Endogenous timing game of mixed duopoly with partial foreign ownership; increasing marginal costs*

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Abstract

This study examines whether the results derived from previous researches hold under the endogenous timing game of mixed duopoly with partial foreign ownership and increasing marginal costs. This study shows the following results. When the difference of a production technology between the public and the private firms is large, public leadership is always a risk-dominant equilibrium. When the difference of the production technology between them is small, for (not) small foreign ownership rate, private-leadership (public-leadership) is a risk-dominant equilibrium. From the viewpoint of social welfare, for the large (small) foreign ownership rate, private leadership (public leadership) is socially preferable. Wherein, in the middle rate of foreign ownership, although private leadership is socially preferable, public leadership becomes a risk-dominant equilibrium.

Keywords: partial foreign ownership, increasing marginal costs, risk-dominant equilibrium

1 Introduction

Even now, many countries including Japan have various mixed duopoly (oligopoly) markets. For example, in Japanese local cities, both a public and private firms serve transportation services. A natural gas industry is also good example. Among others, the industries of railway, electricity, and postal services are also mixed duopoly (oligopoly) markets.

Given various examples like the above, many studies discussed various problems of the mixed duopoly markets (e.g., Defraja and Delbono 1989, 1990). Among these studies, many studies tacitly assumed simultaneous game. On the other hand, there exist some studies to clearly assume Stackelberg-competition (e.g., Chang 2005).

Here, we have an important question; which game, simultaneous game or sequential game, is rational? Because the difference of the type of the game brings the difference of the firm's profit and that of social welfare, we should examine this important problem. With regard to this problem, Pal (1998) demonstrates existence of multiple equilibria assuming the constant marginal costs for the public firm. That is, both private leadership and public leadership are equilibria. In addition, assuming the increasing marginal costs, Tomaru and Kiyono (2010) conclude the same results as Pal (1998).

After their studies, some studies tried to select the equilibrium by using an equilibrium concept "risk-dominant equilibrium" formulated by Harsanyi and Selten (1988). For example, Matsumura and Ogawa (2010) show the same result obtained in Pal (1998) and derive the risk-dominant equilibrium. As a result, they conclude that private leadership is the risk-dominant equilibrium. Matsumura and Ogawa (2017) introduce (i) product differentiation and (ii) foreign private firm, and demonstrate that when the private firm is a foreign ownership firm, public leadership is the risk-dominant equilibrium[1].

In the above mentioned two studies, the constant marginal costs were assumed. However, like Tomaru and Kiyono (2010), both the public and the private firms do not always have the constant marginal costs. In addition, we are not sure that the public firm's production technology is not inferior to the private firm's one. However, no study examines whether the results obtained in Matsumura and Ogawa (2010,

[1]When the private firm is a domestic firm, the risk-dominant equilibrium changes depending on the degree of product differentiation.

2017) still hold under the increasing marginal costs. Additionally, although Matsumura and Ogawa (2017) mention that the risk-dominant equilibrium changes at a certain value of foreign ownership rate, the characteristics of its critical value is not discussed. In addition, we want to know whether the risk-dominant equilibrium is socially preferable equilibrium.

Therefore, this study assumes both the public and private firms have the increasing marginal costs. At the same time, this study assumes that the public firm's size of marginal costs is larger than the private firm's one. In addition, this study introduces a partial foreign ownership rate with regard to the private firm. When the private firm is a fully domestic firm, its profit is included into the domestic social welfare. When the private firm is a fully foreign firm, its profit is not included into the domestic social welfare. When the private firm is partially owned by foreigners, only a part of its profit is included into the domestic social welfare. Considering the above situation, we first examine whether the results obtained in Matsumura and Ogawa (2010, 1017) are robust. In the following, we analyze whether the risk-dominant equilibrium is socially preferable.

Here, we mention other studies to address the endogenous timing game of mixed duopoly market. Pal (1998), Matsumura and Ogawa (2010, 2017), Tomaru and Kiyono (2010) adopts the observal delay game formulated by Hamilton and Slutsky (1990), wherein, Matsumura (2003) uses a two-production-period model formulated by Saloner (1987) and concludes that only private leadership is an equilibrium. Tomaru and Saito (2010) introduce a subsidy into Tomaru and Kiyono (2010) and demonstrate that private leadership cannot be an equilibrium. Lu (2006) introduces both the domestic private firms and the foreign private firms simultaneously and argues that the public firm never chooses quantities simultaneously with the domestic private firms[2].

