Delegation and the choice of strategic variable in oligopoly games

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Abstract

This paper considers a two-stage game. In the first stage, firms commit to a marketing strategy, represented by a willingness to discount their preferred price in order to achieve a target sales quantity. The polar cases are Bertrand strategies in which firms fix a price, and sell the quantity demanded at that price, and Cournot strategies in which firms fix a quantity and sell it at the market-clearing price. In the second stage of the game, a stochastic demand parameter is realized and observed by the firms, which then specify their target price and quantity. In equilibrium, these will clear the market and each will constitute a best response to the strategies of the other firms. We show that, for the case of constant marginal costs, for some parameter range, there will exist two symmetric equilibrium solutions. In addition to the Bertrand outcome, for some parameter range, there exists another symmetric equilibrium where firms behave less competitively than the Bertrand benchmark and earn positive profits. With an appropriate parametrization, the possible alternative equilibria span the space from Bertrand to Cournot.

JEL Classification: L1, L2, L4.

Key-words: multiple equilibria, oligopolistic competition.

1 Introduction

Ever since the study of oligopoly began in the 19th century, the choice of equilibrium concept has been debated. The terms of the debate were set by Bertrand's

(1883) critical review of Cournot's (1838) analysis, which had been largely ignored in the intervening decades. Nearly 150 years after Bertrand's paper, most studies of oligopolistic markets use one or other of these equilibrium concepts.

There have, however, been numerous attempts at a more general treatment of the problem. Two strands of literature are of particular interest.

The first strand of literature model price determination in oligopolistic markets as a two-stage game. One approach (Kreps and Scheinkman, 1983, Grant and Quiggin, 1996) has focused on precommitment to the choice of capital. In the seminal paper by Kreps and Scheinkman (1983), the determination of capital stocks in the first stage results in Cournot-like outcomes which are considerably less competitive than Bertrand competition, even when price is the second-stage strategic variable.[1] In Grant and Quiggin (1996), firms compete in supply function schedules in the second stage. They show that the equilibrium outcome becomes more competitive as the share of capital in the production function decreases.[2] An alternative approach, based on the idea of delegation (Fershtman and Judd (1987), Fumas (1992), Miller and Pazgal, (2001), Sklivas (1987), Vickers (1985)) involves a separation of ownership and control between owners and managers. In the first-stage game, owners determine a remuneration rule for managers. In the second stage, managers play an oligopoly game, with price or quantity as the strategic variable. By manipulating the incentive contract for managers, the owners can commit to strategies in the product market which are not profit-maximising and gain strategic advantages over the rivals. Most papers include the firm's own profit, revenue, and/or output in the incentive contract (Fershtman and Judd (1987), Sklivas (1987), Vickers, (1985)). They find that with strategic choice of incentive contracts before quantity competition in the product market competition, the equilibrium is more competitive than the benchmark one stage Cournot game. On the other hand, if prices are the strategic variable, the owners set the incentive weighting such that the market is less competitive than the one stage Bertrand outcome.[3] Fumas (1992) and Miller and Pazgal (2001) on the other hand, include the rivals' profit in the construction of the incentive scheme. With this relative performance incentive contract,[4] Fumas (1992) analyses the trade-off between risk sharing and strate-

[1] See also extension by Boccard and Wauthy (2000).

[2] Also see Dixon (1985, 1986) and Vives (1986).

[3] Vickers (1985) does not consider price competition.

[4] The term 'relative performance incentive' is most naturally applied to a scheme where the manager is rewarded (punished) for a profit exceeding (falling short of) that of other firms in the industry. This would imply a negative weight on rivals' profit. As shown by Fumas and by Miller and Pazgal, however, the equilibrium weight may be negative, implying an element

gic competition. Miller and Pazgal (2001) show that the same equilibrium is obtained regardless of the strategic variable in the product market.

The second strand of literature are models which expand the space of potential strategies to include a large set of supply schedules. See, for example, Grossman (1981), Robson (1981), Klemperer and Meyer (1989) and Kao, Menezes, and Quiggin (2014). Klemperer and Meyer (1989) prove the negative result that, if the strategy space consists of all possible supply curves, any individually rational market equilibrium may be sustained as a Nash equilibrium. However, a unique Nash equilibrium may be obtained by introducing demand uncertainty. Klemperer and Meyer (1989), following Robson (1981) show that a unique equilibrium may be obtained if uncertainty is resolved after the choice of supply schedules. As shown by Robson, this approach yields the Bertrand zero-profit as the unique equilibrium in the case of constant cost. Kao, Menezes, and Quiggin (2014) consider the case where uncertainty is resolved before the choice of supply schedules, and obtain a unique equilibrium by assuming that the slope of the supply schedule is considered as a measure of the competitiveness of the market and determined exogenously. The resulting class of models range from Cournot (vertically sloped supply curves) to Bertrand (horizontal supply curves). This approach allows for positive oligopoly profits even in the case of zero costs. However, the question of how the slope of the supply curve is determined, that is, how competitive is the market, is left unresolved.

The aim of this paper is to develop a delegation model with second-stage oligopolistic competition in supply schedules. We show that competition in supply schedules can be rationalised by a delegation game, with costly monitoring, between the owner and the retailer. In this framework, owners are assumed to set incentives that define a space of marketing strategies. These incentives determine the intensity of competition, represented by a willingness to discount their preferred price in order to achieve a target sales quantity.

The polar cases are Bertrand strategies in which firms fix a price, and sell the quantity demanded at that price, and Cournot strategies in which firms fix a quantity and sell it at the market-clearing price. Between these two polar extremes are a range of possible marketing strategies, specified by a 'willingness to discount' parameter, β, ranging from zero (Cournot) to infinity (Bertrand).

We assume that firms act as wholesalers giving instructions to dealers who undertake the sales and return the associated revenue to the firm. In the case of a Bertrand strategy, the wholesalers' instructions are self-enforcing. The dealer must pay to the firm an amount equal to the value of the goods sold at the

of co-operation rather than rivalry.

specified price. In the Cournot case, by contrast, revenue depends on the state of demand, which determines the price at which the specified quantity may be sold. Hence, the firm must engage in costly monitoring to ensure that the dealer is repaying the full revenue. More generally, we assume that the associated monitoring cost is inversely related to β. The smaller β is, the greater the benefit to the dealer from cheating, and the greater the associated monitoring cost.

