Privatization, Profit and Welfare in a Differentiated Network Duopoly

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Abstract: We examine the influence of demand-side network externalities and product differentiation on the decisions of consumers and firms, and see how the governments will determine its privatization policy for a state-owned enterprise under Cournot and Bertrand competition. Under Cournot competition, we show that the optimal privatization policy is partial privatization, while under Bertrand competition, the optimal privation policy is fully nationalization. Furthermore, we demonstrate that the optimal choice of public and private firm is Bertrand competition, and social welfare is lower under Cournot competition than under Bertrand competition.

Keywords: network externalities; product differentiation; mixed markets; partial privatization.
JEL classifications: L13, L32, L33.

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1. Introduction

In a few industries, such as in the banking, liquor, steal and petroleum industries that consist of both state-owned enterprises (SOEs) and private enterprises in different countries, the firms compete and produce differentiated goods without encountering the restriction of technical compatibility for the consumers exhibiting positive externalities in the final goods market. In this paper, we are aiming to examine the influence of demand-side network externalities and product differentiation on the decisions of consumers and firms under Cournot and Bertrand competition, and see how the governments will determine its privatization policy for a state-owned enterprise.

There is a proliferation of literature on the privatization issue in a mixed oligopoly. Matsumura (1998) in mixed duopoly showed that neither full privatization nor complete nationalization is optimal in the absence of product differentiation. Matsumura and Kanda (2005) further demonstrated that partial privatization is the optimal policy in the short-run; full nationalization is always optimal in the long run with free entry among private firms. Brandão and Castro (2007) extended the framework by Matsumura and Kanda (2005), and argued that the presence of a public enterprise can be an alternative to direct regulation to avoid the excess entry problem. Wang and Chen (2010) demonstrated that partial privatization is always the best policy for the public firm in long-run equilibrium in the presence of cost gap and that long-run degree of privatization is larger than the short-run one. Wang et al. (2014) examined privatization policy and entry regulation in a mixed oligopoly market with foreign competitors and free entry of private firms. They demonstrated that if the number of domestic private firms is large, an import tariff is imposed and the optimal privatization policy is partial privatization and entry is socially excessive.

Instead of assuming final product is homogeneous, Cremer et al. (1991) and Anderson et al. (1997) are the earlier works exploring the implications of product differentiation in the mixed market. Anderson et al. (1997) incorporated mixed oligopoly into a model of product differentiation in which a representative consumer has a CES subutility function for love of variety considering a price-setting game. They showed the possibility of welfare-deteriorating privatization policy by proving that full nationalization is the best policy in the short-run with exogenous number of firms.[1] Fujiwara (2007) in particular developed a differentiated mixed oligopoly model to establish what implication product differentiation has for the optimal privatization policy by employing a quadratic subutility specification for love of variety and a quantity-setting game. He showed that the short-run optimal policy is non-monotonic in the degree of love of variety, while the optimal degree of privatization is monotonically increasing in the consumer's preference for variety in the long run.

Recently, the endogenous choice of competition modes in mixed oligopoly has gained most attention. Focusing on the heterogeneity of ownership structures, Ghosh and Mitra (2010) showed that price competition gives rise to the higher profitability

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[1] There has been growing literature on mixed oligopoly with differentiated products. See, for example, Barcena-Ruiz and Garzon (2003), Matsumura and Matsushima (2003; 2004), and Li (2006), among many others.

for both private and public firms than quantity competition. Matsumura and Ogawa (2012) then showed that it is a dominant strategy for both firms to choose their prices regardless of whether the products are substitutes or complements. Choi (2012) investigated the choice of competition modes when unions are present and showed that there exists a dominant strategy only for the public firm that chooses price competition irrespective of whether the goods are substitutes or complements; there is no dominant strategy for a private firm. Scrimitore (2013) showed that quantity can constitute dominant strategy equilibrium by introducing firm subsidization. By allowing for partially privatization of a state-controlled firm, Scrimitore (2014) showed that, irrespective of the mode of competition, the ownership of the controlled firm is irrelevant when firms play simultaneously; it matters when firms compete sequentially, requiring the leader to be publicly-owned for an optimal subsidy to restore the first-best. The above divergent results pointed out that the dominant strategy in mixed markets depend not only the objective functions and market environments, but also the choice of policy interventions.

To see how network externality would play a role in the privatization decision-making of the government, Wang and Chiou (2015) considered the technical compatibility for the homogeneous good and showed that the optimal degree of privatization depends crucially on the scale of network externality, the degree of compatibility and the cost type.[2] A positive network externality will decrease the

[2] See, Baake and Boom (2001) examined vertical product differentiation, network externalities, and compatibility decisions in oligopoly model. Willner (2006) examined the issue of privatisation and liberalisation in an industry with network externalities, but it did not consider the optimal privatization policy or compatibility. Recently, Wei and Wang (2016, forthcoming) showed that with cost

optimal degree of privatization, providing the rate of increase of marginal cost remains below a threshold. The impact of compatibility on optimal privatization is less straightforward: if the scale of network externality is low, a higher compatibility will increase optimal privatization if the rate of increase in marginal cost is also low. They only considered the network externality without product differentiation in Cournot competition for deciding privatization policy. The presence of product differentiation in Bertrand competition may significantly influence the desirability of privatization policy.

There are many other factors being equally important in the mixed market, such as cost-efficiency gap, product differentiation and network externalities for the government to decide whether the public firm should be privatized. For example, in many industries like banking market, there exist public and private firms, and the standard for these markets are same among the whole world, so firms in these markets should not encounter the restriction of technical compatibility. In these industries, consumers may care more about the quality and price of their products and services and firms can via consumption network to get their differentiated products more available for their clients. Hence, product differentiation and network externalities will help firms improve their competitiveness and expand their market share. It is essential that markets in which firms produce differentiated goods and consumers

asymmetry, stronger network externalities and less differentiated goods, the uniform tariff rate set by the government of the importing country increases and the optimal discriminatory-tariff gap widens.

exhibit positive externalities without encountering the restriction of technical compatibility should not be ignored when coming to the issue of privatizing state-owned enterprises.

When we incorporate product differentiation and network externalities into mixed market, some mechanism have been changed. Network externalities increase market size and consumers' demands, which will improve social welfare; the government should then lower the degree of privatization. Product differentiation will make products become less substitution for each other. In the absence of network externalities, firms encounter lessen market competition and make more profit, and in this case, government should choose partial privatization to stimulate private firm to expand its production for improving social welfare. When we concurrently consider product differentiation and network externalities in market for reality, the government also chooses partial privatization but its privatization degree should be lower compared to the market without network externalities.