This study, using a model mentioned before, demonstrates following results. When the difference of the production technology between the public and the private firms is large, public leadership is always the risk-dominant equilibrium, which result is different from Matsumura and Ogawa (2010). Because the public firm wants to avoid excess production, it always prefers a leader to a follower. When the difference of the production technology between them is small, for (not) the small foreign ownership rate, private

[2]Among others, Choi and Lu (2009) and Ye (2016) also analyze the endogenous timing game given various conditions.

leadership (public leadership) is the risk-dominant equilibrium, which results almost correspond with Matsumura (2010, 2017). From the viewpoint of social welfare, for the large (small) foreign ownership rate, private leadership (public leadership) is socially preferable. Wherein, in the middle rate of foreign ownership, although private leadership is socially preferable, public leadership becomes the risk-dominant equilibrium.

This study is organized as follows. The following section 2 setups the model. Section 3 analyzes the simultaneous game, section 4 analyzes public leadership game, and section 5 analyzes private leadership game. Based on the results obtained in sections 3, 4, and 5, section 6 compares the equilibrium quantity with each game. Section 7 derives the equilibrium of the timing game. In section 8, we analyze social welfare. Section 9 concludes this study and mention future research.

2 Model

We consider an economy with two firms (firm 0 and firm 1). We assume that firm 0 is a public firm and firm 1 is a private firm. Both firms produce a homogeneous good and supply it to consumers. Therefore, this study considers the homogeneous Cournot competition model. In the following, we term the number of goods supplied by firm i $(= 0, 1)$ as q_i. The consumers have a following demand function.

$$p = a - (q_0 + q_1) \tag{1}$$

Therefore, the consumer surplus is $CS = \frac{(q_0+q_1)^2}{2}$. Following Matsumura and Shimizu (2010), this study assumes that the production technology of the public firm is inferior to that of the private firm[3]. Therefore, the production cost of the public firm is higher than that of the private firm. We assume that the cost function of each firm is

$$C_i(q_i) = \frac{c_i}{2} q_i^2. \tag{2}$$

[3]Even if this assumption is relaxed, our main results can hold.

Here, $c_0 = c(\geq 1)$ and $c_1 = 1$. Therefore, each firm's profit function is

$$\pi_i = pq_i - \frac{c_i}{2}q_i^2. \tag{3}$$

This study considers that the private firm is partially owned by the foreigners. We term the ratio of foreign ownership of the private firm as $s(\in [0,1])$. From this assumption, domestic social welfare (hereafter, social welfare) includes only $(1-s) \times 100\%$ of the firm 1's profit. Among others, social welfare includes consumer surplus and the firm 0's profit. Therefore, we can express social welfare as follows.

$$SW = CS + \pi_0 + (1-s)\pi_1. \tag{4}$$

In the following, we call s as "a foreign-ownership rate."

Firm 0 decides its quantity of production to maximize social welfare. Firm 1 decides its quantity of production to maximize its profit.

In the following, we derive the equilibrium outcomes in the following three cases.

(i) Both firms simultaneously decide the quantity of production.

(ii) Firm 0 first decides the quantity of production (or, acts as a leader), and then firm 1 decides it (or, acts as a follower).

(iii) Firm 1 first decides the quantity of production (or, acts as a leader), and then firm 0 decides it (or, acts as a follower).

The game of this study runs as follows. In the first stage, firm $i(= 0, 1)$ simultaneously chooses whether to act as a leader or a follower. Then, the basic game is played using simultaneous play if both firms choose the same period (i.e., simultaneous game), and sequential play otherwise (i.e., private (public) leadership). In other words, We adopt the observable delay game formulated by Hamilton and Slutsky (1990).

