The Political Economy of Optional Public Service

by

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Abstract

We characterize the optional public service policy that minimizes the expected cost of delivering the requisite public service (e.g., jury or military service). We then examine whether a majority rule voting procedure will implement the optional public service policy (OPS) whenever it entails lower cost than mandatory public service (MPS). We find that majority rule often is biased against OPS in the sense that it implements MPS when OPS would secure the requisite public service at lower cost.

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1 Introduction

Societies often require citizens to perform important "public service" activities (e.g., jury service or military service). In particular, citizens typically are not permitted to pay a fee to secure an exemption from performing these activities. This is the case even though such an exemption policy could admit Pareto gains. Specifically, an individual for whom the public service is particularly onerous might be willing to pay for an exemption from the service more than another individual would require in order to willingly perform the service.

Many reasons have been offered for why an individual's ability to purchase an exemption from public service might be limited in practice. For instance, such ability might unfairly favor relatively wealthy individuals. Furthermore, a public service like jury service might play a valuable role in informing citizens about the merits of the jury system.[1] Mandatory jury service might also help to ensure a "trial by peers" or increase the likelihood that the most capable jurors actually perform jury service (e.g., Bowles, 1980).[2]

We add to this list of potential explanations for the prevalence of mandated public service another, seemingly more subtle, explanation. We demonstrate that a majority voting procedure can entail a systematic bias against policies that allow individuals to pay a fee to avoid performing the service. This bias can arise even when none of the considerations identified above are present, so the only costs associated with a public service policy are the administrative costs of implementing the policy and the direct personal costs that individuals incur when they perform the service.[3]

For concreteness, suppose the public service in question is jury service and a society must choose between an optional jury service policy (OJS) and a mandatory jury service policy

[1] The Department of Justice in Western Australia (2018) observes that "Undertaking jury duty can ... provide a positive appreciation of the court system and is an opportunity to learn how the justice system works."

[2] Bowles (1980, p. 371) also notes that "Juries [may] serve the useful function of reinforcing public confidence in the operation of the legal system." Hans (2008) notes the potential value of jury service in "educating citizens about legal concepts and legal procedures, promoting a sense of procedural justice, legitimizing the justice system, and increasing civic engagement."

[3] These personal costs include opportunity costs, and can be negative if individuals derive pleasure from performing the public service.

(MJS). We specify conditions under which majority voting entails a bias against OJS in the sense that a majority of citizens will vote for MJS even though expected welfare would be higher under OJS. We consider self-financing OJS policies which specify: (i) an exemption fee that any individual can pay to avoid jury service; and (ii) a payment that is made to any individual who voluntarily performs jury service. All jury service payments and policy administrative and implementation costs are financed by the stipulated exemption fee under a self-financing OJS policy.

The bias against OJS arises when it is more costly to implement and administer than MJS. We show that if OJS and MJS entail the same administrative costs, an exemption fee and jury service payment can be designed to ensure that all individuals prefer OJS to MJS. Consequently, all individuals will vote for OJS rather than MJS, so majority voting always implements the welfare-maximizing jury service policy (i.e., OJS).

OJS cannot secure Pareto gains when it is more costly to administer than MJS.[4] Consequently, as long as the additional administrative cost of OJS is not too pronounced, some individuals will prefer an optimal (welfare-maximizing) OJS policy to MJS and others will prefer MJS to OJS. The individuals who prefer OJS are those with the highest and the lowest personal costs (c) of performing jury service. Individuals with the highest c's gain when they are permitted to avoid jury service by paying the exemption fee. Individuals with the lowest c's gain when they are paid to perform jury service. In contrast, individuals with moderate c's will prefer MJS to OJS because such c's will both exceed the optimal jury service payment and be exceeded by the optimal exemption fee.[5]

Even when OJS cannot secure Pareto gains, majority voting is not necessarily biased

[4] The extra costs of implementing OJS include the costs of collecting and processing exemption fees and delivering jury service payments.

[5] Epple and Romano (1996) identify corresponding equilibrium preferences in a setting where majority rule determines the quality of a public good (e.g., public education) that will be implemented and financed with a proportional income tax. In their model, individuals with moderate income favor a relatively high level of quality for the public good. In contrast, individuals with the lowest and the highest incomes prefer lower levels of quality. The preference of low-income individuals reflects their limited wealth. The preference of high-income individuals reflects in part their equilibrium consumption of a substitute good (e.g., private education).

against OJS. To illustrate, we show that majority rule always implements the welfare-maximizing form of jury service when the distribution of c in the population is uniform. In this case, the expected personal gains from OJS exactly offset the corresponding losses both for individuals who elect to perform jury service and for those who choose not to perform jury service. Consequently, majority rule implements OJS if and only if OJS increases aggregate expected welfare in this special case.

More generally, though, majority voting often entails a bias against OJS. This bias arises, for instance, when the number of individuals eligible to perform jury service (N) is sufficiently large relative to the number of trials (T) and when individuals with the lowest c's are relatively common in the population (e.g., when the mean value of c (c^e) exceeds the median value of c (c^d)). In this case, nearly all individuals choose not to perform jury service under OJS, so the personal gain from OJS increases with c for nearly all c realizations. In this case, when the individuals who prefer MJS (primarily those with the lowest c realizations) are relatively common in the population, a majority will vote for MJS even when expected welfare would be higher under OJS.

We show that the bias against OJS often persists when N/T assumes moderate, empirically relevant values,[6] and when c has standard, symmetric bell-shaped densities. The bias against OJS that arises in these settings stems in part from the aforementioned fact that the individuals who benefit the most from OJS are those with the highest and the lowest c realizations. Under many relevant conditions, majority voting does not adequately capture the intensity of these individuals' preferences for OJS, so a majority will vote for MJS even when expected welfare would be higher under OJS.

We cannot claim that the bias against optional public service policies that we document is the primary reason for the limited use of such policies in practice. Rather, the bias against

[6] Jester (2017) estimates that approximately 1.5 million individuals perform jury service annually in the U.S., so that on average, the likelihood that an individual performs jury service in a given year is approximately 0.09%. Collinson (2016) reports that 179,200 individuals (approximately 0.3% of the population) served on juries in England and Wales in 2015. Journalonline (2016) estimates that the likelihood of being called for jury service is approximately three times greater in Scotland than in England and Wales.

optional policies that we identify may best be viewed as a heretofore under-appreciated additional potential explanation for the limited popularity of these policies in practice.

The ensuing discussion focuses on jury service, for concreteness. However, the analysis is relevant in other settings where a jurisdiction decides whether to make the performance of an important public service mandatory or optional. As noted above, the service in question might be military service, which presently is optional in the United States. However, the service has been mandatory in the past, and individuals have at times been permitted to pay a fee to avoid military service.[7] Alternatively, the public service might entail the provision of health care services in under-served regions of a country. In many countries, newly-trained medical professionals are required to practice their specialty for a specified period of time in regions where relatively few established professionals choose to serve. In some of these countries (e.g., Bolivia, Ethiopia, Ghana, India, Indonesia, Peru, South Africa, and Thailand), affected individuals can pay a fee to avoid this service. To illustrate, the fee in the Meghalaya and Tamil Nadu States in India is 1 million rupees, which is approximately $21,000$ U.S. dollars (Frehywot et al., 2010).[8]

Our analysis of the choice between mandatory and optional public service proceeds as follows. Section 2 explains our formulation of MJS and OJS. Section 3 describes the properties of a welfare-maximizing OJS policy. Section 4 examines when a majority rule voting procedure introduces a bias for or against OJS. Section 5 considers a modified form of OJS in which requests for exemption from jury service are not always granted. Section 6 concludes and suggests directions for future research. The proofs of the formal conclusions are presented in the Appendix.[9]

[7] This was the case during the Civil War (Warner and Asch, 2001; Perri, 2013). Ross (1994) and Asal et al. (2017), for example, analyze the choice between mandatory and optional military service in other countries.

[8] Wibulpolprasert and Pengpaibon (2003) discuss corresponding fees (whose magnitudes have varied over time) in Thailand.

[9] The proofs of the formal conclusions in Sections 3 and 4 are presented in Appendix A. Appendix B provides additional conclusions. The proofs of the formal conclusions in Section 5 are presented in Bose et al. (2018).

2 Mandatory and Optional Jury Service Programs

We consider a setting in which N individuals are eligible to serve as jurors on $T \geq 1$ trials. For simplicity, each trial is assumed to require a single juror.[10] The $N > T$ individuals differ in their personal cost ($c \in [\,\underline{c}, \overline{c}\,]$) of performing jury service. This cost might reflect a financial opportunity cost of performing jury service, for example.[11] $\underline{c}$ can be negative, reflecting the possibility that some individuals may enjoy serving as a juror or derive personal pleasure from performing what they regard to be their civic duty. We model each individual's personal cost of performing jury service (c) as an independent draw of a random variable with distribution function $G(c)$ and density function $g(c)$.

Under *mandatory jury service* (MJS), each of the N individuals must serve on a jury when called upon to do so. Jurors are summoned randomly, so the probability that an individual performs jury service under MJS is $\frac{T}{N}$. The compensation paid to an individual who performs jury service under MJS is normalized to 0. Consequently, the expected welfare of an individual with personal cost c is $W_M(c) = \frac{T}{N}[-c\,]$ under MJS. The corresponding level of expected social welfare is:

$$W_M^e \;=\; -\,\frac{T}{N}\,c^e,\ \text{where}\ c^e \;\equiv\; \int_{\underline{c}}^{\overline{c}} c\; dG(c)\,. \tag{1}$$

Under *optional jury service* (OJS), each of the N potential jurors can either "opt in" or "opt out" of jury service. An individual who opts in agrees to be called to perform jury service. If $N_i \in [\,T, N\,]$ individuals opt in under OJS, the probability that any particular one of these individuals is called to perform jury service is $\frac{T}{N_i}$. An individual who performs jury service under OJS receives the fixed payment $w > 0$. An individual who opts in but is not called to perform jury service receives no payment (and makes no payment). Therefore, the expected welfare of an individual with personal cost c who opts in under OJS

[10] If each trial requires R jurors, then the ensuing analysis remains valid if T is replaced by $\widetilde{T} = RT$.

[11] Bowles (1980) suggests that self-employed individuals may face a relatively high opportunity cost of serving on juries because they do not have an employer who continues to pay normal wages while jury service is being performed.

is $W_i(c) = \frac{T}{N_i}[\, w - c\,]$.