Our paper contributes to the literature in three major ways. Firstly, to our best knowledge, this is the first paper to show the influence of demand-side network externalities and product differentiation on the decisions of consumers and firms[3], and see how the governments will determine its privatization policy for a state-owned enterprise under Cournot and Bertrand competition. We extend the previous results,

[3] In an infinitely repeated Cournot game with trigger strategy punishment, Pal and Scrimitore (2016) demonstrated that the relationship between market concentration and collusion sustainability depends on the strength of network externalities. The latter is shown to interact with the number of firms and to affect the profitability of cooperation vs. competition, that lower market concentration can make collusion more stable. However, the analytical result is derived under Cournot competition in the absence of product differentiation.

but some previous results are not hold. In particular, under Cournot competition with the differentiated products exhibit network externalities, we show that 1. The optimal policy for the government to determine is partial privatization; 2. When the network effects increase, the government should slow down the path of privatization; 3. The impact of product differentiation on optimal degree of privatization is non-monotonic and depends crucially on the degree of network externality and the heterogeneity of the differentiated good. Secondly, previous literature on mixed market mostly considered Cournot competition, however, when we investigate mixed markets with network externalities, Bertrand competition may be more common in these markets such as telecommunications industry and banking industry; so, it is important to also consider these markets under Bertrand competition. We show that under Bertrand competition, when the differentiated products exhibit network externalities, the optimal privation policy is fully nationalization. Thirdly, we further explore the endogenous choice of competition modes in mixed oligopoly under product differentiation and network externalities. We find that the optimal choice for public and private firm is Bertrand competition, and social welfare is lower under Cournot than under Bertrand competition which is consistent with the convention wisdom in the private market.

The paper proceeds as follows. Section 2 constructs a basic model. Section 3 provides the equilibrium analysis and optimal privatization policy under Cournot competition. Section 4 carries out the analysis under Bertrand competition. Section 5 compares the results of the two competition modes and provides main results. The final section concludes this paper.

2. The Model

We consider a mixed duopolistic model in a network goods sector with two firms, one public firm and one private firm, represented by firm 1 and 2, respectively. They produce a horizontal differentiated good with network externalities.[4] Following the original specification of Hoernig (2012) which is used in Pal (2014), Bhattacharjee and Pal (2014), Ghosh and Pal (2014), and Pal (2015), we consider the representative utility function as follows:

$$U[q_1,q_2,y_1,y_2]=$$
$$m+\frac{a(q_1+q_2)}{1-\gamma}-\frac{(q_1^{\,2}+2\gamma q_1q_2+q_2^{\,2})}{2(1-\gamma^2)}+n(\frac{(y_1+\gamma y_2)q_1+(y_2+\gamma y_1)q_2}{1-\gamma^2}-\frac{y_1^{\,2}+2\gamma y_1y_2+y_2^{\,2}}{2(1-\gamma^2)}) \quad (1)$$

where q_i is the output produced by firm i, y_i denotes the consumers expectation about firm i's total sales; m is the consumption of all other goods measured in terms of money, and $a>0$ is the market scale; $n\in[0,1)$ represents the network effects, a larger n indicates a larger network effect and a higher willing to pay for the product; and $\gamma\in[0,1)$ indicates the heterogeneity of the differentiated goods, Lower value of γ corresponds to the case of higher degree of product differentiation. So the inverse

[4] Griva and Vetta (2011) examined price competition under product-specific network effects in a simple Hotelling model, where the products are differentiated both horizontally and vertically. In this paper, the role of consumers' expectations formation is emphasized. In particular, when expectations are not influenced by prices, the market may be shared but shares must be equal unless product qualities differ or one firm, possibly even the low-quality one, may capture the entire market. But, when expectations are influenced by prices, which would be the case when there is commitment, competition becomes more intense and the high-quality firm tends to capture a larger market share. Furthermore, under strong network effects there is a continuum of equilibria and the higher the prices, the smaller the difference between those prices can be.

and direct market demands for the two firms can be derived as follows:

$$p_i = \frac{\partial U}{\partial q_i} = \frac{a}{1-\gamma} - \frac{q_i + q_j \gamma}{(1-\gamma^2)} + \frac{n(y_i + y_j \gamma)}{1-\gamma^2} \tag{2}$$

$$q_i = a - p_i + \gamma p_j + n y_i \tag{3}$$

The consumer surplus is calculated as follows:

$$CS = U(.) - p_1 q_1 - p_2 q_2 - m = \frac{q_1{}^2 + q_2{}^2 + 2\gamma q_1 q_2 - n(y_1{}^2 + y_2{}^2 + 2\gamma y_1 y_2)}{2(1-\gamma^2)} \tag{4}$$

The two firms are assumed to share identical technology given by the quadratic cost function $c_i = k\frac{q_i{}^2}{2}$, where $k > 0$ is a constant representing the degree with which the marginal cost increases. Here we assume $k = 1$ to simplify the calculation without loss of the qualitative analysis, the profit of firm i, given the consumers' expectations, is that

$$\pi_i = p_i q_i - \frac{q_i{}^2}{2} = q_i\left(\frac{a}{1-\gamma} - \frac{q_i + \gamma q_j}{(1-\gamma^2)} + \frac{n(y_i + y_j\gamma)}{1-\gamma^2}\right) - \frac{q_i{}^2}{2} \tag{5}$$

Social welfare is the sum of the two firms' profits and the consumer surplus:

$$SW = \pi_1 + \pi_2 + CS = (p_1 q_1 - \frac{q_1{}^2}{2}) + (p_2 q_2 - \frac{q_2{}^2}{2}) + \frac{q_1{}^2 + q_2{}^2 + 2\gamma q_1 q_2 - n(y_1{}^2 + y_2{}^2 + 2\gamma y_1 y_2)}{2(1-\gamma^2)} \tag{6}$$

When the government privatizes the public firm, the public firm is concerned not only about its own profit, π_1, but also the level of social welfare, SW. The optimization problem for the semi-public firm is maximizing

$$\Omega = \theta\pi_1 + (1-\theta)SW = \frac{2q_1(a - q_2\gamma + a\gamma + n(y_1 + y_2\gamma)) - q_1{}^2(2-\gamma^2+\theta)}{2(1-\gamma^2)}$$
$$+ \frac{(2q_2(ny_2 + a + ny_1\gamma + a\gamma) + q_2{}^2(2-\gamma^2) - n(y_1{}^2 + y_2{}^2 + 2y_1 y_2\gamma))(1-\theta)}{2(1-\gamma^2)} \tag{7}$$

where θ is the weight assigned to the profit in the decision-making process of the firm, and $0 \le \theta \le 1$. Following Matsumura (1998), the government can indirectly control θ through its shareholding. The fully privatized firm only seeks the profit if $\theta = 1$; while a fully nationalized firm maximizes social welfare if $\theta = 0$. The larger the θ, the more public firm is concerned about its profit.