3 Simultaneous game

This section analyzes the simultaneous game. First, we derive each reaction function. When solving each maximization problem, we obtain

$$q_0 = \frac{a-(1-s)q_1}{1+c}, \tag{5}$$

$$q_1 = \frac{a-q_0}{3}. \tag{6}$$

As a result, the following equilibrium quantity is obtained. Here, we term this equilibrium quantity as q_i^{sim}.

$$q_0^{sim} = \frac{a(2+s)}{5+9c+s3} \tag{7}$$

$$q_1^{sim} = \frac{3ac}{5+9c+3s} \tag{8}$$

Then, the equilibrium profit of firm 1 is

$$\pi_1^{sim} = \frac{27a^2c^2}{2(5+9c+3s)^2}, \tag{9}$$

and the equilibrium social welfare is

$$SW^{sim} = \frac{a^2(5+3s+c(4+3s))}{2(5+9c+3s)^2}. \tag{10}$$

When comparing q_0^{sim} and q_1^{sim}, we obtain Lemma 1.

Lemma 1 *When c is smaller than 2 (larger than 3), $q_0^{sim} > (<) q_1^{sim}$ always holds. When $2 \leq c \leq 3$, $q_0^{sim} \geq (<) q_1^{sim}$ holds for $s \geq (<) -2+c$.*

Firm 0 decides the quantity of production by considering social welfare; firm 1 decides it by considering only its profit. Therefore, firm 0 has an incentive to produce more. On the other hand, because of the inefficient production technology of firm 0, it has also an incentive to produce less. When c is smaller

than 2 (larger than 3), the former incentive is larger (smaller) than the latter incentive. Therefore, $q_0^{sim} > (<) q_1^{sim}$ always holds. Suppose that $2 < c < 3$. For large s, because social welfare does not include almost the profit of firm 1, firm 0 considers to let the firm 1's revenues be low. Therefore, firm 0 produces more and as a result, firm 1 produces less due to the strategic substitute. For small s, because social welfare includes almost the profit of firm 1, firm 0 considers to let firm 1 produces more in order to save the production costs. Therefore, firm 0 produces less, and as a result firm 1 produces more.

4 Public leadership game

This section analyzes the public leadership game. The reaction function of firm 1 is expressed by eq. (6). When substituting eq. (6) into the firm 0's object function (i.e., social welfare function) and solving the social welfare maximization problem, we obtain the following equilibrium quantity produced by firm 0. Here, we expresses the equilibrium quantity of firm 0 as q_0^{pul}.

$$q_0^{pul} = \frac{a(5+3s)}{5+9c+3s} \tag{11}$$

Therefore, the equilibrium quantity produced by firm 1, which is expressed by q_1^{pul}, becomes

$$q_1^{pul} = \frac{3ac}{5+9c+3s}. \tag{12}$$

Then, the equilibrium profit of firm 1 is

$$\pi_1^{pul} = \frac{27a^2c^2}{2(5+9c+3s)^2}, \tag{13}$$

and the equilibrium social welfare is

$$SW^{pul} = \frac{a^2(5+3s+c(4-3s))}{2(5+9c+3s))}. \tag{14}$$

When comparing q_0^{pul} and q_1^{pul}, we obtain Lemma 2.

Lemma 2 *When c is smaller than $\frac{5}{3}$ (larger than $\frac{8}{3}$), $q_0^{pul} > (<) q_1^{pul}$ always holds. When $\frac{5}{3} \leq c \leq \frac{8}{3}$, $q_0^{sim} \geq (<) q_1^{sim}$ holds for $s \geq (<) \frac{-5+3c}{3}$.*

The mechanism of Lemma 2 is very similar to that of Lemma 1. Therefore, we omit the detailed discussion here.

5 Private leadership game

This section analyzes the private leadership game. The reaction function of firm 0 is expressed by eq. (5). When substituting eq. (5) into the firm 1's profit function and solving the profit maximization problem, we obtain the following equilibrium quantity produced by firm 1. Here, we expresses the equilibrium quantity of firm 0 as q_0^{prl}.

$$q_1^{prl} = \frac{ac}{1+3c+2s} \tag{15}$$

Therefore, the equilibrium quantity produced by firm 0, which is expressed by q_0^{pul}, becomes

$$q_0^{prl} = \frac{3-2t+c(3-s)}{(1+c)(1+3c+2s)}. \tag{16}$$

Then, the equilibrium profit of firm 1 is

$$\pi_1^{prl} = \frac{a^2c^2}{2(1+c)(1+3c+2s)}, \tag{17}$$

and the equilibrium social welfare is

$$SW^{prl} = \frac{a^2\{c^3(4-3s)+c^2(10+3s-3s^2)+6c(1+2s)+(1+2s)^2\}}{2(1+c)(1+3c+2s)^2}. \tag{18}$$

When comparing q_0^{prl} and q_1^{prl}, we obtain Lemma 3.