In the second stage of the game, a stochastic demand parameter is realized and observed by the dealers. The dealers report the demand state to the owners, who then specify their target price and quantity. In equilibrium, with costly monitoring, the dealers report the true demand state, and the specified target quantities and prices will clear the market and each will constitute a best response to the strategies of the other firms.

We first show that, for the case of constant marginal costs, there will, in general, exist two symmetric equilibrium solutions. As in Grossman (1981), Robson (1981), Klemperer-Meyer (1989), and Grant and Quiggin (1996), the Bertrand outcome remains an equilibrium. However, we show that there exists another symmetric equilibrium where firms behave less competitively than the Bertrand benchmark and earn positive profits. With an appropriate parametrization, the possible alternative equilibria span the space from Bertrand to Cournot.

2 The Model

The market demand is represented by $D(P,\varepsilon)$ with $D_1 < 0$, $D_{11} \leq 0$, and $\varepsilon \in \mathbb{R}$ a stochastic shock with $E[\varepsilon] = 0$ and $Var[\varepsilon] = \sigma_\varepsilon^2$. We model an n firm oligopoly game where each firm is characterised by a pair of wholesaler and dealer. Depending on the price quantity target and the permissable discount, the owner incurs some monitoring cost. We show that such price quantity target, together with the willingness to discount variable, give rise to competition in supply function with the price slope chosen in the first stage, followed by the second stage quantity competition.[5] The strategy space for firm i consists of vectors $\left(\beta_i, \widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ where $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ is the 'target' price-quantity pair, and β_i is a variable which may be referred to as 'competitiveness' or willingness to discount. The vector $\left(\beta_i, \widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ represents the instructions given by the firm, as the wholesaler, to its dealers. The firm incurs a monitoring

[5] With competiton in supply function, given the chosen slopes of other firms, the firm maximises profit by choosing one point on its residual demand curve. Choosing quantity and choosing price give the same outcome.

cost M_i. Taking account of production costs, we assume the total cost to be $TC_i(\beta_i, q_i) = M_i(\beta_i) + C_i(q_i)$ with $\beta_i \geq 0$, $q_i > 0$, $M_i^{'} < 0$, $M_i^{''} \geq 0$, $C_i' \geq 0$, and $C_i'' \geq 0$. Hence, profit for firm i, conditional on the choice of β_i and the demand shock ε is

$$\pi_i(q_i; \beta_i, \varepsilon) = Pq_i - TC_i(\beta_i, q_i).$$

The introduction of monitoring costs is novel. The tighter the sales target, and, therefore, the higher the permissible discount, the greater the incentive the dealer has to cheat, by falsely reporting negative values of ε, and hence the greater the monitoring cost to the firm. The monitoring cost decreases in β. We present the game between the owner and the dealer in Appendix A and show that in order to induce truth telling of the demand state, the monitoring cost decreases in β. For $\beta = 0$, the firm wants the retailer to sell exactly the target output. This corresponds to Cournot competition with a fixed output, independent of the market price. For $\beta \to \infty$, the firm allows any deviation from the target output and thus does not incur monitoring cost. For any $P \geq \widetilde{P}_i(\varepsilon)$, the firm is willing to supply any quantity. This corresponds to Bertrand type behavior.

In the Bertrand case, the dealers have no opportunity for cheating, since their incentives are aligned with those of the firm - each sale at a fixed price yields the dealer a known margin over the wholesale price.[6] By contrast, in the Cournot case, the dealer's instructions are to sell the exact target quantity at whatever price they can get.

The timing is as follows: in the first stage, firms simultaneously choose a β_i, representing the desired willingness of dealers to discount price in order to achieve a target sales quantity. The choice of β_i commits firms to a particular degree of competitiveness. The demand shock ε is then realised and observed by the retailers. The retailers than report the demand state to the owners. In the second stage of the game, knowing $\beta_j \forall j$, and *given the reported* demand condition ε, firms simultaneously choose $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$. In equilibrium, the common price is $P(\varepsilon) = \widetilde{P}_i(\varepsilon)$, $\forall i$, firms sell their selected output $\widetilde{q}_i(\varepsilon)$, and the market clears with $D(P(\varepsilon), \varepsilon) = \sum_i \widetilde{q}_i(\varepsilon)$.

The timing of the game differs from that of Klemperer and Meyer (1989), where firms must specify a supply schedule before learning the value of ε, and where it is assumed that the range of values of ε is sufficient to determine a complete supply schedule. In our case, the pair $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ depends on the reported state of demand. Since firms are not price-takers, their desired price

[6] The fixed profit margin is not modelled explicitly here and is included in $C_i(q_i)$.

and quantity will, in general, increase with demand. The resulting locus of equilibrium prices and quantities has many of the properties of a supply curve, but must be interpreted differently. This point is developed further below.

3 The main result

We solve the game backwards to get the subgame perfect Nash equilibrium. That is, we first derive the market equilibrium $(P(\varepsilon, \boldsymbol{\beta}), \mathbf{q}(\varepsilon, \boldsymbol{\beta}))$ where ε is the observed shock, $\boldsymbol{\beta} = (\beta_1, ..., \beta_n)$ is the vector of first round strategy choices and $\mathbf{q} = (q_1, ..., q_n)$ is the vector of equilibrium output quantities. We derive first-order conditions for the general case, and then a closed-form solution for the case where firms with zero marginal production cost compete in an industry with linear inverse demand curve, subject to an additive demand shock:

3.1 The second stage equilibrium

We first describe the second stage game in more detail. The marketing strategy of firm i is specified by $\left(\beta_i, \widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ where $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ represents the desired price-quantity pair for demand state ε, and β_i represents the willingness of firm i to discount the preferred price $\widetilde{P}_i(\varepsilon)$ in order to achieve the desired quantity $\widetilde{q}_i(\varepsilon)$. That is, for given ε, the firm instructs the retailer to offer a locus of price-quantity pairs (P, q) passing through $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$, and such that, evaluated at $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$,

$$\frac{\partial \widetilde{P}_i(\varepsilon)}{\partial \widetilde{q}_i(\varepsilon)} = \frac{1}{\beta_i}. \tag{1}$$

