = 1$ means Deter. Hence, the strategy of player i is $(p_i(0), p_i(1), \dots)$.

Under the situation that the final disposal has not done at t , we calculate the value estimated at 0 from eliminating bad loans at t . The player's payoff estimated at 0 is $F(t), L(t)$ or $B(t)$ if any player compromises at t .

When both players compromise, we define $B(t)$ as the payoff of each player. When player i compromises and player j ($i \neq j$) deters, we define $L(t)$ as the

payoff of player i and $F(t)$ as one of player j . These payoffs include the sunk cost at t . So, we can represent these payoff as follows:

$$\begin{aligned}
F(t) &= -\frac{1 - (\gamma\delta)^t}{1 - \gamma\delta} \frac{a}{2} D_0, \\
L(t) &= -\frac{1 - (\gamma\delta)^t}{1 - \gamma\delta} \frac{a}{2} D_0 - (\gamma\delta)^t b D_0 \\
&= F(t) - (\gamma\delta)^t b D_0, \\
B(t) &= -\frac{1 - (\gamma\delta)^t}{1 - \gamma\delta} \frac{a}{2} D_0 - \frac{1}{2} (\gamma\delta)^t b D_0 \\
&= F(t) - \frac{1}{2} (\gamma\delta)^t b D_0.
\end{aligned}$$

Now, we assume that the subgame is war of attrition. Hence,

Assumption .2 $L(n) > B(n + 1) \Leftrightarrow 2 - \gamma\delta < \frac{a}{b}$.

If this assumption is violated, the subgames become prisoner's dilemma.

2.2 Equilibrium

Now, we investigate Subgame Perfect Nash Equilibrium in the infinite war of attrition. For the simplicity, we set the assumption as follows:

Assumption .3 *No player takes the strategy that he/she takes a stochastic action at any t and a nonstochastic action at the other t .*

At first, we investigate Pure Strategy Equilibrium. In the infinite war of attrition, the two stationary equilibrium are

- (1) $p_A(t) = 1$ and $p_B(t) = 0, \forall t \geq 0$,
- (2) $p_A(t) = 0$ and $p_B(t) = 1, \forall t \geq 0$.

In the former equilibrium, player B compromises at $t = 0$. So, player A does not bear at any cost. Even if player B does not compromise, player A deters infinitely. Therefore, the deposit is procrastinated and player B bear the loss, $\frac{a}{2} D_0 > 0$ at each period. For the minimization of these cost, player B compromises as soon as possible. And the game is finished at $t = 0$. If that helps, we can find some nonstationary equilibrium. In these equilibrium, either player compromises at $t = 0$ and the game is finished at $t = 0$.

At second, we investigate Mixed Strategy Equilibrium. There is a symmetric equilibrium where each player chooses the same strategy because this game is symmetric. We write the mixed strategy in this symmetric equilibrium as $(\hat{p}(0), \hat{p}(1), \dots, \hat{p}(t), \dots)$. We can calculate $\hat{p}(t)$ used by the property that the profit of player i is not changed whether he/she compromises or deters in the mixed strategy equilibrium.

(a) **When the player i compromises,** If player j compromises, the player i 's profit is $B(t) = F(t) - \frac{1}{2}(\gamma\delta)^t bD_0$.

If player j deters, the player i 's profit is $L(t) = F(t) - (\gamma\delta)^t bD_0$.

Because the probability player j compromises is $1 - \hat{p}(t)$, The expected profit of player i is

$$\begin{aligned} E[V(t)|a_i(t) = C] &= [1 - \hat{p}(t)][F(t) - \frac{1}{2}(\gamma\delta)^t bD_0] + \hat{p}(t)[F(t) - (\gamma\delta)^t bD_0] \\ &= F(t) - \frac{1}{2}[1 + \hat{p}(t)](\gamma\delta)^t bD_0. \end{aligned}$$

(we write the action of player i at t as $a_i(t)$.)

(b) **When the player i deters,** If player j compromises, the player i 's profit is $F(t)$.

If player j deters, the final deposit is procrastinated.

From the property of the mixed strategy equilibrium,

$$\begin{aligned} E[V(t+1)|a_i(t+1) = D] &= E[V(t+1)|a_i(t+1) = C] \\ &= F(t+1) - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0. \end{aligned}$$

$$\begin{aligned} E[V(t)|a_i(t) = D] &= [1 - \hat{p}(t)]F(t) + \hat{p}(t)E[V(t+1)|a_i(t+1) = D] \\ &= [1 - \hat{p}(t)]F(t) + \hat{p}(t)[F(t+1) - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0]. \end{aligned}$$

From (a) and (b),

$$\begin{aligned} F(t) - \frac{1}{2}[1 + \hat{p}(t)](\gamma\delta)^t bD_0 &= [1 - \hat{p}(t)]F(t) + \hat{p}(t)[F(t+1) - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0] \\ &\Leftrightarrow \\ F(t) - \frac{1}{2}[1 + \hat{p}(t)](\gamma\delta)^t bD_0 &= [1 - \hat{p}(t)]F(t) + \hat{p}(t)[F(t) - (\gamma\delta)^t \frac{a}{2} D_0 - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0] \\ &\Leftrightarrow \\ -\frac{1}{2}[1 + \hat{p}(t)](\gamma\delta)^t bD_0 &= \hat{p}(t)[-(\gamma\delta)^t \frac{a}{2} D_0 - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0] \\ &\Leftrightarrow \\ -[1 + \hat{p}(t)](\gamma\delta)^t &= \hat{p}(t)[-(\gamma\delta)^t \frac{a}{b} - [1 + \hat{p}(t+1)](\gamma\delta)^{t+1}] \\ &\Leftrightarrow \\ -[1 + \hat{p}(t)] &= \hat{p}(t)[-\frac{a}{b} - [1 + \hat{p}(t+1)](\gamma\delta)] \\ &\Leftrightarrow \\ \gamma\delta\hat{p}(t)\hat{p}(t+1) - [1 - \gamma\delta - \frac{a}{b}]\hat{p}(t) - 1 &= 0 \end{aligned}$$

This suggests the following proposition.

Proposition .1 *In the infinite war of attrition, there are stationary mixed strategy equilibrium and pure strategy equilibrium. That is to say, nonstationary mixed strategy equilibrium does not exist.*

Proof: see Appendix.

From Proposition.1, we can write a stationary mixed strategy equilibrium is (p^*, p^*, \dots) . And p^* satisfied $\gamma\delta p^{*2} - [1 - \gamma\delta - \frac{a}{b}]p^* - 1 = 0$. From assumption 2, the quadratic equation $\gamma\delta p^2 - [1 - \gamma\delta - \frac{a}{b}]p - 1 = 0$ has the real solution from 0 to 1.

$$\text{Therefore, } p^* = \frac{1 - \gamma\delta - \frac{a}{b} + \sqrt{(1 - \gamma\delta - \frac{a}{b})^2 + 4\gamma\delta}}{2\gamma\delta}.$$

Needless to say, the socially best period to deposit is $t = 0$. But the probability to deposit at $t = 0$ is $1 - (p^*)^2$. So, we do not always achieve the social optimality. And the expected period to deposit (EPD) is

$$(1 - p^{*2}) \times 0 + p^{*2}(1 - p^{*2}) \times 1 + \dots + p^{*2t}(1 - p^{*2}) \times t + \dots = \frac{p^{*2}}{1 - p^{*2}}.$$

3 Comparative Statics -Change the Cost of having or eliminating NPLs-

In this section, we show the effect that the financial authority taxes or subsidizes the cost of having or eliminating NPLs. In the model, we assume that the cost of having NPLs is $aD(t)$ and that the cost of eliminating NPLs is $bD(t)$. From the above, we can think that the period to deposit, the probability of deterring and the other outcome in this game are affected by the ratio between the cost of existing bad loans and the cost of eliminating them : $\frac{a}{b}$.