The timing of the game is as follows: in the first stage, the government decides the degree of privatization to maximize social welfare; in the second stage, both the firms compete, with respect to either quantities or price, in the product market. We use backward induction to derive the sub-game perfect Nash equilibrium (SPNE).

3. Cournot Competition

We first analyze the optimal choice on privatization under Cournot competition in a differentiated mixed duopoly. In the second stage, the objective function of private firm is π_2. Choosing the output, the first-order conditions are as follows:

$$\frac{\partial \Omega}{\partial q_1} = \frac{(a - q_2\gamma + a\gamma + n(y_1 + y_2\gamma)) - q_1(2 - \gamma^2 + \theta)}{(1-\gamma^2)} = 0 \qquad (8.a)$$

$$\frac{\partial \pi_2}{\partial q_2} = -q_2 + \frac{a}{1-\gamma} - \frac{q_2}{1-\gamma^2} - \frac{2q_2 + 2q_1\gamma}{2(1-\gamma^2)} + \frac{n(y_2 + y_1\gamma)}{1-\gamma^2} = 0 \qquad (8.b)$$

The second-order conditions hold: $\frac{\partial^2 \Omega}{\partial q_1{}^2} = -\frac{2-\gamma^2+\theta}{1-\gamma^2} < 0$ and $\frac{\partial^2 \pi_2}{\partial q_2{}^2} = -1 - \frac{2}{1-\gamma^2} < 0$. We follow Katz and Shapiro (1985) and impose additional rational expectation conditions by setting $y_i = q_i$, the equilibrium outputs in stage 2 of the game are expressed as follows:

$$q_1{*} = \frac{a(1+\gamma)(3 - n - \gamma + n\gamma - \gamma^2)}{\gamma^4 + n^2(1-\gamma^2) - n(5 - 4\gamma^2 + \theta) + 3(2+\theta) - \gamma^2(6+\theta)} \qquad (9.a)$$

$$q_2{}^* = \frac{a(1+\gamma)(2-n-\gamma+n\gamma-\gamma^2+\theta)}{\gamma^4+n^2(1-\gamma^2)-n(5-4\gamma^2+\theta)+3(2+\theta)-\gamma^2(6+\theta)} \tag{9.b}$$

From Eqs. (9.a) and (9.b), we can see that $q_1{}^*$ and $q_2{}^*$ are always positive.

We then investigate the impacts on the outputs of privatization degree, network externalities, and the heterogeneity of the differentiated goods. A few remarks are in order. Firstly, Under Cournot competition, the higher degree of privatization leads to a decrease in the public firm's output and the total output, but will increase the private firm's output, that is, $\frac{\partial q_1{}^*}{\partial \theta}<0$, $\frac{\partial q_2{}^*}{\partial \theta}>0$ and $\frac{\partial (q_1{}^*+q_2{}^*)}{\partial \theta}<0$. Secondly, an increase in the degree of network externalities increases both firms' outputs and the total output, and the difference in output between two firms increases too; that is, $\frac{\partial q_1{}^*}{\partial n}>\frac{\partial q_2{}^*}{\partial n}>0$. Finally, an increase in heterogeneity of the differentiated goods raises the public firm's output, the total output and the difference in output between the firms. But, the impact on private firm is ambiguous: if privatization degree and network externalities are small, private firm's outputs will increase in heterogeneity of the differentiated goods when γ is small, and will decrease when γ is large; otherwise, an increase in heterogeneity of the differentiated goods increases the private firm's output, $\frac{\partial q_2{}^*}{\partial \gamma}>0$.

We then explore the decision on optimal privatization policy in stage 1. We substitute Eqs. (9.a)-(9.b) into social welfare function and take the first-order derivative with respect to θ,

$$\frac{\partial SW}{\partial \theta}=\frac{a^2(1+\gamma)(n+\gamma-n\gamma+\gamma^2-3)(\gamma^2(1+n^2-2n(1-\theta)-6\theta)}{(1-\gamma)(\gamma^4+n^2(1-\gamma^2)-n(5-4\gamma^2+\theta)+3(2+\theta)-\gamma^2(6+\theta))^3}$$
$$\frac{+\gamma^4\theta-(1-n)\gamma(2-n+4\theta-n\theta))+(3-n)^2\theta+(1-n)\gamma^3(1+\theta))}{(1-\gamma)(\gamma^4+n^2(1-\gamma^2)-n(5-4\gamma^2+\theta)+3(2+\theta)-\gamma^2(6+\theta))^3} \tag{10}$$

We get

$$\theta^*=\frac{(1-n)(2-n+\gamma)(1-\gamma)\gamma}{9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3)} \tag{11}$$

Note that the second-order condition is satisfied, and $\theta^*\in[0,1)$. Please see Appendix A.1 for the proof. From Eq. (11), we have the following Lemma 1.

Lemma 1. *When the differentiated products exhibit network externalities and two firms compete in quantities, full nationalization is optimal if the two goods are completely differentiated. If* $\gamma\neq 0$*, then the optimal policy for government is partial privatization.*

In the absence of network externality, the degree of optimal privatization reduces to $\theta^*=\frac{(2+\gamma)(1-\gamma)\gamma}{9-4\gamma-6\gamma^2+\gamma^3+\gamma^4}$, which depends only on the heterogeneity of the differentiated goods. The optimal policy for the government is partial privatization when the number of firms is exogenously given, which is also obtained by Fujiwara (2007) in a quantity-setting game without considering network externalities: the optimal privatization policy is non-monotonic in the degree of love of variety, the government should obtain the optimal fraction of the public firm's share depending on the degree of love of variety. The optimal privatization policy in our work depends crucially on the scale of network externality and the heterogeneity of the differentiated goods.