Without loss of generality,[7] we can confine attention to the linear locus given by the equation

$$\beta_i \left(\hat{P} - \widetilde{P}_i(\varepsilon)\right) = (q_i - \widetilde{q}_i(\varepsilon)),$$

or

$$q_i\left(\hat{P}, \gamma, \varepsilon\right) = \widetilde{q}_i(\varepsilon) + \beta_i \left(\hat{P} - \widetilde{P}_i(\varepsilon)\right), \tag{2}$$

[7] As shown by Kao, Menezes and Quiggin (2014), if $\left(\widetilde{P}(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$, $i = 1, ..., n$, is an equilibrium loci of the form (2), conditional on demand $D(P, \varepsilon)$, it is an equilibrium for any game in which, for all i, the strategy space is given by loci $\hat{q}_i(P, \beta, \varepsilon)$ satisfying 1 for each $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$. All such strategies may be summarized by the pair $(\gamma_i, \widetilde{q}_i(\varepsilon))$ such that 2 holds in a neighborhood of $\widetilde{P}_i$.

where $q_i\left(\hat{P};\beta_i,\varepsilon\right)$ is the quantity sold by the dealer and $\hat{P}$ is the market clearing price with $D\left(\hat{P};\varepsilon\right)=\sum_{i=1}^{n}q_i\left(\hat{P};\beta_i,\varepsilon\right)$. The case $\beta_i=0\ \forall i$ gives a family of vertical supply schedules. This corresponds to Cournot competition with a fixed output and thus requires high monitoring cost. The case $\beta\to\infty$ gives horizontal supply schedules. This permits any deviation from the target output and thus corresponds to Bertrand type behavior. This does not incur any monitoring cost.

For known ε and β_j, and given choices of $\left(\widetilde{P}_j(\varepsilon),\widetilde{q}_j(\varepsilon)\right)$,$\forall j\neq i$, the residual demand facing firm i is given by

$$D_i\left(\hat{P};\varepsilon\right)=D\left(\hat{P};\varepsilon\right)-Q_{-i}\left(\hat{P};\beta_{-i},\varepsilon\right)$$

where

$$Q_{-i}\left(\hat{P};\beta,\varepsilon\right)=\sum_{j\neq i}q_j\left(\hat{P};\beta_{-i},\varepsilon\right)$$

is the aggregate output of competitors. Firms choose a set of pairs $\left(\widetilde{P}_i(\varepsilon),\widetilde{q}_i(\varepsilon)\right)$ such that for each firm, the choice maximizes net profit

$$\left(\widetilde{P}_i(\varepsilon),\widetilde{q}_i(\varepsilon)\right)\in\arg\max\widetilde{P}_iD_i\left(\hat{P}\left(\widetilde{P}_i\right),\varepsilon\right)-C_i\left(D_i\left(\hat{P}\left(\widetilde{P}_i\right),\varepsilon\right)\right).$$

In equilibrium, $\hat{P}(\varepsilon)=\widetilde{P}_i(\varepsilon)$, $\forall i$, and that the market clears with $D\left(\hat{P}(\varepsilon),\varepsilon\right)=\sum_i\widetilde{q}_i(\varepsilon)$.

Each firm takes its residual demand curve as given, and acts as a profit-maximizing monopolist. The observation of a Delta airlines exexutive, cited by Klemperer and Meyer (1989, footnote 5) that

> 'We don't have to know if a balloon race in Albuquerque or a rodeo in Lubbock is causing an increase in demand for a flight'

is apposite here. Note that the observation remains true if the increased demand for Delta services is caused by a reduction in the number of flights offered by, say, Southwest.

Market clearing gives

$$D\left(\hat{P};\varepsilon\right)=\sum_{j=1}^{N}q_j\left(\hat{P};\beta_i,\varepsilon\right)=q_i\left(\hat{P};\beta_i,\varepsilon\right)+Q_{-i}\left(\hat{P};\beta,\varepsilon\right)$$

Differentiate with respect to q_i:[8]

$$D'\left(\hat{P};\varepsilon\right)\frac{\partial \hat{P}}{q_i} = 1 + Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\frac{\partial \hat{P}}{\partial q_i}.$$

Thus, firm i's output $q_i(\varepsilon)$ affects the market clearing price by:

$$\frac{\partial \hat{P}}{\partial q_i} = \frac{1}{D'\left(\hat{P};\varepsilon\right) - Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)}. \tag{3}$$

The residual demand facing firm i in the second stage is

$$D_i\left(\hat{P};\varepsilon\right) = D\left(\hat{P};\varepsilon\right) - \sum_{j\neq i}^{N} q_j\left(\hat{P};\beta_j,\varepsilon\right).$$

Firm i solves

$$\max_{q_i} \pi_i\left(q_i;\beta_i,\varepsilon\right) = D_i\left(\hat{P};\varepsilon\right)\hat{P} - C_i\left(q_i\right).$$

The FOC gives

$$\hat{P}\left(D'\left(\hat{P};\varepsilon\right) - Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)\frac{\partial P}{\partial q_i} + D_i\left(\hat{P};\varepsilon\right)\frac{\partial \hat{P}}{\partial q_i} \leq C'_i\left(q_i\right)$$

with equality for interior solutions.

After substituting in Equation 3, in an interior solution, the optimal q_i^* in the second stage is implicitly defined by

$$\frac{\left(D'\left(\hat{P}\right) - Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)}{D'\left(\hat{P}\right) - Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)}\left(\hat{P} - C'_i\left(q_i\right)\right) + \frac{q_i^*}{D'\left(\hat{P}\right) - Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)} = 0 \tag{4}$$

Note that this is just the familiar inverse elasticity pricing rule with the residual demand facing firm i adjusted by the supply function. At $\hat{P}$, the residual demand elasticity, ξ, is

$$\xi = \frac{\frac{\partial D q_i}{q_i^*}}{\frac{\partial P}{\hat{P}}} = \frac{\partial D_i}{\partial P}\frac{\hat{P}}{q_i^*} = \left(D'\left(\hat{P};\varepsilon\right) - Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)\frac{\hat{P}}{q_i^*}.$$

Simple re-arrangement of Equation 4 gives

$$\frac{\hat{P} - C'_i\left(q_i\right)}{\hat{P}} = -\frac{q_i^*}{\left(D'\left(\hat{P}\right) - Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)\hat{P}} = -\frac{1}{\xi}. \tag{5}$$

[8] Choosing $\left(\widetilde{P}_i\left(\varepsilon\right), \widetilde{q}_i\left(\varepsilon\right)\right)$, combined with the constraint $\hat{P} = \widetilde{P}_i\left(\varepsilon\right)$ in equilibrium, gives us the same solution as choosing the supply schedule $q_i\left(\hat{P};\beta,\varepsilon\right)$.