We define function $f(p) \equiv \gamma\delta p^2 - [1 - \gamma\delta - \frac{a}{b}]p - 1$. From the previous section, $f(p^*) = 0$. Now, $0 < p < 1 \Rightarrow \frac{\partial f(p)}{\partial \frac{a}{b}} = p > 0$. So, $\frac{\partial p^* (\frac{a}{b}, \gamma\delta)}{\partial \frac{a}{b}} < 0$. To rein in the procrastination, the financial authority must make $\frac{a}{b}$ be higher. Thus, the authority must tax the existence of NPLs and subsidize the disposal of NPLs.

In Japan, the government strengthened the legal structure on the bankruptcy, debt forgiveness and the liquidation of real estate and loans. We think these policies become the subsidy for the disposal of NPLs and promote the final disposal.

The taxation of the existence of NPLs was not done in Japan. But some countries adopt this policy. For example, the deposit insurance rate in Canada is decided by the asset quality. So, the rate of the bank which has large impaired assets is higher than one of the bank which has a little impaired assets. Deposit Insurance Corporation of Japan studied the variable deposit insurance rate. But the deposit insurance rate has been uniformly in Japan.

4 Setup of the Deadline

Is the policy to force the final disposal by the deadline decided by the financial authority effective? The government in Japan came out with the urgent economic package in April 2001. Many economists think that this package promoted the final disposal and recovered Japanese economy. Some of them believe that the setup of deadline is effective because the setup of deadline of the final disposal is a part of the package.

4.1 Simple Deadline

We investigate the policy to set the deadline. At first, we define deadline is at n as that the final deposit is taken by act when $n + 1$ if neither player has compromised till n . That is to say, the payoff profile at $t = n + 1$ is $(B(n + 1), B(n + 1))$ when each player has deterred till n .

We show that there are two pure strategy equilibria as follows:

- (1) $p_A(t) = 1$ and $p_B(t) = 0, \forall t \geq 0$,
- (2) $p_A(t) = 0$ and $p_B(t) = 1, \forall t \geq 0$.

We investigate mixed strategy equilibrium. We focus on a symmetric equilibrium where each player chooses the same strategy. Now, we write $\hat{p}_s(t, n)$ as the probability the player deters at t in the case that the deadline is n . As in the infinite war of attrition, we show that

$$\gamma\delta\hat{p}_s(t, n)\hat{p}_s(t + 1, n) - [1 - \gamma\delta - \frac{a}{b}]\hat{p}_s(t, n) - 1 = 0 \text{ if } t < n.$$

Next, we calculate $\hat{p}_s(n, n)$.

- (a) **When the player i compromises**, If player j compromises, the player i 's profit is $B(n) = F(n) - \frac{1}{2}(\gamma\delta)^n bD_0$.

If player j deters, the player i 's profit is $L(n) = F(n) - (\gamma\delta)^n bD_0$.

Because the probability player j compromises is $1 - \hat{p}_s(n, n)$, the expected profit of player i is

$$\begin{aligned} E[V(n)|a_i(n) = C] &= [1 - \hat{p}_s(n, n)][F(n) - \frac{1}{2}(\gamma\delta)^n bD_0] + \hat{p}_s(n, n)[F(n) - (\gamma\delta)^n bD_0] \\ &= F(n) - \frac{1}{2}[1 + \hat{p}_s(n, n)](\gamma\delta)^n bD_0. \end{aligned}$$

- (b) **When the player i deters**, If player j compromises, the player i 's profit is $F(n)$.

If player j deters, the final deposit is taken by act at $t = n + 1$.

So, the player i 's profit is $B(n + 1) = F(n + 1) - \frac{1}{2}(\gamma\delta)^{n+1} bD_0$

Because the probability player j compromises is $1 - \hat{p}_s(n, n)$,

$$E[V(n)|a_i(n) = D] = [1 - \hat{p}_s(n, n)]F(n) + \hat{p}_s(n, n)[F(n + 1) - \frac{1}{2}(\gamma\delta)^{n+1} bD_0].$$

From (a) and (b),

$$\begin{aligned}
F(n) - \frac{1}{2}[1 + \hat{p}_s(n, n)](\gamma\delta)^n bD_0 &= [1 - \hat{p}_s(n, n)]F(n) + \hat{p}_s(n, n)[F(n+1) - \frac{1}{2}(\gamma\delta)^{n+1}bD_0] \\
&\Leftrightarrow \\
F(n) - \frac{1}{2}[1 + \hat{p}_s(n, n)](\gamma\delta)^n bD_0 &= [1 - \hat{p}_s(n, n)]F(n) + \hat{p}_s(n, n)[F(n) - (\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}bD_0] \\
&\Leftrightarrow \\
-\frac{1}{2}[1 + \hat{p}_s(n, n)](\gamma\delta)^n bD_0 &= \hat{p}_s(n, n)[-(\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}bD_0] \\
&\Leftrightarrow \\
-[1 + \hat{p}_s(n, n)](\gamma\delta)^n b &= \hat{p}_s(n, n)[-(\gamma\delta)^n a - (\gamma\delta)^{n+1}b] \\
&\Leftrightarrow \\
-[1 + \hat{p}_s(n, n)] &= \hat{p}_s(n, n)[-\frac{a}{b} - (\gamma\delta)] \\
&\Leftrightarrow \\
\hat{p}_s(n, n) &= \frac{-1}{1 - \gamma\delta - \frac{a}{b}}.
\end{aligned}$$

Because we solve the game with deadline by backward induction,

$$\hat{p}_s(t, n) = \hat{p}_s(t-1, n-1) = \dots = \hat{p}_s(0, n-t).$$

For the convenience, we define x as $\gamma\delta$ and y as $-1 + \gamma\delta + \frac{a}{b}$. We have that

$$x\hat{p}_s(t, n)\hat{p}_s(t+1, n) + y\hat{p}_s(t, n) - 1 = 0.$$

Then, we can represent $\hat{p}_s(t, n) = \frac{c_{n-t}}{c_{n-t+1}}$ as $c_k = \alpha^k X + \beta^k Y$. We can show that $\alpha = \frac{y + \sqrt{y^2 + 4x}}{2}, \beta = \frac{y - \sqrt{y^2 + 4x}}{2}, X = \frac{\alpha}{\alpha - \beta}, Y = \frac{-\beta}{\alpha - \beta}$. So,

$$\hat{p}_s(n-t, n) = \frac{\alpha^t X + \beta^t Y}{\alpha^{t+1} X + \beta^{t+1} Y} = \frac{\frac{1}{\alpha} + \frac{1}{\alpha} \left(\frac{\beta}{\alpha}\right)^t \frac{Y}{X}}{1 + \left(\frac{\beta}{\alpha}\right)^{t+1} \frac{Y}{X}} = \frac{1}{\alpha} \frac{1 - \left(\frac{\beta}{\alpha}\right)^{t+1}}{1 - \left(\frac{\beta}{\alpha}\right)^{t+2}} = p^* \frac{1 - \left(\frac{\beta}{\alpha}\right)^{t+1}}{1 - \left(\frac{\beta}{\alpha}\right)^{t+2}}.$$

This equation suggests the following results.

