# Production Substitution of Goods Within a Firm and Between Firms in a Multi-Product Duopoly

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**Abstract**

We consider the product line strategies of duopolistic firms, each of that can supply two vertically differentiated products under non-negative output constraints and the expectation of their rivals' product line reactions. Considering a game of firms with heterogeneous unit costs for high-quality but homogeneous for low-quality products, we derive the equilibria for the game and we conduct a comparative statics of the equilibria outcomes on the relative superiority of a high-quality product and on the relative cost-efficiency. In two of the equilibria that we have found, where the cost inefficient firm supplies a high-quality good, social welfare can worsen as the unit cost of the high-quality good decreases. Furthermore, we characterize the result using production substitutions of differentiated goods within a firm and a high-quality good between firms.

9Keywords: Multi-product firm; Duopoly; Production substitution; Vertical product differentiation;

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# 1 Introduction

Real-world economies often include oligopolistic competition in the same market segment, in which firms supply multiple vertically differentiated products. In the mobile phone market, for example, Apple supply iPhone X to the first line segment and Samsung competes with Apple by supplying Galaxy S9 to business. In the second line segment, Apple sends iPhone 8 to the segment and Samsung responds by forwarding the Galaxy S8. However, there are few studies of oligoplolisitic competition in these markets in the economics literature.

In the existing literature on vertical product differentiation, the quality of the goods that the firms produce is treated as an endogenous variable. For example, in Bonanno (1986) and Motta (1993), firms initially choose the quality level and they then compete in a Cournot or Bertrand fashion in an oligopolistic market. Shaked and Sutton (1987) consider a two-stage game model in which each of horizontally and vertically differentiated multi-product firms pays a fixed sunk cost for R&D or advertising expenditure to improve (perceived) quality of its products in the first stage and chooses its respective prices in the second stage.

For a horizontally differentiated multi–product model, Bental and Spiegel (1984) consider an optimal set of product varieties in a monopoly and they analyze the relationship between the degree of differentiation between any two varieties and variety price, or the cost of installing an additional variety. Shaked and Sutton (1990) consider a two-stage price game model in which each of horizontally differentiated multi-product firms (potential entrants) chooses in the first stage which product(s) it will produce and it incurs a "sunk cost" per produced entered and chooses its respective prices in the second stage.

Then they *graphically* characterize the market structure at equlibria by two parameters which measure expansion and competition effects.

However, none of these studies consider the case where firms sell multiple products differentiated in terms of quality (vertically) in the same market. Ellison's (2005) study, which is closely related to the present study, analyzes a market where each firm sells a high-end and low-end version of the same product. Although each firm produces two differentiated goods, the two goods are sold in different markets—each with different types of consumers.

In the markets where firms supply multiple vertically differentiated products, they sometimes compete with rivals that supply one or some vertically differentiated products (i.e., the rival chooses a single product line) to the same market segment. Therefore, Kitamura and Shinkai (2015a) pay attention to competition between two firms that can choose a product line of two vertically differentiated products in the same market segment. However, few previous studies have addressed an oligopolistic market where multi-product firms producing multiple goods that are differentiated in terms of quality (see, for example Johnson and Myatt 2003).[1]

According to Johnson and Myatt (2003), firms that sell multiple quality-differentiated products frequently change their product lines when a competitor enters the market. They provided an explanation for the common strategies of using "fighting brands" and "pruning" their product lines. In particular, they endogenized not only the quality level of each good but also the number of goods that each firm supplied to the market.

Unlike most preceding studies, both the quality level and the number of differentiated goods that each firm supplies are exogenously given and we also do not explicitely consider the stage of product line choice with a fixed "sunk cost" as Shaked and Sutton (1987,

[1] For the sake of simplicity, in this study we focus on a duopoly model.

1990) did in our model (in Kitamura and Shinkai(2015a) and in this study). This setting seems to be appropriate to explore the relationship between production substitution and the difference in the potential value of the goods according to the consumers or to the unit costs of the two goods. Our study's results are also related to those of marketing studies on product segmentation and product distribution strategies. Calzada and Valletti (2012) study a model of film distribution and consumption. They consider a film studio that can release two versions of one film—one for theaters and one for video—, although they do not consider oligopolistic competition between film studios. They show that the optimal strategy for the studio is to introduce versioning (the simultaneous release of their film with one version for theaters and another version for video) if their goods are not close substitutes for each other.

In Kitamura and Shinkai (2015a), we considered a game that includes heterogeneous unit production costs between firms for high-quality goods but homogeneous costs for low-quality products. We described the firms' product line strategies based on the relative quality of the products and on the cost-efficiency ratios of the firms of the high-quality good. We first derived equilibria by assuming that, in any equilibrium, each rival firm chooses positive outputs for both the high- and the low-quality good. Consequently, these equilibria included cases in which a firm chooses negative outputs for one of the goods for some parameter ranges (firms' relative quality ratio or cost inefficiency ratio for the high-quality good). We then retroactively excluded the ranges of parameters in equilibria that result in any negative outputs and we graphically describe the firms' product line strategies based on the relative quality of the products and on the cost-efficiency ratios between the firms in the case of high-quality goods. In Kitamura and Shinkai (2015a), we did not describe the firms' equilibrium profits and equilibrium welfare. However, we also established a result that indirectly supports the result of Calzada and Valletti (2012). In their model, "versioning" and "sequencing" correspond to the simultaneous supply and sequential supply, respectively, of high- and low-quality goods, as in our model. In the case of sequential supply, the film studio supplies a high-quality film version to theaters and then launches a low-quality DVD version in the same market.

Although Kitamura and Shinkai(2015a) assumed that each rival firm chooses positive outputs for both goods in a duopolistic competition, it is crucial that each firm considers its rivals' product line strategies when choosing its own strategy. In these cases, it is important that each firm chooses its own product line strategies for multiple products, given their expectations of the rivals' product line reactions. Therefore, in this study, we consider the product line strategies of duopolistic firms that each supply two vertically differentiated products under non-negative output constraints and an expectation of their rivals' product line reactions. This study differs from Kitamura and Shinkai (2015a) in many points.

First, in this study, we *explicitly* examine the product line strategies of duopolistic firms that supply two vertically differentiated products under a non-negative output constraint and an expectation with regard to rivals' product line reactions. We show that there are five non-trivial equilibria with positive outputs for one or both products and also show that both firms have positive profits in each equilibrum. In these equilibria, the ranges of the two ratio parameters for which positive equilibrium outputs exist for the two firms differ. We graphically describe the firms' product line strategies in equilibrium, based on the relative quality of the products and on the firm's relative cost efficiency for the high-quality good (Figure 1).

Second, at every one of the non-trivial equilibria, comparing the equilibrium total outputs and profits of the two firms, we graphically describe the differences between the two firms' total outputs and profits in the five equilibria, based on the relative product quality and their relative cost efficiency ratios. We then conduct a comparative statics analysis on these two ratio parameters (Figure 2)[2].

We also derive social welfare in every equilibrium. In addition, in a multi-product Cournot duopoly, there exist two equilibria in which a reduction of the relative marginal cost inefficiency decreases social welfare. In one of them (which we derive in case C), the result that we derive is similar to the result that Lahiri and Ono (1988) found in a single-product Cournot oligopoly. Lahiri and Ono (1988) show that a cost reduction in a firm with a sufficiently low share decreases social welfare but a cost reduction in any firm always increases the social welfare if the market *share is same* among all firms. However, we find an equilibrium in which a reduction of the relative marginal cost inefficiency decreases social welfare, even when both firms have the *same* market *shares* (Case E in equilibrium in Figure 3). Through comparative statics based on the firms' relative cost inefficiency ratio for the high-quality good, we also find another equilibrium (which we derive in case E) in which production substitution between the high-quality and the low-quality good occurs within each firm but never occurs between the cost-efficient firm and the cost-inefficient firm for the high-quality good. Consequently, in this equilibrium, the two firms' total outputs remain exactly the same as the relative marginal cost inefficiency ratio decreases. That is, we also derive the different result from that Lahiri and Ono (1988) found. The reason why our result differs from that for Lahiri and Ono's (1988) result is that our result is caused by the production substitution from the high-quality

[2] Professor John Sutton suggested this analysis to us in his comment on our presentation of an earlier version of this study, Shinkai and Kitamura (2015b), at EARIE 2015, the Annual Conference of the European Association for Research Industrial Economics, in Munich, Germany. His comment and suggestion has much improved our study. Therefore, we wish to express our gratitude to Professor John Sutton.

to the low-quality good *within the* cost-inefficient firm and the subsequent production substitution of good H *between firms* by means of a strategic substitute. However, Lahiri and Ono's (1988) result is *only* caused by the production substitution *between the cost-efficient and inefficient firm,* as the cost of the inefficient firm decreases.

The rest of this paper is organized as follows. In Section 2, we present our model. In Section 3, we derive the duopoly equilibria with two vertically differentiated products in the same market under a non-negative output constraint and an expectation with regard to rivals' product line reactions. Furthermore, we graphically describe the firms' product line strategies in equilibrium, based on the relative quality of the products and on the firms' relative cost efficiency for the high-quality good (Figure 1). By comparing the equilibrium total profits of the two firms at every equilibrium, we graphically describe the differences between the two firms' total outputs and the profits in five equilibria, based on the relative product quality and their relative cost efficiency ratios. We conduct comparative statics on these two ratio parameters (Figure 2) in Section 3. In Section 4, we derive the social welfare in every equilibrium derived in Section 2. We then conduct the comparative statics based on the relative product quality and the firms' relative cost inefficiency ratio for the high-quality good (Figure 3). Finally, Section 5 concludes this paper.

## 2 The Model and the Equilibria of the Game

Suppose there are two firms ($i = 1, 2$) in a duopoly, each of which produces two goods ($H$ and $L$), which differ in terms of quality. We assume a continuum of consumers, represented by a taste parameter, $\theta$, which is uniformly distributed between 0 and $r$ ($> 0$), with density 1. We further assume that a consumer is of type $\theta \in [0, r]$, for $r > 0$.

The consumers' preferences are the standard Mussa and Rosen preferences. Thus, the utility (net benefit) of consumer $\theta$ who buys good $\alpha$ $(= H, L)$ from firm $i$ $(= 1, 2)$ is given by

$$U_{i\alpha}(\theta) = V_{\alpha}\theta - p_{i\alpha} \qquad i =, 1, 2 \quad \alpha = H, L. \tag{1}$$

To maximize his/her surplus, each consumer decides whether to buy nothing or one unit of good $\alpha$ from firm $i$.

