Ownership Structure, Shadow Cost of Public Funds and Optimal Privatization

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Abstract

We examine how the shadow cost of public funds will affect the privatization policy in the presence of strategic tax/subsidy policies in a mixed oligopoly model with foreign ownership. We show that (1) The tax/subsidy policies could be adopted and the optimal privatization policy is partial privatization in the presence of shadow cost of public funds and foreign ownership; (2) The optimum subsidy rate is decreasing in the shadow cost of public funds and the degree of privatization is increasing in the shadow

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cost of public funds; (3) The optimum subsidy rate is decreasing in the shareholding ratio of the foreigners, while the degree of privatization is irrelevance of the shareholding ratio of the foreigners; (4) The profit of the privatized firm and private firms, the consumer surplus and social welfare are decreasing in the shadow cost of public funds and the equity share held by foreign investors.

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1. Introduction

Over the past few decades, proliferated theoretical literatures involve the exploration of privatization. De Fraja and Delbono (1989) in a mixed oligopoly model showed that the privatization of welfare-maximizing public firms may improve social welfare. Matsumura (1998) explicitly considered the possibility of partial privatization. In a free-entry market, Matsumura and Kanda (2005) assessed the welfare implications of partial privatization in a homogeneous oligopoly, and can be an alternative to direct regulation to avoid the excess entry problem. Wang and Chen (2010) highlighted the importance of cost efficiency gap between public and private firm, and showed the relation of cost efficiency gap and foreign competition with optimal privatization at free entry market.

The literature on the optimal subsidy in mixed oligopoly, White (1996) showed that the same subsidy rate yields the first-best outcome in both mixed and private oligopoly in his Cournot setting (privatization neutrality theorem). This privatization neutrality theorem (PNT) was supported by Tomaru (2006), who showed that the optimal subsidy, all firms' output, profits and social welfare are identical regardless of the share in a state-owned enterprise (SOE), and Matsumura and Okumura (2013), who also showed with the optimal output floor regulation, privatization does not affect welfare regardless of the time structure and the degree of privatization. Wang and Chen

(2011a) and Matsumura and Tomaru (2013) introduce an excess-taxation burden to describe the violation of the PNT. The violation of the PNT means privatization can work to enhance welfare. Tomaru and Wang (2018) then examined the optimal privatization policy. They showed that the optimal policy is partial privatization when the technical improvement is small. Matsumura (1998) showed that partial privatization is optimal in a mixed duopoly. A crucial difference between them is that Tomaru and Wang (2018) considered subsidization and a technical improvement resulting from privatization. Subsidization adjusts the production allocation, and privatization plays an auxiliary role in enhancing welfare by further adjusting the production allocation and improving the technology of the privatized firm.

We have observed that beside the open-door policy in product marker, recent capital liberalization that is prevalent globally has enabled not only domestic investors but also foreign investors to own domestic private firms in many mixed markets. To see how the foreign penetration affects the privatization policy, Wang and Chen (2011a) showed that in the short-run, the government should increase the degree of privatization when the equity share held by foreign investor is increasing which increases all domestic private firms' profit and social welfare. Cato and Matsumura (2012) investigated how foreign penetration in the domestic market affects the privatization policy and showed that the optimal degree of privatization is increasing in foreign penetration in the long-run. The implications of Wang and Chen (2011a) and Cato and Matsumura (2012) are that the open capital market policy and privatization are

complementary whether there is no entry barrier exists or entry or exit is possible in the market. The country with a more open capital market should privatize the firms more, even though this temporally reduces welfare. The above papers did not consider the implication of industrial policy on whether the government should privatize the public firm in the free entry equilibrium.

Matsumura and Tomaru (2012) found that under the optimal tax-subsidy policy, the government's privatization decision depends on how many private firms in the product market and the equity share held by foreign investor. In particular, when the equity share held by the foreign investors is low, the government should privatize the public firm in the absence of free entry of private firms. Wang and Lee (2013) examined how the order of the firm's moves affects the social efficiency with foreign ownership and free entry in a mixed oligopoly market. In particular, they showed that when the foreign shareholding ratio is low, the entry of private followers will lead to a lower consumer welfare and higher social welfare, but the profit of the incumbent nationalized firm is higher under entry than under no entry. Further, they found that there always exists the problem of excessive entry under public leadership regardless of the degree of foreign ownership which has important implications for industrial and market-opening policies. Wang and Tomaru (2015) showed that partial privatization is optimal for small extent of foreign penetration and the optimal degree of privatization is not monotonically related to foreign penetration. This result is in sharp contrast to the existing works which suggest either the positive or negative relationship. Xu *et al*.