An individual who opts out under OJS makes payment $F > 0$ and is thereby exempted from jury service. The expected welfare of such an individual is $W_o(c) = -F$. An individual with cost c will prefer to opt in than to opt out under OJS if:

$$W_i(c) \;>\; W_o(c) \quad \Leftrightarrow \quad \frac{T}{N_i}[\, w - c\,] \;>\; -F\,. \tag{2}$$

$\widehat{c}$ will denote the cost realization for which an individual is indifferent between opting in and opting out under OJS. Expression (2) implies that if $\widehat{c} \in (\, \underline{c}, \overline{c}\,)$, then $\widehat{c}$ is defined by:

$$W_i(\widehat{c}\,) \;=\; W_o(\widehat{c}\,) \quad \Leftrightarrow \quad \widehat{c} \;=\; w + \frac{F\,N_i}{T}\,. \tag{3}$$

Expressions (2) and (3) imply that individuals with $c \in [\,\underline{c}, \widehat{c}\,]$ opt in and individuals with $c \in (\,\widehat{c}, \overline{c}\,]$ opt out under OJS.[12] Thus, as might be expected, the individuals who find jury service to be relatively onerous are the ones who opt out.

$A \geq 0$ will denote the additional social cost of implementing OJS rather than MJS. This cost includes the additional administrative cost associated with contacting citizens to determine which ones wish to opt out, collecting and processing payments from those who do opt out, and delivering payments to individuals who perform jury service.[13] We consider OJS policies that are self-financing in the sense that the sum of the incremental cost A and payments to jurors ($T\,w$) cannot exceed the expected fees paid by individuals who are exempted from jury service. These expected fees are the product of the fee (F), the number of eligible individuals (N), and the fraction of these individuals who opt out $(\,1 - G(\widehat{c}\,)\,)$. Consequently, the *financing constraint* under OJS is:

$$T\,w + A \;\leq\; [\,1 - G(\widehat{c}\,)\,]\,N\,F\,. \tag{4}$$

We also consider OJS policies that induce an adequate number of individuals to volunteer to perform jury service. In particular, we require an OJS policy to satisfy the following

[12] For expositional ease, we assume that an individual opts in when he is indifferent between opting in and opting out under OJS.

[13] This additional cost might also include expenses associated with establishing the legality of and securing citizen participation in a new policy.

adequate jury pool constraint, which states that at least T individuals are expected to opt in:[14]

$$N\,G(\widehat{c}) \;\geq\; T\,. \tag{5}$$

An optimal OJS policy is one that maximizes expected welfare while satisfying the financing and adequate jury pool constraints. Expected welfare under OJS is:

$$W_O^e \;=\; \int_{\underline{c}}^{\widehat{c}} \frac{T}{N_i}\,[\,w - c\,]\,dG(c) - [\,1 - G(\widehat{c}\,)\,]\,F \;-\, A\,. \tag{6}$$

3 Properties of an Optimal OJS

We now identify the properties of an optimal OJS policy. Welfare is maximized by allowing as many high-cost individuals as possible to opt out of jury service. Therefore, as Lemma 1 reports, $\widehat{c}$ is set to ensure there are just enough jurors to cover the number of trials, in expectation. Consequently, as the number of potential jurors increases (holding constant the number of trials), a larger fraction of potential jurors opt out of performing jury service.

Lemma 1. *The financing and adequate jury pool constraints bind under an optimal OJS. Because* $G(\widehat{c}\,) \;=\; \frac{T}{N}\,,\; \frac{\partial\,\widehat{c}}{\partial N} \;<\; 0$ *and* $\widehat{c} \rightarrow \underline{c}$ *as* $N/T \;\rightarrow\; \infty$.

If OJS is relatively costly ($A > 0$), some individuals experience a reduction in welfare when OJS replaces MJS. However, if A is not too large, two groups of individuals benefit when an optimal OJS replaces MJS. First, individuals with the highest personal costs of performing jury service benefit when OJS allows them to pay F to avoid jury service. Second, individuals with the lowest personal costs benefit when they are paid w to perform jury service, which is not particularly onerous for them. The individuals whose welfare declines when OJS replaces MJS are those with intermediate c's, as Lemma 2 reports.

[14] The key qualitative conclusions drawn below would persist if OJS were designed to ensure that the probability that at least T individuals opt in exceeds a specified minimum level.

Lemma 2. *When an optimal OJS replaces MJS: (i) welfare increases for individuals with the lowest c's ($c \in [\underline{c}, c_1]$ where $c_1 \equiv \widehat{c} - \frac{A}{N-T}$) and the highest c's ($c \in [c_2, \overline{c}]$ where $c_2 \equiv \widehat{c} + \frac{A}{T}$); whereas (ii) welfare declines for individuals with intermediate c's ($c \in (c_1, c_2)$).*

Figure 1 illustrates how an individual's net gain from OJS varies with his personal cost of performing jury service (c). This net gain is $W_O(c) - W_M(c)$, where $W_O(c) \equiv \max\{W_i(c), W_o(c)\}$.

[**Figure 1 here**]

For individuals who opt in under OJS (i.e., those with $c \in [\underline{c}, \widehat{c})$), the net gain from OJS declines with c ($W'_{\Delta i}(c) < 0$) because jury service becomes more onerous as c increases and the probability of performing jury service after opting in under OJS is higher than under MJS (i.e., $1 > \frac{T}{N}$). For individuals who opt out under OJS (i.e., for $c \in (\widehat{c}, \overline{c}]$), the net gain from OJS increases with c ($W'_{\Delta o}(c) > 0$) because the personal cost that is avoided by opting out of jury service increases with c.

Lemma 3 identifies the slopes of the two linear segments in Figure 1.

Lemma 3. *Under an optimal OJS, the rate at which an individual's expected increase in welfare from OJS (relative to MJS) varies with c is:*

$$W'_{\Delta i}(c) = W'_i(c) - W'_M(c) = -\frac{N-T}{N} < 0 \quad \text{for } c \in [\underline{c}, \widehat{c}); \tag{7}$$

$$W'_{\Delta o}(c) = W'_o(c) - W'_M(c) = \frac{T}{N} > 0 \quad \text{for } c \in (\widehat{c}, \overline{c}]. \tag{8}$$

Corollary 1. $$|W'_{\Delta i}(c)| \gtreqless |W'_{\Delta o}(c)| \Leftrightarrow \frac{N-T}{N} \gtreqless \frac{T}{N} \Leftrightarrow N \gtreqless 2T. \tag{9}$$

Corollary 1 indicates that the declining line segment in Figure 1 is more steeply sloped than the increasing segment when $N > 2T$. In this case, the probability of performing jury service under MJS is less than $\frac{1}{2}$. Consequently, due to the relatively large increase in the probability of performing jury service for an individual who opts in under OJS, his net benefit from OJS declines relatively rapidly with c.[15]

Before analyzing the jury service policy that will be adopted under a majority rule voting procedure, we consider as a benchmark the special case in which OJS is no more costly to implement than MJS.

Lemma 4. *If $A = 0$, then OJS can be designed to ensure that every individual secures at least the level of expected welfare he secures under MJS, and that nearly all individuals secure strictly higher levels of expected welfare.*

OJS can secure the welfare gains identified in Lemma 4 by setting $F = \frac{T}{N}\,\widehat{c}$ and $w = \left[\frac{N-T}{N}\right]\widehat{c}$, where $G(\widehat{c}) = \frac{T}{N}$. This policy effectively charges $\frac{T}{N}\,\widehat{c}$ for an exemption from jury service and sets the payment for performing jury service to satisfy the financing constraint (in expression (4)) as an equality.[16] The identified values of w and F leave the individual with $c = \widehat{c}$ indifferent between OJS and MJS and indifferent between opting in and opting out under OJS.[17] Individuals with $c > \widehat{c}$ secure strict gains from OJS because these gains increase with c. (Recall expression (8).) Individuals with $c < \widehat{c}$ also secure strict gains from OJS because these gains increase as c declines (recall expression (7)) due to the higher probability of performing jury service after opting in under OJS.

When $A > 0$, OJS can still enhance welfare by reducing the cost at which the requisite jury service is performed. However, the limited set of policy instruments under OJS (i.e., w and F) does not allow compensation for performing jury service or payments for avoiding jury service to finely match the associated individual costs and benefits. Consequently, whenever OJS is more costly to implement than MJS, the cost savings from the more efficient delivery

[15] When $N = 2T$, the probability of performing jury service under MJS is $\frac{1}{2}$. Therefore, for an individual who opts in under OJS, the probability of performing jury service increases by $\frac{1}{2}$. For an individual who opts out under OJS, the probability of performing jury service declines by $\frac{1}{2}$.

[16] If $A = 0$, $F = \frac{T}{N}\,\widehat{c}$, $w = \left[\frac{N-T}{N}\right]\widehat{c}$, and $G(\widehat{c}) = \frac{T}{N}$, then $T\,w + A = T\left[\frac{N-T}{N}\right]\widehat{c} = \left[1 - \frac{T}{N}\right]N\left[\frac{T}{N}\right]\widehat{c} = \left[1 - G(\widehat{c})\right]N\,F$.

[17] When the individual with $c = \widehat{c}$ opts in under this policy, his welfare is $w - \widehat{c} = \left[\frac{N-T}{N}\right]\widehat{c} - \widehat{c} = -\frac{T}{N}\,\widehat{c}$. This is precisely the welfare he secures if he opts out under OJS ($-F = -\frac{T}{N}\,\widehat{c}$) and his expected welfare under MJS.

of jury services under OJS cannot be allocated to generate Pareto gains. This conclusion is recorded formally in Lemma 5.

Lemma 5. *Suppose $A > 0$. Then an OJS policy that secures a strict increase in expected welfare for some individuals (relative to MJS) necessarily reduces the expected welfare of some other individuals.*

4 Majority Voting for Jury Policy

We now examine any biases that may arise when the choice between MJS and OJS is determined by a majority rule voting procedure ("majority rule"). Proposition 1 identifies a setting in which majority rule always implements the welfare-maximizing jury service policy. This is the setting in which OJS and MJS are equally costly to implement. Recall from Lemma 4 that an optimal OJS policy increases the welfare of (nearly) all individuals in this case, so (nearly) all individuals will vote for the policy that ensures the highest level of expected welfare.

Proposition 1. *Suppose $A = 0$. Then all individuals (weakly) prefer an optimal OJS policy to MJS, and majority rule always implements the (welfare-maximizing) OJS policy.*

When OJS is more costly to implement than MJS (so $A > 0$), OJS cannot be designed to ensure that all individuals prefer OJS to MJS. (Recall Lemma 5.) When A is large, a relatively large number of individuals prefer MJS to OJS because OJS entails a relatively small w and a relatively large F to cover the high cost of implementing OJS.[18] However, when A is small, a relatively large number of individuals may prefer OJS to MJS because OJS secures the requisite jury service at lower personal cost for jurors.

To assess whether majority rule ensures the adoption of the jury service policy that maximizes expected welfare, let A_m denote the largest value of A for which a majority of

[18] When A is sufficiently large, all individuals will prefer MJS to OJS. The ensuing analysis considers settings where $A > 0$ is sufficiently small that some individuals prefer an optimal OJS policy to MJS.

individuals prefer (an optimally designed) OJS to MJS. Also let A_w denote the value of A for which expected welfare is the same under OJS and MJS. If $A_m < A_w$, then for all $A \in (A_m, A_w)$, a majority will vote for MJS even though welfare is maximized under OJS. In this sense, a majority rule voting procedure is biased against OJS. If $A_w < A_m$, then for all $A \in (A_w, A_m)$, a majority will vote for OJS even though welfare is maximized under MJS. In this sense, majority rule is biased in favor of OJS.