Let $V_H$ and $V_L$ denote the quality of the high-quality and the low-quality good, respectively. Then, the maximum amount that consumers are willing to pay for each good is assumed to be $V_H = \mu V_L = \mu > V_L = 1$. Thus, for simplicity, we normalize the quality of the low-quality good by setting $V_L = 1$ and assume the quality of the high-quality good is $\mu$ times that of the low-quality good. Good $\alpha$ $(= H, L)$ is assumed to be homogeneous for all consumers. Suppose that there always exists a consumer $\underline{\theta}_{iL}, i = 1, 2$ who is indifferent between purchasing good L and purchasing nothing in a monopoly or a duopoly. For this consumer, $\underline{\theta}_{iL}$ satisfies

$$\begin{aligned} & U_{iL}(\underline{\theta}_L) = 0 \\ \Leftrightarrow \ & \underline{\theta}_{iL} = \frac{p_{iL}}{V_L} = p_{iL}, i = 1, 2. \end{aligned} \tag{2}$$

We can derive the demand for good $H$ as $Q_H = r - \widehat{\theta}$, and that for good $L$ as $Q_L = \widehat{\theta} - \underline{\theta}_{iL}$, as shown in Figure 1, where $Q_{\alpha} = q_{i\alpha} + q_{j\alpha}$, for $\alpha = H, L$ and $j = 1, 2$. Without loss of generality, we set $r = 1$. Here, $\widehat{\theta}$, the threshold between the demand for $H$ and that for $L$, is given by

$$\widehat{\theta} = (p_H - p_L)/(\mu - 1). \tag{3}$$

Then, as in Kitamura and Shinkai (2015a), we derive the following inverse demand functions:

$$\begin{cases} p_H = V_H(1 - Q_H) - Q_L = \mu(1 - Q_H) - Q_L \\ p_L = V_L - Q_H - Q_L = 1 - Q_H - Q_L, \end{cases} \tag{4}$$

where $Q_\alpha = q_{i\alpha} + q_{j\alpha}$ and $p_\alpha$ and $q_{i\alpha}$ denote the price of good $\alpha$ and firm $i$' output of good $\alpha$, respectively, for $\alpha = H, L$ and $i, j = 1, 2$.

Moreover, suppose that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L = 0$, where $c_{i\alpha}$ is firm $i$'s marginal and average cost of good $\alpha$. This implies that a high-quality good incurs a higher cost of production than that of a low-quality good. Here, without loss of generality, we assume $c_{2H} > c_{1H} = 1 > c_{iL} = 0$, which means that firm 1 is more efficient than firm 2. Under these assumptions, each firm's profit is defined in the following manner:

$$\pi_i = (p_H - c_{iH})q_{iH} + p_L q_{iL} \qquad i = 1, 2. \tag{5}$$

Firm $i(= 1, 2)$ chooses the outputs for $H$ and $L$ to maximize its profit function in Cournot fashion under non-negative output constraints, provided that firm $j(\neq i)$ chooses *any given* product line strategy $\mathbf{s}_j \in \mathbf{S}_j \equiv \{(0, 0), (+, 0), (0, +), (+, +)\}$, where $(0, 0)$ implies $(q_{jH} = 0, q_{jL} = 0)$, $(+, 0)$ implies $(q_{jH} > 0, q_{jL} = 0)$, and so on. Thus, for any given $\mathbf{s}_j \in \mathbf{S}_j$

$$\max_{q_{iH},q_{iL}} \ \pi_i = \{\mu(1 - q_{iH} - q_{jH}) - q_{iL} - q_{jL} - c_{iH})q_{iH} + (1 - q_{iH} - q_{jH} - q_{iL} - q_{jL})q_{iL} \quad (6)$$

$$s.t. \ \ q_{iH} \geq 0, q_{iL} \geq 0, \ i \neq j, \ i, j = 1, 2.$$

The necessary and complementary conditions for this maximization problem are

$$\frac{\partial \pi_i}{\partial q_{iH}} \leq 0, \ \frac{\partial \pi_i}{\partial q_{iL}} \leq 0, \quad (7)$$

$$q_{iH} \cdot \frac{\partial \pi_i}{\partial q_{iH}} = q_{iL} \cdot \frac{\partial \pi_i}{\partial q_{iL}} = 0, \quad (8)$$

$$q_{iH} \geq 0, \ q_{iL} \geq 0, \ \ i = 1, 2. \quad (9)$$

Each firm chooses its product line strategy for the two vertically differentiated products,; that is, whether to produce positive (zero) quantities of product $H$ and $L$, given the rival firm's product line strategy.

Note that each inequality $\partial \pi_i / \partial q_{i\alpha} \leq 0$ in (7) and the corresponding complementary slackness condition $q_{i\alpha} \cdot \partial \pi_i / \partial q_{i\alpha} = 0$ in (8) imply that if the marginal revenue of firm $i$ for product $\alpha (= H, L)$ is below (the same as) its marginal cost, then firm $i$ does not produce (does produce) a positive quantity of the product.

In the following, we present the equilibria of a Cournot duopoly Game, in which each firm can choose its product line and outputs for the two vertically differentiated goods. The firms operate under a non-negative output constraint. After presenting the equilibrium, we describe the firms' product line strategies based on the products' relative quality and on the firms' relative cost efficiency with respect to the high-quality good in equilibrium.

There are 15 cases to be solved, based on each firm's product line strategies, given the firm's expectation of its rival's product line strategies, except for the trivial case in which neither firm produces $H$ or $L$. After performing lengthy calculations and checking the non-negative constraints for the outputs in each equilibrium, we find that 10 of the 15 cases have no equilibrium in the corresponding games. Owing to space limitations, we omit these calculations and the proofs of our results. Thus, we examine the following five cases:

- **Case A**: $\mathbf{s}_1 = (0, +),\ \mathbf{s}_2 = (0, +)$

  In this case, a duopoly market of the low-quality good is realized in equilibrium:

$$(q_{1H}^{*A}, q_{1L}^{*A}, q_{2H}^{*A}, q_{2L}^{*A}) = \left(0,\ \frac{1}{3},\ 0,\ \frac{1}{3}\right) \quad if\ 1 < \mu \leq 2, \tag{10}$$

  where the last inequality must hold due to the necessary condition. In Figure 2, area A corresponds to this case. The relative superiority of the high-quality product $H, \mu$, is too small compared to the firms' relative cost efficiency of the high-quality good, $c_{2H}$. Therefore, neither firm produces product $H$ and instead each firm produces only the low-quality product $L$.

  From (4), (5), and (10), each firm's equilibrium price and profit are

$$(p_H^{*A}, p_L^{*A}) = \left(\frac{3\mu - 2}{3},\ \frac{1}{3}\right) \tag{11}$$

$$(\pi_1^{*A}, \pi_2^{*A}) = \left(\frac{1}{9}, \frac{1}{9}\right). \tag{12}$$

- **Case B**: $\mathbf{s}_1 = (+,0),\ \mathbf{s}_2 = (0,+)$

In this case, each firm specializes in the product that is more cost efficient for the firm. Thus, we obtain

$$(q_{1H}^{*B}, q_{1L}^{*B}, q_{2H}^{*B}, q_{2L}^{*B}) = \left(\frac{2\mu-3}{4\mu-1}, 0,\ 0,\ \frac{\mu+1}{4\mu-1}\right) \tag{13}$$

$$if\ \ 4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}), \tag{14}$$

where the last inequality must hold, due to the necessary condition. In Figure 1, area B corresponds to this case. In area B, the relative cost inefficiency of the high-quality good of firm 2, $c_{2H}$, is relatively strong compared to $\mu$, the relative quality superiority of the high-quality product $H$. From (4), (5), and (10), we obtain the corresponding equilibrium price and the profit of each firm:

$$(p_H^{*B}, p_L^{*B}) = \left(\frac{(\mu+1)(2\mu-1)}{4\mu-1}, \frac{\mu+1}{4\mu-1}\right) \tag{15}$$

$$(\pi_1^{*B}, \pi_2^{*B}) = \left(\frac{\mu(2\mu-3)^2}{(4\mu-1)^2}, \frac{(\mu+1)^2}{(4\mu-1)^2}\right). \tag{16}$$

We also find that $q_{1H}^{*B} - q_{2L}^{*B} = \frac{\mu-4}{4\mu-1} \geq 0 \Leftrightarrow q_{1H}^{*B} \geq q_{2L}^{*B}$ and that

$$\pi_1^{*B} - \pi_2^{*B} = \frac{1}{4\mu-1}\left(\mu^2 - 3\mu + 1\right) > 0 \text{ for } \frac{1}{2}\sqrt{5} + \frac{3}{2} < 4 < \mu. \tag{17}$$

- **Case C**: $\mathbf{s}_1 = (+, 0),\ \mathbf{s}_2 = (+, +)$

  In case C, firm 2 (which has a higher unit cost for the high-quality product $H$) produces both products but firm 1, which is efficient in the production of product $H$, specializes in product $H$:

$$(q_{1H}^{*C}, q_{1L}^{*C}, q_{2H}^{*C}, q_{2L}^{*C}) = \left( \frac{\mu + c_{2H} - 2}{3\mu},\ 0,\ \frac{2\mu^2 + (1 - 4\mu)c_{2H} - 2}{6\mu(\mu - 1)},\ \frac{c_{2H}}{2(\mu - 1)} \right) \tag{18}$$

where

$$q_{1H}^{*C} > q_{2H}^{*C},\ q_{2L}^{*C} > 0 \text{ and } q_{2H}^{*C} \gtreqless q_{2L}^{*C} \Leftrightarrow \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) \lesseqgtr \mu, \tag{19}$$

and

$$\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu \Leftrightarrow q_{2H}^{*C} > 0 \tag{20}$$

hold. Furthermore, we obtain

$$c_{2H} \geq 2 \text{ and } \mu > 4. \tag{21}$$

For $q_{1H}^{*C} > 0$, the inequality $\mu > 2 - c_{2H}$ holds because $c_{2H} \geq 2$. In Figure 1, areas C.1 and C.2 correspond to this case. In area C.1, the quality superiority $\mu$ of the high-quality product is high compared to the relative cost inefficiency, $c_{2H}$, of the high-quality good for firm 2. Moving from area C.1 to area C.2, the relative quality superiority $\mu$ decreases and becomes small compared to the relative cost inefficiency,

$c_{2H}$, of good $H$ for firm 2. Hence, firm 2 substitutes the production of high-quality good $H$ for that of low-quality $L$. The corresponding equilibrium price and profit for each firm are

$$(p_H^{*C}, p_L^{*C}) = \left( \frac{\mu + c_{2H} + 1}{3}, \; \frac{2\mu - c_{2H} + 2}{6\mu} \right), \tag{22}$$

$$\begin{aligned} \pi_1^{*C} &= \frac{(\mu + c_{2H} - 2)^2}{9\mu}, \\ \pi_2^{*C} &= \frac{4\mu^3 - 4(4c_{2H} - 1)\mu^2 + 4(2c_{2H} - 1)(2c_{2H} + 1)\mu - (7c_{2H} - 2)(c_{2H} - 2)}{36\mu(\mu - 1)}. \end{aligned} \tag{23}$$

$$\Delta\pi^{*C} = \frac{1}{12\mu(\mu - 1)} (4(c_{2H} - 1)\mu(2\mu - c_{2H} - 3) + (c_{2H} + 2)(c_{2H} - 2)). \tag{24}$$

When $\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu, c_{2H} \geq 2$, we see that $\Delta\pi^{*C} = \pi_1^{*C} - \pi_2^{*C} > 0$.