(2017) investigated the impact of the timing of privatization and liberalization policies on the degree of privatization and number of entering firms in free-entry mixed markets. They formulated two models: ex-post privatization and ex-ante privatization. In the former, the government liberalizes the market and then privatizes the public firm, whereas the order of the policies is reversed in the latter. They showed that ex-ante privatization yields a higher (lower) level of privatization and a larger (smaller) equilibrium number of entering private firms when foreign ownership in private firms is high (low).

In the literature on mixed oligopolies, Capuano and De Feo (2010), Wang and Chen (2011b) and Matsumura and Tomaru (2013) have tackled the policy burden issue by examining the welfare effect of a change in a public firm's objective function when the government takes into account the shadow cost of public funds (or, henceforth, excess taxation burden, ETB). Wang and Chen (2011b) considered only the case of Cournot competition with cost efficiency gap, while Matsumura and Tomaru (2013) compared the optimal subsidies and the resulting welfare levels among four regimes: mixed and private Cournot duopolies and Stackelberg competition with public and private leaderships. Further, Matsumura and Tomaru (2015) examined the effect of product differentiation, while Xu *et al*. (2016) and Lee and Wang (2018) considered the relation with foreign competition. Looking at the influence of environmental policies on privatization in a free-entry mixed markets taking account of excess burden of taxation, Xu and Lee (2018) showed that when the excess burden of taxation is small

(large), ex-post taxation imposes a lower (higher) tax level than ex-ante taxation, which induces a larger (smaller) number of firms and a higher (lower) environmental damage. They also showed that the excess burden of taxation can increase the welfare, but ex-ante taxation always yields higher welfare than ex-post taxation.

In this paper, we examine how the foreign ownership will affect the privatization policy in the presence of strategic tax/subsidy policies under Cournot competition. We show that (1) The tax/subsidy policies could be adopted and the optimal privatization policy is partial privatization in the presence of shadow cost of public funds and foreign ownership; (2) The optimum subsidy rate is decreasing in the shadow cost of public funds and the degree of privatization is increasing in the shadow cost of public funds; (3) The optimum subsidy rate is decreasing in the shareholding ratio of the foreigners, while the degree of privatization is irrelevance of the shareholding ratio of the foreigners; (4) The profit of the privatized firm and private firms, the consumer surplus and social welfare are decreasing in the shadow cost of public funds and the equity share held by foreign investors.

This paper is organized as follows. Basic modeling is provided in Section 2. Section 3 explores how the ETB and foreign penetration will affect the privatization policy in the presence of strategic tax/subsidy policies. Section 4 provides conclusions.

2. Basic Model

Consider a domestic market for a homogeneous good produced by one public firm, and n domestic firms. The linear demand function is specified as $P = a - Q$. The

supply equation is given by $Q = q_0 + \sum_{i=1}^{n} q_i$, where q_0, and q_i denote, respectively, the output of public firm, and domestic firms. As in many existing studies on mixed oligopoly, we assume that all firms use an identical technology and have the increasing marginal cost function: $\frac{q_0^2}{2}$, and $\frac{q_i^2}{2}$, respectively.[1]

The profits of domestic firms and foreign firms are given by:

$$\pi_0 = (a - q_0 + \sum_{i=1}^{n} q_i + s)q_0 - \frac{q_0^2}{2} \tag{1}$$

$$\pi_i = (a - q_0 + \sum_{i=1}^{n} q_i + s)q_i - \frac{q_i^2}{2} \tag{2}$$

where s is the unit subsidy rate.