Proposition .2 *If k is odd, $\hat{p}_s(n-k, n) < p^*$. If k is even or 0, $\hat{p}_s(n-k, n) > p^*$.*

Proof: see Appendix.

Proposition .3 $\lim_{n \rightarrow \infty} \hat{p}_s(t, n) = p^*$ for all $t < n$.

Proof: see Appendix.

We can represent the expected period to deposit in the game with simple deadline whose terminal node is n as $EPD_s(n)$.

Proposition .4 $\lim_{n \rightarrow \infty} EPD_s(n) = \frac{p^{*2}}{1 - p^{*2}}$.

Proof: see Appendix.

$\frac{p^{*2}}{1-p^{*2}}$ is the expected period to deposit (EPD) in the infinite game. So, we show that the case in the simple deadline is closed to the infinite case if n is close to the infinity by Proposition.3 and .4. This phenomenon may be consistent with the intuition which many persons have.

However, $\hat{p}_s(t, n)$ vibrates around p^* . Many persons may feel that setting the deadline enlarges the probability of compromising. This vibration shows that the intuition is wrong. We explain this vibration and conversion in the next subsection.

4.2 Why does the sequence of $\hat{p}_s(t, n)$ vibrate?

We feel that setting the deadline enlarges the probability of compromising intuitively. However, Proposition 2 shows that this intuition is wrong.

We try the intuitive explanation of this phenomenon.

We focus on the payoff matrixes.

	Compromise	Deter
Compromise	$F(t) - \frac{1}{2}bD_0(\gamma\delta)^t$ $F(t) - \frac{1}{2}bD_0(\gamma\delta)^t$	$F(t)$ $F(t) - bD_0(\gamma\delta)^t$
Deter	$F(t) - bD_0(\gamma\delta)^t$ $F(t)$	$F(t+1) - \frac{1}{2}bD_0(1 + \hat{p}(t+1))(\gamma\delta)^{t+1}$ $F(t+1) - \frac{1}{2}bD_0(1 + \hat{p}(t+1))(\gamma\delta)^{t+1}$

Table 1: the payoff matrix in the infinite war of attrition

	Compromise	Deter
Compromise	$F(n) - \frac{1}{2}bD_0(\gamma\delta)^n$ $F(n) - \frac{1}{2}bD_0(\gamma\delta)^n$	$F(n)$ $F(n) - bD_0(\gamma\delta)^n$
Deter	$F(n) - bD_0(\gamma\delta)^n$ $F(n)$	$F(n+1) - \frac{1}{2}bD_0(\gamma\delta)^{n+1}$ $F(n+1) - \frac{1}{2}bD_0(\gamma\delta)^{n+1}$

Table 2: the payoff matrix in the war of attrition with deadline ($t = n$)

Two matrixes have the property of chicken game. They are the same matrixes except (D,D). And the payoff of (D,D) at $t = n$ in the game with deadline is larger than one at steady state in the infinite game.

Because the expected payoff taking C must be the same as the payoff taking D, the probability of deterring at $t = n$ in the game with deadline: $\hat{p}_s(n, n)$ must become larger than the probability at steady state in the infinite game p^* (see Figure 1).

To put it another way, because compulsory disposal is done at $t = n + 1$, each player has the incentive not to bear all of the disposal cost at $t = n$ but to shift blame the half of the disposal cost to each other.

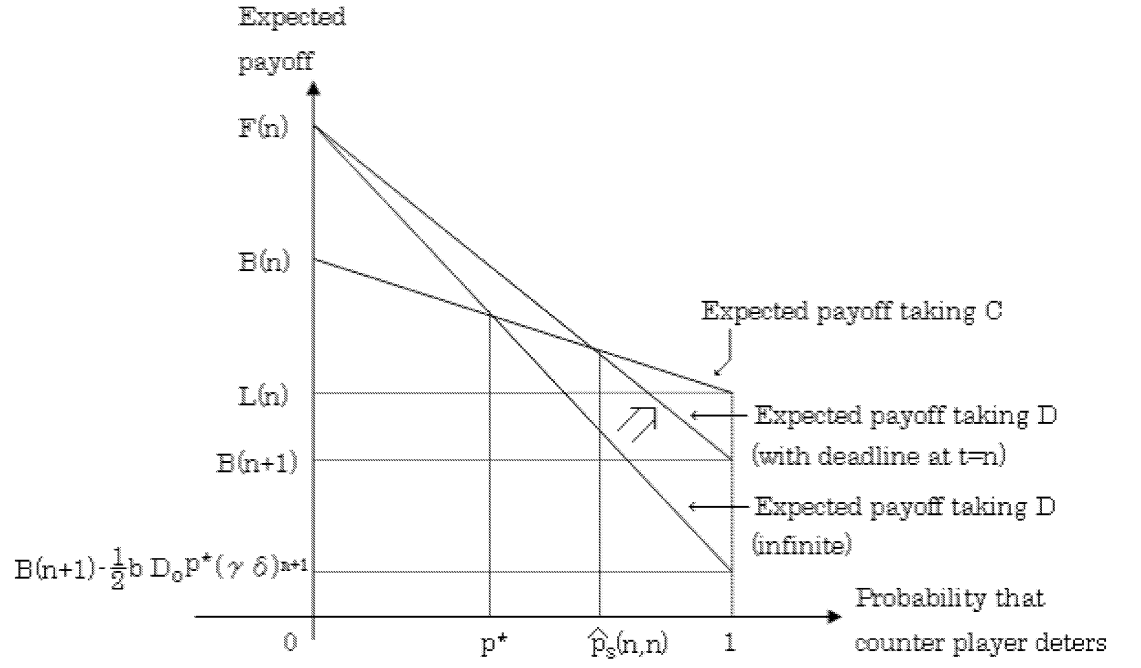


Figure 1: Expected payoff and probability that counter player deters

	Compromise	Deter
Compromise	$\begin{matrix} F(t) - \frac{1}{2}bD_0(\gamma\delta)^t \\ F(t) - \frac{1}{2}bD_0(\gamma\delta)^t \end{matrix}$	$\begin{matrix} F(t) \\ F(t) - bD_0(\gamma\delta)^t \end{matrix}$
Deter	$\begin{matrix} F(t) - bD_0(\gamma\delta)^t \\ F(t) \end{matrix}$	$\begin{matrix} F(t+1) - \frac{1}{2}bD_0(1 + \hat{p}_s(t+1, n))(\gamma\delta)^{t+1} \\ F(t+1) - \frac{1}{2}bD_0(1 + \hat{p}_s(t+1, n))(\gamma\delta)^{t+1} \end{matrix}$

Table 3: the payoff matrix in the war of attrition with deadline ($t < n$)

Next, the payoff matrix at $t < n$ is Table 3. Table 1 and Table 3 are the same matrixes excepted the probability at (D,D). When $\hat{p}_s(t+1, n) > p^*$, the payoff in the game with deadline is lower than one at steady state in the infinite game if the player deters. Therefore, the probability of deterring $\hat{p}_s(t, n)$ must drop than p^* .

In fact, each player has more incentive to dispose at this period because the probability of deterring at the next period is up.

Vice versa, when $\hat{p}_s(t+1, n) < p^*$, the payoff in the game with deadline is higher than one at steady state in the infinite game if the player deters. Therefore, the probability of deterring $\hat{p}_s(t, n)$ must come up than p^* (see Figure 2).