Lemma 3 *When c is smaller than $\frac{1+\sqrt{5}}{2}$ (larger than 3), $q_0^{prl} > (<) q_1^{prl}$ always holds. When $\frac{1+\sqrt{5}}{2} \leq c \leq 3$, $q_0^{prl} \geq (<) q_1^{prl}$ holds for $s \geq (<) \frac{c^2-c-1}{2+c}$.*

The mechanism of Lemma 3 is also very similar to that of Lemma 1. Therefore, we omit the detailed discussion here.

6 Comparison of equilibrium quantity

At first, we compare the equilibrium quantity produced by firm 0 with three cases. As a result, we can obtain following Lemma 4.

Lemma 4 *$q_0^{sim} > q_0^{prl} > q_0^{pul}$ holds.*

In the following, we compare the equilibrium quantity produced by firm 1 with three cases. As a result, we can obtain following Lemma 5.

Lemma 5 *If $s \geq \frac{2}{3}$, $q_1^{pul} > q_1^{prl} > q_1^{sim}$ holds. Otherwise, $q_1^{prl} > q_1^{pul} > q_1^{sim}$ holds.*

The following figure 1 shows one of the comparison results. The equilibrium quantity under simultaneous game is the intersection point of reaction functions. The equilibrium quantity under private leadership is the point of tangent between the firm 0's reaction function and the firm 1's iso-profit function. The equilibrium quantity under public leadership is the point of tangent between the firm 1's reaction function and the firm 0's iso-social welfare function.

Figure 1

Under simultaneous game, firm 1 produces less because firm 1 worries to decrease the price. Therefore, q_1^{sim} is the smallest among three games. On the contrary, because of the strategic substitutes, q_0^{sim} is the largest among three cases. In the following, we discuss the comparison results between private leadership and public leadership. From the viewpoint of firm 0, owning to the inefficient production technology, firm 0 does not want to produce more. Under private leadership, firm 1 guesses that firm 0 produces more to decrease the price if firm 1 produces less. Therefore, firm 1 produces more in order to obtain more profits even if the price of the goods is low. Because firm 1 produces the sufficient number of goods, firm 0 can decrease its production. Under public leadership, firm 0 expects that if firm 0 produces more,

firm 1 produces less due to the strategic substitution. Here, because the production technology of firm 0 is inferior to that of firm 1, it is socially preferable that firm 1 produces more and firm 0 produces less. Therefore, in order to let firm 1 produces more, firm 0 chooses a leader and produces less. As a result, q_0^{pul} is the smallest among three games. On the other hand, from the viewpoint of firm 1, the order of quantity between private leadership and public leadership depends on the degree of the foreign-ownership rate s.

When s is large, firm 0 has an incentive to produce much more to not lose the domestic rents (or, to reduce the revenues of firm 1). Therefore, under private leadership, firm 1 cannot produce more. Under public leadership, firm 0 produces less. When considering that q_1 and q_0 are strategic substitute, firm 1 can produce more. Therefore, q_1^{pul} becomes larger than q_1^{prl}. When s is small, firm 0 wants to let firm 1 produces more. Under private leadership, because firm 1 guesses that firm 0 produces less when firm 1 produces more, firm 1 has an incentive to produce much more products. Although firm 1 has also an incentive to produce more under public leadership, private leadership gives an incentive to produce much more than public leadership[4]. Consequently, q_1^{prl} is larger than q_1^{pul}.

7 Derivation of Nash-equilibrium

This section derives the Nash-equilibrium of timing game. At first, we investigate the best response of each firm.

Given that firm 1 chooses "leader", if firm 0 also chooses "leader", because its game is a simultaneous game, SW^{sim} comes true; if firm 0 chooses "follower", because its game is a private leadership game, SW^{prl} comes true. Then, deriving the difference between SW^{sim} and SW^{prl}, we obtain

$$SW^{sim} - SW^{prl} = -\frac{c^2(s-1)(1+6c^2+6s-s^3+c(6-3(-3+s)s))}{2(1+c)(2+3c+s)^2(1+3c+2s)^2} < 0. \tag{19}$$

Therefore, given that firm 1 chooses "leader", the best response of firm 0 is "follower".