The social welfare is defined as,

$$SW = CS + (1 - \alpha) \sum_{i=1}^{n} \pi_i - (1 + \lambda)(s \sum_{i=1}^{n} q_i + sq_0 - \pi_0) \tag{3}$$

where the consumer surplus is given by $CS = Q^2/2$, α is the shareholding ratio of the foreigners, and λ signifies the shadow cost of public funds for representing administrative inefficiency of government bureaucracy.[2] As explained in Matsumura and Tomaru (2012), when all private firms are symmetric and α denotes the share of foreign investors in the private firms, then there are αn foreign private firms and $(1 - \alpha)n$ domestic private firms. As such, two formulations yield exactly the same equilibrium outcomes.[3] It is important to be aware that privatization or share release is

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1 See Matsumura and Kanda (2005), Wang *et al.* (2009) and Wang and Chen (2010) for using the specification of increasing marginal costs (decreasing returns to scale technology) in mixed oligopolies. In the current paper, we use a homogeneous demand function with decreasing returns to scale technology, which is not a general specification of the demand and the cost side of the model. Once the two assumptions are relaxed, our results may qualitatively differ.

2 The similar specification can be found in Capuano and De Feo (2010), Wang and Chen (2011b), and Matsumura and Tomaru (2013).

3 Foreign ownership of public firms is not considered in this paper. Lin and Matsumura (2012) also

not always the best policy option when a government tries to improve the efficiency of public firms. We assume that $\lambda \in [0, \infty)$. As pointed out in Matsumura and Tomaru (2013), the welfare can be decomposed into the welfare without excess taxation burden and the distortion due to taxation. Moreover, we can rewrite equation (3) to obtain

$$
\begin{aligned}
SW(\lambda) &= CS + (1 - \alpha) \sum_{i=1}^{n} \pi_i - (1 + \lambda)(s \sum_{i=1}^{n} q_i + sq_0 - \pi_0) \\
&= [CS + (1 - \alpha) \sum_{i=1}^{n} \pi_i - s \sum_{i=1}^{n} q_i - sq_0 + \pi_0] \\
&\quad - \lambda(s \sum_{i=1}^{n} q_i + sq_0 - \pi_0) \\
&= SW(\lambda = 0) + \lambda(\pi_0 - s \sum_{i=1}^{n} q_i - sq_0)
\end{aligned}
$$

The right-hand side of the equation states that the excess burden applies on the subsidy paid to the private firms. As easily inferred from this welfare, an increase in λ makes the official put greater emphasis on the profit of SOE and tariff revenue.

The government sells all or a part of shares in firm 0 in the first stage (shares-selling stage). This means that the revenue from selling the shares is fixed in the later stage, where the output-setting stage follows the shares-selling stage. The government finances the subsidies for the firms from the partial profits of the privatized firm, and the revenue from selling the stocks of firms. Then, the government sets s and θ to maximize the following welfare:

$$
\begin{aligned}
W &= CS + (1 - \alpha) \sum_{i=1}^{n} \pi_i + \theta \pi_0 - V \\
&\quad + (1 + \lambda)[(1 - \theta)\pi_0 + V - sq_0 - s \sum_{i=1}^{n} q_{di}]
\end{aligned} \tag{4}
$$

where V is the revenue from selling the shares in firm 0. Then, the government maximizes the welfare W, expecting (i) the equilibrium result in the subgames and (ii)

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investigated the presence of foreign investors in privatized firms and confirmed Wang and Chen's finding that an increase in the stockholding ratio of foreign investors in a privatized firm increases the optimal degree of privatization, whereas an increase in the penetration of foreign firms in product markets reduces it. These results imply that the degree of openness of financial markets and that of product markets have contrasting implications for the optimal privatization policy.

$V = \theta \pi_0$ due to the private investors' rationality and the assumption of perfect stock markets.

When government privatizes the public firm partially, the optimization problem for the semi-public firm is:

$$\max_{\{q_0\}} . \Omega = \theta \pi_0 + (1 - \theta)W \tag{5}$$

where θ is the weight assigned to the profits in the decision-making process of the firm, and $1 > \theta > 0$.[4] Following Matsumura (1998), the government can indirectly control θ through its shareholding. The fully privatized firm only seeks the profit if $\theta = 1$; contrarily, a fully nationalized firm maximizes the social welfare if $\theta = 0$. The government chooses the subsidy rate and the degree of privatization to maximize social welfare.

We construct a two-stage game. In the first stage of the game, the government decides the subsidy rate and the degree of privatization. In the second stage, the firms engage in Cournot competition. The backward induction is used to derive the *sub-game perfect Nash equilibrium* (SPNE).