We further consider the influence of network effect and the heterogeneity of the differentiated goods on the degree of optimal privatization. The results are summarized in Proposition 1.

Proposition 1. *Under Cournot competition,*

(i) *When the network effect gets stronger, the government should lower the degree of privatization.*

(ii) *The impact of product differentiation on optimal degree of privatization is non-monotonic. There exist critical value of* $\overline{\gamma}$ *, the optimal degree of privatization will increase in* γ *when* $0<\gamma<\overline{\gamma}$ *, and will decrease when* $\overline{\gamma}<\gamma<1$*. The critical value* $\overline{\gamma}$ *will depend on network effect. When it satisfies* $0<n<\overline{n}_1$*, we have* $\overline{\gamma}=\overline{\gamma_1}$ *; when* $\overline{n}_1<n<1$*, then* $\overline{\gamma}=\overline{\gamma_2}$ *, and we have* $\overline{\gamma_1}<\overline{\gamma_2}$ *.*

Proof: See Appendix A.2.

The reasoning behind Proposition 1(i) is straightforward: with a higher network externality, the market scale is expanding and consumer surplus increases faster with output, the government should then lower down the degree of privatization for maintaining a larger output and emphasizing more on social welfare.

The impact of heterogeneity of the differentiated goods on the optimal degree of privatization is illustrated in Fig. 2.

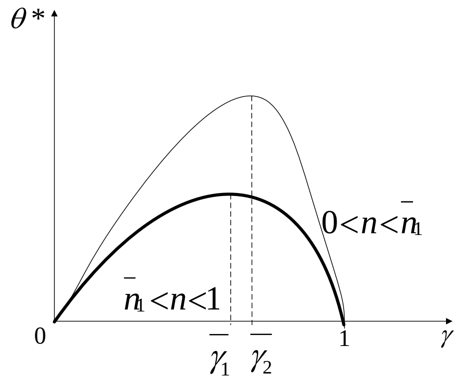

Figure 1. Optimal privatization policy and the heterogeneity of the differentiated goods

Proposition 1(ii) can be interpreted as follows. When the product market competition is soften and the difference of these two products is large, an increase in the market competition will make both firms increase their outputs; however, the low degree of privatization will reduce the increased amount of private firm due to the output-substitution effect, and the high difference of these two products make the increase for total output cannot bring a larger increase for consumer surplus. In this case, the government should increase the degree of optimal privatization to raise the profits for both firms. When γ is sufficiently high, these two products become closer substitutes and consumer surplus will increase faster with total output. Hence, the government should lower down the degree of privatization to maintain a larger total output and higher consumer surplus.

Note that a higher network externality will soften the competition in the product market and make consumer surplus increase faster with output. There are two effects affecting the degree of optimal privatization. When the scale of network externalities is small, on the one hand, a rise in the heterogeneity of the differentiated goods will be associated with a higher degree of privatization due to the lower increase of consumer surplus with output. On the other hand, the heterogeneity of the differentiated goods will need to reach a higher level and the government will then lower the degree of privatization for the public firm. Hence, whether the government should increase or decrease the degree of privatization depending on the strength of the network externality and the degree of product heterogeneity.

4. Bertrand Competition

In this section, we examine the optimal choice of privatization under Bertrand competition in a differentiated mixed duopoly. In the second stage, both firms choose price simultaneously, then the equilibrium prices and quantities of both firms can be derived as

$$p_1* = \frac{a(3(1+\theta)+\gamma(3+\theta)+n^2(\gamma+2\theta)-n(2+5\theta+\gamma(3+\theta)))}{3(2+\theta)-\gamma^2(3+\theta)-n^2(\gamma^2-2(1+\theta))-n(7+5\theta-\gamma^2(3+\theta))} \tag{12.a}$$

$$p_2* = \frac{a(2-n)(2+\theta+\gamma(1+\theta)-n(1+\theta+\gamma\theta))}{3(2+\theta)-\gamma^2(3+\theta)-n^2(\gamma^2-2(1+\theta))-n(7+5\theta-\gamma^2(3+\theta))} \tag{12.b}$$

$$q_1* = \frac{a(3-\gamma^2(1-\theta)+\gamma(1+\theta)-n(2-\gamma^2(1-\theta)+\gamma\theta))}{3(2+\theta)-\gamma^2(3+\theta)-n^2(\gamma^2-2(1+\theta))-n(7+5\theta-\gamma^2(3+\theta))} \tag{12.c}$$

$$q_2* = \frac{a(2+\theta+\gamma(1+\theta)-n(1+\theta+\gamma\theta))}{3(2+\theta)-\gamma^2(3+\theta)-n^2(\gamma^2-2(1+\theta))-n(7+5\theta-\gamma^2(3+\theta))} \tag{12.d}$$

From Eqs. (12.a), (12.b), (12.c) and (12.d), all the equilibrium values are always positive.

We then investigate the impacts on the outputs and prices of privatization degree, network externalities, and the heterogeneity of the differentiated goods. A few remarks are in order. Firstly, Under Bertrand competition, the higher degree of privatization leads to an increase in the prices of both firms and the private firm's output, but will decrease the public firm's output and the total output; that is, $\frac{\partial p_1*}{\partial\theta} > \frac{\partial p_2*}{\partial\theta} > 0$, $\frac{\partial q_1*}{\partial\theta} < 0$, $\frac{\partial q_2*}{\partial\theta} > 0$ and $\frac{\partial(q_1*+q_2*)}{\partial\theta} < 0$. Secondly, an increase in the degree of network externalities increases both firms' prices, outputs and the total output, the difference in output between two firms increases when network externalities are small, and decreases when network externalities are large; that is, $\frac{\partial p_1*}{\partial n} > 0$, $\frac{\partial p_2*}{\partial n} > 0$, $\frac{\partial q_1*}{\partial n} > 0$ and $\frac{\partial q_2*}{\partial n} > 0$. Finally, an increase in heterogeneity of the differentiated goods raises both firms' prices, outputs and the total output, but decreases the difference in output between the firms; that is, $\frac{\partial p_1*}{\partial\gamma} > \frac{\partial p_2*}{\partial\gamma} > 0$ and $\frac{\partial q_2*}{\partial\gamma} > \frac{\partial q_1*}{\partial\gamma} > 0$.