Given that firm 1 chooses "follower", if firm 0 chooses "leader", because its game is a public leadership

[4]Note that under private leadership, for large s, firm 0 does not let firm 1 produce more; for small s, firm 0 lets firm 1 produce more.

game, SW^{pul} comes true; if firm 0 chooses "follower" because is game is a simultaneous game, SW^{sim} comes true. Then, deriving the difference between SW^{sim} and SW^{pul}, we obtain

$$SW^{sim} - SW^{pul} = -\frac{c^2}{2(2+3c+s)^2(5+9c+3s)} < 0. \tag{20}$$

Therefore, given that firm 1 chooses "follower", the best response of firm 0 is "leader".

Given that firm 0 chooses "leader", if firm 1 also chooses "leader", π_1^{sim} comes true; if firm 1 chooses "follower", π_1^{pul} comes true. Then, deriving the difference between π_1^{sim} and π_1^{pul}, we obtain

$$\pi_1^{sim} - \pi_1^{pul} = -\frac{3c^2(11+18c+6s)}{2(2+3c+s)^2(5+9c+3s)^2} < 0 \tag{21}$$

Therefore, given that firm 0 chooses "leader", the best response of firm 1 is "follower".

Given that firm 0 chooses "follower", if firm 1 chooses "leader", π_1^{prl} comes true; if firm 1 chooses "follower", π_1^{sim} comes true. Then, deriving the difference between π_1^{sim} and π_1^{prl}, we obtain

$$\pi_1^{sim} - \pi_1^{prl} = -\frac{c^2(-1+s)^2}{2(1+c)(1+3c+2s)(2+3c+s)^2} < 0 \tag{22}$$

Therefore, given that firm 0 chooses "follower", the best response of firm 1 is "leader".

From the above analysis, we demonstrate following Proposition 1.

Proposition 1 *The Nash-equilibrium of timing game is that the public firm is "leader" and the private firm is "follower", and that the private firm is "leader" and the public firm is "follower".*

Proposition 1 corresponds with various previous studies about the timing game of public and private firms.

In the following, in order to select the equilibrium of the above timing game, we adopt the "risk-dominant equilibrium" formulated by Harsanyi and Selten (1988).

First, we define

$$\Delta_{pul} = (SW^{pul} - SW^{sim})(\pi_1^{pul} - pi_1^{sim}), \tag{23}$$

$$\Delta_{prl} = (SW^{prl} - SW^{sim})(\pi_1^{prl} - pi_1^{sim}). \tag{24}$$

When deriving the difference between Δ_{pul} and Δ_{prl}, we obtain

$$\begin{aligned}\Delta_{pul} - \Delta_{prl} = \{&c^4(-23 + 162c^4 - 73s + 471s^2 - 97s^3 - 305s^4 + 153s^5 + 54s^6 - 27s^7 \\ &+ 9c^3(-13 + 192s - 162s^2 + 54s^3) - 3c^2(125 - 533s - 366s^2 + 918s^3 - 540s^4 + 81s^5) \\ &- 3c(61 - 66s - 653s^2 + 586s^3 + 27s^4 - 216s^5 + 54s^6))\} \\ &/\{4(1+c)^2(1+3c+2s))^3(5-92+3s)^3(2+3c+s)^2\}\end{aligned} \tag{25}$$

Here, because of a complex calculation result, we perform a simulation analysis, and as a result, we obtain figure 2.

Figure 2 here

When $\Delta_{pul} > (<)\Delta_{prl}$, public leadership (private leadership) is the risk-dominant equilibrium. As a result, we obtain following Proposition 2.

Proposition 2 *When the difference of the production technology between the public and the private firms is large, public leadership is always the risk-dominant equilibrium. When its difference is small, for small the foreign ownership rage, private leadership is the risk-dominant equilibrium; for not small the foreign ownership rate, public leadership is the risk-dominant equilibrium.*

When c is not large (i.e., the difference of the production technology between the public and private firms is small), this study's result almost corresponds with the result obtained in Matsumura and Ogawa (2010, 2017). When c is large, we obtain the reverse result of Matsumura and Ogawa (2010). In the following, we discuss this mechanism.