- **Case D**: $\mathbf{s}_1 = (+, +), \; \mathbf{s}_2 = (0, +)$

In this case, in contrast to case C, firm 1 is efficient in producing product $H$ and supplies both products. However, the inefficient firm 2 specializes in product $L$.

$$(q_{1H}^{*D}, q_{1L}^{*D}, q_{2H}^{*D}, q_{2L}^{*D}) = \left( \frac{\mu - 2}{\mu - 1}, \; \frac{4 - \mu}{6(\mu - 1)}, \; 0, \; \frac{1}{3} \right) \text{ if } 2 < \mu < 4 \, and \, \mu \leq 2c_{2H}. \tag{25}$$

where the last inequalities must hold due to both the positive output condition and the necessary condition.

In addition, we have $q_{1L}^{*D} \gtreqless q_{1H}^{*D} \Leftrightarrow \mu \lesseqgtr 5/2$. Areas D.1 and D.2 correspond to this case. The relative superiority, $\mu$, of the high-quality good is relatively small compared to the relative cost inefficiency, $c_{2H}$, of the high-quality good for firm 2, especially in case D.2.

From (4), (5), and (10), the corresponding equilibrium price and profit for each firm are

$$(p_H^{*D}, p_L^{*D}) = \left(\frac{3\mu+2}{6}, \frac{1}{3}\right) \tag{26}$$

$$(\pi_1^{*D}, \pi_2^{*D}) = \left(\frac{9\mu^2 - 32\mu + 32}{36(\mu-1)}, \frac{1}{9}\right). \tag{27}$$

Here,

$$\pi_1^{*D} - \pi_2^{*D} = \frac{1}{4(\mu-1)}(\mu-2)^2 > 0, \text{for } \mu \leq 2c_{2H},\ 2 < \mu < 4. \tag{28}$$

- **Case E**: $\mathbf{s}_1 = (+,+),\ \mathbf{s}_2 = (+,+)$

In case E, both firms produce both products,

$$(q_{1H}^{*E}, q_{1L}^{*E}, q_{2H}^{*E}, q_{2L}^{*E}) = \left(\frac{\mu + c_{2H} - 3}{3(\mu-1)}, \frac{2-c_{2H}}{3(\mu-1)}, \frac{\mu - 2c_{2H}}{3(\mu-1)}, \frac{2c_{2H}-1}{3(\mu-1)}\right) \tag{29}$$

$$if\ 1 < c_{2H} < 2 \text{ and } \min\{3 - c_{2H},\ 2c_{2H}\} < \mu,$$

where the last inequalities must hold due to both the positive output and the necessary condition. In addition, we obtain

$$q_{1H}^{*E} \gtreqless q_{1L}^{*E} \Leftrightarrow \mu \gtreqless 5 - 2c_{2H},\ q_{2H}^{*E} \gtreqless q_{1L}^{*E} \text{ and } q_{2L}^{*E} \gtreqless q_{1H}^{*E} \Leftrightarrow \mu \lesseqgtr c_{2H} + 2.$$

Furthermore, we show that

$$q_{2H}^{*E} \gtreqless q_{2L}^{*E} \Leftrightarrow \mu \gtreqless 4c_{2H} - 1.$$

In this case, $c_{2H}$ is very small compared to $\mu$. In Figure 1, this case corresponds to areas E.1, E.2, E.3, and E.4. Moving from area E.4 to E.1, the relatively inefficient firm 2 reduces its output of high-quality product $H$.

Then, we obtain the corresponding equilibrium price and profit of each firm from (4), (5), and (10):

$$(p_H^{*E}, p_L^{*E}) = \left( \frac{\mu + c_{2H} + 1}{3},\ \frac{1}{3} \right) \tag{30}$$

$$\begin{aligned} \pi_1^{*E} &= \frac{1}{9(\mu - 1)} \left( \mu^2 + (2c_{2H} - 5)\,\mu + (c_{2H} - 2)(c_{2H} - 4) \right), \\ \pi_2^{*E} &= \frac{1}{9(\mu - 1)} \left( \mu^2 - (4c_{2H} - 1)\,\mu + 4c_{2H}^2 - 1 \right). \end{aligned} \tag{31}$$

$$\begin{aligned} \text{For } 1 &< c_{2H} < 2, \mu \geq 2c_{2H} > \frac{1}{2}(c_{2H} + 3), \\ \pi_1^{*E} - \pi_2^{*E} &= \frac{1}{3\,(\mu - 1)} (c_{2H} - 1)\,(2\mu - c_{2H} - 3) > 0. \end{aligned} \tag{32}$$

By combining these five cases, we obtain the following proposition. Furthermore, we show the product line strategy of the duopoly game under the rival's non-negative output belief in the $c_{2H}$-$\mu$ plane in Figure 1.

[Insert Figure 1 here]

**Proposition 1** *In the duopoly equilibrium of the game, given the rival's expectation of a non-negative quantity, the following inequalities hold for the outputs of the high- and low-quality good for each firm:*

$$\begin{aligned} 0 &< q_{2H}^{*E} < q_{1H}^{*E} \leq q_{1L}^{*E} < q_{2L}^{*E} \\ \text{for } (c_{2H}, \mu) &\in \{(c_{2H}, \mu) \in R^{2++} \mid \mu > 2c_{2H}, \mu \leq 5 - 2c_{2H} \text{ and } 1 < c_{2H} < \frac{5}{4}\} \text{ (E.1)}, \end{aligned}$$

$$\begin{aligned} 0 &< q_{2H}^{*E} < q_{1L}^{*E} < q_{1H}^{*E} < q_{2L}^{*E} \text{ for } (c_{2H}, \mu) \in \\ \{(c_{2H}, \mu) &\in R^{2++} \mid \mu > 2c_{2H}, \mu > 5 - 2c_{2H}, \mu < c_{2H} + 2 \text{ and } 1 < c_{2H} < 2\} \text{ (E.2)}, \end{aligned}$$

$$\begin{aligned} 0 &< q_{1L}^{*E} \leq q_{2H}^{*E} < q_{2L}^{*E} < q_{1H}^{*E} \text{ for } (c_{2H}, \mu) \in \\ \{(c_{2H}, \mu) &\in R^{2++} \mid \mu \geq c_{2H} + 2, \mu < 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \text{ (E.3)}, \end{aligned}$$

$$
\begin{aligned}
0 &< q_{1L}^{*E} < q_{2L}^{*E} \leq q_{2H}^{*E} < q_{1H}^{*E} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \mu \geq 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \text{ (E.4)}.
\end{aligned}
$$

$$
\begin{aligned}
q_{1L}^{*C} &= 0 < q_{2L}^{*C} < q_{2H}^{*C} < q_{1H}^{*C} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \mu > \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) > 4, c_{2H} \geq 2\} \text{ (C.1)}, \\
q_{1L}^{*C} &= 0 < q_{2H}^{*C} \leq q_{2L}^{*C} < q_{1H}^{*C} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) > \mu \geq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) > 4 \\
, c_{2H} &\geq 2\} \text{ (C.2)}.
\end{aligned}
$$

$$
\begin{aligned}
q_{1H}^{*B} &\geq q_{2L}^{*B} > q_{1L}^{*B} = q_{2H}^{*B} = 0 \\
\text{for } (c_{2H}, \mu) &\in \{(c_{2H}, \mu) \in R^{2++} \mid 4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}), \mu \geq \frac{5}{2} \text{ (B)}.
\end{aligned}
$$

$$
\begin{aligned}
q_{2L}^{*D} &= \frac{1}{3} > q_{1H}^{*D} > q_{1L}^{*D} > q_{2H}^{*D} = 0 \text{ when } \frac{5}{2} < \mu < 4,\ \mu \leq 2c_{2H} \text{ (D.1)}, \\
q_{2L}^{*D} &= \frac{1}{3} > q_{1L}^{*D} \geq q_{1H}^{*D} > q_{2H}^{*D} = 0 \text{ when } 1 < \mu \leq \frac{5}{2}, \mu \leq 2c_{2H}, \text{(D.2)}.
\end{aligned}
$$

$$
q_{1H}^{*A} = q_{2H}^{*A} = 0 < q_{1L}^{*A} = q_{2L}^{*A} = \frac{1}{3} \text{ when } 1 < \mu \leq 2 \text{ (A)}.
$$

*where the A, B, C.1, C.2, D.1, D.2, E.1, E.2, E.3 and E.4 indicate the area in the* $c_{2H}$-$\mu$ *plane in Figure 1.*

Note that each equilibrium output presented in Proposition 1 is that of a duopoly game, given the firms' expectations about their rival's non-negative output(s).

The result presented in Proposition 1 leads firms to infer *correctly* the quality superiority and the relative cost-efficiency ratios *ex post* by observing the output strategies in equilibrium.

Note that we assume $c_{2H} > c_{1H} = 1$ and $V_H = \mu V_L = \mu > V_L = 1$. Thus, the horizontal and vertical axes in Figure 1 show the relative cost ratio $c_{2H}$ and the quality ratio $\mu$, respectively. At any point $(c_{2H}, \mu)$ in areas E.1, E.2, E.3, and E.4 in Figure 1, the relative cost ratio $c_{2H}$ is between one and two. Thus, the difference between the unit costs of the two firms is small. The equilibrium in case E corresponds to these areas. In areas E.1, E.2, and E.3, the relative superiority of the high-quality good $\mu$ is not very high. Thus, both firms are likely to supply high- and low-quality goods. However, as the quality ratio $\mu$ becomes sufficiently high and the relative cost ratio $c_{2H}$ is sufficiently low in area E.4, the inefficient firm 2 produces far more of the high-quality good which has a higher cost than of the low-quality good (with no production cost). Naturally, the efficient firm 1 produces more of the high-quality good H than of the low-quality good L because its production cost for H are lower than that of the rival firm. Its marginal revenue from good H is also high because its quality superiority $\mu$ is very high. From these illustration, we find a substitution of production from the low-quality good to the high-quality good in both firms as the point $(c_{2H}, \mu)$ moves from Area E.1 to Areas E.2, E.3, and E.4 in Figure 1. Note that this substitution is stronger for the efficient firm than it is for the inefficient firm.