Now, we turn to the optimal privatization policy in stage 1. Substituting Eqs. (12.a)-(12.b) into social welfare function and take the first-order derivative with respect to θ,

$$\begin{aligned}\frac{\partial SW}{\partial\theta} &= -a^2(1-n)^2(1-\gamma)^2(3+2\gamma-n(2+\gamma))((2-n)\gamma^3(1-\theta)+(3-2n)^2\theta+(2-n)\gamma^2(3-n-\theta)+\\ &(2-n)\gamma(2-n+4\theta-3n\theta))/(3(2+\theta)-\gamma^2(3+\theta)-n^2(\gamma^2-2(1+\theta))-n(7+5\theta-\gamma^2(3+\theta)))^3=0\end{aligned} \tag{13}$$

Then we have:

$$\theta* = -\frac{(2-n)(2-n+\gamma)\gamma(1+\gamma)}{9+8\gamma-2\gamma^2-2\gamma^3+n^2(4+3\gamma)-n(12+10\gamma-\gamma^2-\gamma^3)} \leq 0 \tag{14}$$

Note that the second-order condition is satisfied, and $\theta* \leq 0$. Please see Appendix A.3 for the proof. From Eq. (14), we have the following Proposition 2.

Proposition 2. *When the differentiated products exhibit network externalities and* *firms compete in price, the optimal privatization policy is fully nationalization.*

The optimal privatization policy for the government is full nationalization when the firms compete only in price with horizontal differentiated products, which is similar as the finding obtained by Ishibashi and Kaneko (2008) in a standard Hotelling spatial model of duopoly in the absence of quality competition. Without considering quality competition and network externalities, the optimal choice of government is fully nationalization; however, when they take quality competition into consideration, their result support a completely mixed objective between welfare and profit maximizations or partial privatization of the public firm.

5. Comparisons between Cournot and Bertrand Competition

In a seminal paper, Singh and Vives (1984) showed that, in a differentiated duopoly, Cournot competition entails higher prices and profits than Bertrand competition, whereas both firms' output and social welfare are higher under Bertrand competition. We have examined the Cournot competition and Bertrand competition in section 3 and section 4. In this section, we compare the equilibrium results and social welfare of both competitions. The result is summarized in the following Proposition 3.

Proposition 3. *In the presence of network externalities and the heterogeneity of the differentiated goods, the optimal choice of public firm and private firm is Bertrand competition,* $\Omega^B > \Omega^C$ *and* $\pi_2{}^B > \pi_2{}^C$.

Proof: See Appendix A.4.

Proposition 3 revisits the conventional wisdom. Each firm earns higher profit under Bertrand competition than under Cournot competition. Note that even in the absence of network externality and product differentiation, the above inequalities remains intact, and the only sub-game perfect Nash equilibrium entails that both firms choose to offer a price contract, and the optimal privatization policy for government is fully nationalization. Our result is in line with Ghosh and Mitra (2010) who showed that price competition gives rise to the higher profitability for both private and public firms than quantity competition. Matsumura and Ogawa (2012) also showed that it is a dominant strategy for both firms to choose their prices regardless of whether the products are substitutes or complements. We have generalized the choice of price contract as a dominant strategy under mixed duopoly with the presence of network externalities and the heterogeneity of the differentiated goods.

Note that in the network duopoly without having public firm, Pal (2014) also showed that in the case of strong network externalities and imperfect-substitute goods, the classical profit-ranking of Cournot and Bertrand equilibria is reversed-each firm earns higher profit under Bertrand competition than under Cournot competition. His result is extended in the mixed network duopoly market.

Proposition 4. *Comparing the equilibrium results under Cournot and Bertrand competition,* $p_2{}^B < p_2{}^C$; $q_1{}^B < q_1{}^C$; $q_2{}^B > q_2{}^C$; $(q_1{}^B - q_2{}^B) < (q_1{}^C - q_2{}^C)$.

Proof: See Appendix A.5.

Proposition 4 shows that under Bertrand competition, private firm yields lower prices, which is obviously that under Bertrand competition, private firm must decrease its price to get more profit, but for the public firm, price change is ambiguous when comparing these two types of competition. In addition, we can see that the equilibrium output of public firm is lower under Bertrand competition, while the output of private firm is larger than that under Cournot competition. It is because private firm will produce more under Bertrand competition, and the public firm will decrease its output due to the output-substitution effect. Hence, the public firm will be less aggressive under Bertrand competition and the difference in output between two firms is lower than that under Cournot competition.

Notably, we seek to make the welfare comparison under different competition modes considering network externality and the heterogeneity of the differentiated goods. The finding is summarized by the following proposition.

Proposition 5. *When the products exhibit network externalities, social welfare is lower under Cournot than under Bertrand competition,* $SW^B > SW^C$.

Proof: See Appendix A.6.

Our Proposition 5 indicates that Bertrand competition yields higher social welfare at equilibrium than under Cournot competition, which is consistent with those in the literatures under both private and mixed duopoly.

6. Conclusions

We examined the influence of demand-side network externalities and product differentiation on the decisions of consumers and firms, and see how the governments will determine its privatization policy for a state-owned enterprise under Cournot and Bertrand competition. The previous results are extended, but some results are not hold. In particular, under Cournot competition with the differentiated products exhibit network externalities, we showed that 1. The optimal policy for the government to determine is partial privatization; 2. When the network effects increase, the government should slow down the path of privatization; 3. The impact of product differentiation on optimal degree of privatization is non-monotonic and depends crucially on the degree of network externality and the heterogeneity of the differentiated good. We also showed that under Bertrand competition, when the differentiated products exhibit network externalities, the optimal privation policy is fully nationalization. Furthermore, we demonstrated that the optimal choice of public and private firm is Bertrand competition, and social welfare is lower under Cournot than under Bertrand competition which confirms the convention wisdom.