Proposition 2 considers the bias that majority rule introduces when the number of potential jurors is large relative to the number of trials.[19]

Proposition 2. *When N/T is sufficiently large, majority rule is biased against OJS when $c^e > c^d$, biased in favor of OJS when $c^e < c^d$, and entails no bias for or against OJS when $c^e = c^d$ (i.e., $A_m \gtreqqless A_w \Leftrightarrow c^e \lesseqqgtr c^d$).*

Proposition 2 reflects the fact that as $N/T \to \infty$, nearly all individuals choose not to perform jury service under an optimal OJS (i.e., $\widehat{c} \to \underline{c}$, from Lemma 1). The primary difference between OJS and MJS in this case is that those who opt out make a small payment in return for the slightly reduced probability of being called to perform jury service. Because the benefit of a reduced likelihood of performing jury service increases with c, individuals with the highest c's prefer OJS to MJS whereas nearly all individuals with the lower c's prefer MJS to OJS. Consequently, when individuals with the lowest c's are relatively common in the population (so $c^d < c^e$), majority rule may implement MJS when aggregate expected welfare would be higher under OJS. Conversely, when individuals with the highest c's are relatively common in the population (so $c^d > c^e$), majority rule may implement OJS when aggregate expected welfare would be higher under MJS.[20]

A different conclusion emerges when the number of potential jurors is very close to the number of trials.

[19] Proposition 2 refers to the mean value of c (c^e) and the median value of c (c^d).

[20] The mean income in a country typically exceeds the median income. Therefore, if the distribution of c is similar to the distribution of income, a majority rule voting procedure will often be biased against OJS in the limiting settings where $N/T \to \infty$.

Proposition 3. *When N/T is sufficiently close to 1, majority rule is biased in favor of OJS when $c^e > c^d$, biased against OJS when $c^e < c^d$, and entails no bias for or against OJS when $c^e = c^d$ (i.e., $A_m \lesseqgtr A_w \Leftrightarrow c^e \lesseqgtr c^d$).*

Proposition 3 reflects the fact that as $N/T \to 1$, nearly all individuals must opt in under OJS to ensure that a juror is available for every trial. The primary difference between OJS and MJS in this case is that those who opt in receive a small payment in return for the slight increase in their probability of performing jury service. Because jury service becomes more onerous as c increases, individuals with the smallest c's prefer OJS to MJS whereas nearly all individuals with the larger c's prefer MJS to OJS. Consequently, when individuals with the lowest c's are relatively common in the population (so $c^d < c^e$), majority rule may implement OJS when aggregate expected welfare would be higher under MJS. Conversely, when individuals with the highest c's are relatively common in the population (so $c^d > c^e$), majority rule may implement MJS when aggregate expected welfare would be higher under OJS.

In practice, N/T typically exceeds 1 and is finite. Therefore, it is important to determine whether majority rule introduces a bias for or against OJS when $N/T > 1$ is finite. Propositions 2 and 3 suggest that majority rule may not entail any bias for or against OJS when the distribution of c is symmetric, so $c^e = c^d$. Proposition 4 confirms the absence of bias for the special symmetric distribution in which all c's are equally likely.

Proposition 4. *Majority rule introduces no bias for or against OJS (so $A_m = A_w$) when $g(c)$ is the uniform density.*

In essence, a uniform density for c eliminates any bias for or against OJS by ensuring that when $A = A_m$, the expected personal gains from OJS exactly offset the corresponding losses, both for individuals who opt in and for those who opt out. Consequently, majority rule implements OJS if and only if it increases aggregate expected welfare.

Two initial observations help to explain this conclusion. First, recall from Lemma 2 that under an optimal OJS, the individuals who prefer OJS to MJS are those with c's in $[\,\underline{c},\, c_1\,] \cup [\,c_2, \overline{c}\,]$, whereas the individuals who prefer MJS to OJS are those with c's in (c_1, c_2). Therefore, when $A = A_m$, the two regions $[\,\underline{c},\, c_1\,] \cup [\,c_2,\, \overline{c}\,]$ and $(c_1,\, c_2)$ must contain the same (expected) number of individuals. Second, for any $\widehat{c} \in (\,\underline{c}, \overline{c}\,)$,[21] the two regions $\left[\,\underline{c},\, \frac{1}{2}\,(\,\underline{c} + \widehat{c}\,)\right] \cup \left[\,\frac{1}{2}\,(\,\widehat{c} + \overline{c}\,)\,,\, \overline{c}\,\right]$ and $\left[\,\frac{1}{2}\,(\,\underline{c} + \widehat{c}\,)\,,\, \frac{1}{2}\,(\,\widehat{c} + \overline{c}\,)\right]$ contain the same (expected) number of individuals when $g(c)$ is the uniform density.[22] These two observations imply that, as depicted in Figure 1, $c_1 = \frac{1}{2}\,[\,\underline{c} + \widehat{c}\,]$ and $c_2 = \frac{1}{2}\,[\,\widehat{c} + \overline{c}\,]$ when $A = A_m$ and $g(c)$ is the uniform density.

The personal gains from OJS decline with c at the constant rate $W'_{\Delta i}(c) = -\frac{N-T}{N}$ for all individuals with $c \in [\,\underline{c},\, \widehat{c}\,]$ because all of these individuals opt in under OJS. (Recall expression (7).) Consequently, gains and losses from OJS are symmetric around c_1, a c realization at which the individual is indifferent between OJS and MJS. This symmetry, the fact that c_1 is the midpoint of $[\,\underline{c},\, \widehat{c}\,]$, and the fact that $g(c)$ is the uniform density imply that the increase in expected welfare associated with OJS on $[\,\underline{c}, c_1\,]$ is equal to the reduction in expected welfare associated with OJS on $[\,c_1,\, \widehat{c}\,]$.

Similarly, the personal gains from OJS increase with c at the constant rate $W'_{\Delta o}(c) = \frac{T}{N}$ for all individuals with $c \in [\,\widehat{c},\, \overline{c}\,]$ because all of these individuals opt out under OJS. (Recall expression (8).) Consequently, gains and losses from OJS are symmetric around c_2, a c realization at which the individual is indifferent between OJS and MJS. This symmetry, the fact that c_2 is the midpoint of $[\,\widehat{c},\, \overline{c}\,]$, and the fact that $g(c)$ is the uniform density imply that the reduction in expected welfare associated with OJS on $[\,\widehat{c},\, c_2\,]$ is equal to the increase in expected welfare associated with OJS on $[\,c_2,\, \overline{c}\,]$. Therefore, expected welfare gains from OJS (relative to MJS) are equal to expected welfare losses from OJS when $A = A_m$, which ensures $A_m = A_w$ (and the absence of any bias for or against OJS under majority rule).

[21] Recall that the individual with c realization $\widehat{c} \in (c_1,\, c_2)$ is indifferent between opting in and opting out under OJS.

[22] This is the case because when $g(c)$ is the uniform density, the intervals $\left[\,\underline{c},\, \frac{1}{2}\,(\underline{c} + \widehat{c}\,)\right]$ and $\left[\,\frac{1}{2}\,(\underline{c} + \widehat{c}\,)\,,\, \widehat{c}\,\right]$ contain the same (expected) number of individuals, as do the intervals $\left[\,\widehat{c},\, \frac{1}{2}\,(\widehat{c} + \overline{c}\,)\right]$ and $\left[\,\frac{1}{2}\,(\widehat{c} + \overline{c}\,)\,,\, \overline{c}\,\right]$.

We now consider whether the absence of bias for or against OJS persists more generally when $N/T > 1$ is finite. We focus on settings where the most extreme realizations of c are relatively unlikely, so the density of c has the common bell (or inverted-U) shape.[23] We begin by considering symmetric densities with $c^e = c^d$. For analytic tractability, we initially consider piecewise linear densities. Formally, we normalize $[\, \underline{c}, \overline{c}\,]$ to be $[\, 0, 2\,]$ without loss of generality,[24] and assume that for $a \in [\, 0, \frac{1}{2})$:

$$g(c) \;=\; \begin{cases} a + [\, 1 - 2\, a\,]\, c & \text{if} \quad 0 \le c \le 1 \\ a + [\, 1 - 2\, a\,]\, [\, 2 - c\,] & \text{if} \quad 1 \le c \le 2\,. \end{cases} \tag{10}$$

This density increases at the constant rate $1 - 2\, a > 0$ on $[\, \underline{c}, c^e\,]$ and declines at the corresponding rate on $[\, c^e, \overline{c}\,]$. Proposition 5 reports that majority rule entails a systematic bias against OJS when $g(c)$ has this inverted-V shape.[25]

Proposition 5. *For any finite $N/T > 1$ and $a \in [\, 0, \frac{1}{2}\,)$, majority rule is biased against OJS (so $A_m < A_w$) when $g(c)$ is as specified in expression (10).*

Numerical solutions reveal a corresponding systematic bias against OJS when $g(c)$ has an inverted-U shape or a bell shape rather than an inverted-V shape.[26] We employ the Beta density to derive numerical solutions because it can assume a wide variety of shapes as its parameters (α, β) change. The Beta density, which has positive support on $[\, 0, 1\,]$, is:

$$g(c) \;=\; \frac{c^{\alpha-1}\, [\, 1 - c\,]^{\beta-1}}{B(\alpha, \beta)} \;\; \text{for } c \in [\, 0, 1\,]\,, \text{ where } B(\alpha, \beta) \;\equiv\; \int_0^1 x^{\alpha-1}\, [\, 1 - x\,]^{\beta-1}\, dx.$$

This density has an inverted-U shape when $\alpha = \beta \in (\, 1, 2\,]$ and a bell shape when $\alpha > 2$ and

[23] Additional findings for U-shaped densities are presented in Appendix B.

[24] The proof of Proposition 5 demonstrates that this normalization is without loss of generality.

[25] Observe that as $\alpha \to \frac{1}{2}$, the density in expression (10) converges to the uniform density. Recall from Proposition 4 that $A_m = A_w$ if $g(c)$ is the uniform density. Proposition 5A in the Appendix demonstrates that $A_m \to A_w$ as $\alpha \to \frac{1}{2}$ when $g(c)$ is as specified in expression (10).

[26] General analytic conclusions are difficult to derive for nonlinear $g(c)$ densities in part because when $N/T >$ 1 is finite, a non-trivial set of individuals with the smallest c realizations prefer OJS to MJS, as do individuals with the highest c realizations. Consequently, the properties of the entire distribution of c, rather than just the mean and median of the distribution, can determine whether a majority rule voting procedure will be biased for or against OJS.

$\beta > 2$. As Figure 2 illustrates, this density is symmetric about its mean ($\frac{\alpha}{\alpha+\beta}$) when $\alpha = \beta$, skewed to the left (with $c^e > c^d$) when $\beta > \alpha > 1$, and skewed to the right (with $c^e < c^d$) when $\alpha > \beta > 1$.