This result is consistent with that of Calzada and Valletti (2012), where the optimal strategy for a film studio is to introduce versioning if their goods are not close substitutes. Thus, when the quality of the high-quality good H is large compared to that of good L, we can conclude that they are not close substitutes. Then, the result in the above proposition confirms that it would be better for both firms to supply both goods in the market; that is, to obey the "versioning strategy" of Calzada and Valletti (2012).

At any point $(c_{2H}, \mu)$ in areas C.1 and C.2, the relative superiority $\mu$ is large compared to the relative cost ratio $c_{2H}$. Thus, the margin of the efficient firm 1 for the high-quality good H, $p_H^{*C} - 1$ is very high, and the firm substitutes the production of good L by that of good H. In other words, the efficient firm 1 specializes in good H, with its relatively large margin compared to that for the low-quality good L (that is $p_L^{*C}$) . Moving from area C.1 to area C.2, the inefficient firm 2 loses its incentive to supply the high-quality good more because $c_{2H}$ increases but $\mu$ decreases. Thus, it reduces its output of the high-quality good and increases its output of the low-quality good. The equilibrium in case C corresponds to these areas. In area B, the relative superiority $\mu$ is at a moderate level but is smaller than those in areas C.1 and C.2, and the relative cost ratio $c_{2H}$ is larger than those in areas C.1 and C.2. Hence, firm 2, with its inefficient production technology for the high-quality good, stops producing good H and specializes in the low-quality good L. Two monopoly markets appear in this case. The equilibrium in case B corresponds to this area. As the relative superiority $\mu$ decreases from the point $(c_{2H}, \mu)$ in area D.1 to that in area D.2, firm 1 with efficient production technology for the high-quality good reduces its output of good H and increases its output of the low-quality good, thus substituting production of the high-quality good by that of the low-quality good. As the relative superiority $\mu$ decreases further in the equilibrium in case A (area A), firm 1 ceases to produce the high-quality good H and specializes in the low-quality good. Consequently, the market in the equilibrium becomes a duopoly of the low-quality good.

Next, we provide a lemma on the equilibrium profits of the firms for the five non-trivial equilibria.

**Lemma 1** *In the duopoly equilibrium of the game under the expectation of the rival firm's non-negative quantities, the following equality and inequalities hold for the profits of each firm:*

*In case A, the equilibrium profits of both firms are identical:* $\pi_1^{*A} = \pi_2^{*A}$. *In the equilibria for cases B, C, D and E , firm 1—which is efficient in producing the high-quality good—earns more than firm 2—which has inefficient technology for producing the high-quality good. Thus,* $\pi_1^{*k} > \pi_2^{*k}$, $k = B, C, D,$ *and* $E$.

Note that the firm that is cost efficient in producing the high-quality product earns more than the inefficient firm does at all equilibria except that of case A.

In case A, taking into account the results of Proposition 1 and Lemma 1, we find that the relative superiority $\mu$ of the high-quality good is too small compared to the unit costs for H. Thus, both firms specialize in good L and the market for good L becomes a Cournot duopoly. Hence, the two firms' equilibrium profits are the same.

# 3 Comparative statics of the difference of in total equilibrium outputs and profits based on $\mu$ and $c_{2H}$

In this section, we conduct a comparative statics analysis of the differences of total outputs and profits on the two ratios $\mu$ and $c_{2H}$ in the five equilibria.

First, we investigate how the change of the relative superiority of the high-quality good $\mu$ or the relative cost ratio $c_{2H}$ has an effect on the difference in the total output between the cost efficient firm 1 and the inefficient firm 2 at equilibrium $k(=A, B, C, D, E)$.

We denote by $Q_i^{*k} = q_{iL}^{*k} + q_{iH}^{*k}$ and $\Delta Q_{12}^{*k} \equiv Q_1^{*k} - Q_2^{*k}$, the total output of firm $i(=1,2)$ and the difference in total output of the high-quality good H between the efficient firm 1 and the inefficient firm 2, respectively, at equilibrium $k(=A, B, C, D, E)$.

From (10), (13), (14), (18), (21), (25), and (29), we see that

$$Q_1^{*A} = Q_2^{*A} = \frac{1}{3}, \Delta Q_{12}^{*A} = 0, \tag{33}$$

$$Q_1^{*B} = \frac{2\mu - 3}{4\mu - 1}, Q_2^{*B} = \frac{\mu + 1}{4\mu - 1}, \Delta Q_{12}^{*B} = \frac{\mu - 4}{4\mu - 1} > 0, \tag{34}$$

$$Q_1^{*C} = \frac{1}{3\mu}\left(\mu + c_{2H} - 2\right), Q_2^{*C} = \frac{1}{6\mu}\left(2\mu - c_{2H} + 2\right), \Delta Q_{12}^{*C} = \frac{1}{2\mu}\left(c_{2H} - 2\right) > 0, \tag{35}$$

$$Q_1^{*D} = \frac{1}{6\left(\mu - 1\right)}\left(5\mu - 8\right), Q_2^{*D} = \frac{1}{3}, \Delta Q_{12}^{*D} = \frac{1}{2\left(\mu - 1\right)}\left(\mu - 2\right) > 0 \tag{36}$$

and

$$Q_1^{*E} = Q_2^{*E} = \frac{1}{3}, \Delta Q_{12}^{*E} = 0. \tag{37}$$

From (33), we see that $\frac{\partial}{\partial\mu}\Delta Q_{12}^{*A} = \frac{\partial}{\partial c_{2H}}\Delta Q_{12}^{*A} = 0$. From (34), we can easily show that $\frac{d}{d\mu}\Delta Q_{12}^{*B} > 0$ and $\frac{\partial}{\partial c_{2H}}\Delta Q_{12}^{*B} = 0$.

From (35), we see that $\frac{\partial}{\partial\mu}\Delta Q_{12}^{*C} \leq 0$ for $c_{2H} \geq 2$ and $\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu$.

From (36), we have $\frac{d\Delta Q_{12}^{*D}}{d\mu} > 0$ for $2 < \mu < 4$. From (37), we obviously see that $\Delta Q_{12}^{*E} = 0$ and $\frac{\partial \Delta Q^{*E}}{\partial \mu} = 0$.

Next, we investigate the effects of the difference in the profit of the cost efficient firm 1 and the inefficient firm 2 for the high-quality good H at every equilibrium when the relative superiority of the high-quality good $\mu$ and the relative cost ratio $c_{2H}$ of the high-quality good *change*.

We denote by $\Delta\pi^{*k} \equiv \pi_1^{*k} - \pi_2^{*k}$ the difference in the profit of the efficient firm 1 for the high-quality good H and the inefficient firm 2 at equilibrium $k(= A, B, C, D, E)$.

Note that $\mu$ stands for the relative consumers' monetary estimate of the high-quality good over the low-quality good, $V_H/V_L = \mu > 1 = V_L$. Let $\Delta\mu$ ($\Delta c_{2H}$) denote the variation of $\mu$ ($c_{2H}$). $\Delta\mu >$ (<) 0 implies that the firms' product innovation of the high-quality good succeeds in improving (fails to improve) the relative consumers' monetary estimate of the high-quality good compared to their estimate of the low-quality one. $\Delta c_{2H} >$ (<) 0 implies that the process innovation on the high-quality good of the efficient firm 1 (the inefficient firm 2) succeeds in undermining the relative cost efficiency of the high-quality good for the inefficient firm 2 (the efficient firm 1).

Because $\Delta\pi^{*A} = 0$, we obviously see that $\frac{\partial}{\partial\mu}\Delta\pi^{*A} = \frac{\partial}{\partial c_{2H}}\Delta\pi^{*A} = 0$. From (17), we have $\frac{d}{d\mu}\Delta\pi^{*B} = \frac{d}{d\mu}(\frac{1}{4\mu-1}(\mu^2 - 3\mu + 1)) > 0$ if $1 \leq \mu$ and $\frac{\partial}{\partial c_{2H}}\Delta\pi^{*B} = 0$. From (24), we can show that $\frac{\partial\Delta\pi^{*C}}{\theta\mu} > 0 \Leftrightarrow$For $2 < c_{2H}$,then$\frac{\partial\Delta\pi^{*C}}{\theta\mu} = \frac{1}{12\mu^2(\mu-1)^2}(c_{2H}^2 - 4)(4\mu^2 - 2\mu - 1) > 0 \Leftrightarrow \mu > 4$. From (28), $\frac{d\Delta\pi^{*D}}{d\mu} > 0 \Leftrightarrow \mu > 2$ because $2 < \mu < 4$. From (32), $\frac{\partial\Delta Q_{12}^{*E}}{\partial\mu} = 0$ and $\frac{\partial\Delta\pi^{*E}}{\partial\mu} > 0$ for $\mu > c_{2H} + 1$, $1 < c_{2H} < 2$.

In summary, we can obtain the next proposition on the effect of changing $\Delta Q_{12}^{*k}$ and $\Delta\pi^{*k}, k = A, B, C, D, E$, when the relative superiority of the high-quality good $\mu$ and the relative cost ratio $c_{2H}$ of high-quality good H change. The proof of the proposition is provided in the appendix.