In fact, each player has less incentive to dispose at this period because the probability of deterring at the next period is down.

This is the intuition of Proposition 2.

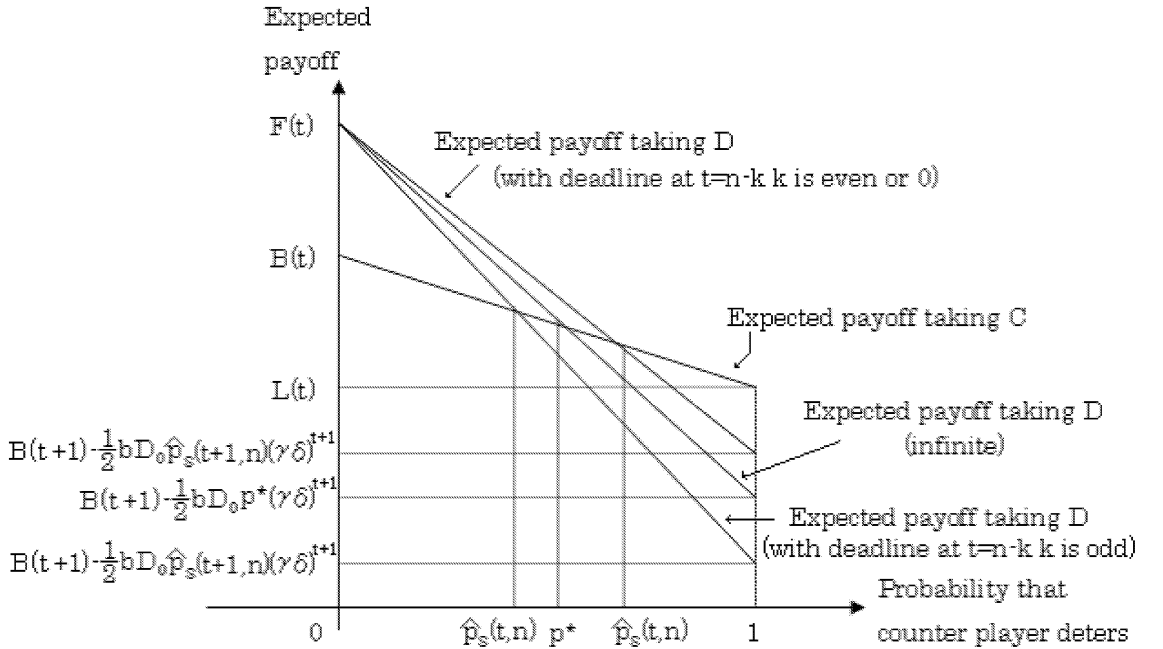


Figure 2: Expected payoff and probability that counter player deters

4.3 Is $EPD_s(n)$ larger or smaller than $\frac{p^{*2}}{1-p^{*2}}$?

It is hard to answer this question analytically because $EPD_s(n)$ is complex. (Please see the Proof of Proposition .4 in Appendix.)

Now, we try a simulation. We set $\delta = 0.2$, $\gamma = 3$ and $\frac{a}{b} = 1.41$. We have that $x = 0.6$, $y = 1.01$ and $p^* \approx 0.69946$. So, $\alpha \approx 1.43$, $\beta \approx -0.42$, $X \approx 0.77$, $Y \approx 0.23$.

We have that $\frac{p^{*2}}{1-p^{*2}} \approx 0.957879752$, $EPD_s(0) \approx 0.980296049$, $EPD_s(1) \approx 0.769642824$, $EPD_s(2) \approx 0.923807412$, $EPD_s(3) \approx 0.923255652$, $EPD_s(4) \approx 0.946256876, \dots$.

We show this as Figure.3.

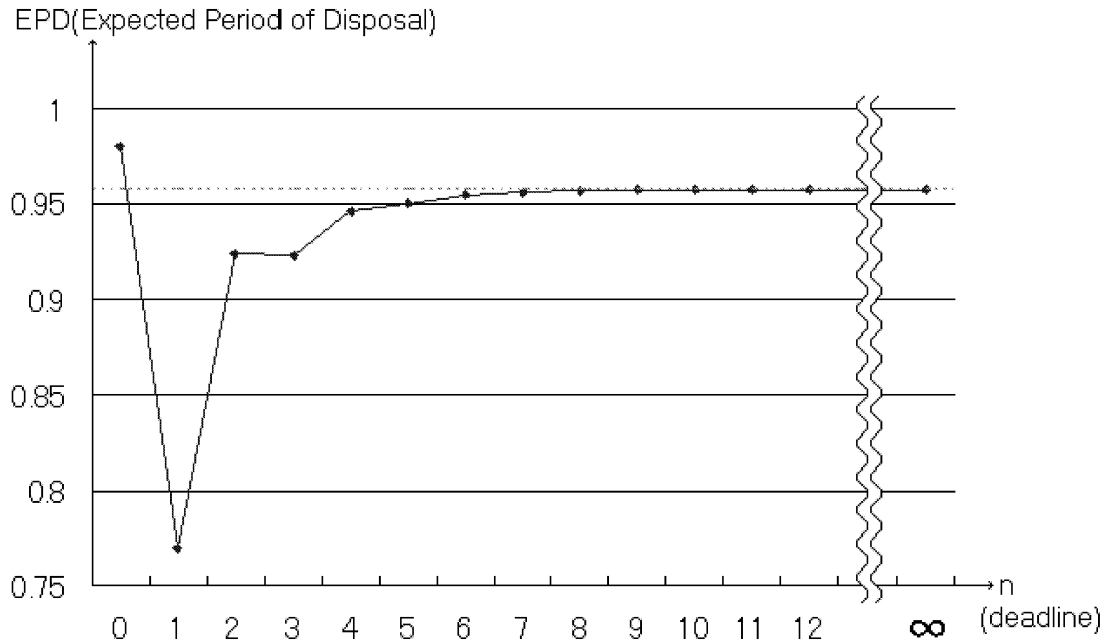


Figure 3: Expected period to deposit in simple deadline and infinite game

The above example shows that the regulation that the compulsory disposal will be done at the next period if the final disposal is not done at this period can be counterproductive. $\frac{p^{*2}}{1-p^{*2}} < EPD_s(0)$ means that the setup of the deadline encourage the procrastination.

The setting the deadline has two effects. The one is that the deadline prohibits the continued existence of the game. The other is that the deadline changes the incentive for the final disposal.

The latter effect at the odd period before the deadline is to encourage the procrastination. So, at the one period before the deadline, the extense of the deadline encourages the procrastination.

Now, we focus on the magnitude correlation between $EPD_s(0)$ and $\frac{p^{*2}}{1-p^{*2}}$.

$$\begin{aligned}
EPD_s(0) &> \frac{p^{*2}}{1-p^{*2}} \\
&\Leftrightarrow \\
&\frac{-y^2 + 2x + y\sqrt{y^2 + 4x}}{2} - 1 > 0 \\
&\Leftrightarrow \\
&\frac{-(-1 + \gamma\delta + \frac{a}{b})^2 + 2\gamma\delta - 1 + \gamma\delta + \frac{a}{b}\sqrt{(-1 + \gamma\delta + \frac{a}{b})^2 + 4\gamma\delta}}{2} - 1 > 0 \quad (1)
\end{aligned}$$

We show this area as Figure 4.