[**Figure 2 here**]

Initially consider the symmetric Beta density with $\alpha = \beta = 2$, so $c^e = c^d = \frac{1}{2}$.[27] Table 1 identifies for the setting where $A = A_m$: (i) the cost realization, $\widehat{c}$, at which the individual is indifferent between opting in and opting out under OJS; and (ii) the boundaries of the regions of c realizations in which individuals prefer OJS to MJS ($[\underline{c}, c_1] \cup [c_2, \overline{c}]$) and the region in which individuals prefer MJS to OJS ((c_1, c_2)). The Table also identifies the values of A_m and A_w for a range of values of $N/T > 1$.[28] Table 1 reports a systematic bias of majority rule against OJS (i.e., $A_m < A_w$ for all $N/T > 1$).[29]

N	T	$\widehat{c}\,(A_m)$	$c_1\,(A_m)$	$c_2\,(A_m)$	A_m	A_w	$\frac{A_m}{A_w}$
1,001	1,000	0.981639	0.499368	0.982121	0.482	0.488	0.98871
1,010	1,000	0.941395	0.494350	0.945866	4.470	4.611	0.96946
1,100	1,000	0.814010	0.459152	0.849496	35.486	37.820	0.93828
2,000	1,000	0.500000	0.326352	0.673648	173.648	187.500	0.92610
2,500	1,000	0.432931	0.293355	0.642295	209.364	226.017	0.92632
5,000	1,000	0.287141	0.214110	0.579263	292.122	314.238	0.92962
10,000	1,000	0.195800	0.157078	0.544296	348.495	371.916	0.93702
50,000	1,000	0.084038	0.075329	0.510781	426.743	444.391	0.96029
100,000	1,000	0.058903	0.054399	0.505702	446.799	460.932	0.96934
1,000,000	1,000	0.018370	0.017888	0.500632	482.262	487.772	0.98870

Table 1. Outcomes for the Beta Density with $\alpha = \beta = 2$.

[27] This density is illustrated in Figure 2.

[28] Table 1 takes the number of trials to be 1,000, for expositional ease. Mize et al. (2007) report there were 148,558 state jury trials and 5,463 federal jury trials in the U.S. in 2006. If the values of N and T in Table 1 were to be multiplied by $k > 0$, the values of A_m and A_w would also be multiplied by k. The entries in the other columns in the table would not change.

[29] Table 1 also reports that the value of A_m/A_w varies non-monotonically with N/T. This finding reflects Propositions 2 and 3, which conclude that when $c^e = c^d$, $A_m/A_w \to 1$ as $N/T \to \infty$ and as $N/T \to 1$. The value of A_m/A_w in the last column of Table 1 (and in the last column of Table 2 below) reflects the actual values of A_m and A_w, not the rounded values that appear in the two preceding columns.

This bias persists in other symmetric Beta densities. Numerical solutions reveal that $A_m < A_w$ for all $N/T > 1$, regardless of the value of $\alpha = \beta$.[30]

Majority voting also often introduces a bias against OJS when $g(c)$ is an asymmetric density with $c^e > c^d$. Although this bias does not prevail as $N/T \to 1$ (recall Proposition 3), Table 2 indicates that the bias often arises whenever N/T is even slightly larger than 1. The table records outcomes for the setting in which $g(c)$ is the Beta density with parameters $\alpha = 2$ and $\beta = 3$. This density has a bell shape and is skewed to the left (with $c^e > c^d$), as illustrated in Figure 2.[31]

N	T	$\widehat{c}\,(A_m)$	$c_1\,(A_m)$	$c_2\,(A_m)$	A_m	A_w	$\frac{A_m}{A_w}$
1,000.01	1,000	0.986381	0.385722	0.986387	0.00601	0.00600	1.01842
1,000.7565	1,000	0.941753	0.385304	0.942174	0.42095	0.42095	1.00000
1,010	1,000	0.859618	0.380607	0.864408	4.790	85.962	0.96664
1,100	1,000	0.690691	0.348258	0.724935	34.243	37.269	0.91881
2,000	1,000	0.385728	0.236127	0.535329	149.601	165.533	0.90375
2,500	1,000	0.329167	0.210774	0.506756	177.589	196.258	0.90488
5,000	1,000	0.212317	0.152349	0.452189	239.872	264.366	0.90735
10,000	1,000	0.142559	0.111415	0.422854	280.295	307.478	0.91159
50,000	1,000	0.060140	0.053309	0.394839	334.699	360.327	0.92888
100,000	1,000	0.041999	0.038478	0.390564	348.566	372.203	0.93650
1,000,000	1,000	0.013023	0.012649	0.386268	373.245	391.337	0.95377

Table 2. Outcomes for the Beta Density with $\alpha = 2$ and $\beta = 3$.

Table 3 reports corresponding conclusions for other Beta densities with $c^e > c^d$. The first two columns of the table specify the relevant values of the parameters of the Beta density, α and β. The third and fourth columns present the mean and median of the density. The last column reports the smallest value of N/T ($\frac{\widetilde{N}}{T}$) for which $\frac{A_m}{A_w} > 1$ for $N/T \in (1, \frac{\widetilde{N}}{T})$ and $\frac{A_m}{A_w} < 1$ for $N/T > \frac{\widetilde{N}}{T}$ (so majority rule introduces a bias against OJS when $N/T > \frac{\widetilde{N}}{T}$).

[30] This conclusion reflects numerical solutions that consider all values of $\alpha = \beta \in (1, 25]$ in increments of 1, all values of $N/T \in (1, 2]$ in increments of 0.001, all values of $N/T \in [2, 10]$ in increments of 0.01, all values of $N/T \in [10, 1000]$ in increments of 1, and all values of $N/T \in [1,000, 10,000]$ in increments of 10.

[31] The mean of this density is 0.4. The median of this density is 0.385728.

Table 3, like Table 2, reports that majority rule often introduces a bias against OJS whenever N/T is even slightly larger than 1.

α	β	c^e	c^d	$\frac{\widetilde{N}}{T}$
1	2	0.33333	0.29289	1.03030
1	3	0.25000	0.20630	1.00660
1	4	0.20000	0.15910	1.00180
2	3	0.40000	0.38573	1.00070
2	4	0.33333	0.31381	1.00040
2	5	0.28571	0.26445	1.00015
3	4	0.42857	0.42141	1.00002
3	5	0.37500	0.36412	1.00002
3	6	0.33333	0.32052	1.00001

Table 3. Bias Against OJS for $N/T > \frac{\widetilde{N}}{T}$.

To understand the bias against OJS that often arises under majority rule when $c^e \geq c^d$ and N/T is finite, recall from Proposition 2 that this bias prevails when $N/T \to \infty$.[32] In this case, nearly all individuals opt out of OJS, so $\widehat{c} \to \underline{c}$ and the private gain from OJS increases with c at a constant rate for nearly all individuals. Now consider the changes that arise as N/T becomes finite. A non-negligible fraction of individuals now opt in under OJS, so $\widehat{c}$ increases above $\underline{c}$ and a nontrivial region $[\,\underline{c}, c_1\,]$ arises in which individuals prefer OJS to MJS under an optimal OJS policy.

To assess the impact of these changes, suppose $A = A_m$, so one-half of the population prefers OJS to MJS. When $[\,\underline{c}, c_1\,]$ expands as N/T declines, $[\,c_2, \overline{c}\,]$ contracts sufficiently to ensure that exactly half of the population continues to prefer OJS to MJS. Let S_O denote the set of individuals who prefer OJS to MJS. The expansion of $[\,\underline{c}, c_1\,]$ adds to S_O individuals whose net gain from OJS is relatively sensitive to c. The contraction of $[\,c_2, \overline{c}\,]$ removes from S_O individuals whose net gain from OJS is relatively insensitive to c. (Recall from Corollary 1 that $|\,W'_{\Delta i}(c)\,| > |\,W'_{\Delta o}(c)\,|$ for all $N/T > 2$.) When $g(c)$ has an inverted-U shape or a bell shape, the lowest c realizations are relatively unlikely. Consequently, to ensure S_O

[32] The bias against OJS is strict in the sense that $A_m < A_w$ when $c^e > c^d$. There is no bias (so $A_m = A_w$) when $N/T \to \infty$ and $c^e = c^d$.

includes exactly half of the population as N/T declines, $[\,\underline{c}, c_1\,]$ expands by more than $[\,c_2, \overline{c}\,]$ contracts. The net effect is to introduce greater variation in the net gain from OJS (i.e., greater variation in the intensity of preference for OJS) within S_O.

The outcome of majority rule depends only on the number of individuals who prefer OJS to MJS, not the intensity of the preference for OJS. Therefore, majority rule fails to account fully for the increased intensity of preference for OJS as N/T declines toward 2, promoting a bias against OJS (i.e., $A_m < A_w$).

Before proceeding to consider modified OJS policies, we briefly consider settings in which $c^e < c^d$. Proposition 2 suggests that majority rule may introduce a bias in favor of OJS in these settings when N/T is relatively large. As Table 4 indicates, the requisite magnitude of N/T varies with the extent to which c^d exceeds c^e. Table 4 reports the same variables that are reported in Table 3, this time for settings where the Beta density has $c^e < c^d$.

α	β	c^e	c^d	$\frac{\widetilde{N}}{T}$
2	1	0.66667	0.70711	34
3	2	0.60000	0.61427	1,323
3	1	0.75000	0.79370	151
4	3	0.57134	0.57859	44,320
4	2	0.66667	0.68619	2,424
4	1	0.80000	0.84090	531
5	4	0.55556	0.55985	1,766,300
5	3	0.62500	0.63588	48,320
5	2	0.71429	0.73555	5,883
5	1	0.83333	0.87055	1,754

Table 4. Bias in Favor of OJS for $N/T > \frac{\widetilde{N}}{T}$.

Table 4 indicates that when c^d/c^e is close to 1 (e.g., when $\alpha = 4$ and $\beta = 3$ or when $\alpha = 5$ and $\beta = 4$), a bias in favor of OJS emerges only when N/T is pronounced (e.g., greater than

40,000). In contrast, when c^d/c^e is considerably larger than 1 (e.g., when $\alpha = 2$ and $\beta = 1$), a bias in favor of OJS can arise for substantially smaller values of N/T (e.g., 34).

5 More Limited Exemption from Jury Service

We have shown that the individuals with the highest c's will all choose not to perform jury service under a welfare-maximizing OJS policy. Some might view this outcome to be "unfair." This outcome might also reduce the performance of the jury system if individuals with the highest c's are the best jurors. In addition, if individuals with similar c's are deemed to be the closest peers, then the identified outcome might be viewed as limiting the likelihood of a "trial by peers" for individuals with high c's.[33]

Consider, then, a modified OJS policy in which an individual's request for exemption from jury service is approved with probability $p \in (0, \overline{p}\,]$, where $\overline{p} < 1$. The individual pays the fee $F > 0$ if (and only if) his request is approved. If the request is denied, the individual joins those who agree to perform jury service in the pool of eligible jurors. The expected number of individuals in this pool is $N_i = N\,[\,1 - p\,(\,1 - G(\widehat{c}\,)\,)\,]$.[34] An individual in this pool is paid $w > 0$ if and only if he is called upon to perform jury service. Each individual in the pool is summoned to perform jury service with probability $\frac{T}{N_i} \in (0, 1\,]$.