**Proposition 2** *At equilibrim k , we have* $\Delta Q_{12}^{*k} > 0$, *k=B, C, D, however* $\Delta Q_{12}^{*A} = \Delta Q_{12}^{*E} = 0$. *At equilibrium A, changing the relative consumers' monetary estimate of the high-quality good over the low-quality one* $\mu$ *and the relative cost ratio of high-quality good* $c_{2H}$ *has no effect on* $\Delta Q_{12}^{*A}$ and $\Delta\pi^{*A}$ . *At equilibria B and D,* $\Delta Q_{12}^{*B}$, $\Delta\pi^{*B}$, $\Delta Q_{12}^{*D}$, and $\Delta\pi^{*D}$ *increase as the relative consumers' monetary estimate of high-quality good* $\mu$ *increases but there is no effect on* $\Delta Q_{12}^{*B}$, $\Delta\pi^{*B}$, $\Delta Q_{12}^{*D}$, and $\Delta\pi^{*D}$ *when the relative cost ratio of the high-quality good* $c_{2H}$ *changes. At equilibrium C ,* $\Delta Q_{12}^{*C}$ *decreases but* $\Delta\pi^{*C}$ *increases as the relative consumers' monetary estimate of high-quality good* $\mu$ *increases. However, both* $\Delta Q_{12}^{*C}$ *and* $\Delta\pi^{*C}$ *increase as* $c_{2H}$ *increases. At equilibrium E, changing the extent of product innovation* $\mu$ *and the relative cost ratio of the high-quality good* $c_{2H}$ *has no effect on* $\Delta Q_{12}^{*E}$, *but* $\Delta\pi^{*E}$ *increases as the relative consumers' monetary estimate of high-quality good* $\mu$ *or the relative cost ratio of high-quality good* $c_{2H}$ *increases.*

The results in Proposition 2 are summarized graphically in Figure 2.

[Insert Figure 2 here]

In equilibrium A, the relative consumers' monetary estimate of the high-quality good $\mu$ is too low, so both firms supply the same output, consisting of only low-quality good L, to the market, irrespective of the relative cost ratio of the high-quality good $c_{2H}$. Hence, any change of $\mu$ and $c_{2H}$ in area A has no effect on the difference in the total output and profit between firms, since $\Delta Q_{12}^{*A} = 0$ and $\Delta\pi^{*A} = 0$.

In equilibrium D, the relative consumers' monetary estimate of the high-quality good $\mu$ is relatively low but the relative cost ratio of the high-quality good $c_{2H}$ is not as low. The inefficient firm 2 stops producing the high-quality good and specializes in supplying the low-quality good L but the efficient firm 1 supplies high-quality good H to the market, as well as low-quality good L. Hence, $\Delta\mu > 0$ in the area D brings production substitution from the low-quality good L to the high-quality-good H in firm 1's product line, but there is no change in firm 2's product line. However, the total output $Q_1^{*D}$ of firm 1 for both goods L and H increases because the increase in the output of good H surpasses the decrease in the output of good L. Consequently, if the relative consumers' monetary estimate of high-quality good $\mu$ increases, then $\Delta Q_{12}^{*D}$ and $\Delta\pi^{*D}$ increase. However, $\Delta c_{2H}$ has no effect because the inefficient firm 2 never produces the high-quality good H in equilibrium D.

In equilibrium B, $\mu$, the relative superiority of consumers' estimate of the high quality good is higher and the relative cost efficiency $c_{2H}$ is not as high as in the D equilibrium. Hence, firm 1 stops producing the low-quality good L and specializes in producing the high-quality good H. In contrast, the inefficient firm 2 continues to specialize in producing good L because its relative cost inefficiency of good H compared to that of firm 1 is strong in this area. However, when $\mu$ is sufficiently high, firm 1 stops supplying good L and increases its output of good H, so firm 2 increases its output of good L. The increment of the former surpasses that of the latter so that $\Delta Q_{12}^{*B}(> 0)$ is increasing in $\mu$. In addition, the difference in the profits of the two firms, $\Delta\pi^{*B}$also increases as $\mu$ increases because the markup of the high-quality good H is larger than that of the low-quality one .

In equilibrium C, $\mu$, the relative superiority of consumer's monetary estimate of the high quality good is much higher but the relative cost inefficiency $c_{2H}$ is not higher than that in equilibrium B. Therefore, the efficient firm 1 keeps specializing in supplying good H but the inefficient firm 2 supplies the high-quality good H, as well as good L, because the margin of good H is large enough for the inefficient firm 2 to produce good

H. The difference in the firms' total output, $\Delta Q_{12}^{*C}$, decreases (increases) as $\mu$ ($c_{2H}$) increases because the decrease in the output of good H (the total output of firm 1) by firm 1 outweighs the increase of the resultant total putput of firm 2 by the production substitution from good L (H) to good H (L) in firm 2. In consequent, the difference in the firms' total market shares $\Delta Q_{12}^{*C}$ shrinks. The difference in the firms' profits $\Delta\pi^{*C}$ is increasing (decreasing) in $\mu$ ($c_{2H}$) because the increase of $\mu$ ($c_{2H}$) expands (shrinks) the mark-ups from good H of both firms.

In equilibrium E, the relative cost inefficiency $c_{2H}$ is very low. Hence, firm 1 begins to supply low-quality good L to the market, as well as good H. In this equilibrium, $\Delta Q_{12}^{*E} = 0$ because the quantities of production substitution from one good to another offset each other as $\mu$ ($c_{2H}$) increases. Note that in equilibrium E, $Q_1^{*E} = Q_2^{*E} = 1/3$, from (29). This implies that both good H and good L are perfectly substituted in each firm, so changing $\mu$ or $c_{2H}$ causes *production substitution* between good H and good L *only within* each firm, but, *subsequently*, the *production substitution of each good between the cost-efficient firm 1 and the cost-inefficient firm 2*. However, the difference in the firms' profit $\Delta\pi^{*E}$ as $\mu$ ($c_{2H}$) increases because the increase of both $\mu$ and $c_{2H}$ enhances the cannibalization from good L to good H in both firms and efficient firm 1's markup on good H for producing good H is larger than that of the inefficient firm 2.

# 4 Welfare Analysis

In this section, we first define social welfare. We then present the social welfare in the equilibria derived in the preceding section and we compare the equilibrium social welfare in the five cases. We define social welfare $W^{*k}$, for $k = A, B, C, D$, and $E$, and the social surplus as the sum of the consumer surplus $CS^{*k}$ and the producer surplus $PS^{*k}$:

$$W^{*k} = CS^{*k} + PS^{*k}, k = A, B, C, D \text{ and } E.$$

We define $CS^{*k}$ and $PS^{*k}$ as

$$\begin{aligned} CS^{*k} &\equiv \int_{\underline{\theta}^{*k}}^{\widehat{\theta}^{*k}} (\theta - p_L^{*k}) d\theta + \int_{\hat{\theta}^{*k}}^{1} (\mu\theta - p_H^{*k}) d\theta \\ &= \frac{1}{2}\left[\mu + (1-\mu)(\widehat{\theta}^{*k})^2 - (\underline{\theta}^{*k})^2\right] - p_L^{*k}(\widehat{\theta}^{*k} - \underline{\theta}^{*k}) - p_H^{*k}(1 - \widehat{\theta}^{*k}) \end{aligned} \quad (38)$$

and

$$\begin{aligned} PS^{*k} &\equiv \pi_1^{*k} + \pi_2^{*k} \\ &= (p_H^{*k} - c_{1H}^{*k})q_{1H}^{*k} + p_{1L}^{*k}Q_{1L}^{*k} + (p_H^{*k} - c_{2H}^{*k})q_{2H}^{*k} + p_{2L}^{*k}Q_{2L}^{*k}. \end{aligned} \quad (39)$$

Then, from (38) and (39), the social surplus is defined as

$$\begin{aligned} W^{*k}(\hat{\theta}^{*k}) &\equiv \int_{\underline{\theta}^{*k}}^{\hat{\theta}^{*k}} \theta d\theta + \int_{\hat{\theta}^{*k}}^{1} \mu\theta d\theta - c_{iH}q_{iH}^{*k} - c_{jH}^{*k}q_{jH}^{*k} \quad (40) \\ &= -\frac{\mu-1}{2}\left(\hat{\theta}^{*k}\right)^2 + \frac{\mu}{2} - \frac{1}{2}(\underline{\theta}^{*k})^2 - c_{iH}q_{iH}^{*k} - c_{jH}^{*k}q_{jH}^{*k}, i, j = 1, 2, j \neq i. \end{aligned}$$

For case A, from (3), (2), (39), (12), (38), and (40), we have

$$\widehat{\theta}^{*A} = 1, \underline{\theta}^{*A} = p_L^{*A} = \frac{1}{3}, CS^{*A} = \int_{\frac{1}{3}}^{1} \left(\theta - \frac{1}{3}\right) d\theta = \frac{2}{9}$$

and

$$W^{*A}(\hat{\theta}^{*A}) = PS^{*A} + CS^{*A} = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}. \tag{41}$$

For case B, from (3), (2), (39), (16), (38), and (40), we obtain

$$\widehat{\theta}^{*B}(\mu) = \frac{2(\mu+1)}{4\mu-1}, \underline{\theta}^{*B}(\mu) = P_L^{*B}(\mu) = \frac{\mu+1}{4\mu-1}, CS^{*B} = \frac{1}{2\left(4\mu-1\right)^2}\left(4\mu^3 - 7\mu^2 + 9\mu - 5\right)$$

and

$$W^{*B}(\mu) = W^{*B}(\hat{\theta}^{*B}(\mu)) = \frac{1}{2\left(4\mu-1\right)^2}\left(12\mu^3 - 29\mu^2 + 31\mu - 3\right). \tag{42}$$

For case C, from (3), (2), (39), (23), (38), and (40), we have

$$\begin{aligned}
\widehat{\theta}^{*C}(\mu, c_{2H}) &= \frac{1}{6\mu(\mu-1)}\left(2\mu^2 + 2c_{2H}\mu + c_{2H} - 2\right), \underline{\theta}_L^{*C} = P_L^{*C} = \frac{1}{6\mu}\left(2\mu - c_{2H} + 2\right), \\
CS^{*C}(\mu, c_{2H}) &\equiv CS^{*C}(\widehat{\theta}^{*C}(\mu, c_{2H}) \\
&= \frac{1}{72\mu\left(\mu-1\right)}\left(16\mu^3 - 16(c_{2H}+2)\mu^2 + 4\left(c_{2H}+5\right)\left(c_{2H}+1\right)\mu + \left(5c_{2H}+2\right)\left(c_{2H}-2\right)\right)
\end{aligned}$$

and

$$\begin{aligned}
W^{*C}(\mu, c_{2H}) &\equiv W^{*C}(\hat{\theta}^{*C}(\mu, c_{2H})) \\
&= \frac{1}{72\mu\left(\mu-1\right)}(32\mu^3 - 32(c_{2H}+2)\mu^2 + 4(11c_{2H}^2 - 6c_{2H} + 19)\mu \\
&\quad - \left(17c_{2H} - 22\right)\left(c_{2H} - 2\right)).
\end{aligned} \tag{43}$$