3.2 First stage equilibrium

We now consider the determination of β in the first stage of the game. In the first stage, firm i chooses β_i to maximise

$$\max_{\gamma_i} E\left[q_i^*\left(\beta_i\right)\hat{P}\left(q_i^*\left(\beta_i\right), q_{-i}^*\right) - TC_i\left(\beta_i, q_i^*\right)\right].$$

The FOC gives

$$E\left[\frac{\partial q_i^*\left(\beta_i\right)}{\partial \beta_i}\left(\hat{P}\left(q_i^*\left(\beta_i\right), q_{-i}^*\right) - Q_{-i}'\left(\hat{P};\beta,\varepsilon\right)\right) + q_i^*\frac{\partial \hat{P}\left(q_i^*\left(\beta_i\right), Q_{-i}^*\right)}{\partial \beta_i} - \frac{\partial M_i\left(\beta_i\right)}{\partial \beta_i}\right] \begin{matrix} < \\ = \\ > \end{matrix} 0, \tag{6}$$

with $<$ for $\beta_i^* = 0$, $=$ for $\beta_i^* \in (0,\infty)$, and $>$ for $\beta_i^* \to \infty$.

We have

Proposition 1 *Assume that* $TC_i\left(0, q_i\right) \equiv cq_i$ *for all* i *and* q_i*. Then* $\beta_i \to \infty, \forall i$ *is a first stage equilibrium, with the corresponding second stage solution* $\hat{P}\left(\varepsilon\right) = c$*,* $Q\left(\varepsilon\right) \equiv D\left(c,\varepsilon\right)$*.*

Proof. Given $\beta_j \to \infty$, $\forall j \neq i$, from Equation 5, we have $\hat{P} = \frac{\partial TC_i}{\partial q_i} = c$ and $\frac{\partial \hat{P}}{\partial \beta_i} = 0$. Hence, With $\frac{\partial M_i(\beta_i)}{\partial \beta_i} < 0$, $\beta_i^* \to \infty$. ■

Proposition 2 *For some range of marginal monitoring cost, there exists a positive profit symmetric equilibrium.*

Proof. Let $\frac{\partial C_i(\beta_i, q_i)}{\partial q_i} = c \forall i$. In an interior solution, the FOC in Equation 6 gives

$$E\left[\frac{\partial q_i^*\left(\beta_i\right)}{\partial \beta_i}\left(\hat{P}\left(q_i^*\left(\beta_i\right), q_{-i}^*\right) - c\right) + q_i^*\frac{\partial \hat{P}\left(q_i^*\left(\beta_i\right), q_{-i}^*\right)}{\partial \beta_i}\right] = E\left[\frac{\partial M_i\left(\beta_i\right)}{\partial \beta_i}\right]. \tag{7}$$

As shown in the proof of Proposition 1, the LHS goes to 0 when $\beta \to \infty$.

At $\beta = 0$, the LHS becomes

$$E\left[\frac{-2D'\left(\hat{P}\right)(n-1)\left(\hat{P}-c\right)^2}{(1+n)D'\left(\hat{P}\right) + nD''\left(\hat{P}\right)\left(\hat{P}-c\right)}\right] < 0.$$

Given that the LHS is continuous in β, for $E\left[\frac{\partial M_i(\beta_i)}{\partial \beta_i}\right] \in \left(E\left[\frac{-2D'\left(\hat{P}\right)(n-1)\left(\hat{P}-c\right)^2}{\left((1+n)D'\left(\hat{P}\right)+nD''\left(\hat{P}\right)\left(\hat{P}-c\right)\right)}\right], 0\right)$, there exists a symmetric positive profit equilibrium such that $\beta^* \in (0,\infty)$, and the FOC at β^* is satisfied for all firm i. ■

3.3 Existence of positive profit equilibrium

Proposition 1 establishes that Bertrand behavior is supported in equilibrium in the two-stage game with constant marginal costs. This replicates the results in Grossman (1981), Robson (1981), Klemperer-Meyer (1989), and Grant and Quiggin (1996). The question remains whether this game structure supports any other type of oligopolistic behaviour.

The purpose of this section is to demonstrate, by example, the existence of positive profit equilibria. To this end, we will focus on the simple case where n firms with zero marginal production cost compete in an industry with linear inverse demand curve, subject to an additive demand shock. We assume that the monitoring cost, $M_i = \frac{\theta}{\beta_i}$, where θ is a common parameter for all firm.

$$P = 1 - b\sum_{i=1}^{n} q_i + \varepsilon. \tag{8}$$

With these simplifications, we derive the closed form solution to the second stage game and characterise the first stage β choices. We have

Proposition 3 *For $0 < \theta < \frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2 n^3 (n-1)}$, there exists, in addition to the Bertrand equilibrium, a unique symmetric equilibrium $\beta = \beta(\theta) \in (0,\infty)$, with second stage equilibrium*

$$q_i^* = \frac{(1+\varepsilon)\left(\frac{1}{b} + (n-1)\beta\right)}{n+1+b(n-1)n\beta}, Q = \frac{(1+\varepsilon)\left(\frac{n}{b} + n(n-1)\beta\right)}{n+1+b(n-1)n\beta}, P = \frac{(1+\varepsilon)}{n+1+b(n-1)n\beta}.$$

In equilibrium,

$$\pi^* = \frac{\left(1+\sigma_\varepsilon^2\right)\left(\frac{1}{b} + (n-1)\beta\right)}{\left(n+1+b(n-1)n\beta\right)^2} - \frac{\theta}{\beta}.$$

Given the FOC condition (Equation 12)on β and the implied relationship between β and θ, $\pi^* > 0$.