Throughout the ensuing analysis, we assume $\overline{p}$ is sufficiently small relative to N/T that the adequate jury pool requirement, $N_i \geq T$, is not constraining when identifying the welfare-maximizing (optimal) OJS policy.[35] It is readily shown that $p = \overline{p}$ under a welfare-maximizing OJS policy in this case. Thus, requests for exemption from jury service are granted with the highest permissible probability. This outcome allows jury service to be performed by those for whom it is least costly to the greatest extent possible. It is

[33] As more individuals with high c's opt out of jury service, defendants with low c's will be more likely to receive a "trial by peers" to the extent that individuals with similar c realizations are "peers."

[34] $\widehat{c} \in [\underline{c}, \overline{c}\,]$ continues to represent the c realization for which the individual is indifferent between opting in (i.e., agreeing to perform jury service when called upon to do so) and opting out (i.e., requesting to be exempted from performing jury service).

[35] This assumption helps to ensure that the findings reported below in Propositions 6 and 7 do not simply reflect the forces that underlie the conclusions in Propositions 4 and 5. The adequate jury pool requirement will not be constraining when $\overline{p} \leq 1 - \frac{T}{N}$.

also readily shown that under a welfare-maximizing OJS policy, the individuals who prefer OJS to MJS are those for whom $c \in [\underline{c}, \overline{c}_1] \cup [\overline{c}_2, \overline{c}]$, where $\overline{c}_1 = \widehat{c} - \frac{A}{T} \frac{N_i}{\overline{p}[1-G(\widehat{c})]N}$ and $\overline{c}_2 = \widehat{c} + \frac{A}{T} \frac{N_i}{\overline{p}G(\widehat{c})N}$. Individuals with $c \in (\overline{c}_1, \overline{c}_2)$ prefer MJS to OJS. Furthermore, $\frac{\partial \widehat{c}}{\partial \overline{p}} < 0$ under an optimal OJS policy, so as a request for exemption from jury service becomes less likely to be granted, fewer individuals request an exemption.

Propositions 6 and 7 report that the qualitative conclusions drawn in Propositions 4 and 5 persist under this modified OJS policy. Specifically, Proposition 6 reports that majority rule introduces no bias for or against this modified OJS policy when $g(c)$ is the uniform density. Proposition 7 reports that majority rule introduces a systematic bias against the modified OJS policy when $g(c)$ has an inverted-V shape.

Proposition 6. *Majority rule entails no bias for or against the modified OJS policy (so $A_m = A_w$) for all $\overline{p} \in (0,1)$ if $g(c)$ is the uniform density.*

Proposition 7. *Majority rule is biased against the modified OJS policy (so $A_m < A_w$) for all $\overline{p} \in (0,1)$ if $g(c)$ is as specified in expression (10).*

Numerical solutions reveal that majority rule also introduces a systematic bias against the modified OJS policy when $g(c)$ is a bell-shaped Beta density with $c^e \geq c^d$.[36] Therefore, the key qualitative conclusions drawn above persist under alternative forms of OJS.[37]

6 Conclusions

We have characterized the optional public service policy that maximizes the expected welfare of individuals who are eligible to perform the service, where an individual's welfare is the difference between the payment he receives and the personal cost (c) he incurs in

[36] This conclusion reflects numerical solutions that consider all values of $\alpha \in [1, 15]$ in increments of 1, all values of $\beta \in [\alpha, 25]$ in increments of 1 (except $\alpha = \beta = 1$), and all values of $\overline{p} \in [0.1, 0.9]$ in increments of 0.01.

[37] The analysis in Appendix B reveals that the bias against OJS that arises when $g(c)$ is U-shaped can be mitigated to some extent when $\overline{p}$ is sufficiently small. This is the case because as $\overline{p}$ declines, the welfare gains and losses from OJS become less sensitive to c (because the individuals with the highest c's who request exemption from jury service are not ensured of exemption). Consequently, any deficiency of majority voting in fully reflecting the intensity of preferences for OJS is ameliorated.

performing the service. We have also identified conditions under which a majority rule voting procedure will implement the optional service policy more or less often than would a social planner who maximizes expected welfare. We found that majority rule often introduces a bias against optional service in the sense that optional service often is not implemented under majority rule even when it would secure greater expected welfare than mandatory service. This is the case, for instance, when the most extreme realizations of c are relatively unlikely, so the density for c has the common bell shape.

The failure of a majority rule voting procedure to systematically implement the least costly public service policy reflects in part the limited set of policy instruments under consideration. Individuals in our model were only permitted to opt in or opt out of the public service in return for a single associated fixed payment. In principle, optional service policies could admit varying probabilities of being exempted from service, p, in return for payments that increase with p.[38] Although such policies likely would be more costly to design, implement, and administer, they have the potential to more closely align private and social benefits under an optional service policy, and therefore might enhance the ability of majority rule to systematically implement the welfare-maximizing public service policy.

A complete assessment of the likelihood that common voting procedures will implement the welfare-maximizing public service policy must account for potential divergence between the private and social benefits of performing public service.[39] As noted in the Introduction, the private and social costs of performing a public service can diverge if an individual's personal cost of performing the service is correlated with his efficacy in performing the service.[40] These private and social costs may also diverge to the extent that the income an

[38] See Panzar and Sibley (1978) and Wilson (1993), for example, for corresponding analyses in settings where individuals can pay higher fees to reduce the probability that their electricity supply will be interrupted.

[39] Such a complete assessment must also account for benefits and costs of mandatory and optional public service programs that were not included in our streamlined model. Several additional potential benefits of mandatory programs are noted in the Introduction. Optional programs may provide additional benefits if they are able to: (i) reduce wasteful, costly lobbying to avoid public service; or (ii) induce governments to manage productive resources more efficiently (Martin, 1972).

[40] Diamond and Rose (2005, p. 256) report that "The impact of demographic characteristics ... on jury decision making is often dramatically overestimated, especially given the far stronger effect that the weight

individual foregoes when he substitutes public service for his normal employment activities differs from the corresponding reduction in the social value of the activity.[41] These extensions of our analysis await further research.

of the evidence has in determining verdict outcomes."

[41] Bowles (1980, p. 371) notes that the costs of jury service "lie mainly in the value of production lost to the economy as a result of the absence of jurors from work." Koch and Birchenall (2016) identify conditions under which social welfare increases when a voluntary military enlistment policy replaces a draft, thereby allowing the most productive individuals to continue to earn high wages and pay high taxes.

Appendix A. Proofs of Formal Conclusions

Proof of Lemma 1. $\widehat{c}$ is defined by:

$$\frac{T}{N\,G(\widehat{c})}[\,w-\widehat{c}\,] \;=\; -\,F \quad\Leftrightarrow\quad T\,[\,w-\widehat{c}\,]+F\,N\,G(\widehat{c}) \;=\; 0$$

$$\Rightarrow\quad \frac{d\widehat{c}}{dF} \;=\; \frac{N\,G(\widehat{c})}{T-F\,N\,g(\widehat{c})} \quad\text{and}\quad \frac{d\widehat{c}}{dw} \;=\; \frac{T}{T-F\,N\,g(\widehat{c})}\,. \tag{11}$$

$$\text{Define}\quad J \;\equiv\; \int_{\underline{c}}^{\widehat{c}} \frac{T}{N\,G(\widehat{c})}[\,w-c\,]\,dG(c)-[\,1-G(\widehat{c})\,]\,F-A$$

$$=\; \frac{T}{N}\,w-\frac{T}{N\,G(\widehat{c})}\int_{\underline{c}}^{\widehat{c}} c\,dG(c)-[\,1-G(\widehat{c})\,]\,F-A \tag{12}$$

$$\Rightarrow\quad \frac{\partial J}{\partial \widehat{c}} \;=\; -\,\frac{T}{N}\left[\frac{G(\widehat{c})\,\widehat{c}\,g(\widehat{c})-g(\widehat{c})\int_{\underline{c}}^{\widehat{c}} c\,dG(c)}{[\,G(\widehat{c})\,]^2}\right]+g(\widehat{c})\,F$$

$$=\; g(\widehat{c})\left[F-\frac{T}{N\,[\,G(\widehat{c})\,]^2}\int_{\underline{c}}^{\widehat{c}}[\,\widehat{c}-c\,]\,dG(c)\right]. \tag{13}$$

An optimal OJS is the solution to the following problem, [P]:

$$\underset{w,\,F}{\text{Maximize}}\;\; J \quad \text{subject to (4) and (5).}$$

Let λ_1 and λ_2 denote the Lagrange multipliers associated with (5) and (4), respectively. Then the necessary conditions for a solution to [P] include:

$$w:\quad \frac{T}{N}[\,1-\lambda_2\,N\,]+\left[\frac{\partial J}{\partial \widehat{c}}+\lambda_1\,N\,g(\widehat{c})-\lambda_2\,F\,N\,g(\widehat{c})\right]\frac{d\widehat{c}}{dw} \;=\; 0\,;\;\;\text{and} \tag{14}$$

$$F:\quad -\,[\,1-G(\widehat{c})\,]\,[\,1-\lambda_2\,N\,]+\left[\frac{\partial J}{\partial \widehat{c}}+\lambda_1\,N\,g(\widehat{c})-\lambda_2\,F\,N\,g(\widehat{c})\right]\frac{d\widehat{c}}{dF} \;=\; 0\,. \tag{15}$$

(11) implies that $\frac{d\widehat{c}}{dF} \overset{s}{=} \frac{d\widehat{c}}{dw}$. Therefore, (14) and (15) imply:

$$\frac{T}{N}[\,1-\lambda_2\,N\,] \;\overset{s}{=}\; -\,[\,1-G(\widehat{c})\,]\,[\,1-\lambda_2\,N\,] \quad\Rightarrow\quad \lambda_2 \;=\; \frac{1}{N} \;>\; 0\,,$$

so the financing constraint binds.