Appendices

Appendix 1. Proof of the second-order condition for the maximization of social welfare under Cournot competition

We substitute Eqs. (9.a)-(9.b) into social welfare function and get the second-order derivatives with respect to θ:

$$\frac{\partial^2 SW}{\partial\theta^2}=((\alpha^2(1+\gamma)(n+\gamma-n\gamma+\gamma^2-3)(\gamma^7+\gamma^8+n^4(1-\gamma)^2(1+\gamma)+\gamma^3(15-14\theta)-$$
$$n^3(1-\gamma)(11+4\gamma-11\gamma^2-\gamma^3+\gamma^4-2\theta)-2\gamma^6(6-\theta)+18\gamma^4(3-\theta)+54(1-\theta)-\gamma^5(7-2\theta)$$
$$-6\gamma(1-4\theta)-9\gamma^2(11-6\theta)+n^2(1-\gamma)(45-12\gamma^3+7\gamma^4+\gamma^5+2\gamma(14-\theta)-18\theta-\gamma^2(42-4\theta))$$
$$-n(81-6\gamma^6+\gamma^7+\gamma^4(53-6\theta)-2\gamma^5(6-\theta)+18\gamma^3(2-\theta)-54\theta-9\gamma^2(15-4\theta)-\gamma(17-38\theta))))$$
$$/((1-\gamma)(\gamma^4+n^2(1-\gamma^2)+3(2+\theta)-\gamma^2(6+\theta)-n(5-4\gamma^2+\theta))^4))$$

The optimal privatization degree is:

$\theta^*=\dfrac{(1-n)(2-n+\gamma)(1-\gamma)\gamma}{9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4+n(6-5\gamma-2\gamma^2+\gamma^3)}$. We then check the sign of $\dfrac{\partial^2 SW}{\partial\theta^2}$ at the optimal degree of privatization:

$$\frac{\partial^2 SW}{\partial\theta^2}\Big|_{\theta=\theta^*}=$$
$$\frac{-\alpha^2(1+\gamma)(9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3))^4}{(1-\gamma)(n+\gamma-n\gamma+\gamma^2-3)^2(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)+n^2(8-9\gamma^2+\gamma^4)-n(21-25\gamma^2+5\gamma^4))^3}<0$$

Appendix 2. Proof of Proposition 1

(i) Take the derivative of θ^* w.r.t. n, yielding:

$$\frac{\partial\theta^*}{\partial n}=-(1-\gamma)\gamma$$
$$\frac{(15+\gamma-13\gamma^2-3\gamma^3+3\gamma^4+\gamma^5+n^2(3-3\gamma-\gamma^2+\gamma^3)-2n(7-3\gamma-5\gamma^2+\gamma^3+\gamma^4))}{(9-n^2(-1+\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3))^2}<0$$

(ii) Take the derivative of θ^* w.r.t. γ, yielding:

$$\frac{\partial\theta^*}{\partial\gamma}=\frac{18+n^4(1-\gamma)^2-18\gamma-11\gamma^2+4\gamma^3+\gamma^4+2\gamma^5+\gamma^6-n^3(1-\gamma)^2(9+2\gamma+\gamma^2)}{(9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3))^2}$$

$$\frac{+n^2(29-44\gamma+6\gamma^2+4\gamma^3+2\gamma^4+2\gamma^5)-n(39-48\gamma-10\gamma^2+8\gamma^3+2\gamma^4+4\gamma^5+\gamma^6)}{(9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3))^2}$$

To calculate the sign of $\frac{\partial\theta^*}{\partial\gamma}$, we get the critical values of $\overline{\gamma_1}$, $\overline{\gamma_2}$ and $\overline{n_1}$ which are:

$$\overline{\gamma_1}=\sqrt{\frac{1}{2}(-3+\sqrt{17})}\ ; \qquad \overline{\gamma_2}=0.7967341474198391$$

$$\overline{n_1}=\frac{-5+5\gamma+\gamma^3+\sqrt{1-10\gamma+21\gamma^2-18\gamma^3+10\gamma^4-4\gamma^5+\gamma^6}}{2(-1+\gamma)}$$

The denominator of $\frac{\partial\theta^*}{\partial\gamma}$ is always positive, and the sign of $\frac{\partial\theta^*}{\partial\gamma}$ will depend on the numerator. The numerator is the function of n, and one of the solutions for this function is $\overline{n_1}$: when $\overline{n_1}\geq 1$ holds, $\frac{\partial\theta^*}{\partial\gamma}$ will be positive for all $0\leq n<1$; when $\overline{n_1}<0$ holds, for $0\leq n<1$ we will have $\frac{\partial\theta^*}{\partial\gamma}<0$; when $0\leq\overline{n_1}<1$ holds, the sign of $\frac{\partial\theta^*}{\partial\gamma}$ will depend on the relative relationship between n and $\overline{n_1}$. We can find two critical values for γ to decide the size of $\overline{n_1}$, which can be summarized as follows:

(1) If $0<\gamma<\overline{\gamma_1}$ holds, $\overline{n_1}\geq 1$; so for all $0\leq n<1$, $\frac{\partial\theta^*}{\partial\gamma}$ will be positive;

(2) If $\overline{\gamma_2}<\gamma<1$, then $\overline{n_1}<0$; so for all $0\leq n<1$, $\frac{\partial\theta^*}{\partial\gamma}<0$;

(3) If $\overline{\gamma_2}<\gamma<\overline{\gamma_1}$, then $0\leq\overline{n_1}<1$ and for all $0\leq n<1$, the sign of $\frac{\partial\theta^*}{\partial\gamma}$ depends

on the scale of network externality. When $0 \le n < \overline{n}_1$, $\dfrac{\partial \theta *}{\partial \gamma} > 0$ and when $\overline{n}_1 < n < 1$, $\dfrac{\partial \theta *}{\partial \gamma} < 0$.

Appendix 3. Proof of the second-order condition for the maximization of social welfare under Bertrand competition

We substitute Eqs. (12.a)-(12.b) into social welfare function and get the second-order derivatives with respect to θ:

$$
\begin{aligned}
\frac{\partial^2 SW}{\partial \theta^2} &= ((a^2(1-n)^2(1-\gamma)^2(3+2\gamma-n(2+\gamma))(\gamma^2(93-30\theta)+\gamma^3(42-28\theta)-4\gamma^4(6-\theta)- \\
&4\gamma^5(3-\theta)-54(1-\theta)-12\gamma(1-4\theta)-n^4(8-10\gamma^2-3\gamma^3-16\theta-12\gamma\theta)+n^3(52+4\gamma^4+\gamma^5+ \\
&\gamma(2-70\theta)-88\theta-2\gamma^3(12-5\theta)-\gamma^2(71-12\theta))+n(27(5-6\theta)+6\gamma^4(7-\theta)+6\gamma^5(3-\theta) \\
&+20\gamma(1-7\theta)-\gamma^3(89-62\theta)-\gamma^2(218-68\theta))+n^2(\gamma^2(188-50\theta)+\gamma^3(70-44\theta)-2\gamma^5(4-\theta) \\
&-\gamma^4(23-2\theta)-18(7-10\theta)-\gamma(11-150\theta))))/(-3(2+\theta)+\gamma^2(3+\theta)+n^2(\gamma^2-2(1+\theta))+ \\
&n(7+5\theta-\gamma^2(3+\theta)))^4)
\end{aligned}
$$