3. Foreign Ownership, Privatization and Welfare

The outputs of the domestic private firms and the public firm are obtained by partially differentiating Eqs. (2) and (5) with respect to q_i, and q_0, the first-order conditions are:

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[4] Public firms may have other different targets, such as maximizing the profit, income, employee's income or management of license, etc. See De Fraja and Delbono (1989), and Pal and White (1998) on the modeling of a public firm as a social welfare maximizer.

$$\frac{\partial \pi_i}{\partial q_i} = a + s - q_0 - 2q_i - nq_i = 0 \tag{6}$$

$$\frac{\partial \Omega}{\partial q_0} = a + s\theta + a\lambda - a\theta\lambda + (-2 - \theta + 3(-1 + \theta)\lambda)q_0$$

$$+ n(-1 + \alpha - \alpha\theta + (-1 + \theta)\lambda)q_i = 0 \tag{7}$$

The second-order conditions are:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -1 - 2n$$

$$\frac{\partial^2 \Omega}{\partial q_0^2} = -2 - \theta + 3(-1 + \theta)\lambda$$

$$\frac{\partial^2 \pi_i}{\partial q_i \, \partial q_0} = -1$$

$$\frac{\partial^2 \Omega}{\partial q_0 \, \partial q_i} = n(-1 + \alpha - \alpha\theta + (-1 + \theta)\lambda)$$

$$|H| = \begin{vmatrix} \dfrac{\partial^2 \Omega}{\partial q_0^2} & \dfrac{\partial^2 \Omega}{\partial q_0 \, \partial q_i} \\[3mm] \dfrac{\partial^2 \pi_i}{\partial q_i \, \partial q_0} & \dfrac{\partial^2 \pi_i}{\partial q_i^2} \end{vmatrix}$$

$$= 2 + \theta + 3\lambda(1 - \theta) + n(3 + \alpha + (2 - \alpha)\theta + 5\lambda(1 - \theta)) > 0$$

Taking total differential，we have:

$$dq_0 \frac{\partial^2 \Omega}{\partial q_0^2} + dq_1 \frac{\partial^2 \Omega}{\partial q_0 \, \partial q_i} = 0$$

$$dq_0 \frac{\partial^2 \pi_i}{\partial q_i \, \partial q_0} + dq_1 \frac{\partial^2 \pi_i}{\partial q_i^2} = 0$$

The slope of the reaction functions are:

$$\left. \frac{dq_1}{dq_0} \right|_{R_0} = -\frac{\dfrac{\partial^2 \Omega}{\partial q_0^2}}{\dfrac{\partial^2 \Omega}{\partial q_0 \, \partial q_i}} = -\frac{2 + \theta + 3(1 - \theta)\lambda}{n(1 - \alpha + \alpha\theta + (1 - \theta)\lambda)} < 0$$

$$\frac{dq_1}{dq_0}\bigg|_{R_1} = -\frac{\frac{\partial^2 \pi_i}{\partial q_i \, \partial q_0}}{\frac{\partial^2 \pi_i}{\partial q_i^2}} = -\frac{1}{2n-1} < 0$$

The stable condition of the game is:

$$\left|\frac{dq_1}{dq_0}\bigg|_{R_0}\right| - \left|\frac{dq_1}{dq_0}\bigg|_{R_1}\right| = \frac{2+\theta+3(1-\theta)\lambda}{n(1-\alpha+\alpha\theta+(1-\theta)\lambda)} - \frac{1}{2n-1} > 0, \text{ if } \lambda > \frac{n(n-5(1-\alpha)-3n\alpha)-2}{3+5n}.$$

The equilibrium outputs are:

$$q_i^*(s,\theta) = \frac{a+2s+a\theta+(2a+3s)(1-\theta)\lambda}{4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda} \tag{8}$$

$$q_0^*(s,\theta) = a + s - \frac{(2+n)(a+2s+a\theta+(2a+3s)(1-\theta)\lambda)}{4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda} \tag{9}$$