Proof. See Appendix. ■

The polar cases of the second-stage solution span the range of (weakly positively sloped) supply-schedule equilbria. As $\beta \to \infty$, P and $\pi_i \to 0$, $q_i \to \frac{1+\varepsilon}{bn}$, $Q \to \frac{1+\varepsilon}{b}$. This is the Bertrand solution. As $\beta \to 0$, $P \to \frac{1+\varepsilon}{n+1}$, $q_i \to \frac{1+\varepsilon}{b(n+1)}$, $Q \to \frac{n(1+\varepsilon)}{b(n+1)}$ and the gross profit $\pi_i + \frac{\theta}{\beta} \to \frac{1+\sigma_\varepsilon^2}{b(n+1)^2}$. This is the Cournot solution.

To show that these polar cases arise as equilibria for the full game, we require

Proposition 4 *As $\theta \to \frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2 n^3 (n-1)}$ from below, $\beta \to \infty$, and the equilibrium outcome converges to the Bertrand solution. As $\theta \to 0$, $\beta \to 0$ and the equilibrium outcome converges to the Cournot solution. More generally, For $0 < \theta < 2\frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2 n^3 (n-1)}$, as θ increases, the interior symmetric equilibrium value β increases.*

In general, in oligopoly problems, consumer surplus and producer surplus move in opposite directions. However, welfare analysis in this model is complicated by the cost $\theta\gamma$ incurred by firms in the first stage. Expected consumer surplus in a interior symmetric equilibrium is

$$ECS = E\left[\frac{1}{2}(1+\varepsilon-P)Q\right] = \frac{b\left(1+\sigma_\varepsilon^2\right)}{2}\left(\frac{\left(\frac{n}{b}+n(n-1)\beta\right)}{n+1+b(n-1)n\beta}\right)^2.$$

The expected total surplus is defined to be the sum of firms' expected profits and expected consumer surplus.Thus

$$ETS = n\left(\frac{\left(1+\sigma_\varepsilon^2\right)\left(\frac{1}{b}+(n-1)\beta\right)}{(n+1+b(n-1)n\beta)^2}-\frac{\theta}{\beta}\right)+\frac{b\left(1+\sigma_\varepsilon^2\right)}{2}\left(\frac{\left(\frac{n}{b}+n(n-1)\beta\right)}{n+1+b(n-1)n\beta}\right)^2.$$

We now present some numerical examples.

Example 1 *For $b=1$, $n=2$, $\theta=0.03$, $\sigma_\varepsilon^2=0.01$ the symmetric equilibrlim gives $\beta\approx 0.952$, $Eq\approx 0.398$, $EP\approx 0.20, E\pi\approx 5.047\times 10^{-2}$, and $ETS\approx 0.421$. For $b=1$, $n=2$, $\theta=0.1$ and , $\sigma_\varepsilon^2=0.01$, the symmetric equilibrium gives $\beta\approx 3.050$, $Eq\approx 0.445$, $EP\approx 0.11$, $E\pi\approx 1.661\times 10^{-2}$, and $ETS=0.433$.*

We futher explore the relationship between the optimal symmetric β, the firm's profit, the market price, the the total surplus and θ in the diagram below. The diagram below is plotted with $\sigma_\varepsilon^2=0.01$.

3.4 Stability and strategic complementarity

In the presence of multiple equilibria, questions of stability arise. We have

Proposition 5 *When there exists another symmetric equilibrium other than Bertrand, this equilibrium with positive profit is a stable equilibrium.*

Proof. See Appendix. ■

Remark 1 *Firms' second stage quantity choices are strategic substitutes.*

Remark 2 *Firms'first stage γ choices are strategic complements for when firms choose close enough γ.*

For any given $j\neq i$:

$$\frac{\partial^2 E\pi_i}{\partial\beta_i\partial\beta_j} = -\frac{2\left(1+\sigma_\varepsilon^2\right)b(n-1)}{\left(1+n+b(n-1)\sum_{i=1}^N\beta_i\right)^3}+\frac{6\left(1+\sigma_\varepsilon^2\right)b\left(1+b\sum_{j\neq i}^N\beta_j\right)(n-1)^2}{\left(1+n+b(n-1)\sum_{i=1}^N\beta_i\right)^4} \tag{9}$$

$\frac{\partial^2 E\pi_i}{\partial\beta_i\partial\beta_j}>0$ *for close enough β_i and β_j and $n>1$.*

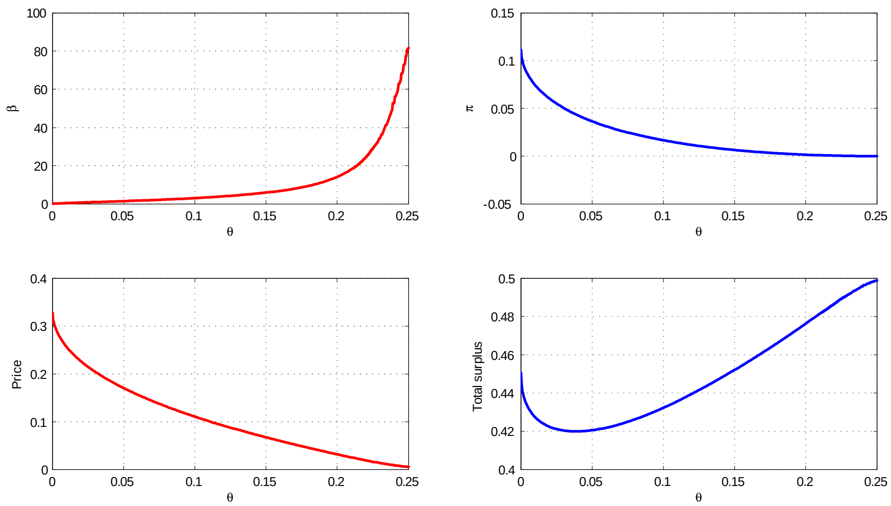

Figure 1: Symmetric Equilibrium β, π, EP, ETS. Plotted with $b = 1$, $n = 2$, $\sigma_\varepsilon^2 = 0.01$.

4 Implications

The existence of multiple equilibria for a non-cooperative oligopoly game, one of which is the Bertrand equilibrium, has significant implications for competition policy.