The optimal privatization degree is:

$\theta^* = -\dfrac{(2-n)(2-n+\gamma)\gamma(1+\gamma)}{9+8\gamma-2\gamma^2-2\gamma^3+n^2(4+3\gamma)-n(12+10\gamma-\gamma^2-\gamma^3)}$. We then check the sign of $\dfrac{\partial^2 SW}{\partial \theta^2}$ at the optimal degree of privatization:

$$
\frac{\partial^2 SW}{\partial \theta^2}\Big|_{\theta=\theta^*} = -\frac{a^2(1-n)^2(1-\gamma)^2(9+8\gamma-2\gamma^2-2\gamma^3+n^2(4+3\gamma)-n(12+10\gamma-\gamma^2-\gamma^3))^4}{(3+2\gamma-n(2+\gamma))^2(18-19\gamma^2+4\gamma^4-n^3(4-3\gamma^2)-n(33-30\gamma^2+4\gamma^4)+n^2(20-16\gamma^2+\gamma^4))^3} < 0
$$

Appendix 4. Proof of Proposition 3

The objective function for both firms under Cournot and Bertrand competition are as follows:

$$\Omega^B = a^2(34+18\gamma-25\gamma^2-13\gamma^3+3\gamma^4+\gamma^5-n^3(7+\gamma-7\gamma^2-\gamma^3+2\gamma^4)$$
$$+n^2(36+10\gamma-31\gamma^2-9\gamma^3+7\gamma^4+\gamma^5)-n(61+25\gamma-47\gamma^2-19\gamma^3+8\gamma^4+2\gamma^5))\,/$$
$$2(1-\gamma)(n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2))^2$$

$$\Omega^C = (\alpha(1+\gamma)((1-n)\alpha(2-n+\gamma)(1-\gamma)^2\gamma(9+9\gamma-8\gamma^2-5\gamma^3+2\gamma^4+\gamma^5+n^2(1+2\gamma-\gamma^2)-$$
$$n(6+7\gamma-5\gamma^2-\gamma^3+\gamma^4))+((3-n-\gamma+n\gamma-\gamma^2)^2(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2$$
$$+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4))(a(30-4n^2(-1+\gamma)-14\gamma-22\gamma^2+4\gamma^3+4\gamma^4+n(-22+18\gamma+8\gamma^2-4\gamma^3))$$
$$-\alpha(13+2n^2(1-\gamma)-6\gamma-10\gamma^2+2\gamma^3+2\gamma^4-2n(5-4\gamma-2\gamma^2+\gamma^3))))\,/\,(9+n^2(1-\gamma)-4\gamma-6\gamma^2$$
$$+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3))))\,/\,(2(1-\gamma)(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)$$
$$+n^2(8-9\gamma^2+\gamma^4))^2)$$

$$\pi_2{}^B = \frac{a^2(3-2n)(2-n+\gamma)^2}{2(n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2))^2}$$

$$\pi_2{}^C = \frac{a^2(1-\gamma)(1+\gamma)(3-\gamma^2)(6+n^2+3\gamma-2\gamma^2-\gamma^3-n(5+\gamma-\gamma^2))^2}{2(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4))^2}$$

The differences of objective functions for both firms under Cournot and Bertrand competition are:

$$\Omega^B-\Omega^C = \frac{1}{2(1-\gamma)}a(a(34+18\gamma-25\gamma^2-13\gamma^3+3\gamma^4+\gamma^5-n^3(7+\gamma-7\gamma^2-\gamma^3+2\gamma^4)+$$
$$n^2(36+10\gamma-31\gamma^2-9\gamma^3+7\gamma^4+\gamma^5)-n(61+25\gamma-47\gamma^2-19\gamma^3+8\gamma^4+2\gamma^5))\,/\,(n(7-3\gamma^2)$$
$$-3(2-\gamma^2)-n^2(2-\gamma^2))^2+((1+\gamma)(((a(1-n)(2-n+\gamma)(1-\gamma)^2\gamma(9+9\gamma-8\gamma^2-5\gamma^3+2\gamma^4+$$
$$\gamma^5+n^2(1+2\gamma-\gamma^2)-n(6+7\gamma-5\gamma^2-\gamma^3+\gamma^4))+a(3-n-\gamma+n\gamma-\gamma^2)^2(17+2n^2(1-\gamma)-8\gamma$$
$$-12\gamma^2+2\gamma^3+2\gamma^4-2n(6-5\gamma-2\gamma^2+\gamma^3))(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+$$
$$5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)))\,/\,(9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3)))))$$
$$/(18-25\gamma^2+9\gamma^4-\gamma^6+n^3(-1+\gamma^2)+n(-21+25\gamma^2-5\gamma^4)+n^2(8-9\gamma^2+\gamma^4))^2)>0$$

$$\pi_2^B-\pi_2^C = \frac{1}{2}a^2(\frac{(3-2n)(2-n+\gamma)^2}{(n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2))^2}+$$
$$\frac{(1-\gamma)(1+\gamma)(3-\gamma^2)(6+n^2+3\gamma-2\gamma^2-\gamma^3-n(5+\gamma-\gamma^2))^2}{(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4))^2})>0$$

Appendix 5. Proof of Proposition 4

The equilibrium results under Bertrand competition are as follows:

$$p_1^{\ B} = \frac{a(3-2n+3\gamma-3n\gamma+n^2\gamma)}{6-3\gamma^2-n(7-3\gamma^2)+n^2(2-\gamma^2)}; \quad p_2^{\ B} = \frac{a(2-n)(2-n+\gamma)}{6-3\gamma^2-n(7-3\gamma^2)+n^2(2-\gamma^2)}$$

$$q_1^{\ B} = \frac{a(3+\gamma-\gamma^2-n(2-\gamma^2))}{3(2-\gamma^2)+n^2(2-\gamma^2)-n(7-3\gamma^2)}; \quad q_2^{\ B} = \frac{a(2-n+\gamma)}{3(2-\gamma^2)+n^2(2-\gamma^2)-n(7-3\gamma^2)}$$

$$Q^B = \frac{a(5+2\gamma-\gamma^2-n(3-\gamma^2))}{3(2-\gamma^2)+n^2(2-\gamma^2)-n(7-3\gamma^2)}; q_1^{\ B}-q_2^{\ B} = \frac{a(1-n)(1-\gamma^2)}{3(2-\gamma^2)+n^2(2-\gamma^2)-n(7-3\gamma^2)}$$