Because $\lambda_2 = \frac{1}{N}$, (13) and (14) imply:

$$\frac{\partial J}{\partial \widehat{c}} + \lambda_1 N g(\widehat{c}) - \lambda_2 F N g(\widehat{c}) = 0$$

$$\Rightarrow \lambda_1 N g(\widehat{c}) = F g(\widehat{c}) - g(\widehat{c}) F + \frac{T g(\widehat{c})}{N [G(\widehat{c})]^2} \int_{\underline{c}}^{\widehat{c}} [\widehat{c} - c] dG(c)$$

$$= \frac{T g(\widehat{c})}{N [G(\widehat{c})]^2} \int_{\underline{c}}^{\widehat{c}} [\widehat{c} - c] dG(c) > 0 \Rightarrow \lambda_1 > 0 .$$

Therefore, the adequate jury pool constraint binds. ■

Proof of Lemma 2. Because $G(\widehat{c}) = \frac{T}{N}$ from Lemma 1, the definition of $\widehat{c}$ implies:

$$w - \widehat{c} = -F \Leftrightarrow F = \widehat{c} - w . \tag{16}$$

By the definition of c_1:

$$w - c_1 = -\frac{T}{N} c_1 \Rightarrow c_1 \left[\frac{N-T}{N} \right] = w_1 \Rightarrow c_1 = \left[\frac{N}{N-T} \right] w . \tag{17}$$

Because the financing constraint binds and $G(\widehat{c}) = \frac{T}{N}$ (from Lemma 1):

$$T w + A = [N-T] F \Rightarrow T w + A = [N-T][\widehat{c} - w]$$

$$\Rightarrow T w + A = [N-T]\widehat{c} - N w + T w \Rightarrow w = \frac{1}{N} [(N-T)\widehat{c} - A] . \tag{18}$$

The second equality in the first line of (18) reflects (16). (17) and (18) imply:

$$c_1 = \left[\frac{N}{N-T} \right] \frac{1}{N} [(N-T)\widehat{c} - A] = \widehat{c} - \frac{A}{N-T} .$$

By the definition of c_2:

$$-F = -\frac{T}{N} c_2 \Rightarrow c_2 = \frac{N F}{T} . \tag{19}$$

(16) and (18) imply:

$$F = \widehat{c} - \frac{1}{N} [(N-T)\widehat{c} - A] = \frac{T}{N} \widehat{c} + \frac{A}{N} . \tag{20}$$

(19) and (20) imply:

$$c_2 = \frac{N}{T} \left[\frac{T}{N} \widehat{c} + \frac{A}{N} \right] = \widehat{c} + \frac{A}{T} . \quad ■$$

Proof of Lemma 3. Because $G(\widehat{c}\,) = \frac{T}{N}$ from Lemma 1:

$$W_{\Delta i}(c) \;=\; W_i(c) - W_M(c) \;=\; w - c - \left(-\frac{T}{N}\,c \right) \;=\; w - \left[\frac{N-T}{N} \right] c$$

$$\Rightarrow \;\; W'_{\Delta i}(c) \;=\; -\frac{N-T}{N} \;<\; 0 \;\; \text{for } c \in [\,\underline{c}, \widehat{c}\,)\,; \text{ and}$$

$$W_{\Delta o}(c) \;=\; W_o(c) - W_M(c) \;=\; -F - \left(-\frac{T}{N}\,c \right) \;=\; -F + \frac{T}{N}\,c$$

$$\Rightarrow \;\; W'_{\Delta o}(c) \;=\; \frac{T}{N} \;>\; 0 \;\; \text{for } c \in (\,\widehat{c}, \overline{c}\,]\,. \;\; \blacksquare$$

Proof of Lemma 4. As demonstrated in the text, the financing and adequate jury pool constraints are satisfied as equalities when $F = \frac{T}{N}\,\widehat{c}$ and $w = \left[\frac{N-T}{N}\right]\widehat{c}$. Furthermore, (3) implies $W_i(\widehat{c}\,) = W_o(\widehat{c}\,)$ because:

$$w + F\,\frac{N_i}{T} \;=\; \left[\frac{N-T}{N} \right] \widehat{c} + \frac{T}{N}\,\widehat{c} \left[\frac{T}{T} \right] \;=\; \widehat{c}\,.$$

It remains to demonstrate that $W_O(\widehat{c}\,) = W_M(\widehat{c}\,)$ and $W_O(c) > W_M(c)$ for all $c \neq \widehat{c}$.

$$W_O(\widehat{c}\,) \;=\; W_M(\widehat{c}\,) \;\;\Leftrightarrow\;\; w - \widehat{c} \;=\; -\frac{T}{N}\,\widehat{c}$$

$$\Leftrightarrow \;\; \left[\frac{N-T}{N} \right] \widehat{c} - \widehat{c} \;=\; -\frac{T}{N}\,\widehat{c} \;\;\Leftrightarrow\;\; -\frac{T}{N}\,\widehat{c} \;=\; -\frac{T}{N}\,\widehat{c}\,.$$

For $c < \widehat{c}$:

$$W_O(c) \;=\; W_i(c) \;>\; W_M(c) \;\;\Leftrightarrow\;\; w - c \;>\; -\frac{T}{N}\,c \;\;\Leftrightarrow\;\; \left[\frac{N-T}{N} \right] \widehat{c} - c \;>\; -\frac{T}{N}\,c$$

$$\Leftrightarrow \;\; \left[\frac{N-T}{N} \right] \widehat{c} \;>\; \left[\frac{N-T}{N} \right] c \;\;\Leftrightarrow\;\; c \;<\; \widehat{c}\,.$$

For $c > \widehat{c}$:

$$W_O(c) \;=\; W_o(c) \;>\; W_M(c) \;\;\Leftrightarrow\;\; -F \;>\; -\frac{T}{N}\,c \;\;\Leftrightarrow\;\; -\frac{T}{N}\,\widehat{c} \;>\; -\frac{T}{N}\,c \;\;\Leftrightarrow\;\; c \;>\; \widehat{c}\,. \;\; \blacksquare$$

Proof of Lemma 5. Under an optimal OJS that ensures $W_O(c) > W_M(c)$ for some $c \in [\,\underline{c}, \overline{c}\,]$, there exists a $\widehat{c} \in (\,\underline{c}, \overline{c}\,)$ defined by:

$$W_i(\widehat{c}\,) \;=\; W_o(\widehat{c}\,) \;\;\Leftrightarrow\;\; w - \widehat{c} \;=\; -F \;\;\Leftrightarrow\;\; w \;=\; \widehat{c} - F\,. \tag{21}$$

Suppose the OJS policy can be designed to ensure $W_O(c) \geq W_M(c)$ for all $c \in [\,\underline{c}, \overline{c}\,]$. Then it must be the case that:

$$W_O(\widehat{c}\,) \;\geq\; W_M(\widehat{c}\,) \;\;\Leftrightarrow\;\; w - \widehat{c} \;\geq\; -\frac{T}{N}\,\widehat{c} \;\;\Leftrightarrow\;\; w \;\geq\; \left[\frac{N-T}{N} \right] \widehat{c} \tag{22}$$

$$\Leftrightarrow \;\; \widehat{c} - F \;\geq\; \left[\frac{N-T}{N}\right]\widehat{c} \;\;\Leftrightarrow\;\; \frac{T}{N}\,\widehat{c} \;\geq\; F \;\;\Leftrightarrow\;\; N\,F \;\leq\; T\,\widehat{c}\,. \tag{23}$$

The first equivalence in (23) reflects (21). Because $G(\widehat{c}) = \frac{T}{N}$ from Lemma 1, the last inequality in (22) implies:

$$[\,1 - G(\widehat{c})\,]\,N\,F - w\,T \;\leq\; [\,1 - G(\widehat{c})\,]\,N\,F - \left[\frac{N-T}{N}\right]\widehat{c}\,T$$

$$= \left[\frac{N-T}{N}\right] N\,F - \left[\frac{N-T}{N}\right]\widehat{c}\,T \;=\; \left[\frac{N-T}{N}\right][N\,F - T\,\widehat{c}\,] \;\leq\; 0\,. \tag{24}$$

The inequality in (24) reflects (23). (24) implies that (4) cannot hold for any $A > 0$. Therefore, it cannot be the case that the OJS policy ensures $W_O(c) \geq W_M(c)$ for all $c \in [\,\underline{c}, \overline{c}\,]$. ■

Proof of Proposition 1. The proof follows immediately from the associated discussion in the text. ■

Proof of Proposition 2. The average expected cost that individuals incur under OJS is $\frac{T}{N}\frac{\int_{\underline{c}}^{\widehat{c}} c\,dG(c)}{G(\widehat{c})} + \frac{A}{N}$. The corresponding average expected cost under MJS is $\frac{T}{N}\,c^e$. Therefore, the average expected net gain from OJS is:

$$\frac{T}{N}\,c^e - \frac{T}{N}\,\frac{\int_{\underline{c}}^{\widehat{c}} c\,dG\,(c)}{G(\widehat{c})} - \frac{A}{N}\,. \tag{25}$$

(25) and the definition of A_w imply:

$$\frac{T}{N}\,c^e - \frac{T}{N}\,\frac{\int_{\underline{c}}^{\widehat{c}} c\,dG\,(c)}{G(\widehat{c})} - \frac{A_w}{N} \;=\; 0 \;\;\Rightarrow\;\; \frac{A_w}{T} \;=\; c^e - \frac{\int_{\underline{c}}^{\widehat{c}} c\,dG\,(c)}{G(\widehat{c})}\,. \tag{26}$$

$$\text{Define:} \quad c_1\,(A) \;=\; \widehat{c} - \frac{A}{N-T} \quad \text{and} \quad c_2\,(A) \;=\; \widehat{c} + \frac{A}{T}\,. \tag{27}$$

Lemma 2 and the definitions of A_m and A_w imply:

$$A_m \;\gtreqless\; A_w \;\;\Leftrightarrow\;\; G\,(c_2\,(A_w)) - G\,(c_1\,(A_w)) \;\lesseqgtr\; \frac{1}{2}, \;\text{ and} \tag{28}$$

$$A_m \;\lesseqgtr\; A_w \;\;\Leftrightarrow\;\; \int_{c_1(A_w)}^{c_2(A_w)} dG\,(c) \;\gtreqless\; \frac{1}{2}\,. \tag{29}$$

From (26):

$$\widehat{c} + \frac{A_w}{T} \;=\; \widehat{c} + c^e - \frac{\int_{\underline{c}}^{\widehat{c}} c\,dG\,(c)}{G(\widehat{c})}\,. \tag{30}$$

L'Hopital's rule implies:

$$\lim_{\widehat{c}\to \underline{c}} \left[\frac{\int_{\underline{c}}^{\widehat{c}} c \, dG(c)}{G(\widehat{c})} \right] = \lim_{\widehat{c}\to \underline{c}} \left[\frac{\widehat{c}\, g(\widehat{c})}{g(\widehat{c})} \right] = \lim_{\widehat{c}\to \underline{c}} \left[\widehat{c} \right] = \underline{c} \, . \tag{31}$$

(30), (31), and Lemma 1 imply that as $N/T \to \infty$:

$$\begin{aligned} \widehat{c} + \frac{A_w}{T} \;\to\; \lim_{\widehat{c}\to \underline{c}} \left[\widehat{c} + \frac{A_w}{T} \right] &= \lim_{\widehat{c}\to \underline{c}} \left[\widehat{c} + c^e - \frac{\int_{\underline{c}}^{\widehat{c}} c \, dG(c)}{G(\widehat{c})} \right] \\ &= \underline{c} + c^e - \underline{c} = c^e \, . \end{aligned} \tag{32}$$