Substituting Eqs. (8) and (9) into Eq. (4) and then differentiating with respect to s

and θ, the optimum subsidy and degree of privatization are given as

$$s^* = \frac{a(1-\alpha(3+6\lambda)-\lambda(2+n+(6+n)\lambda))}{n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)} > 0,$$

$$\text{if } \lambda < \hat{\lambda} \equiv \frac{\sqrt{n(8+n)+4(7+3\alpha(-4+3\alpha))}-2+n+6\alpha}{2(6+n)}, \tag{10}$$

$$\theta^* = \frac{n(1+\lambda)}{6+n+n\lambda} < 1, \text{ for all } \lambda. \tag{11}$$

Lemma 1:

(i) q_i^*, and $q_0^* + nq_i^*$ are increasing in s, and q_0^* is increasing in s if the

shareholding ratio of the foreigners is relatively large.

(ii) q_i^* are increasing in θ while q_0^*, and $q_0^* + nq_i^*$ are decreasing in θ.

Proof:

We evaluate the comparative analysis at $\theta = \theta^*$ and $s = s^*$

$$\frac{\partial q_i^*(s,\theta)}{\partial s} = \frac{2+3(1-\theta)\lambda}{4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda} = \frac{6+n+(9+n)\lambda}{12+6n+n^2+3n\alpha+(3+n)(6+n)\lambda} > 0,$$

$$\frac{\partial q_0^*(s,\theta)}{\partial s} = 1 - \frac{(2+n)(2+3(1-\theta)\lambda)}{4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda} = \frac{n(3\alpha-2(1+\lambda))}{12+6n+n^2+3n\alpha+(3+n)(6+n)\lambda} > 0,$$

If $\alpha > \frac{2(1+\lambda)}{3}$.

$$\frac{\partial Q^*(s,\theta)}{\partial s} = \frac{2\theta+n(1+\alpha+\theta(1-\alpha)+2\lambda(1-\theta))}{4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda} = \frac{n(4+n+3\alpha+(7+n)\lambda)}{12+6n+n^2+3n\alpha+(3+n)(6+n)\lambda} > 0,$$

$$\frac{\partial q_i^*(s,\theta)}{\partial \theta} = \frac{2(a+an\alpha-s(2+n(1-\alpha+\lambda)3\lambda))}{(4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda)^2}$$

$$= \frac{a(1+\lambda)(\alpha+\lambda)(6+n+n\lambda)^2}{2(12+6n+n^2+3n\alpha+(3+n)(6+n)\lambda)(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))} > 0,$$

$$\frac{\partial q_0^*(s,\theta)}{\partial \theta} = -\frac{2(2+n)(a+an\alpha-s(2+n(1-\alpha+\lambda)3\lambda))}{(4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda)^2}$$

$$= -\frac{a(2+n)(1+\lambda)(\alpha+\lambda)(6+n+n\lambda)^2}{2(12+6n+n^2+3n\alpha+(3+n)(6+n)\lambda)(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))} < 0,$$

$$\frac{\partial Q^*(s,\theta)}{\partial \theta} = \frac{4(2+n)s-4(a+n(a+s)\alpha)+4(3+n)s\lambda}{(4+n+n\alpha+2\theta+n\theta(1-\alpha)+2(3+n)(1-\theta)\lambda)^2}$$

$$= -\frac{a(1+\lambda)(\alpha+\lambda)(6+n+n\lambda)^2}{(12+6n+n^2+3n\alpha+(3+n)(6+n)\lambda)(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))} < 0.$$

The output of the private firms and total output are increasing in s, due to the *output-expanding effect*. The output of the privatized firm is increasing in s, if *the shareholding ratio of the foreigners is relatively large*. Due to *output-substitution effect* between the privatized firm and the private firms, the output of the privatized firm is decreasing in θ. The output of the private firms and total output are increasing in θ, due to the output-substitution effect shifting production from the privatized firm to the private firms.

We have the following proposition.

Proposition 1: *The privatization policy is partial privatization which is irrelevance of*

the shadow of the public fund and the shareholding ratio of the foreigners, and the

production tax may be used if the shadow of the public fund is relatively large.