For case D, from (3), (2), (39), (27), (38), and (40), we have

$$
\begin{aligned}
\widehat{\theta}^{*D}(\mu) &= \frac{1}{2}\frac{\mu}{\mu-1}, \underline{\theta}_L^{*D} = P_L^{*D} = \frac{1}{3}, CS^{*D} = \frac{1}{72(\mu-1)}\left(9\mu^2 - 20\mu + 20\right), \\
W^{*D}(\hat{\theta}^{*D}(\mu)) &= \frac{1}{72(\mu-1)}\left(27\mu^2 - 76\mu + 76\right). \qquad (44)
\end{aligned}
$$

For case E, from (3), (2), (39), (31), (38), and (40), we have

$$
\begin{aligned}
\widehat{\theta}^{*E}(\mu, c_{2H}) &= \frac{1}{3\mu-3}(\mu + c_{2H}), \underline{\theta}^{*E} = p_L^{*E} = \frac{1}{3}, \\
CS^{*E}(\mu, c_{2H}) &\equiv CS^{*E}(\widehat{\theta}^{*E}(\mu, c_{2H})) \\
&= \frac{1}{18(\mu-1)}\left(4\mu^2 - 4(c_{2H}+2)\mu + (c_{2H}+5)(c_{2H}+1)\right), \\
W^{*E}(\mu, c_{2H}) &\equiv W^{*E}(\hat{\theta}^{*E}(\mu, c_{2H})) \\
&= \frac{1}{18(\mu-1)}\left(8\mu^2 - 8(c_{2H}+2)\mu + (11c_{2H}^2 - 6c_{2H} + 19)\right). \qquad (45)
\end{aligned}
$$

In observing the non-negativity conditions for the equilibrium outputs of these four equilibria, (10), (25), (14) and (20), we can ensure that the value of the upper bound of the condition in case A is exactly the same as that of the lower bound in case D, that of the upper bound in case D is also the same as that of the lower bound in case B, and so on.

Hence, we obtain the following proposition on the equilibrium welfare in the five cases. The proof is provided in the appendix.

**Lemma 2**

$\frac{\partial}{\partial\mu}W^{*D}(\mu) > 0$ *for* $2 < \mu < 4$ *in case D.* $\frac{\partial}{\partial\mu}W^{*B}(\mu) > 0$ *for* $4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4})$ *in Case B.* $\frac{\partial}{\partial\mu}W^{*C}(\mu, c_{2H}) > 0$ *for* $\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu,\ 2 \leq c_{2H}$*;* $\frac{\partial}{\partial c_{2H}}W^{*C}(\mu, c_{2H}) > 0$ *for* $c_{2H} \geq 2,\ \frac{1}{2}(2 + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) \leq \mu < \frac{11}{8}c_{2H} + \frac{1}{8}\sqrt{121c_{2H}^2 - 134c_{2H} + 121} - \frac{3}{8}$*;* $\frac{\partial}{\partial c_{2H}}W^{*C}(\mu, c_{2H}) \leq 0$ *for* $c_{2H} \geq 2,\ \frac{11}{8}c_{2H} + \frac{1}{8}\sqrt{121c_{2H}^2 - 134c_{2H} + 121} - \frac{3}{8} \leq \mu$*.* $\frac{\partial}{\partial\mu}W^{*E}(\mu, c_{2H}) > 0$ *for* $1 < c_{2H} < 2, \mu > 2c_{2H} > 2$*.* $\frac{\partial}{\partial c_{2H}}W^{*E}(\mu, c_{2H}) > 0$ *for* $\frac{1}{4}(11c_{2H} - 3) > \mu > 2c_{2H}$*,* $\frac{\partial}{\partial c_{2H}}W^{*E}(\mu, c_{2H}) \leq 0$ *for* $2c_{2H} < \frac{1}{4}(11c_{2H} - 3) \leq \mu$*.*

The results presented in Lemma 2 are summarized graphically in Figure 3.

[Insert Figure 3 here]

An increase in the relative superiority of the consumers' estimate of the high quality good $\mu$ leads to an increase in the equilibrium output of good H in the market. This increases both: (i) the sum of consumers' willingness to pay (positive effect on welfare), $\partial\left(\int_{\underline{\theta}^{*k}}^{\hat{\theta}^{*k}} \theta d\theta + \int_{\hat{\theta}^{*k}}^{1} \mu\theta d\theta\right)/\partial\mu$; and (ii) the firms' relative production costs (negative effect on welfare), $-c_{iH}(\partial q_{iH}^{*k}/\partial\mu) - c_{jH}^{*k}(\partial q_{jH}^{*k}/\partial\mu)$, from (40). Lemma 2 implies that the direct effect (i) is stronger than the indirect effect (ii), so that the increase in $\mu$ improves social welfare.

Furthermore, in two cases, C and E, the marginal cost of good H for firm 2 impacts the social welfare because firm 2 produces good H in these cases. Lemma 2 means that, in these combinations of the product line, the social welfare is convex in $c_{2H}$.

The social welfare in this model consists of: (a) the sum of consumers' willingness-to-pay, (b) the production cost of good H for firm 1 ($c_{1H} = 1$), and (c) the production cost of good H for firm 2 ($c_{2H} > 1 = c_{1H}$).

- (a) The sum of consumers' willingness-to-pay (indirect effect)

$$\frac{\partial\left(\int_{\underline{\theta}^{*C}}^{\hat{\theta}^{*C}}\theta d\theta+\int_{\hat{\theta}^{*C}}^{1}\mu\theta d\theta\right)}{\partial c_{2H}}=\frac{4-22\mu^2+18\mu^3-c_{2H}(5+4\mu)}{36\mu(\mu-1)},$$
$$\frac{\partial\left(\int_{\underline{\theta}^{*E}}^{\hat{\theta}^{*E}}\theta d\theta+\int_{\hat{\theta}^{*E}}^{1}\mu\theta d\theta\right)}{\partial c_{2H}}=\frac{-(\mu+c_{2H})}{9(\mu-1)}<0.$$

  In case C, this positively (negatively) affects the social welfare if $c_{2H}$ is sufficiently small (large), whereas it always negatively affects the welfare in case E.

- (b) The production cost of good H for firm 1 (indirect effect)

$$\frac{\partial(-q_{1H}^{*C})}{\partial c_{2H}}=\frac{-1}{3(\mu-1)}<0,\quad \frac{\partial(-q_{1H}^{*E})}{\partial c_{2H}}=\frac{-1}{3\mu}<0.$$

  These negatively affect the social welfare.

- (c) The production cost of good H for firm 2 (direct effect)

$$\frac{\partial(-c_{2H}q_{2H}^{*C})}{\partial c_{2H}}=\frac{1-\mu^2+c_{2H}(4\mu-1)}{3\mu(\mu-1)},\quad \frac{\partial(-c_{2H}q_{2H}^{*E})}{\partial c_{2H}}=\frac{4c_{2H}-\mu}{3(\mu-1)}.$$

  In both cases, this negatively (positively) affects the social welfare if $c_{2H}$ is sufficiently small (large).

This implies that, in both cases (C and E), the direct effect of $c_{2H}$ (the third factors) is stronger than any other effects; that is, the social welfare $W^{*C}(\mu, c_{2H})$ and $W^{*E}(\mu, c_{2H})$ decrease (increase) in $c_{2H}$ when $c_{2H}$ is small (large) enough from Lemma 2. Thus, the social welfare in these cases is a convex function in $c_{2H}$.[3] For these two cases, we have

[3] The result of case E corresponds to Proposition 5 in Kitamura and Shinkai (2015b).

the following proposition:

**Proposition 3** *In the multi-product Cournot duopoly equilibria C and E, a reduction of the relative cost inefficiency can decrease social welfare. This may occur even when both firms have an equal market share of the total output of the two goods.*

In Figure 3, in the equilibrium of both cases, the social welfare has a tendency to decrease (increase) when $c_{2H}$ decreases, if $\mu$ is sufficiently small (large) in case E or if $c_{2H}$ is sufficiently large (small) in case C. The reason is that in case E, a decrease in the relatively high (low)-cost good H for firm 2 makes the inefficient firm 2 for production substitution from good L to good H *within* firm 2. It then causes the efficient firm 1's production substitution from good H to good L *between* firm 1 and firm 2, and that makes the social welfare worse. In case C, when $c_{2H}$ is sufficiently large, the social welfare becomes worse as $c_{2H}$ falls because of the production substitution between two firms for high-quality good; that is, relatively inefficient firm 2 sells more good H and efficient firm 1 sells less one.

Lahiri and Ono (1988) show that, in a single-product Cournot oligopoly, reducing the marginal cost of a minor firm with a sufficiently low share can decrease welfare by production substitution. Moreover, they find that, if all the firms have an equal market share, then a cost reduction in any firm improves social welfare. In case C, from proposition 2, $\Delta Q_{12}^{C}$ increases when $\mu$ is small or $c_{2H}$ is large, so that the market share of inefficient firm 2 decreases as $c_{2H}$ decreases. In this case, whereas Proposition 3 states the same as the former result of Lahiri and Ono (1988), the cause of our result differs from that of their results. Our result is caused by both a stronger production substitution from good L to good H within firm *2* and, subsequently, a weaker production substitution of good H between the cost-efficient and inefficient firm by means of a strategic substitute, from (18) as large $c_{2H}$ decreases. However, Lahiri and Ono's (1988) result is only caused by production substitution between the cost-efficient and inefficient firm as the inefficient firm's cost decreases. Meanwhile, in case E, $\Delta Q_{12}^{E} = 0$ means that the two firms have the same market share. In this case, Proposition 3 states that, in our model, a cost reduction can decrease social welfare even though both firms have the same market share; that is, $Q_1^{*E} = Q_2^{*E} = 1/3$, from (33). This implies that both goods H and L are perfectly substituted in each firm, so the changing $\mu$ or $c_{2H}$ causes not only production substitution between good H and good L within each firm but also causes the production substitution of each good between the two firms.

We account for the mechanism of how reducing the relative marginal cost inefficiency decreases social welfare. In the equilibrium that we derive in this study, both goods H and L are perfectly substituted in each firm. Therefore, changing $\mu$ or $c_{2H}$ causes not only production substitution between good H and good L within each firm but also causes the production substitution of each good between the two firms. This occurs because we consider a model in which each firm can afford to produce and supply vertically differentiated multi-products. However, Lahiri and Ono's (1988) result is caused only by the production outputs of each good between the cost-effective and cost ineffective firm because they consider the model in which firms can supply a homogeneous single product but firms have asymmetric production costs.