4.1 More competitors or more competition

An important question in competition policy is whether it is more important to increase the number of competitors in an industry or to promote more competitive behavior. An increase in the number of competitors may be promoted through structural remedies such as divestiture, while interventions aimed at promoting more competitive behavior may be described as behavioral remedies. Using this terminology, the European Commission, for example, states a preference for behavioral remedies. See, for example, the discussion in van Koten and Ortmann (2013).

A large literature has developed from the proposition of Allaz and Vila (1993) that, in the presence of forward markets, firms engaged in Cournot competition may increase total output, thereby reducing prices and profits and enhancing consumer welfare. Van Koten and Ortmann (2013) summarize this literature and provide experimental evidence supporting the claims of Allaz and Vila for the case of convex production costs and competition in quantities.

Menezes and Quiggin (2012) consider the case where firms are constrained to compete in supply schedules with exogenously determined slopes, and provide conditions under which the benefits of a flatter (more elastic) supply schedule will outweigh those of a larger number of firms.

The present paper takes this point further by making the slope of the supply schedule endogenous. The existence of two equilibria, one competitive (or Bertrand) and one oligopolistic implies that any behavioral remedy that ensures the selection of the competitive equilibrium (for example, by allowing firms to revise prices downwards, but not upwards, after observing their competitors' initial offers) will yield benefits greater than those of a structural remedy that increases the number of firms in the oligopolistic equilibrium.

4.2 The equilibrium locus

The equilibrium locus (Busse (2012), Menezes and Quiggin (2013a)) consists of the set of pairs $\{(P(\varepsilon), Q(\varepsilon)) : \varepsilon \in \mathbb{R}\}$ where

$$Q(\varepsilon) = D(P(\varepsilon), \varepsilon) = \sum_i \widetilde{q}_i(\varepsilon)$$

The key idea is that, in equilibrium, each firm acts as a monopolist, choosing the optimal price-quantity combination from a residual demand curve determined by a given observation of market demand and the (equilibrium) strategies of the other firms. Aggregating across firms, we can determine an equilibrium relationship between price and quantity for any particular realization of the demand shock.

For the symmetric zero cost case, we have as above

$$q_1 = q_2 = \frac{(1+\varepsilon)(\beta+1)}{2\beta+3}$$

$$P = \frac{1+\varepsilon}{2\beta+3}$$

or

$$\begin{aligned} q_1 &= q_2 = (1+\beta) P \\ Q &= 2(1+\beta) P. \end{aligned}$$

This provides a useful way to characterize equilibrium behavior as a function of the competitiveness parameter γ.

4.3 Mavericks

Maverick firms play a prominent role in the examination of coordinated effects in merger analysis. In particular, competition authorities are often concerned with the elevated risk of tacit collusion if a merger eliminates a maverick firm. The U.S. Merger guidelines, for example, defines a maverick as a firm that constitutes an unusually disruptive force in the market place. This is not particularly illuminating definition, which is not helped by the lack of a substantive body of theory and empirical evidence identifying maverick firms.[9]

This paper provides a possible way of modelling a maverick firm by assuming that it has a high first-stage cost θ. That is, for reasons of organizational structure, or firm culture, the firm finds it difficult to commit to a strategy involving a low value of β, which entails a stable market share. The presence of such a firm will reduce the profitability of the non-Bertrand equilibrium for all others, and may therefore reduce the likelihood that this equilibrium will emerge.

4.4 Mergers

Mergers are common in oligopolistic industries, and would be more common if it were not for anti-trust policies that prohibit anti-competitive mergers. Yet, ever since the analysis of losses from horizontal merger put forward by Salant, Switzer and Reynolds (1983), it has been known that a merger between two firms in a Cournot–Nash oligopoly with constant returns to scale will reduce the profitability of both firms. Only under stringent conditions (four out of five firms merging) will such mergers be profitable.

On this basis, Salant, Switzer and Reynolds conclude that when mergers are endogenous, socially injurious mergers (those that do not give rise to scale economies sufficient to offset the reduction in competition) will not take place and therefore 'need not be guarded against'. Indeed, since some socially beneficial mergers will not take place, the policy problem is one of too few mergers rather than too many.

Noting the counterintuitive nature of their results, Salant, Switzer and Reynolds consider and reject the idea that the solution is to replace the Cournot–Nash solution concept. They observe their result is robust to various modifications of the simple Nash equilibrium.

In all the cases considered by Salant, Switzer and Reynolds, the equilibrium is unique. By contrast the present model is characterized by multiple equilibria.

[9] Breunig and Menezes (2008) and Engle and Ocknefls (2014) provide a review of the existing literature on mavericks.

There are several reasons why we might expect a reduction in the number of firms to increase the likelihood that the positive-profit equilibrium would replace Bertrand.

First, as discussed in more detail below, if a single firm credibly commits to Bertrand behavior, then only the Bertrand equilibrium is feasible.

Second, the smaller the number of firms, the easier it is for firms to communicate a commitment to an upward sloping supply schedule, and therefore the more sustainable is the positive profit equilibrium.

Menezes and Quiggin (2013b) provide conditions under which a merger leading to a higher equilibrium value of γ will be profitable. In particular, a merger will always be profitable in the limiting case where the post-merger equilibrium is a Cournot duopoly.

5 Appendix A: Monitoring game

We assume that the owner of firm i can observe the quantity sold q_i and the strategic choice of the of the other firm γ_j but (in the absence of costly monitoring) not the demand shock ε or the price P. Hence, the manager has an incentive to report a low price $\check{P}$ and retain the surplus $\left(\check{P} - \check{P}\right) q_i$.

Assume the owner of the firm prescribes a supply curve with slope β. The truthful equilibrium for given ε, is as described here $\left(\widetilde{P}\left(\varepsilon\right), q_i\left(\widetilde{P}, \beta, \varepsilon\right), \varepsilon; \beta_i, \beta_j\right)$.