This proposition is different from the corresponding result of De Fraja and Delbono (1989), who found that the complete privatization of the public firm is desirable in terms of social welfare when the number of private firms is large, and is not desirable when the number of private firms is small. Proposition 1 suggests that regardless of the number of domestic private firms, the partial privatization of a public firm is always desirable from a welfare point of view in the presence of excess taxation burden. This is in sharp contrast to the existing literature. Capuano and De Feo (2010) demonstrated that with nil or large efficiency gains, an inefficient public firm that maximizes welfare may still be preferred where there exists excess burden of taxation in the government's objective function. Wang and Chen (2011b) found that for an imposition of the optimal subsidy, the level of welfare with privatization depends on the level of the cost-efficiency gap and the excess burden of taxation. Comparing privatization with mixed duopoly, Matsumura and Tomaru (2013) investigated optimal tax-subsidy policies with the excess burden of taxation. They focused on both the optimal tax-subsidy policies with endogenous timing of production, as well as the PNT. They showed that mixed oligopoly reducing welfare does not hold with excess burden of taxation. Hence, the PNT does not hold even when they compared the resulting outcomes before and after privatization. However, they did not extend the study to the situation in which the public firm competes with multiple private firms as we do in this

paper.

We further show that the production tax may be used if *the shadow of the public fund* is relatively large. This result coincides with Matsumura and Tomaru (2013): *s* is not always positive (i.e., the government may impose a production tax). When λ is small, $s > 0$. However, when λ is large, *s* is negative. Naturally, a higher cost of public funding reduces the optimal subsidy rate, and it becomes negative if λ is large.

In our modelling, there are two distortions thereby needing two policy instruments concurrently to deal with these: firstly, the domestic market is under oligopolistic competition and output of domestic private firm is less than the one under perfect competition, and it needs output subsidy to increase the private firm's output; secondly, for the public firm, it cares about social welfare, which leads to more output and higher marginal cost needing higher degree of privatization to curtail the output of public firm.

Although we assume $\lambda \in [0, \infty)$ in our analysis, here we consider what would happen if $\lambda \to \infty$. Taking limit of θ^*, and s^* we have

$$\lim_{\lambda \to \infty} \theta^* = \lim_{\lambda \to \infty} \frac{n(1+\lambda)}{6+n+n\lambda} = 1.$$

$$\lim_{\lambda \to \infty} s^* = \lim_{\lambda \to \infty} \frac{a(1-\alpha(3+6\lambda)-\lambda(2+n+(6+n)\lambda))}{n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)} = -\frac{a}{2}.$$

The optimal degree of privatization is 1, and the tariff rate is $\frac{a}{2}$. The output subsidy is negative, $-\frac{a}{2}$; that is, a production tax is adopted. Due to officials putting a greater emphasis on the profit of SOE, they do not care about consumer surplus anymore. The government's objective reduces to maximize their revenue (production tax and profit of

the privatized firm), and full privatization is chosen.

Taking differentiation of s^* and θ with respect to λ, we obtain:

$$\frac{\partial s^*}{\partial \lambda} = -\frac{a(15+n(7+n)+3n\alpha+9\alpha^2+2(4+n)(6+n)\lambda+(6+n)(7+n-3\alpha)\lambda^2)}{(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^2} < 0, \qquad (12)$$

$$\frac{\partial \theta^*}{\partial \lambda} = \frac{6n}{(6+n+n\lambda)^2} > 0. \qquad (13)$$

We have the following proposition.

Proposition 2: *The optimum subsidy rate is decreasing in the shadow cost of public funds, and the degree of privatization is increasing in the shadow cost of public funds.*

When the social cost of public fund is more severe, and the degree of privatization and production subsidy are complementary instruments for a given degree of foreign ownership; the degree of privatization and the production subsidy should decrease in order to mitigate the policy distortion.

This result coincides with Lee and Wang (2018): *The optimal degree of privatization is increasing in the* excess *taxation burden if the excess taxation burden is relatively large.* In their framework, a higher shadow cost of public funds will cause a lower subsidy rate and a higher degree of privatization, due to the subsidy policy being more costly. When λ is relatively small, in this case, the government does care more about the consumer surplus. Thus, a lower degree of privatization is needed to enhance the total market output. When λ is relatively large, in this case, the government does care less about the consumer surplus. Thus, a higher degree of privatization is adopted to enhance the total revenue of the government regardless of the degree of foreign

ownership.