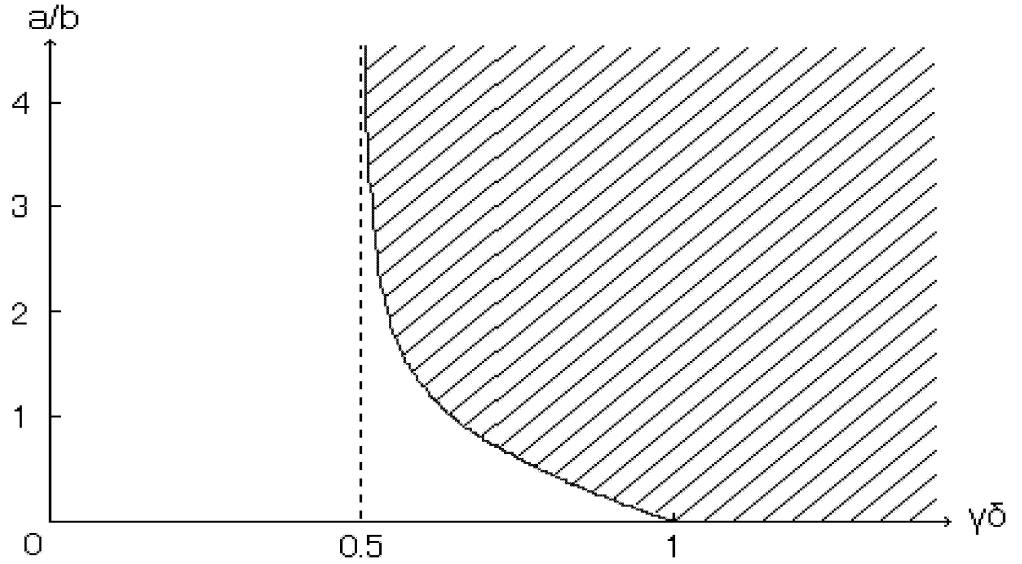


Figure 4: Expected payoff and probability that counter player deters

From Equation (1), When $\gamma\delta > 1$, the setting of the deadline always encourages the procrastination. However, the setting of the deadline always inhibits the procrastination when $\gamma\delta \leq 0.5$.

Now, we investigate the reason why the increases of $\gamma\delta$ has the propensity that the setting of the deadline encourages the procrastination. Plainly speaking, $\gamma\delta$ is the real growth rate of debt. The increase of the real growth rate of debt blocks the procrastination. But, the effect without deadline is more than one with deadline. Therefore, the increases of $\gamma\delta$ has the propensity that the setting of the deadline encourages the procrastination.

Now, we calculate the case of $n = 1$.

$$F(0) = 0, B(0) = -\frac{1}{2}bD_0, B(1) - \frac{1}{2}bD_0p^*\gamma\delta = -\frac{a}{2}D_0 - \frac{1}{2}bD_0\gamma\delta(1 + p^*), \\ L(0) = -bD_0.$$

If $\gamma\delta$ enlarges, p^* becomes small but $\hat{p}(0, 0)$ does not change. So, the expected period to deposit at the infinite game becomes small. But the expected period to deposit at the finite game does not change. Therefore, the increases of $\gamma\delta$ has the propensity that the setting of the deadline encourages the procrastination.

4.4 Punitive deadline

In the above, we assume that the eliminating cost is split when the financial authority forces the final disposal. We show that the setup of deadline does not necessarily imply that the procrastination is prevented in this situation. But, it is ordinary that the authority will punish the bank if the bank has done nothing by the deadline which the authority set. So, we assume that the authority punish all the bank that have deterred by the deadline.

We set b as follows:

$$b = \begin{cases} b_0 & \text{if } t < n + 1 \\ b_0 + b_p & \text{if } t = n + 1 \end{cases} \quad (b_0 > 0, b_p > 0)$$

We consider b_p as the punishment from the authority. Needless to say, if $b_p = 0$, b is the same as the simple deadline.

Now, we replace Assumption .1 and .2. We set the assumption as follow.

Assumption .4 $2 - \gamma\delta < \frac{a}{b_0}$.

This assumption is the same as Assumption .1 and .2.

We write $\hat{p}_p(t, n)$ as the probability the player deters at t in the case that the punitive deadline is n . We show that

$$\gamma\delta\hat{p}_p(t, n)\hat{p}_p(t + 1, n) - [1 - \gamma\delta - \frac{a}{b_0}]\hat{p}_p(t, n) - 1 = 0 \text{ if } t < n.$$

We calculate $\hat{p}_p(n, n)$ as follows by using the property that the expected payoff when the player compromises is the same as one when the player deters at n :

(a) When the player i compromises, If player j compromises, the player i 's profit is $B(n) = F(n) - \frac{1}{2}(\gamma\delta)^n b_0 D_0$.

If player j deters, the player i 's profit is $L(n) = F(n) - (\gamma\delta)^n b_0 D_0$.

Because the probability player j compromises is $1 - \hat{p}_p(n, n)$, the expected profit of player i is

$$E[V(n)|a_i(n) = C] = [1 - \hat{p}_p(n, n)][F(n) - \frac{1}{2}(\gamma\delta)^n b_0 D_0] + \hat{p}_p(n, n)[F(n) - (\gamma\delta)^n b_0 D_0] \\ = F(n) - \frac{1}{2}[1 + \hat{p}_p(n, n)](\gamma\delta)^n b_0 D_0.$$

(b) **When the player i deters**, If player j compromises, the player i 's profit is $F(n)$.

If player j deters, the final deposit is taken by act at $t = n + 1$.

So, the player i 's profit is $F(n + 1) - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0$

Because the probability player j compromises is $1 - \hat{p}_p(n, n)$,

$$\begin{aligned}
E[V(n)|a_i(n) = D] &= [1 - \hat{p}_p(n, n)]F(n) + \hat{p}_p(n, n)[F(n + 1) - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&= [1 - \hat{p}_p(n, n)]F(n) + \hat{p}_p(n, n)[F(n) - (\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&= F(n) + \hat{p}_p(n, n)[-(\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0].
\end{aligned}$$

From (a) and (b),

$$\begin{aligned}
F(n) - \frac{1}{2}[1 + \hat{p}_p(n, n)](\gamma\delta)^n b_0 D_0 &= F(n) + \hat{p}_p(n, n)[-(\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&\Leftrightarrow \\
-\frac{1}{2}[1 + \hat{p}_p(n, n)](\gamma\delta)^n b_0 D_0 &= \hat{p}_p(n, n)[-(\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&\Leftrightarrow \\
-[1 + \hat{p}_p(n, n)](\gamma\delta)^n b_0 &= \hat{p}_p(n, n)[-(\gamma\delta)^n a - (\gamma\delta)^{n+1}(b_0 + b_p)] \\
&\Leftrightarrow \\
-[1 + \hat{p}_p(n, n)]b_0 &= \hat{p}_p(n, n)[-a - (\gamma\delta)(b_0 + b_p)] \\
&\Leftrightarrow \\
\hat{p}_p(n, n) &= \frac{-b_0}{b_0 - \gamma\delta(b_0 + b_p) - a}.
\end{aligned}$$

The recurrence formula in ‘‘punitive deadline’’ is the same as one in ‘‘simple deadline.’’ However, $\hat{p}_s(n, n) \neq \hat{p}_p(n, n)$ if $b_p > 0$.