Hence, the equilibrium locus for firm i (Busse (2012), Menezes and Quiggin (2013a)) consists of the set of pairs $\left\{\widetilde{P}\left(\varepsilon, \beta_i, \beta_j\right), q_i\left(\widetilde{P}, \beta, \varepsilon\right) : \varepsilon \in \mathbb{R}\right\}$. The equilibrium locus is *not* the second-stage equilibrium supply schedule for some given ε. Rather it is the locus of price-quantity pairs traced out as ε varies. In particular, for the case of Cournot, the equilibrium locus is upward-sloping rather than vertical, reflecting the fact that as demand increases so do the equilibrium prices and quantities.

In the absence of monitoring, the agent can report $\check{\varepsilon}\left(\varepsilon\right) < \varepsilon$, and choose a point $\left(\check{P}, \check{q}_i\right)$on the residual demand curve with

$$\begin{aligned} q_i &= q_i\left(\widetilde{P}\left(\check{\varepsilon}, \beta_i, \beta_j\right), \beta, \check{\varepsilon}\right) \\ \check{P}\left(\check{\varepsilon}\right) &< \widetilde{P}\left(\varepsilon\right) \end{aligned}$$

The market-clearing price P^* in the presence of cheating will depend on the response of the other player,but will satisfy $P^* \geq \check{P}$.

The agent chooses $\check{\varepsilon}\left(\varepsilon\right)$ to maximize the illicit surplus

$$S\left(\varepsilon, \check{\varepsilon}, \gamma\right) = \left(P^* - \check{P}\left(\varepsilon\right)\right) \check{q}_i\left(\widetilde{P}\left(\check{\varepsilon}\right), \beta_i, \beta_j, \check{\varepsilon}\right)$$

Thus, in the absence of monitoring, the agent receives an expected payoff from cheating

$$\bar{S}\left(\gamma_i, \gamma_j\right) = E_\varepsilon S\left(\varepsilon, \check{\varepsilon}, \beta_i, \beta_j\right)$$

We observe

Lemma 1 *$\bar{S}\left(\beta_i, \beta_j\right)$ is decreasing in β_i*

Suppose the owner can monitor the agent with probability π at a cost $\theta(\pi)$ and can levy a penalty Π if the agent is caught cheating. The monitoring cost increases with the probability of investigation, π. Then incentive compatibility requires $\pi\Pi \geq \bar{S}\left(\beta_i, \beta_j\right)$. Assuming equality, the monitoring cost is $\theta_i\left(\frac{\bar{S}\left(\beta_i,\beta_j\right)}{\Pi}\right)$ which is decreasing in β_i.

6 Appendix B

Proof. Proof of Proposition 3

Assuming zero production costs, the first-order condition 4 becomes ■

$$\left(\frac{1}{b} + \sum_{j\neq i}\beta_j\right)\left(1 - b\sum_{j=1}^{n} q_j + \varepsilon\right) - q_i = 0 \tag{10}$$

yielding the best response

$$q_i = \frac{\left(\frac{1}{b} + \sum_{j\neq i}\beta_j\right)\left(1 - b\sum_{j\neq i}^{n} q_j + \varepsilon\right)}{\left(2 + b\sum_{j\neq i}\beta_j\right)}.$$

From FOC **??**, we have

$$q_i = \left(\frac{1}{b} + \sum_{j\neq i}^{N}\beta_j\right)\left(1 + \varepsilon - bQ\right).$$

Summing up the n, $n = 1, ..., n$, FOCs we have

$$Q = \left(\frac{n}{b} + (n-1)\sum_{i=1}^{N}\beta_i\right)\left(1 - bQ + \varepsilon\right).$$

This gives

$$Q = \frac{(1+\varepsilon)\left(\frac{n}{b} + (n-1)\sum_{i=1}^{N}\beta_i\right)}{\left(1 + n + b\,(n-1)\sum_{i=1}^{N}\beta_i\right)}$$

and

$$P = \frac{(1+\varepsilon)}{n+1+b\,(n-1)\sum_{i=1}^{N}\beta_i}.$$

$$q_i^* = \frac{(1+\varepsilon)\left(\frac{1}{b}+\sum_{j\neq i}^{N}\beta_j\right)}{1+n+b\,(n-1)\sum_{i=1}^{N}\beta_i}.$$

Remark 3 *In a symmetric equilibrium, we have we have* $\beta_1 = \beta_2 = ... = \beta_n = \beta$ *and*

$$q_i^* = \frac{(1+\varepsilon)\left(\frac{1}{b}+(n-1)\,\beta\right)}{1+n+b\,(n-1)\,n\beta}$$

$$Q = \frac{(1+\varepsilon)\left(\frac{n}{b}+n\,(n-1)\,\beta\right)}{1+n+b\,(n-1)\,n\beta}$$

$$P = \frac{(1+\varepsilon)}{1+n+b\,(n-1)\,n\beta}.$$

We now consider the choice of β_i in the first stage. Given the second stage outcome, firm i soloves

$$\max_{\beta_i} E\left[\pi_i\left(\beta_i,\boldsymbol{\beta}_{-i},\varepsilon\right)\right] = \left(\frac{(1+\varepsilon)\left(\frac{1}{b}+\sum_{j\neq i}^{N}\beta_j\right)}{1+n+b\,(n-1)\sum_{i=1}^{N}\beta_i}\right)\left(\frac{(1+\varepsilon)}{n+1+b\,(n-1)\sum_{i=1}^{N}\beta_i}\right)-\frac{\theta}{\beta_i}.$$

The FOC gives

$$\frac{\partial E\left[\pi_i\left(\beta_i,\boldsymbol{\beta}_{-i},\varepsilon\right)\right]}{\partial\beta_i} = -\frac{2\left(1+\sigma_\varepsilon^2\right)\left(1+b\sum_{j\neq i}^{N}\beta_j\right)(n-1)}{\left(1+n+b\,(n-1)\sum_{i=1}^{N}\beta_i\right)^3}+\frac{\theta}{\beta_i^2}\leq 0 \qquad (11)$$

with equality for an interior solution.

In a symmetric interior solution

$$\theta = \frac{2\left(1+\sigma_\varepsilon^2\right)(1+b\,(n-1)\,\beta)\,(n-1)\,\beta^2}{(1+n+b\,(n-1)\,n\beta)^3}. \qquad (12)$$

The RHS is equal to 0 if $\beta = 0$ and is equal to $\frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2n^3(n-1)}$ as $\beta\to\infty$. Furthermore, the RHS increases in β for $n\geq 1$. Given this monotonicity, for each given $\theta\in\left(0,\frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2n^3(n-1)}\right)$, there is a unique symmetric β solution.