Taking differentiation of s^* and θ^* with respect to α, we obtain that

$$\frac{\partial s^*}{\partial \alpha} = -\frac{3a(1+\lambda)^2(4+n+(6+n)\lambda)}{\left(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)\right)^2} < 0, \tag{13}$$

$$\frac{\partial \theta^*}{\partial \alpha} = 0. \tag{14}$$

We have the following proposition.

Proposition 3: *The optimum subsidy rate is decreasing in the shareholding ratio of the foreigners, while the degree of privatization is irrelevance of the shareholding ratio of the foreigners.*

In mixed oligopoly with partial privatization, the production level of each private firm is too low for domestic welfare. Thus, the government has an incentive to raise s so as to stimulate the production of private firms. On the other hand, an increase in s raises the outflow of surplus to the foreign investors and reduces domestic welfare. The latter effect becomes more significant when α is large. Thus, we have the optimum subsidy rate is decreasing in the shareholding ratio of the foreigners.

In mixed oligopoly, the public firm cares about social welfare, which leads to more output and higher marginal cost needing higher degree of privatization to curtail the output of public firm. The effect of partial privatization amends the cost inefficiency caused by over-production by public firm. Due to the cost structure of both the private firms and the privatized firm are irrelevance of the shareholding ratio of the foreigners. Thus, the degree of privatization is irrelevance of the shareholding ratio of the

foreigners.

Substituting Eqs. (9) and (10) into Eqs. (1)-(3), we have the following equilibrium

outcomes of the SPNE with excess taxation burden:

$$q_0 = \frac{a(1+\lambda)(1+3\alpha+4\lambda)}{n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)}, \quad q_i = \frac{a(1+\lambda)^2}{n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)},$$

$$Q = \frac{a(1+\lambda)(1+n+3\alpha+(4+n)\lambda)}{n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)},$$

$$P^* = a - \frac{a(1+\lambda)(1+n+3\alpha+(4+n)\lambda)}{n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)},$$

$$\pi_0^* = -\frac{3a^2(-1+\alpha)(1+\lambda)^2(1+3\alpha+4\lambda)}{2(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^2},$$

$$\pi_i^* = \frac{3a^2(1+\lambda)^4}{2(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^2},$$

$$CS^* = \frac{a^2(1+\lambda)^2(1+n+3\alpha+(4+n)\lambda)^2}{2(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^2},$$

$$W^* = \frac{a^2(1+\lambda)^2(1+n+3\alpha+(4+n)\lambda)}{2(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))}.$$

Taking differentiation of π_0^*, π_i^*, CS^*, and W^* with respect to λ, we obtain that

$$\frac{\partial \pi_0^*}{\partial \lambda} = -\frac{3a^2(1-\alpha)(1+\lambda)(5+9\alpha^2+2\lambda(17+2n(1+\lambda)^2+\lambda(31+12\lambda))+6\alpha(n(1+\lambda)^2+3(1+\lambda(3+\lambda))))}{(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^3} < 0,$$

$$\frac{\partial \pi_i^*}{\partial \lambda} = -\frac{3a^2(1+\lambda)^3(7+n-3\alpha+(13+n-9\alpha)\lambda)}{(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^3} < 0,$$

$$\frac{\partial CS^*}{\partial \lambda} = -\frac{a^2(1+\lambda)(1+n+3\alpha+(4+n)\lambda)(n(5+3\alpha(1-\lambda)+11\lambda)(1+\lambda)+(1+\lambda)^2+(1+3\alpha+4\lambda)^2)}{(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^3} < 0,$$

$$\frac{\partial W^*}{\partial \lambda} = \frac{a^2(1+\lambda)(2n^2\lambda(1+\lambda)^2+(1+3\lambda)(1+3\alpha+4\lambda)^2+2n(1+\lambda)(-1+3\alpha+3\lambda+9\alpha\lambda+10\lambda^2))}{2(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^2} < 0.$$

We have the following proposition.