From (26):

$$\widehat{c} - \frac{A_w}{N-T} = \widehat{c} - \left[\frac{T}{N-T} \right] \frac{A_w}{T} = \widehat{c} - \frac{T}{N-T} \left[c^e - \frac{\int_{\underline{c}}^{\widehat{c}} c \, dG(c)}{G(\widehat{c})} \right] . \tag{33}$$

(31), (33), and Lemma 1 imply that as $N/T \to \infty$:

$$\begin{aligned} \widehat{c} - \frac{A_w}{N-T} \;&\to\; \lim_{\substack{\widehat{c}\to \underline{c} \\ N/T\to\infty}} \left[\widehat{c} - \frac{A_w}{N-T} \right] \\ &= \lim_{\substack{\widehat{c}\to \underline{c} \\ N/T\to\infty}} \left[\widehat{c} - \left(\frac{T}{N-T} \right) \left(c^e - \frac{\int_{\underline{c}}^{\widehat{c}} c \, dG(c)}{G(\widehat{c})} \right) \right] = \underline{c} - [\,0\,]\,[\, c^e - \underline{c} \,] = \underline{c} \, . \end{aligned} \tag{34}$$

(28), (32), and (34) imply that when N/T is sufficiently large:

$$A_m \gtreqqless A_w \;\Leftrightarrow\; G(c^e) - G(\underline{c}) \lesseqqgtr \frac{1}{2} \;\Leftrightarrow\; G(c^e) \lesseqqgtr G\left(c^d\right) \;\Leftrightarrow\; c^e \lesseqqgtr c^d \, . \quad \blacksquare$$

<u>**Proof of Proposition 3**</u>. As $N/T \to 1$, nearly all individuals must opt in under OJS to ensure that every trial has a juror. Consequently:

$$\widehat{c} \;\to\; \overline{c} \;\text{ as }\; N/T \;\to\; 1 \, . \tag{35}$$

Therefore, (26) implies that as $N/T \to 1$:

$$\begin{aligned} \frac{A_w}{T} = c^e - \frac{\int_{\underline{c}}^{\widehat{c}} c \; g(c) \, dc}{G(\widehat{c})} \;\to\; c^e - \frac{\int_{\underline{c}}^{\overline{c}} c \; g(c) \, dc}{G(\overline{c})} = c^e - \frac{c^e}{1} = 0 \\ \Rightarrow\; A_w \;\to\; 0 \;\text{ as }\; N/T \;\to\; 1 \, . \end{aligned} \tag{36}$$

(27) and the definition of A_m imply:

$$G\left(\widehat{c} + \frac{A_m}{T} \right) - G\left(\widehat{c} - \frac{A_m}{N-T} \right) = \frac{1}{2} \, . \tag{37}$$

(35) and (36) imply:

$$G\left(\widehat{c}+\frac{A_m}{T}\right) \;\rightarrow\; 1 \;\text{ as }\; N/T \;\rightarrow\; 1\,. \tag{38}$$

(27), (35), (36), and (38) imply:

$$c_2\left(A_w\right) \;\rightarrow\; \overline{c} \;\text{ as }\; N/T \;\rightarrow\; 1\,. \tag{39}$$

(27), (37), and (38) imply:

$$G\left(\widehat{c}-\frac{A_m}{N-T}\right) \;=\; G\left(c_1\left(A_m\right)\right) \;\rightarrow\; \frac{1}{2} \;\Rightarrow\; c_1\left(A_m\right) \;\rightarrow\; c^d \;\text{ as }\; N/T \;\rightarrow\; 1\,. \tag{40}$$

From the definition of A_w:

$$\int_{\underline{c}}^{c_1(A_w)} c\, g(c)\, dc + \int_{c_2(A_w)}^{\overline{c}} c\, g(c)\, dc \;=\; \int_{c_1(A_w)}^{c_2(A_w)} c\, g(c)\, dc\,. \tag{41}$$

(39) and (41) imply that as $N/T \rightarrow 1$:

$$\int_{\underline{c}}^{c_1(A_w)} c\, g(c)\, dc + \int_{\overline{c}}^{\overline{c}} c\, g(c)\, dc \;\rightarrow\; \int_{c_1(A_w)}^{\overline{c}} c\, g(c)\, dc$$

$$\Rightarrow\; \int_{\underline{c}}^{c_1(A_w)} c\, g(c)\, dc \;\rightarrow\; \int_{c_1(A_w)}^{\overline{c}} c\, g(c)\, dc \;\Rightarrow\; c_1\left(A_w\right) \;\rightarrow\; c^e\,. \tag{42}$$

From (28):

$$A_w \;\gtreqless\; A_m \;\Leftrightarrow\; G\left(c_2\left(A_w\right)\right) - G\left(c_1\left(A_w\right)\right) \;\gtreqless\; \frac{1}{2}\,. \tag{43}$$

(39) and (42) imply that as $N/T \rightarrow 1$:

$$G\left(c_2\left(A_w\right)\right) - G\left(c_1\left(A_w\right)\right) \;\rightarrow\; 1 - G\left(c^e\right)\,. \tag{44}$$

(43) and (44) imply that as $N/T \rightarrow 1$:

$$A_w \;\gtreqless\; A_m \;\Leftrightarrow\; 1 - G\left(c^e\right) \;\gtreqless\; \frac{1}{2} \;\Leftrightarrow\; G\left(c^e\right) \;\lesseqgtr\; \frac{1}{2} \;=\; G\left(c^d\right) \;\Leftrightarrow\; c^e \;\lesseqgtr\; c^d\,. \quad \blacksquare$$

Proof of Proposition 4. For expositional ease and without loss of generality,[42] suppose $\underline{c}=0$, so $g(c)=\frac{1}{\overline{c}}$. Because $G\left(\widehat{c}\right)=\frac{T}{N}$ from Lemma 1:

$$\frac{\widehat{c}}{\overline{c}} \;=\; \frac{T}{N} \;\Rightarrow\; \widehat{c} \;=\; \frac{T}{N}\,\overline{c}\,. \tag{45}$$

[42] See the proof of Proposition 5 below.

From (26):

$$\frac{A_w}{T} \;=\; c^e - \frac{\int_0^{\widehat{c}} c \, dG(c)}{G(\widehat{c})} \;=\; \frac{\overline{c}}{2} - \frac{\frac{(\widehat{c})^2}{2}}{\widehat{c}} \;=\; \frac{1}{2}\,[\,\overline{c} - \widehat{c}\,]\,. \tag{46}$$

(27) and (46) imply:

$$c_2(A_w) \;=\; \widehat{c} + \frac{A_w}{T} \;=\; \widehat{c} + \frac{1}{2}\,[\,\overline{c} - \widehat{c}\,] \;=\; \frac{1}{2}\,[\,\overline{c} + \widehat{c}\,]\,. \tag{47}$$

(27) and (46) also imply:

$$\begin{aligned} c_1(A_w) \;&=\; \widehat{c} - \frac{A_w}{N-T} \;=\; \widehat{c} - \frac{A_w}{T}\left[\frac{T}{N-T}\right] \;=\; \widehat{c} - \frac{1}{2}\,[\,\overline{c} - \widehat{c}\,]\,\frac{T}{N-T} \\ &=\; \widehat{c}\left[1 + \frac{1}{2}\left(\frac{T}{N-T}\right)\right] - \frac{\overline{c}\,T}{2\,[\,N-T\,]} \;=\; \frac{\widehat{c}\,[\,2\,N - T\,] - \overline{c}\,T}{2\,[\,N-T\,]}\,. \end{aligned} \tag{48}$$

(47) and (48) imply:

$$\begin{aligned} \int_{c_1(A_w)}^{c_2(A_w)} dG(c) \;&=\; \frac{1}{\overline{c}}\,[\,c_2(A_w) - c_1(A_w)\,] \;=\; \frac{1}{\overline{c}}\left[\frac{[\,N-T\,]\,[\,\overline{c} + \widehat{c}\,] - \widehat{c}\,[\,2\,N - T\,] + \overline{c}\,T}{2\,[\,N-T\,]}\right] \\ &=\; \frac{\widehat{c}\,[\,N - T - 2\,N + T\,] + \overline{c}\,N}{2\,\overline{c}\,[\,N-T\,]} \;=\; \frac{N\,[\,\overline{c} - \widehat{c}\,]}{2\,\overline{c}\,[\,N-T\,]}\,. \end{aligned} \tag{49}$$

(28) and (49) imply:

$$\begin{aligned} A_m \;\lesseqgtr\; A_w \;\;&\Leftrightarrow\;\; \frac{N\,[\,\overline{c} - \widehat{c}\,]}{2\,\overline{c}\,[\,N-T\,]} \;\gtreqless\; \frac{1}{2} \;\;\Leftrightarrow\;\; N\,[\,\overline{c} - \widehat{c}\,] \;\gtreqless\; \overline{c}\,[\,N-T\,] \\ &\Leftrightarrow\;\; N\,\widehat{c} \;\lesseqgtr\; T\,\overline{c} \;\;\Leftrightarrow\;\; \widehat{c} \;\lesseqgtr\; \frac{T}{N}\,\overline{c}. \end{aligned} \tag{50}$$

(45) and (50) imply $A_m = A_w$. ■

Proof of Proposition 5. As noted in the text, $[\,\underline{c}, \overline{c}\,]$ can be normalized to $[\,0, 2\,]$ without loss of generality. To prove this assertion, consider a random variable X that is distributed on $[\,\underline{c}, \overline{c}\,]$ with cumulative distribution function G_X. Define a random variable $Y = \frac{2\,[\,X - \underline{c}\,]}{\overline{c} - \underline{c}}$. (Observe that Y is distributed on $[\,0, 2\,]$.) Let G_Y be the cumulative distribution function for Y. Then, by definition:

$$\begin{aligned} G_Y\left(\frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right) \;&=\; P\left[Y \;\le\; \frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right] = \; P\left[\frac{2\,[\,X - \underline{c}\,]}{\overline{c} - \underline{c}} \;\le\; \frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right] \\ &=\; P\,[\,X \;\le\; x\,] \;=\; G_X(x)\,. \end{aligned} \tag{51}$$

Define $\widehat{c}_X^*$ and $\widehat{c}_Y^*$ by:

$$G_X(\widehat{c}_X^*) \;=\; \frac{T}{N} \quad \text{and} \quad G_Y(\widehat{c}_Y^*) \;=\; \frac{T}{N} \;\;\Rightarrow\;\; G_X(\widehat{c}_X^*) \;=\; G_Y(\widehat{c}_Y^*)\,. \tag{52}$$

(51) and (52) imply:

$$\widehat{c}_Y^* \;=\; \frac{2\,[\,\widehat{c}_X^* - \underline{c}\,]}{\overline{c} - \underline{c}}\,. \tag{53}$$