Remark 4 *The second order condition is always satisfied for symmetric equilibrium with finite positive* β*.*

Substititue the θ value for an interior β solution from Equation 11,

$$\frac{\partial^2 E\pi_i\left[\pi_i\left(\beta_i,\boldsymbol{\beta}_{-i}^*\right)\right]}{\partial\beta_i^2}=2\left(1+\sigma_\varepsilon^2\right)(n-1)\left(1+b\sum_{j\neq i}^{N}\beta_j\right)\left(\frac{b(n-1)\beta_i-2(1+n)-2b(n-1)\sum_{j\neq i}^{N}\beta_i}{\beta_i\left(1+n+b(n-1)\sum_{i=1}^{N}\beta_i\right)^4}\right). \tag{13}$$

$\frac{\partial^2 E\pi_i\left[\pi_i\left(\gamma_i,\gamma_{-i},\varepsilon\right)\right]}{\partial\beta_i^2}<0$ *for close enough β_i and β_{-i}.*

Proof. Proof of Proposition 5

This proof follows Dixit (1986), Martin (2002, p. 30), and Seade (1980), adapted to our setup. Let $\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)$ be an equilibrium of the first stage game. Suppose that if firms choose $\left(\beta_i,\boldsymbol{\beta}_{-i}^*\right)$ in the neighbourhood of $\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)$, firm i changes β_i over time at a rate proportional to its marginal profitability,

$$\frac{d\beta_i}{dt}=k_i\frac{\partial E\pi_i\left(\beta_i,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i}, \tag{14}$$

for $k_i>0$. That is, if it is profitable to increase β_i, firm i increases β_i at a rate which is proportional to marginal profitability.

Take a local linear approximation to Equation 14 around $\left(\beta_i^*,\beta_{-i}^*\right)$:

$$\frac{d\beta_i}{dt}=k_i\frac{\partial E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i}+k_i\left[\frac{\partial^2 E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i^2}\left(\beta_i-\beta_i^*\right)+\sum_{j\neq i}\frac{\partial^2 E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i\partial\beta_j}\left(\beta_{-i}-\beta_j^*\right)\right].$$

At an interior $\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)$, $\frac{\partial E\pi_i\left(\beta_i,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i}=0$. Repeat the analysis for each firm j, $j\neq i$, the system of adjustment equations can be written as

$$\begin{pmatrix}\frac{d\beta_1}{dt}\\ \frac{d\beta_2}{dt}\\ \frac{d\beta_n}{dt}\end{pmatrix}=\begin{pmatrix}k_1 & 0...0 & 0\\ 0 & k_2 & 0\\ 0 & 0...0 & k_n\end{pmatrix}\begin{pmatrix}\frac{\partial^2 E\pi_1\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_1^2} & \frac{\partial^2 E\pi_1\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_1\partial\beta_2}... & \frac{\partial^2 E\pi_1\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_1\partial\beta_n}\\ \frac{\partial^2 E\pi_2\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_2\partial\beta_1} & \frac{\partial^2 E\pi_2\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_2^2}... & \frac{\partial^2 E\pi_2\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_2\partial\beta_n}\\ \frac{\partial^2 E\pi_n\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_n\partial\beta_1} & \frac{\partial^2 E\pi_n\left(\gamma_i^*,\boldsymbol{\gamma}_{-i}^*\right)}{\partial\beta_n\partial\beta_2}... & \frac{\partial^2 E\pi_n\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_n^2}\end{pmatrix}\begin{pmatrix}\beta_1-\beta_1^*\\ \beta_2-\beta_2^*\\ \beta_n-\beta_n^*\end{pmatrix}.$$

Note that the matrix in the LHS is a $n\times 1$ matrix. The RHS are $n\times n$, $n\times n$, and $n\times 1$ matrices respectively. One necessary condition for stability is for the Jacobian matrix to have a negative trace. This is true given that the second order condition is satisfied in equilibrium. Another necessary condition is that the determinant of the Jacobian matrix should have the same sign as $(-1)^n$. The determinant of the matrix can be computed as (see Dixit, 1986, and Seade, 1980):

$$\prod_{i=1,j\neq 1}^{n}\left(\frac{\partial^2 E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i^2}-\frac{\partial^2 E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i\partial\beta_j}\right)\left(1+\sum_{i=1,j\neq i}^{n}\frac{\frac{\partial^2 E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i\partial\beta_j}}{\frac{\partial^2 E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i^2}-\frac{\partial^2 E\pi_i\left(\beta_i^*,\boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i\partial\beta_j}}\right),$$

the necessary condition is thus

$$(-1)^n \left[\prod_{i=1, j\neq 1}^{n} \left(\frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i^2} - \frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i \partial \beta_j} \right) \left(1 + \sum_{i=1, j\neq i}^{n} \frac{\frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i \partial \beta_j}}{\frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i^2} - \frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i \partial \beta_j}} \right) \right] > 0. \tag{15}$$

The simplest set of sufficient condition is obtained by requiring diagonal dominance in the matrix:

$$\left| \frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i^2} \right| > (n-1) \left| \frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i \partial \beta_j} \right|. \tag{16}$$

With symmetry, Equations 13 and 9 become

$$\frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i^2} = 2\left(1+\sigma_\varepsilon^2\right)(n-1)\left(1+b\left(n-1\right)\beta\right)\left(\frac{b\left(n-1\right)\beta - 2\left(1+n\right) - 2b\left(n-1\right)^2\beta}{\beta\left(1+n+b\left(n-1\right)n\beta\right)^4} \right).$$

$$\frac{\partial^2 E\pi_i \left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i \partial \beta_j} = -\frac{2\left(1+\sigma_\varepsilon^2\right)b\left(n-1\right)}{\left(1+n+b\left(n-1\right)n\beta\right)^3} + \frac{6\left(1+\sigma_\varepsilon^2\right)b\left(1+b\left(n-1\right)\beta\right)\left(n-1\right)^2}{\left(1+n+b\left(n-1\right)n\beta\right)^4}.$$

Condition 16 is satisfied for $n > 1$. ■

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