Proposition 4: *The profit of the privatized firm and private firm, the consumer surplus and social welfare are decreasing in the shadow cost of public funds.*

From the viewpoint of firms' objectives, Tomaru (2006) showed the robustness by adopting the partial privatization approach formulated by Matsumura (1998). Kato and Tomaru (2007) considered non-profit-maximizing private firms and showed that the theorem holds true under various payoff functions of private firms. These works demonstrated that the PNT is quite robust. Wang and Chen (2011b), and Matsumura and Tomaru (2013) casted doubt on the optimal tax-subsidy policies in mixed and private oligopolies with excess burden of taxation. Matsumura and Tomaru (2013) compared the optimal subsidies and the resulting welfare levels among four regimes: mixed and private Cournot duopolies and Stackelberg competition with public and private leaderships. They show that under general demand and cost functions, all four regimes yield the same equilibrium welfare under the optimal subsidy policies if and only if $\lambda = 0$. In other words, the PNT holds only when there is no excess burden of taxation.

Our proposition points out that the profit of the privatized firm and private firm, the consumer surplus and social welfare are decreasing in the social cost of public fund. In other words, the higher the excess burden of taxation, the lower for all the important equilibrium outcomes. The intuition is that the larger the social cost of public fund, the smaller the subsidy rate and the higher the degree of privatization. The social cost of public fund makes the private firms produce less and the privatized firm produces more, leading an inefficiency of production and lower the total output of the industry.

We further want to see how the change of foreign ownership (liberalization of

capital market) will affect all the important equilibrium outcomes.

Taking differentiation of π_0^*, π_i^*, CS^*, and W^* with respect to α, we obtain that

$$\frac{\partial \pi_0^*}{\partial \alpha} = \frac{3a^2(1+\lambda)^3(n(1+2\lambda)(1-3\alpha-2\lambda)-2(2+3\lambda)(1+3\alpha+4\lambda))}{(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda))^3} > 0, \text{ if } n > \frac{2(2+3\lambda)(1+3\alpha+4\lambda)}{(1+2\lambda)(1-3\alpha-2\lambda)}$$

$$\frac{\partial \pi_i^*}{\partial \alpha} = -\frac{9a^2(1+\lambda)^4(2+3\lambda)}{\left(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)\right)^3} < 0,$$

$$\frac{\partial CS^*}{\partial \alpha} = -\frac{3a^2n(1+\lambda)^4(1+n+3\alpha+(4+n)\lambda)}{\left(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)\right)^3} < 0,$$

$$\frac{\partial W^*}{\partial \alpha} = -\frac{3a^2n(1+\lambda)^4}{2\left(n(1+\lambda)(1+2\lambda)+(2+3\lambda)(1+3\alpha+4\lambda)\right)^2} < 0.$$

We have the following proposition.

Proposition 5: *The profit of the private firms, the consumer surplus and social welfare are decreasing in the equity share held by foreign investors, and the profit of the privatized firm decreasing in the equity share held by foreign investors if the number of the private firm is sufficiently large.*

As in proposition 3, the optimum subsidy rate is decreasing in the shareholding ratio of the foreigners, while the degree of privatization is irrelevance of the shareholding ratio of the foreigners. A lower subsidy leads a lower output and market share of the private firms. Thus, the profit of the private firms will decrease in the shareholding ratio of the foreigners. A lower subsidy also leads a lower output of the privatized firm, but the direction of the market share is depending on the number of the private firms. If the number of the private firms is relative large, a lower subsidy leads a higher market share and profit of the privatized firm. Because the total output is decreasing in the output subsidy, we have the consumer surplus is decreasing in the shareholding ratio of the foreigners. When the shareholding ratio of the foreigners is

increasing, the direct effect raises the outflow of surplus to the foreign investors and reduces domestic welfare. The indirect effect decline the profit of the private firms and consumer surplus, and reduces domestic welfare, too.

4. Conclusions

We used a mixed oligopoly model with foreign penetration examining how the shadow cost of public funds will affect the privatization policy in the presence of strategic tax/subsidy policies. We showed that (1) The tax/subsidy policies could be adopted and the optimal privatization policy is partial privatization in the presence of shadow cost of public funds and foreign ownership; (2) The optimum subsidy rate is decreasing in the shadow cost of public funds and the degree of privatization is increasing in the shadow cost of public funds; (3) The optimum subsidy rate is decreasing in the shareholding ratio of the foreigners, while the degree of privatization is irrelevance of the shareholding ratio of the foreigners; (4) The profit of the privatized firm and private firms, the consumer surplus and social welfare are decreasing in the shadow cost of public funds and the equity share held by foreign investors.

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