The equilibrium results under Cournot competition are as follows:

$$p_1^{\ C} = \frac{a(9+7\gamma-9\gamma^2-5\gamma^3+2\gamma^4+\gamma^5+n^2(1+\gamma-\gamma^2)-n(6+4\gamma-6\gamma^2-\gamma^3+\gamma^4))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)}$$

$$p_2^{\ C} = \frac{a(2-\gamma^2)(6+n^2+3\gamma-2\gamma^2-\gamma^3-n(5+\gamma-\gamma^2))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)}$$

$$q_1^{\ C} = \frac{a(1+\gamma)(9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)}$$

$$q_2^{\ C} = \frac{a(1-\gamma)(1+\gamma)(6+n^2+3\gamma-2\gamma^2-\gamma^3-n(5+\gamma-\gamma^2))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)}$$

$$Q^C = \frac{\alpha(1+\gamma)(15+2n^2(1-\gamma)-7\gamma-11\gamma^2+2\gamma^3+2\gamma^4-n(11-9\gamma-4\gamma^2+2\gamma^3))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)}$$

$$q_1^{\ C}-q_2^{\ C} = \frac{\alpha(1+\gamma)(3-n-\gamma+n\gamma-\gamma^2)}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)}$$

Comparing the equilibrium prices of both firms are:

$$p_1^B-p_1^C = \frac{a(3-2n+3\gamma-3n\gamma+n^2\gamma)}{6-3\gamma^2-n(7-3\gamma^2)+n^2(2-\gamma^2)} - \frac{a(9+7\gamma-9\gamma^2-5\gamma^3+2\gamma^4+\gamma^5+n^2(1+\gamma-\gamma^2)-n(6+4\gamma-6\gamma^2-\gamma^3+\gamma^4))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)+n^2(8-9\gamma^2+\gamma^4)-n(21-25\gamma^2+5\gamma^4)}$$

$$p_2^B-p_2^C = \frac{a(2-n)(2-n+\gamma)}{6-3\gamma^2-n(7-3\gamma^2)+n^2(2-\gamma^2)} - \frac{a(2-\gamma^2)(6+n^2+3\gamma-2\gamma^2-\gamma^3-n(5+\gamma-\gamma^2))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)+n^2(8-9\gamma^2+\gamma^4)-n(21-25\gamma^2+5\gamma^4)} > 0$$

Comparing the equilibrium outputs of both firms are:

$$q_1^B - q_1^C = -\frac{a(3+\gamma-\gamma^2-n(2-\gamma^2))}{n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2)} -$$
$$\frac{a(1+\gamma)(9+n^2(1-\gamma)-4\gamma-6\gamma^2+\gamma^3+\gamma^4-n(6-5\gamma-2\gamma^2+\gamma^3))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)+n^2(8-9\gamma^2+\gamma^4)-n(21-25\gamma^2+5\gamma^4)} < 0$$

$$q_2^B - q_2^C = -\frac{a(2-n+\gamma)}{n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2)} -$$
$$\frac{a(1-\gamma)(1+\gamma)(6+n^2+3\gamma-2\gamma^2-\gamma^3-n(5+\gamma-\gamma^2))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)+n^2(8-9\gamma^2+\gamma^4)-n(21-25\gamma^2+5\gamma^4)} < 0$$

Comparing the equilibrium total output and the difference in output between two firms are:

$$Q^B - Q^C = -\frac{a(5+2\gamma-\gamma^2-n(3-\gamma^2))}{n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2)} -$$
$$\frac{a(1+\gamma)(15+2n^2(1-\gamma)-7\gamma-11\gamma^2+2\gamma^3+2\gamma^4-n(11-9\gamma-4\gamma^2+2\gamma^3))}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)+n^2(8-9\gamma^2+\gamma^4)-n(21-25\gamma^2+5\gamma^4)}$$

$$(q_1^B - q_2^B) - (q_1^C - q_2^C) = -\frac{a(1-n)(1-\gamma^2)}{n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2)} -$$
$$\frac{a(1+\gamma)(3-n-\gamma+n\gamma-\gamma^2)}{18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)+n^2(8-9\gamma^2+\gamma^4)-n(21-25\gamma^2+5\gamma^4)} < 0$$

Appendix 6. Proof of Proposition 5

The social welfare under Cournot and Bertrand competition are as follows:

$$SW^B = a^2(34+18\gamma-25\gamma^2-13\gamma^3+3\gamma^4+\gamma^5-n^3(7+\gamma-7\gamma^2-\gamma^3+2\gamma^4)$$
$$+n^2(36+10\gamma-31\gamma^2-9\gamma^3+7\gamma^4+\gamma^5)-n(61+25\gamma-47\gamma^2-19\gamma^3+8\gamma^4+2\gamma^5))\,/$$
$$2(1-\gamma)(n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2))^2$$

$$SW^C = a(1+\gamma)(a(30+4n^2(1-\gamma)-14\gamma-22\gamma^2+4\gamma^3+4\gamma^4-n(22-18\gamma-8\gamma^2+4\gamma^3))$$
$$-a(13+2n^2(1-\gamma)-6\gamma-10\gamma^2+2\gamma^3+2\gamma^4-2n(5-4\gamma-2\gamma^2+\gamma^3)))\,/$$
$$2(1-\gamma)(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4))$$

The difference of social welfare under Cournot and Bertrand competition is:

$$SW^B - SW^C = a^2(2-n+\gamma)^2(8n^3(1-\gamma^2)^2-n^4(1-\gamma^2)^2+\gamma^2(8-2\gamma^2-4\gamma^4+\gamma^6)-n^2(18-37\gamma^2+$$
$$15\gamma^4+4\gamma^6-\gamma^8)+2n(6-16\gamma^2+5\gamma^4+4\gamma^6-\gamma^8))\,/\,2(n(7-3\gamma^2)-3(2-\gamma^2)-n^2(2-\gamma^2))^2$$
$$(18-25\gamma^2+9\gamma^4-\gamma^6-n^3(1-\gamma^2)-n(21-25\gamma^2+5\gamma^4)+n^2(8-9\gamma^2+\gamma^4)) > 0$$

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