Then, we can represent $\hat{p}_p(t, n) = \frac{d_{n-t}}{d_{n-t+1}}$ as $d_k = \alpha^k Z + \beta^k W$. We can show that $Z = \frac{\alpha + x \frac{b_p}{b_0}}{\alpha - \beta}$, $W = \frac{\beta - x \frac{b_p}{b_0}}{\alpha - \beta}$. So,

$$\hat{p}_p(n-t, n) = \frac{\alpha^t Z + \beta^t W}{\alpha^{t+1} Z + \beta^{t+1} W} = \frac{\frac{1}{\alpha} Z + \frac{1}{\alpha} (\frac{\beta}{\alpha})^t W}{Z + (\frac{\beta}{\alpha})^{t+1} W} = \frac{1}{\alpha} \frac{Z + (\frac{\beta}{\alpha})^t W}{Z + (\frac{\beta}{\alpha})^{t+1} W} = p^* \frac{Z + (\frac{\beta}{\alpha})^t W}{Z + (\frac{\beta}{\alpha})^{t+1} W}.$$

This equation suggests the following propositions.

Proposition .5 (a) When $b_p > -\beta \frac{b_0}{x}$

If k is odd, $\hat{p}_p(n - k, n) > p^*$. If k is even or 0, $\hat{p}_p(n - k, n) < p^*$.

(b) When $b_p < -\beta \frac{b_0}{x}$

If k is odd, $\hat{p}_p(n - k, n) < p^*$. If k is even or 0, $\hat{p}_p(n - k, n) > p^*$.

(c) When $b_p = -\beta \frac{b_0}{x}$

$\hat{p}_p(n - k, n) = p^*$.

Proof: see Appendix.

Proposition .6 $\lim_{n \rightarrow \infty} \hat{p}_p(t, n) = p^*$ for all $t < n$.

Proof: see Appendix.

We can represent the expected period to deposit in the game with punitive deadline whose terminal node is n as $EPD_p(n)$.

Proposition .7 $\lim_{n \rightarrow \infty} EPD_p(n) = \frac{p^{*2}}{1 - p^{*2}}$.

Proof: see Appendix.

Proposition .8 If k is odd, $\hat{p}_s(n - k + 1, n) > \hat{p}_p(n - k, n)$. If k is even, $\hat{p}_s(n - k + 1, n) < \hat{p}_p(n - k, n)$.

Proof: see Appendix.

Proposition .9

$$\lim_{b_p \rightarrow \infty} \hat{p}_p(t, n) = \begin{cases} \hat{p}_s(t, n + 1) & \text{if } t \neq n \\ 0 & \text{if } t = n \end{cases}$$

Proof: see Appendix.

From Proposition .6 and .7, there is little point in punishment when n is large. From Proposition .5, the difference between the case in simple deadline and one in punitive deadline is little when b_p is small. These results may be consistent with your intuition.

But, the possibility of deterring does not become small uniformly if b_p becomes large. Even if b_p becomes larger than the threshold $(-\beta \frac{b_a}{x})$, the magnitude relation between $\hat{p}_p(n - k, n)$ and p^* becomes reversed only.

Each player has more incentive to dispose at $t = n$ because the player receives the large punishment if the game has not ended since $t = n$. The possibility of deterring is vibrated as the simple deadline because the recurrence formula in simple deadline is the same as one in punitive deadline. Therefore, the possibility of deterring does not become small uniformly.

From Proposition .9, however serious the punishment be, it move up the players' action only one period.

5 Concluding Remarks

In this paper, we investigate the policy of preventing the procrastination. We show that the taxation of having the NPLs and the subsidy of eliminating NPLs is the effective methods of keeping off the procrastination. And we show that the setup of deadline makes the probability of deterring be up and down every one period even if the financial authority implements the punishment. Therefore,

we conclude that the setup of deadline is not the effective methods of keeping off the procrastination.

Finally, we must admit that our model is restrictive. For example, we assume that the game is symmetric. We think this assumption is moderate in nonperforming loan problems not in main-bank system. However, this assumption may not be moderate in labor strikes or main-bank system. The change of the act (or custom) may affect the result of this paper. ⁴

Extending our model to these directions remains for future research.

A Appendix

A.1 Proof of Proposition .1

We assume that non stationary mixed strategy equilibrium exists. This equilibrium can be represented by $(p(0), p(1), \dots, p(t), \dots)$ (for each t and s , $p(t) \neq p(s)$). For any $p(t)$, $\gamma\delta p(t)^2 - [1 - \gamma\delta - a]p(t) - 1 = 0$.

Then, there is T which $p(T) > 1$. This is contradiction. So, non stationary mixed strategy equilibrium does not exist.

A.2 Proof of Proposition .2

Now, $\beta < 0, \alpha > 0$. $-y - \sqrt{y^2 + 4x} < -y + \sqrt{y^2 + 4x} \Rightarrow -\alpha < \beta < 0$.

Therefore, $-1 < \frac{\beta}{\alpha} < 0$.

a) k is odd

$$\begin{aligned} -\left(\frac{\beta}{\alpha}\right)^{k+1} < 0 < -\left(\frac{\beta}{\alpha}\right)^{k+2} \\ 0 < 1 - \left(\frac{\beta}{\alpha}\right)^{k+1} < 1 - \left(\frac{\beta}{\alpha}\right)^{k+2} \\ \frac{1 - \left(\frac{\beta}{\alpha}\right)^{k+1}}{1 - \left(\frac{\beta}{\alpha}\right)^{k+2}} < 1 \\ \hat{p}_s(n-k, n) = p^* \frac{1 - \left(\frac{\beta}{\alpha}\right)^{k+1}}{1 - \left(\frac{\beta}{\alpha}\right)^{k+2}} < p^*. \end{aligned}$$

b) k is even

$$\begin{aligned} -\left(\frac{\beta}{\alpha}\right)^{k+2} < 0 < -\left(\frac{\beta}{\alpha}\right)^{k+1} \\ 0 < 1 - \left(\frac{\beta}{\alpha}\right)^{k+2} < 1 - \left(\frac{\beta}{\alpha}\right)^{k+1} \\ \frac{1 - \left(\frac{\beta}{\alpha}\right)^{k+1}}{1 - \left(\frac{\beta}{\alpha}\right)^{k+2}} > 1 \end{aligned}$$

⁴Okuno-Fujiwara and Kouno(2007) shows that the role-contingent strategy can be used in the main-bank system and that this strategy makes pure strategy efficient equilibria stable.

$$\hat{p}_s(n-k, n) = p^* \frac{1 - (\frac{\beta}{\alpha})^{k+1}}{1 - (\frac{\beta}{\alpha})^{k+2}} > p^*.$$

A.3 Proof of Proposition .3

$$\lim_{n \rightarrow \infty} \hat{p}_s(t, n) = \lim_{n \rightarrow \infty} p^* \frac{1 + (\frac{\beta}{\alpha})^{n-t+1}}{1 + (\frac{\beta}{\alpha})^{n-t+2}} = p^*$$