First, we should remember that q_0^{prl} is larger than q_0^{pul}. When c is large, firm 0 does not want to produce more because of the high production cost. Therefore, if private leadership comes true, production of firm 0 increases, which is not preferable for firm 0. In order to avoid this situation, firm 0 prefers public leadership. When c is not large, if s is large, it is preferable for firm 0 that firm 0 produces less and firm 1 produces more, which situation comes true in private leadership. In other words, firm 0 prefers to become the follower. As a result, private leadership becomes the risk-dominant equilibrium. If s is not large, it is preferable for firm 0 that firm 0 produces somewhat more in order to keep the domestic rents (or, reduce the firm 1's revenues). In this case, firm 1 is hesitant to be a leader in order to obtain more profits. That is, if firm 1 becomes the leader, firm 0 produces much more products and as a result the price of the goods largely decreases. Then, the profit of firm 1 largely decreases. In order to avoid this situation, firm 1 prefers to be a follower. As a result, public leadership becomes the risk-dominant equilibrium.

8 Social welfare

This section compares social welfare of private leadership and of public leadership. When deriving the difference between SW^{prl} and SW^{pul}, we obtain

$$SW^{prl} - SW^{pul} = \frac{c^2\{1 + c(5 - 6s) + s(5 + s(-10 + 3s))\}}{2(1 + c)(5 + 9c + 3s)(1 + 3c + 2s)^2}. \tag{26}$$

From eq. (26), we obtain the following relationship.

$$SW^{prl} \geq (<) SW^{pul} \Longleftrightarrow c \geq (<) \frac{1 + s(5 + s(-10 + 3s))}{-5 + 6s} \tag{27}$$

This relationship is drawn in Figure 3.

Figure 3

As a result, we can obtain Proposition 3.

Proposition 3 *When s is (not) large, SW^{pul} is larger (smaller) than SW^{prl}.*

For not large s, firm 1's profit is almost included into social welfare. Then, from the viewpoint of social welfare, it is preferable to decrease the total production costs. In other words, firm 0 should produce less and firm 1 should produce more. Thereby, firm 0 threatens firm 1 to produce more by announcing to be a follower. Namely, private leadership is socially preferable. For large s, the firm 1's profit is not almost included into social welfare. If private leadership comes true, firm 0 produces much more in order to decrease the price and reduce the firm 1's revenues. In this case, because of too much production produced by firm 0, the total production costs largely increase, which is not socially preferable. Consequently, public leadership becomes socially preferable.

According to Matsumura and Ogawa (2017), when the private firm is a domestic firm, private leadership is socially preferable; when it is a foreign firm, public leadership is socially preferable. Therefore, Proposition 3 is similar results obtained in Matsumura and Ogawa (2017).

Finally, we analyze whether the risk-dominant equilibrium is socially preferable, which is not analyzed in Matsumura and Ogawa (2017). When summarizing Figures 2 and 3, we can obtain Figure 4.

Figure 4

As a result, we can obtain Proposition 4.

Proposition 4 *When c is large and s is not large and when c is not large and s is medium, although private leadership is socially preferable, public leadership is the risk-dominant equilibrium.*

The most important mechanism of Proposition 4 is the difference of the firm 1's profit between private leadership and public leadership. Under public leadership, firm 1 can act as a monopoly. Therefore, in order to increase the price and increase its profit, firm 1 produces somewhat less. On the other hand, under private leadership, because firm 0 which is follower plans to decrease the price, firm 1 must produce much more and obtain more profit. Consequently, firm 1 can gain more profit under public leadership than under private leadership.

When c is not large and s is medium and when c is large, the difference of the firm 1's profit between

private leadership and public leadership is still large. Therefore, although $SW^{p}rl$ is larger than SW^{pul}, public leadership becomes the risk-dominant equilibrium.

9 Concluding remarks

This study analyzed the endogenous timing game with partial foreign ownership and increasing marginal costs. In the past, although there exist some studies to address either foreign ownership or increasing marginal costs, no studies addressed these factors simultaneously. At the same time, no studies examine whether the results obtained in Matsumura and Ogawa (2010, 2017) hold under the increasing marginal costs. This study analyzed these problems.