This may be interpreted as follows: a part of Proposition 3 mimics Lahiri and Ono's (1988) finding but the other parts of Proposition 3 provide a different result that is caused by production substitution arising from the multi-product duopoly.

# 5 Conclusion

In this study, we consider a duopoly game with two vertically differentiated products under non-negative output constraints and an expectation with regard to the rival's product line strategies. We derive an equilibrium for the game and we describe the firms' product line strategies and their realized profits in each equilibrium, based on the goods' quality superiority and relative cost-efficiency.

We also show that the cost efficient firm producing the high-quality good earns more than the inefficient firm does, except in the special case where the relative superiority of the high-quality good $\mu$ is too small compared to the unit cost of the high-quality good H. In this case, both firms specialize in good L, and the market for good L becomes a Cournot duopoly. Thus, both firms' profits are the same. We also show that the social welfare increases as the relative superiority of consumers' estimate of the high quality good $\mu$ increases in all five cases. However, in cases C and E, where the inefficient firm 2 produces good H, the social welfare is convex in the relative cost inefficiency of good H, $c_{2H}$. Thus, in a multi-product Cournot duopoly, a reduction of the relative marginal cost inefficiency decreases social welfare, whereas Lahiri and Ono (1988) find the same result in a single-product Cournot oligopoly. We also find the reason that our result differs from that of Lahiri and Ono (1988). Our result is caused by both a stronger production substitution from good L to good H within firm 2 and, subsequently, a weaker production substitution of good H between the cost-efficient and inefficient firms by means of a strategic substitute, from (18) as the large $c_{2H}$ decreases (in equilibrium C). However, the result of Lahiri and Ono (1988) is only caused by the production substitution between the cost-efficient and inefficient firm as the cost of inefficient firm decreases. Furthermore, we find that this may occur even when both firms have the same market share. In contrast, Lahiri and Ono

(1988) show that a cost reduction in any firm always increases social welfare if the market share is the same among all firms. In addition, we find the reason why this is counter-intuitive to our result. In equilibrium E in our study, both goods H and L are perfectly substituted in each firm, so changing $\mu$ or $c_{2H}$ causes not only production substitution between good H and good L within each firm but also causes the production substitution of each good between the two firms. This occurs because we consider a model in which each firm can afford to produce and supply vertically differentiated multi-products. In contrast, the result of Lahiri and Ono (1988) is caused only by the production substitution of the goods between the cost-effective and cost ineffective firms because they consider a model in which firms can supply a homogeneous single product but the firms have asymmetric production costs.


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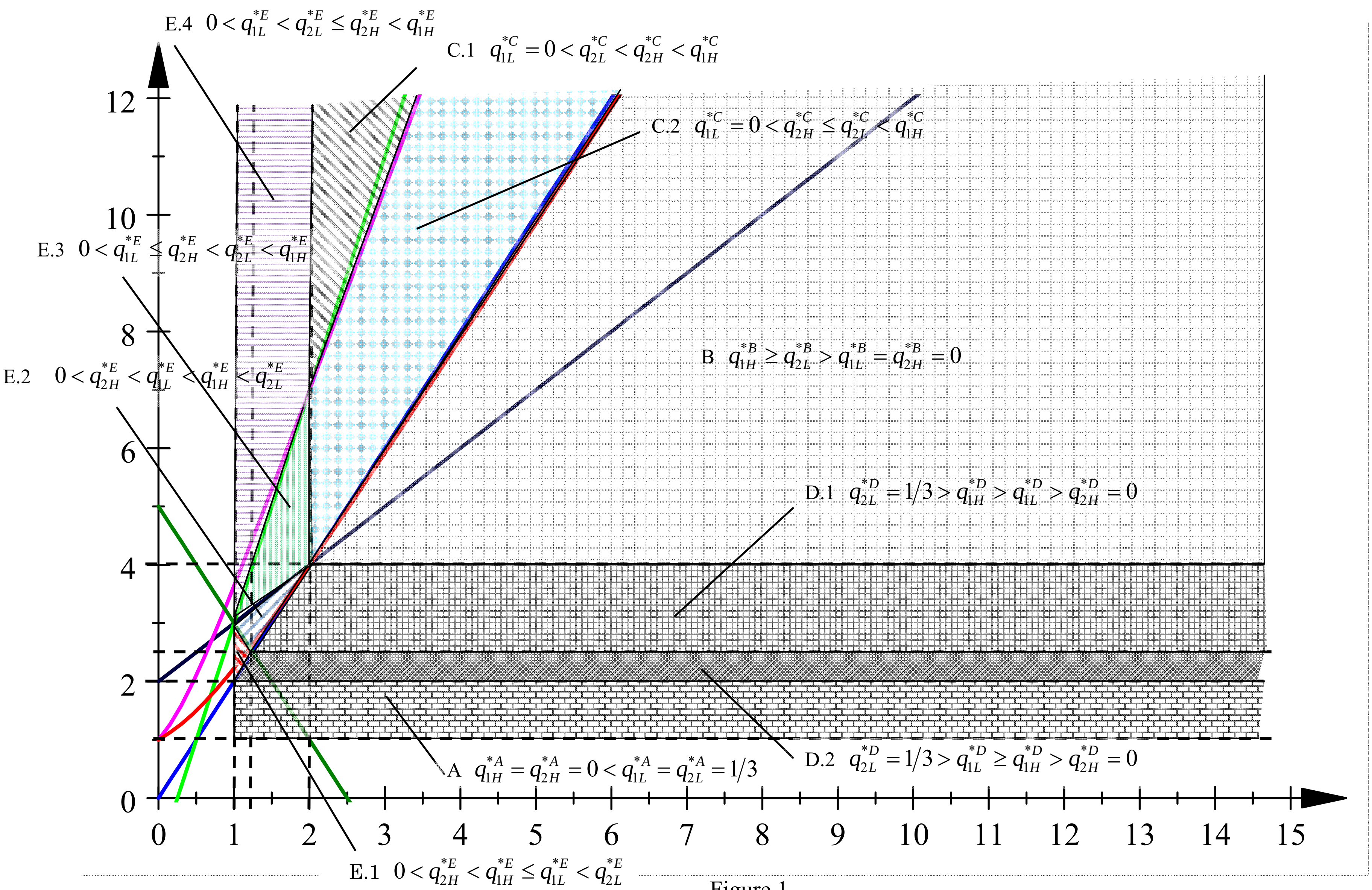
# 6 Appendix

**Proof of Proposition 2**

**Proof:** From (33) and lemma 1, we see that $\Delta Q_{12}^{*A} = 0$ and $\Delta\pi^{*A} = 0$, so $\frac{\partial}{\partial\mu}\Delta Q_{12}^{*A} = \frac{\partial}{\partial c_{2H}}\Delta Q_{12}^{*A} = 0$ and $\frac{\partial}{\partial\mu}\Delta\pi^{*A} = \frac{\partial}{\partial c_{2H}}\Delta\pi^{*A} = 0$. From (34) and (17), we have $\Delta Q_{12}^{*B} = \frac{\mu-4}{4\mu-1}$ and $\Delta\pi^{*B} = \frac{1}{4\mu-1}\left(\mu^2 - 3\mu + 1\right)$, so we can show that $\frac{d}{d\mu}\Delta Q_{12}^{*B} = \frac{d}{d\mu}\left(\frac{\mu-4}{4\mu-1}\right) = \frac{15}{(4\mu-1)^2} > 0$ and $\frac{\partial}{\partial c_{2H}}\Delta Q_{12}^{*B} = 0$ and $\frac{d}{d\mu}\Delta\pi^{*B} = \frac{d}{d\mu}\left(\frac{1}{4\mu-1}\left(\mu^2 - 3\mu + 1\right)\right) = \frac{1}{(4\mu-1)^2}(4\mu^2 - 2\mu - 1) > 0$ if $1 \leq \mu$ and $\frac{\partial}{\partial c_{2H}}\Delta\pi^{*B} = 0$. From (35) and (23), $\Delta Q_{12}^{*C} = \frac{1}{2\mu}\left(c_{2H} - 2\right)$ and $\Delta\pi^{*C} = \frac{1}{12\mu(\mu-1)}(8(c_{2H}-1)\mu^2 - 4(c_{2H}-1)(c_{2H}+3)\mu + (c_{2H}+2)(c_{2H}-2))$, so we can show that $\frac{\partial}{\partial\mu}\Delta Q_{12}^{*C} = -\frac{1}{2\mu^2}\left(c_{2H} - 2\right) \leq 0$ for $c_{2H} \geq 2$ and $4 < \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu$,

$\frac{\partial\Delta\pi^{*C}}{\theta\mu} = \frac{1}{12\mu^2(\mu-1)^2}(c_{2H}^2 - 4)(4\mu^2 - 2\mu - 1) \geq (<)0 \Leftrightarrow \mu \leq \frac{1}{2}(c_{2H} + 1 - \sqrt{c_{2H}^2 + c_{2H} + 1}) < 1$ or $1 < \frac{1}{2}(c_{2H} + 1 + \sqrt{c_{2H}^2 + c_{2H} + 1}) \leq \mu \quad (1 < \mu < \frac{1}{2}(c_{2H} + 1 + \sqrt{c_{2H}^2 + c_{2H} + 1}))$. From

(36) and (28), $\Delta Q_{12}^{*D} = \frac{1}{2(\mu-1)}(\mu-2)$ and $\Delta\pi^{*D} = \pi_1^{*D} - \pi_2^{*D} = \frac{1}{4(\mu-1)}(\mu-2)^2$. Hence, we have $\frac{d\Delta Q_{12}^{*D}}{d\mu} = \frac{d}{d\mu}(\frac{1}{2(\mu-1)}(\mu-2)) = \frac{1}{2(\mu-1)^2} > 0$, $\frac{d\Delta\pi^{*D}}{d\mu} = \frac{1}{4}\frac{\mu}{(\mu-1)^2}(\mu-2) > 0 \Leftrightarrow \mu > 2$, since $2 < \mu < 4$ at the equilibrium D from (25), and the result holds. From (37) and (32), we see that $\Delta Q_{12}^{*E} = 0$ and $\Delta\pi^{*E} = \frac{1}{3(\mu-1)}(c_{2H}-1)(2\mu - c_{2H} - 3)$ .So $\frac{\partial\Delta Q^{*E}}{\partial\mu} = 0$ and $\frac{\partial\Delta\pi^{*E}}{\partial\mu} = -\frac{2}{3(\mu-1)}(c_{2H} - \mu + 1) > 0$ for $\mu > c_{2H}+1$, $1 < c_{2H} < 2$ at the equilibrium E and the result follows. ■