A.4 Proof of Proposition .4

$$\begin{aligned} EPD_s(n) &= (1 - \hat{p}_s(0, n)^2) \times 0 \\ &+ \hat{p}_s(0, n)^2 (1 - \hat{p}_s(1, n)^2) \times 1 \\ &+ \hat{p}_s(0, n)^2 \hat{p}_s(1, n)^2 (1 - \hat{p}_s(2, n)^2) \times 2 \\ &+ \dots \\ &+ \hat{p}_s(0, n)^2 \hat{p}_s(1, n)^2 \dots \hat{p}_s(n-1, n)^2 (1 - \hat{p}_s(n, n)^2) \times n \\ &+ \hat{p}_s(0, n)^2 \hat{p}_s(1, n)^2 \dots \hat{p}_s(n-1, n)^2 \hat{p}_s(n, n)^2 \times (n+1) \\ &= (1 - \frac{c_n^2}{c_{n+1}^2}) \times 0 \\ &+ \frac{c_n^2}{c_{n+1}^2} (1 - \frac{c_{n-1}^2}{c_n^2}) \times 1 \\ &+ \frac{c_n^2}{c_{n+1}^2} \frac{c_{n-1}^2}{c_n^2} (1 - \frac{c_{n-2}^2}{c_{n-1}^2}) \times 2 \\ &+ \dots \\ &+ \frac{c_n^2}{c_{n+1}^2} \frac{c_{n-1}^2}{c_n^2} \dots \frac{c_1^2}{c_2^2} (1 - \frac{c_0^2}{c_1^2}) \times n \\ &+ \frac{c_n^2}{c_{n+1}^2} \frac{c_{n-1}^2}{c_n^2} \dots \frac{c_1^2}{c_2^2} \frac{c_0^2}{c_1^2} \times (n+1) \\ &= \frac{c_{n+1}^2 - c_n^2}{c_{n+1}^2} \times 0 \\ &+ \frac{c_n^2 - c_{n-1}^2}{c_{n+1}^2} \times 1 \\ &+ \frac{c_{n-1}^2 - c_{n-2}^2}{c_{n+1}^2} \times 2 \\ &+ \dots \\ &+ \frac{c_1^2 - c_0^2}{c_{n+1}^2} \times n \\ &+ \frac{c_0^2}{c_{n+1}^2} \times (n+1) \\ &= \frac{c_n^2 + c_{n-1}^2 + \dots + c_1^2 + c_0^2}{c_{n+1}^2} \end{aligned}$$

Now,

$$\begin{aligned} c_n^2 + c_{n-1}^2 + \cdots + c_1^2 + c_0^2 &= (X+Y)^2 + (\alpha X + \beta Y)^2 + \cdots + (\alpha^n X + \beta^n Y)^2 \\ &= \frac{1-\alpha^{2n+2}}{1-\alpha^2} X^2 + \frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta} 2XY + \frac{1-\beta^{2n+2}}{1-\beta^2} Y^2 \end{aligned}$$

$$c_{n+1}^2 = \alpha^{2n+2} X^2 + 2(\alpha\beta)^{n+1} XY + \beta^{2n+2} Y^2$$

So,

$$\begin{aligned} EPD_s(n) &= \frac{\frac{1-\alpha^{2n+2}}{1-\alpha^2} X^2 + \frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta} 2XY + \frac{1-\beta^{2n+2}}{1-\beta^2} Y^2}{\alpha^{2n+2} X^2 + 2(\alpha\beta)^{n+1} XY + \beta^{2n+2} Y^2} \\ &= \frac{\frac{\frac{1}{\alpha^{2n+2}} - 1}{1-\alpha^2} X^2 + \frac{\frac{1}{\alpha^{2n+2}} - (\frac{\beta}{\alpha})^{n+1}}{1-\alpha\beta} 2XY + \frac{\frac{1}{\alpha^{2n+2}} - (\frac{\beta}{\alpha})^{2n+2}}{1-\beta^2} Y^2}{X^2 + 2(\frac{\beta}{\alpha})^{n+1} XY + (\frac{\beta}{\alpha})^{2n+2} Y^2} \end{aligned}$$

$$\lim_{n \rightarrow \infty} EPD_s(n) = \frac{\frac{-1}{1-\alpha^2} X^2}{X^2} = \frac{-1}{1-\alpha^2} = \frac{p^{*2}}{1-p^{*2}}.$$

A.5 Proof of Proposition .5

At first, we must show $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0$.

If $(\frac{\beta}{\alpha})^k \frac{W}{Z} \geq 0$, $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} \geq 1 > 0$.

Otherwise, (1) k is even and $\frac{W}{Z} < 0$ or (2) k is odd and $\frac{W}{Z} > 0$.

(1) k is even and $\frac{W}{Z} < 0$

$$1 + \frac{W}{Z} < 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}$$

$$\text{From } 1 + \frac{W}{Z} = \frac{W+Z}{Z} = \frac{1}{Z} > 0, 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0.$$

(2) k is odd and $\frac{W}{Z} > 0$

$$1 + \frac{\beta W}{\alpha Z} \leq 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}$$

$$\text{From } 1 + \frac{\beta W}{\alpha Z} = \frac{\alpha Z + \beta W}{Z} = \frac{y + x \frac{b_p}{b_0}}{Z} > 0, 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0.$$

So, we show $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0$.

(a) $W < 0$ ($b_p > -\beta \frac{b_0}{x}$) i) k is odd

$$0 < 1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z} < 1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}$$

$$\frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} > 1$$

$$\hat{p}_p(n-k, n) = p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} > p^*$$

ii) k is even

$$0 < 1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z} < 1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}$$

$$\frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} < 1$$

$$\hat{p}_p(n-k, n) = p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} < p^*$$

(b) $W > 0$ ($b_p < -\beta \frac{b_0}{x}$) i) k is odd

$$0 < 1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z} < 1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}$$

$$\frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} < 1$$

$$\hat{p}_p(n-k, n) = p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} < p^*$$

ii) k is even

$$0 < 1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z} < 1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}$$

$$\frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} > 1$$

$$\hat{p}_p(n-k, n) = p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} > p^*$$

(c) $W=0$ ($b_p = -\beta \frac{b_0}{x}$)

$$\hat{p}_p(n-k, n) = p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^k \times 0}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \times 0} = p^*$$

A.6 Proof of Proposition .6

$$\lim_{n \rightarrow \infty} \hat{p}_p(t, n) = \lim_{n \rightarrow \infty} p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^{n-t} \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{n-t+1} \frac{W}{Z}} = p^*$$

A.7 Proof of Proposition .7

$$\begin{aligned}
EPD_p(n) &= (1 - \hat{p}_p(0, n)^2) \times 0 \\
&+ \hat{p}_p(0, n)^2 (1 - \hat{p}_p(1, n)^2) \times 1 \\
&+ \hat{p}_p(0, n)^2 \hat{p}_p(1, n)^2 (1 - \hat{p}_p(2, n)^2) \times 2 \\
&+ \dots \\
&+ \hat{p}_p(0, n)^2 \hat{p}_p(1, n)^2 \dots \hat{p}_p(n-1, n)^2 (1 - \hat{p}_p(n, n)^2) \times n \\
&+ \hat{p}_p(0, n)^2 \hat{p}_p(1, n)^2 \dots \hat{p}_p(n-1, n)^2 \hat{p}_p(n, n)^2 \times (n+1) \\
&= \left(1 - \frac{d_n^2}{d_{n+1}^2}\right) \times 0 \\
&+ \frac{d_n^2}{d_{n+1}^2} \left(1 - \frac{d_{n-1}^2}{d_n^2}\right) \times 1 \\
&+ \frac{d_n^2}{d_{n+1}^2} \frac{d_{n-1}^2}{d_n^2} \left(1 - \frac{d_{n-2}^2}{d_{n-1}^2}\right) \times 2 \\
&+ \dots \\
&+ \frac{d_n^2}{d_{n+1}^2} \frac{d_{n-1}^2}{d_n^2} \dots \frac{d_1^2}{d_2^2} \left(1 - \frac{d_0^2}{d_1^2}\right) \times n \\
&+ \frac{d_n^2}{d_{n+1}^2} \frac{d_{n-1}^2}{d_n^2} \dots \frac{d_1^2}{d_2^2} \frac{d_0^2}{d_1^2} \times (n+1) \\
&= \frac{d_{n+1}^2 - d_n^2}{d_{n+1}^2} \times 0 \\
&+ \frac{d_n^2 - d_{n-1}^2}{d_{n+1}^2} \times 1 \\
&+ \frac{d_{n-1}^2 - d_{n-2}^2}{d_{n+1}^2} \times 2 \\
&+ \dots \\
&+ \frac{d_1^2 - d_0^2}{d_{n+1}^2} \times n \\
&+ \frac{d_0^2}{d_{n+1}^2} \times (n+1) \\
&= \frac{d_n^2 + d_{n-1}^2 + \dots + d_1^2 + d_0^2}{d_{n+1}^2}
\end{aligned}$$