As a result, we obtained following results. When the difference of the production technology between the public and the private firms is large, public leadership is always the risk-dominant equilibrium, which result is different from Matsumura and Ogawa (2010). When the difference of the production technology between them is small, for (not) the small foreign ownership rate, private-leadership (public-leadership) is the risk-dominant equilibrium, which results almost correspond with Matsumura (2010, 2017). From the viewpoint of social welfare, for the large (small) foreign ownership rate, private leadership (public leadership) is socially preferable. Wherein, in the middle rate of foreign ownership, although private leadership is socially preferable, public leadership becomes the risk-dominant equilibrium.

This study did not consider how the decision timing affects the privatization problem. However, some studies analyzes this interesting problem. Therefore, we want to examine this problem. Next, this study assumes a single market. However, like Kawasaki and Naito (2017) and Masuda and Sato (2017), we should consider the multi-market situation. If we allow the multi-market competition, how does the main result change? This will also be an interesting problem. We want to consider these problems in the future.

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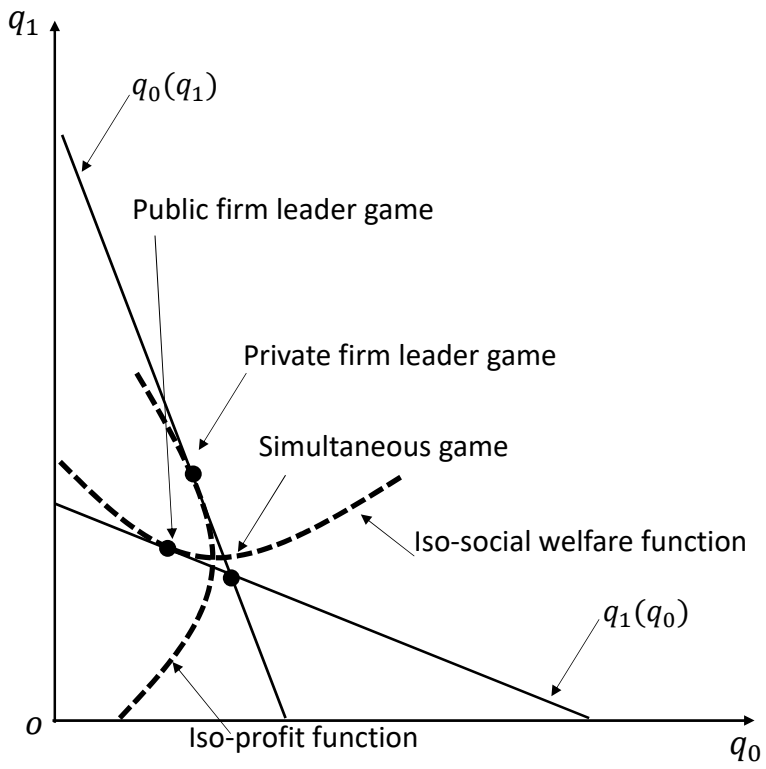