E.4 $0 < q_{1L}^{*E} < q_{2L}^{*E} \le q_{2H}^{*E} < q_{1H}^{*E}$
C.1 $q_{1L}^{*C} = 0 < q_{2L}^{*C} < q_{2H}^{*C} < q_{1H}^{*C}$
C.2 $q_{1L}^{*C} = 0 < q_{2H}^{*C} \le q_{2L}^{*C} < q_{1H}^{*C}$
E.3 $0 < q_{1L}^{*E} \le q_{2H}^{*E} < q_{2L}^{*E} < q_{1H}^{*E}$
B $q_{1H}^{*B} \ge q_{2L}^{*B} > q_{1L}^{*B} = q_{2H}^{*B} = 0$
E.2 $0 < q_{2H}^{*E} < q_{1L}^{*E} < q_{1H}^{*E} < q_{2L}^{*E}$
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E.1 $0 < q_{2H}^{*E} < q_{1H}^{*E} \le q_{1L}^{*E} < q_{2L}^{*E}$
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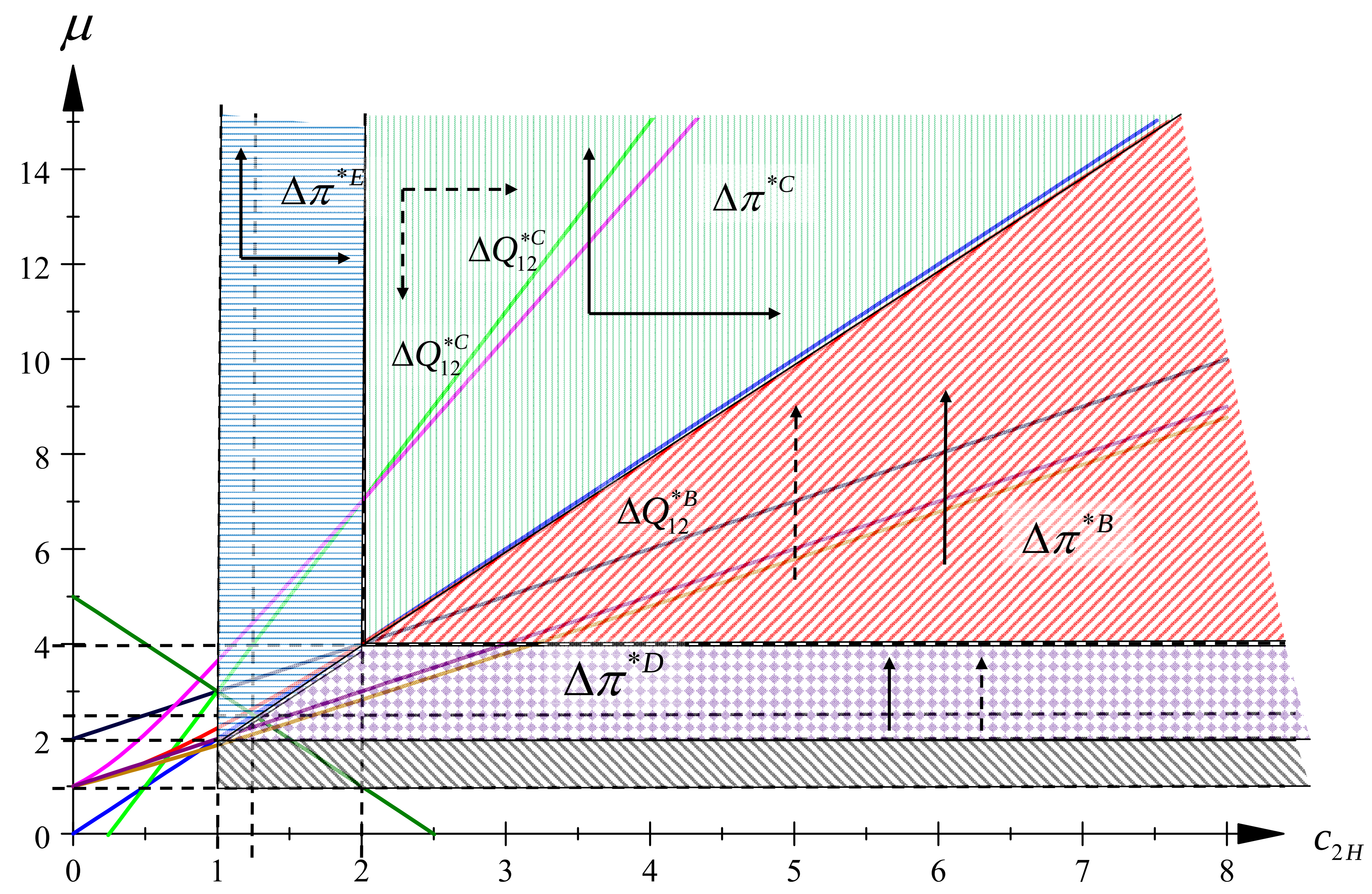
Figure 1




$\Delta\pi^{*k} \equiv \pi_1^{*k} - \pi_2^{*k}, k = A, B, C, D, E.$ $\Delta Q_{12}^{*k} \equiv Q_1^{*k} - Q_2^{*k}, k(=A, B, C, D, E)$ $\Delta\pi^{*A} = 0$ $\Delta Q_{12}^{*A} = \Delta Q_{12}^{*E} = 0, \Delta Q_{12}^{*k} > 0, k = B, C, D.$

↑ ( ↓ ) implies that $\Delta\pi^{*k}$ is increasing (decreasing) in $\mu$. - - - → ( ← - - - ) implies that $\Delta Q_{12}^{*k}$ increasing (decreasing ) in $c_{2H}$ $, k = B, C, D, E.$

Figure 2 Comparative Statics of $\Delta\pi^{*k}$ w.r.t. $\mu$ and $c_{2H}$

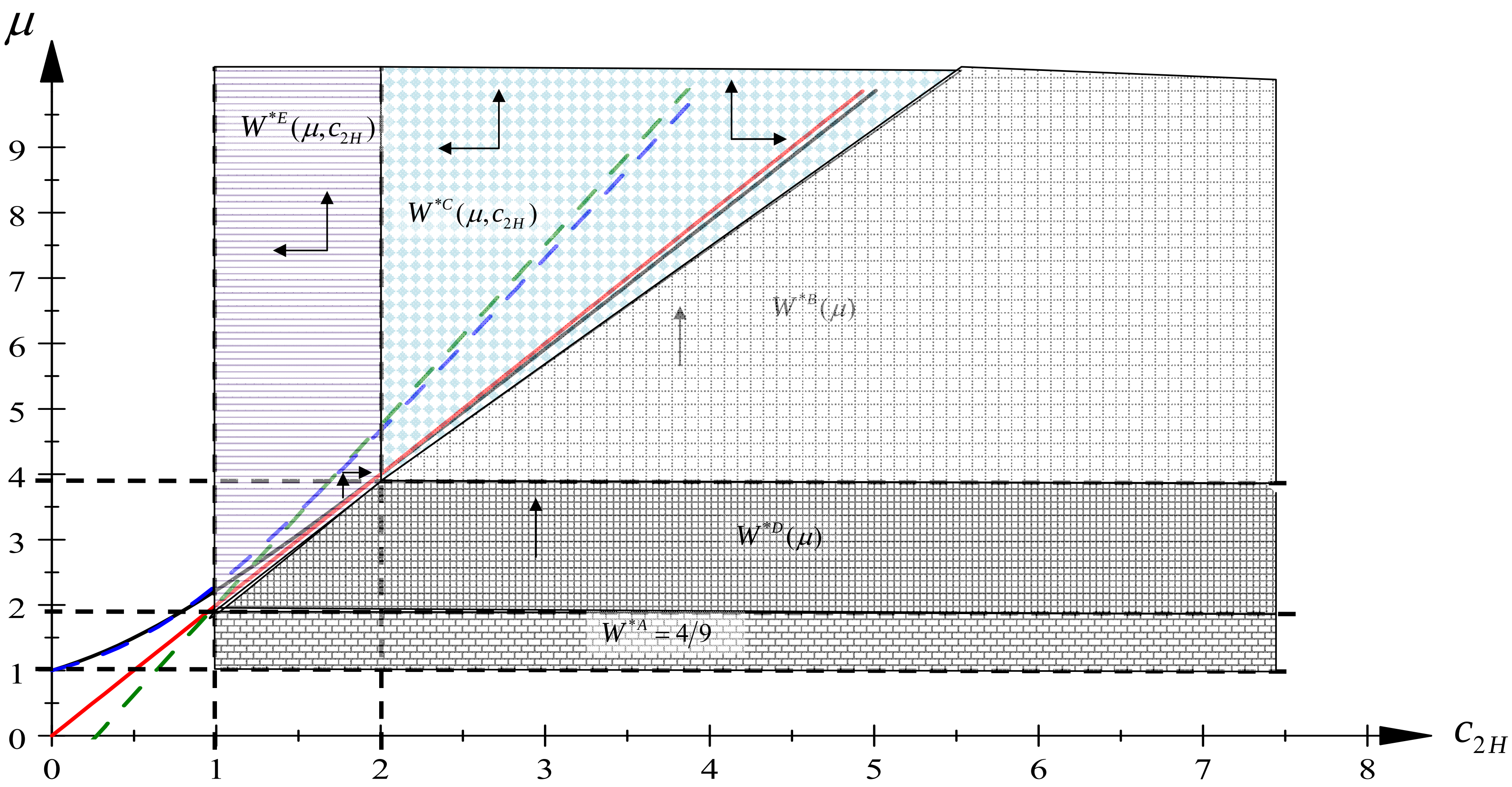



Figure 3 Welfare Comparison and Comparative Statics on $\mu$ and $c_{2H}$

$\mu = \frac{1}{2}(2c + \sqrt{4c^2 - 2c + 4})$ Black ↑ implies that $W^{*k}(\mu)$ or $W^{*k}(\mu, c_{2H})$ increasing in $\mu$. ⟶ implies that $W^{*k}(\mu)$ or $W^{*k}(\mu, c_{2H})$ increasing in $c_{2H}$. ⟵ implies that $W^{*k}(\mu)$ or $W^{*k}(\mu, c_{2H})$ decreasing in $c_{2H}$.

$\mu = 2c$ Red $\mu = \frac{1}{4}(11c - 3)$ Green $\mu = \frac{11}{8}c + \frac{1}{8}\sqrt{121c^2 - 134c + 121} - \frac{3}{8}$ Light Blue