Now,

$$\begin{aligned} d_n^2 + d_{n-1}^2 + \cdots + d_1^2 + d_0^2 &= (Z + W)^2 + (\alpha Z + \beta W)^2 + \cdots + (\alpha^n Z + \beta^n W)^2 \\ &= \frac{1 - \alpha^{2n+2}}{1 - \alpha^2} Z^2 + \frac{1 - (\alpha\beta)^{n+1}}{1 - \alpha\beta} 2ZW + \frac{1 - \beta^{2n+2}}{1 - \beta^2} W^2 \end{aligned}$$

$$d_{n+1}^2 = \alpha^{2n+2} Z^2 + 2(\alpha\beta)^{n+1} ZW + \beta^{2n+2} W^2$$

So,

$$\begin{aligned} EPD_p(n) &= \frac{\frac{1 - \alpha^{2n+2}}{1 - \alpha^2} Z^2 + \frac{1 - (\alpha\beta)^{n+1}}{1 - \alpha\beta} 2ZW + \frac{1 - \beta^{2n+2}}{1 - \beta^2} W^2}{\alpha^{2n+2} Z^2 + 2(\alpha\beta)^{n+1} ZW + \beta^{2n+2} W^2} \\ &= \frac{\frac{\frac{1}{\alpha^{2n+2}} - 1}{1 - \alpha^2} Z^2 + \frac{\frac{1}{\alpha^{2n+2}} - (\frac{\beta}{\alpha})^{n+1}}{1 - \alpha\beta} 2ZW + \frac{\frac{1}{\alpha^{2n+2}} - (\frac{\beta}{\alpha})^{2n+2}}{1 - \beta^2} W^2}{Z^2 + 2(\frac{\beta}{\alpha})^{n+1} ZW + (\frac{\beta}{\alpha})^{2n+2} W^2} \end{aligned}$$

$$\lim_{n \rightarrow \infty} EPD_p(n) = \frac{\frac{-1}{1 - \alpha^2} Z^2}{Z^2} = \frac{-1}{1 - \alpha^2} = \frac{p^{*2}}{1 - p^{*2}}.$$

A.8 Proof of Proposition .8

$$\begin{aligned} -Z - W &= \frac{-\alpha - x \frac{b_p}{b_0} + \beta + x \frac{b_p}{b_0}}{\alpha - \beta} \\ &= \frac{-\alpha + \beta}{\alpha - \beta} = -1 < 0 \end{aligned}$$

From this, $Z > 0$ and $\frac{\beta}{\alpha} < 0 < 1$,

$$\begin{aligned} \frac{\beta}{\alpha} \left(-1 - \frac{W}{Z}\right) &> -1 - \frac{W}{Z} \\ -\frac{\beta}{\alpha} + \frac{W}{Z} &> -1 + \frac{\beta W}{\alpha Z}. \end{aligned}$$

a) k is odd

$$\begin{aligned} 1 - \left(\frac{\beta}{\alpha}\right)^{2k+1} \frac{W}{Z} - \left(\frac{\beta}{\alpha}\right)^{k+1} + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z} &< 1 - \left(\frac{\beta}{\alpha}\right)^{2k+1} \frac{W}{Z} - \left(\frac{\beta}{\alpha}\right)^k + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z} \\ \left(1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}\right) \left(1 - \left(\frac{\beta}{\alpha}\right)^{k+1}\right) &< \left(1 - \left(\frac{\beta}{\alpha}\right)^k\right) \left(1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}\right) \end{aligned}$$

From $1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z} > 0$ and $1 - \left(\frac{\beta}{\alpha}\right)^{k+1} > 0$,

$$\frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} < \frac{1 - \left(\frac{\beta}{\alpha}\right)^k}{1 - \left(\frac{\beta}{\alpha}\right)^{k+1}}.$$

Therefore, $\hat{p}_s(n - k + 1, n) = p^* \frac{1 - \left(\frac{\beta}{\alpha}\right)^k}{1 - \left(\frac{\beta}{\alpha}\right)^{k+1}} > p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} = \hat{p}_p(n - k, n)$

b) k is even

$$\begin{aligned} 1 - \left(\frac{\beta}{\alpha}\right)^{2k+1} \frac{W}{Z} - \left(\frac{\beta}{\alpha}\right)^{k+1} + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z} &> 1 - \left(\frac{\beta}{\alpha}\right)^{2k+1} \frac{W}{Z} - \left(\frac{\beta}{\alpha}\right)^k + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z} \\ \left(1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}\right) \left(1 - \left(\frac{\beta}{\alpha}\right)^{k+1}\right) &> \left(1 - \left(\frac{\beta}{\alpha}\right)^k\right) \left(1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}\right) \end{aligned}$$

From $1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z} > 0$ and $1 - \left(\frac{\beta}{\alpha}\right)^{k+1} > 0$,

$$\frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} > \frac{1 - \left(\frac{\beta}{\alpha}\right)^k}{1 - \left(\frac{\beta}{\alpha}\right)^{k+1}}.$$

Therefore, $\hat{p}_s(n - k + 1, n) = p^* \frac{1 - \left(\frac{\beta}{\alpha}\right)^k}{1 - \left(\frac{\beta}{\alpha}\right)^{k+1}} < p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^k \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{k+1} \frac{W}{Z}} = \hat{p}_p(n - k, n)$

A.9 Proof of Proposition .9

$$\lim_{b_p \rightarrow \infty} \frac{W}{Z} = \lim_{b_p \rightarrow \infty} \frac{-\beta - x \frac{b_p}{b_0}}{\alpha + x \frac{b_p}{b_0}} = \lim_{b_p \rightarrow \infty} \frac{-\frac{\beta}{b_p} - \frac{x}{b_0}}{\frac{\alpha}{b_p} + \frac{x}{b_0}} = \frac{-\frac{x}{b_0}}{\frac{x}{b_0}} = -1$$

Therefore,

$$\begin{aligned} \lim_{b_p \rightarrow \infty} \hat{p}_p(t, n) &= \lim_{b_p \rightarrow \infty} p^* \frac{1 + \left(\frac{\beta}{\alpha}\right)^{n-t} \frac{W}{Z}}{1 + \left(\frac{\beta}{\alpha}\right)^{n-t+1} \frac{W}{Z}} = p^* \frac{1 - \left(\frac{\beta}{\alpha}\right)^{n-t}}{1 - \left(\frac{\beta}{\alpha}\right)^{n-t+1}} \\ &= \begin{cases} \hat{p}_s(t, n + 1) & \text{if } t \neq n \\ 0 & \text{if } t = n \end{cases} \end{aligned}$$

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