Figure 1: Each game's equilibrium

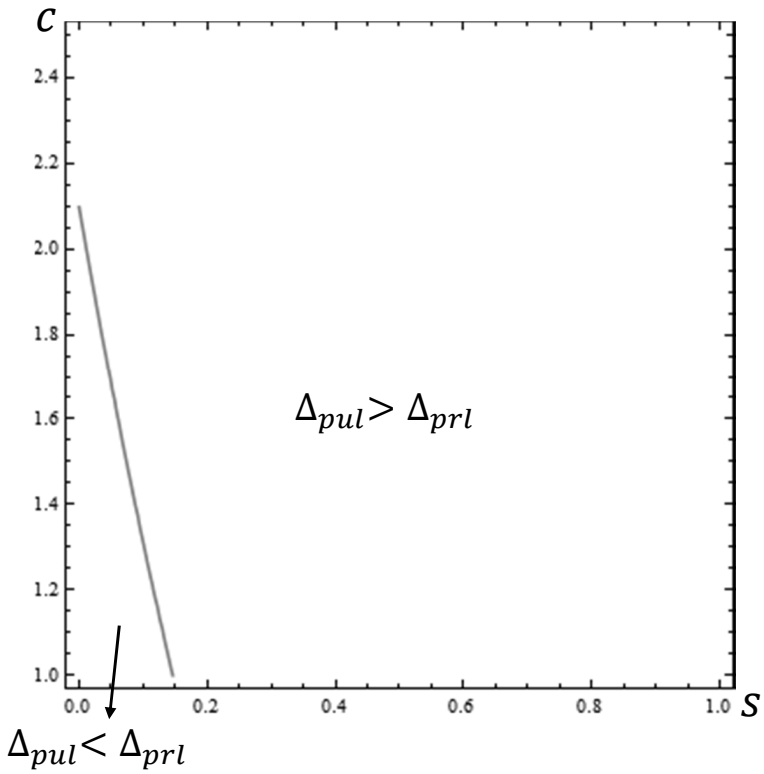

Figure 2: Risk-dominant equilibrium

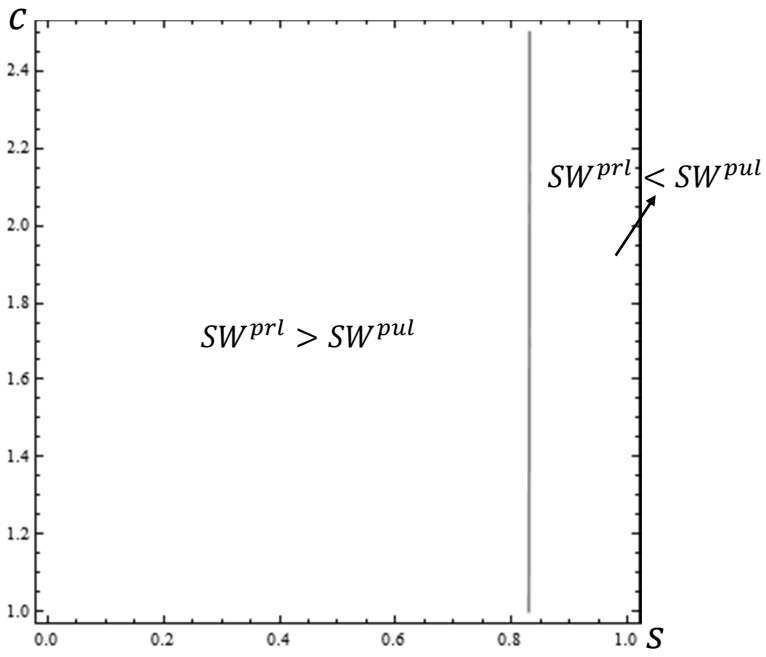

Figure 3: Comparison result of social welfare

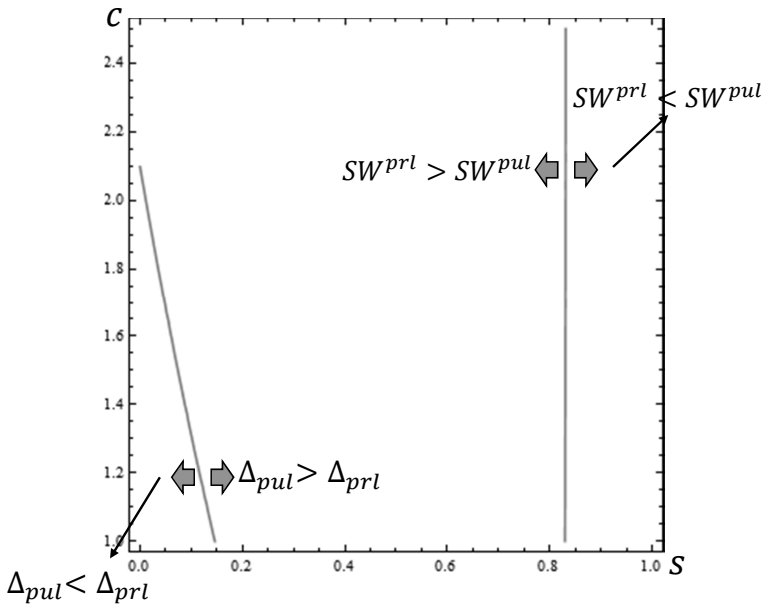

Figure 4: Comparison of the risk-dominant equilibrium and the socially preferable game