Let g_X and g_Y be the density functions for the random variables X and Y, respectively. (51) implies:

$$g_Y\left(\frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right)\left[\frac{2}{\overline{c} - \underline{c}}\right] \;=\; g_X(x) \;\;\Rightarrow\;\; g_Y\left(\frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right) \;=\; \left[\frac{\overline{c} - \underline{c}}{2}\right] g_X(x)\,. \tag{54}$$

Define:

$$\frac{A_w(X)}{T} \;=\; E\,[X] - \frac{\int_{\underline{c}}^{\widehat{c}_X^*} t\, g_X(t)\, dt}{G_X(\widehat{c}_X^*)} \quad \text{and} \quad \frac{A_w\,(Y)}{T} \;=\; E\,[Y] - \frac{\int_0^{\widehat{c}_Y^*} t\, g_Y(t)\, dt}{G_Y(\widehat{c}_Y^*)}$$

$$\Rightarrow\;\; \widehat{c}_Y^* + \frac{A_w(Y)}{T} \;=\; \widehat{c}_Y^* + E\,[Y] - \frac{\int_0^{\widehat{c}_Y^*} t\, g_Y(t)\, dt}{G_Y(\widehat{c}_Y^*)}$$

$$=\; \frac{2\,[\,\widehat{c}_X^* - \underline{c}\,]}{\overline{c} - \underline{c}} + \frac{2\,[\,E\,[X] - \underline{c}\,]}{\overline{c} - \underline{c}} - \frac{\int_0^{\widehat{c}_Y^*} t\, g_Y(t)\, dt}{G_X\,(\widehat{c}_X^*)}\,. \tag{55}$$

$$\text{Define } \; t \;=\; \frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}} \quad \Rightarrow \;\; dt \;=\; \left[\frac{2}{\overline{c} - \underline{c}}\right] dx$$

$$\Rightarrow\;\; \int_0^{\widehat{c}_Y^*} t\, g_Y(t)\, dt \;=\; \int_{\underline{c}}^{\widehat{c}_X^*} \left(\frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right) g_Y\left(\frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right)\left[\frac{2}{\overline{c} - \underline{c}}\right] dx$$

$$=\; \int_{\underline{c}}^{\widehat{c}_X^*} \left(\frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\right)\left[\frac{\overline{c} - \underline{c}}{2}\right] g_X(x) \left[\frac{2}{\overline{c} - \underline{c}}\right] dx \;=\; \int_{\underline{c}}^{\widehat{c}_X^*} \frac{2\,[\,x - \underline{c}\,]}{\overline{c} - \underline{c}}\, g_X(x)\, dx$$

$$=\; \int_{\underline{c}}^{\widehat{c}_X^*} \left[\frac{2\,x}{\overline{c} - \underline{c}}\right] g_X(x) dx - \left[\frac{2\,\underline{c}}{\overline{c} - \underline{c}}\right] G_X\,(\widehat{c}_X^*)\,. \tag{56}$$

The second equality in (56) reflects (54). (55) and (56) imply:

$$\widehat{c}_Y^* + \frac{A_w\,(Y)}{T} \;=\; \frac{2\,[\,\widehat{c}_X^* - \underline{c}\,]}{\overline{c} - \underline{c}} + \frac{2\,[\,E\,[X] - \underline{c}\,]}{\overline{c} - \underline{c}}$$

$$-\; \frac{1}{G_X(\widehat{c}_X^*)}\left[\int_{\underline{c}}^{\widehat{c}_X^*} \left(\frac{2\,x}{\overline{c} - \underline{c}}\right) g_X(x)\, dx - \left[\frac{2\,\underline{c}}{\overline{c} - \underline{c}}\right] G_X(\widehat{c}_X^*)\right]$$

$$=\; \frac{2\,[\,\widehat{c}_X^* - \underline{c}\,]}{\overline{c} - \underline{c}} + \frac{2\,[\,E\,[X] - \underline{c}\,]}{\overline{c} - \underline{c}} - \left[\frac{2}{\overline{c} - \underline{c}}\right] \frac{\int_{\underline{c}}^{\widehat{c}_X^*} x\, g_X(x)\, dx}{G_X(\widehat{c}_X^*)} + \frac{2\,\underline{c}}{\overline{c} - \underline{c}}$$

$$= \frac{2}{\overline{c} - \underline{c}} \left[\widehat{c}_X^* - \underline{c} + E[X] - \frac{\int_{\underline{c}}^{\widehat{c}_X^*} x\, g_X(x)\, dx}{G_X(\widehat{c}_X^*)} \right]$$

$$\Rightarrow \quad G_Y\left(\widehat{c}_Y^* + \frac{A_w(Y)}{T}\right) = P\left[Y \le \widehat{c}_Y^* + \frac{A_w(Y)}{T} \right]$$

$$= P\left[\frac{2[X - \underline{c}]}{\overline{c} - \underline{c}} \le \frac{2}{\overline{c} - \underline{c}} \left[\widehat{c}_X^* - \underline{c} + E[X] - \frac{\int_{\underline{c}}^{\widehat{c}_X^*} x\, g_X(x)\, dx}{G_X(\widehat{c}_X^*)} \right] \right]$$

$$= P\left[X \le \widehat{c}_X^* + E[X] - \frac{\int_{\underline{c}}^{\widehat{c}_X^*} x\, g_X(x)\, dx}{G_X(\widehat{c}_X^*)} \right] = G_X\left(\widehat{c}_X^* + \frac{A_w(X)}{T}\right). \tag{57}$$

Analogous arguments reveal:

$$G_Y\left(\widehat{c}_Y^* - \frac{A_w(Y)}{N-T}\right) = G_X\left(\widehat{c}_X^* - \frac{A_w(X)}{N-T}\right). \tag{58}$$

(57) and (58) imply:

$$G_Y\left(\widehat{c}_Y^* + \frac{A_w(Y)}{T}\right) - G_Y\left(\widehat{c}_Y^* - \frac{A_w(Y)}{N-T}\right)$$

$$= G_X\left(\widehat{c}_X^* + \frac{A_w(X)}{T}\right) - G_X\left(\widehat{c}_X^* - \frac{A_w(X)}{N-T}\right)$$

$$\Rightarrow \quad G_Y\left(\widehat{c}_Y^* + \frac{A_w(Y)}{T}\right) - G_Y\left(\widehat{c}_Y^* - \frac{A_w(Y)}{N-T}\right) \gtreqless \frac{1}{2}$$

$$\Leftrightarrow \quad G_X\left(\widehat{c}_X^* + \frac{A_w(X)}{T}\right) - G_X\left(\widehat{c}_X^* - \frac{A_w(X)}{N-T}\right) \gtreqless \frac{1}{2}. \tag{59}$$

(59) implies that the support of c can be taken to be $[0, 2]$ without loss of generality when assessing the presence of bias for or against OJS.

To specify the distribution function corresponding to $g(c)$, observe from (10) that when $c \in [0, 1]$:

$$G(c) = \int_0^c (a + [1 - 2a]\widetilde{c})\, d\widetilde{c} = \left[a\widetilde{c} + (1 - 2a)\left(\frac{\widetilde{c}^2}{2}\right) \right]_0^c = ac + [1 - 2a]\frac{c^2}{2}.$$

For $c \in [1, 2]$:

$$G(c) = \int_0^1 (a + [1 - 2a]c)\, dc + \int_1^c (a + [1 - 2a][2 - \widetilde{c}])\, d\widetilde{c}$$

$$= \frac{1}{2} + a[c - 1] + [1 - 2a]\left[2\widetilde{c} - \frac{\widetilde{c}^2}{2} \right]_1^c$$

$$= \frac{1}{2} + a\,[\,c-1\,] + [\,1-2\,a\,]\left[2\,(c-1) - \frac{c^2-1}{2}\right]$$

$$= \frac{1}{2} + a\,[\,c-1\,] + \left[\frac{1-2\,a}{2}\right]\left[\,1 - \left(c^2 - 4\,c + 4\right)\,\right]$$

$$= \frac{1}{2} + a\,[\,c-1\,] + \left[\frac{1-2\,a}{2}\right]\left[\,1 - (2-c)^2\,\right].$$

In summary, the distribution function for the density function in (10) is:

$$G\,(c) \;=\; \begin{cases} a\,c + [\,1-2\,a\,]\,\frac{c^2}{2} & \text{if} \quad 0 \,\le\, c \le\, 1 \\ \frac{1}{2} + a\,[\,c-1\,] + \left[\,\frac{1-2\,a}{2}\,\right]\left[\,1-(2-c)^2\,\right] & \text{if} \quad 1 \,\le\, c \,\le\, 2\,. \end{cases} \tag{60}$$

Case A. $N \,\geq\, 2\,T$.

Define $y \equiv \frac{T}{N}$. $\widehat{c} \le 1$ because: (i) $G\,(\,\widehat{c}\,) = y$ from Lemma 1; (ii) $y \,\le\, \frac{1}{2}$ by assumption; and (iii) $G(1) = \frac{1}{2}$ due to the symmetry in (10). Therefore, from (60):

$$a\,\widehat{c} + [\,1-2\,a\,]\,\frac{(\,\widehat{c}\,)^2}{2} \;=\; y \quad\Leftrightarrow\quad [\,1-2\,a\,]\,(\,\widehat{c}\,)^2 + 2\,a\,\widehat{c} - 2\,y \;=\; 0\,. \tag{61}$$

It is apparent from (61) that $\widehat{c} \,=\, 2\,y$ when $a = \frac{1}{2}$. If $a \neq \frac{1}{2}$, then (61) implies:

$$\widehat{c} \;=\; \frac{-\,2\,a + \sqrt{(2\,a)^2 + 8\,y\,[\,1-2\,a\,]}}{2\,[\,1-2\,a]} \;=\; \frac{-\,a + \sqrt{(a)^2 + 2\,y\,[\,1-2\,a\,]}}{1-2\,a}\,.$$

In summary:

$$\widehat{c} \;=\; \begin{cases} \frac{-\,a + \sqrt{a^2 + 2\,y[1-2\,a\,]}}{1-2\,a} & if \quad a \,\neq\, \frac{1}{2} \\ 2\,y & if \quad a \,=\, \frac{1}{2}\,. \end{cases} \tag{62}$$

(10), (26), and (61) imply that when $a \neq \frac{1}{2}$:

$$\frac{A_w}{T} \;=\; c^e - \frac{\int_0^{\widehat{c}} c\,dG(c)}{G\,(\,\widehat{c}\,)} \;=\; 1 - \frac{\int_0^{\widehat{c}} c\,(\,a + [\,1-2\,a\,]\,c\,)\,dc}{y}$$

$$=\; 1 - \frac{1}{y}\left[a\left(\frac{c^2}{2}\right)_0^{\widehat{c}} + (1-2\,a)\left(\frac{c^3}{3}\right)_0^{\widehat{c}}\right] \;=\; 1 - \frac{1}{y}\left[\frac{a}{2}\,(\,\widehat{c}\,)^2 + \frac{[\,1-2\,a\,]\,(\,\widehat{c}\,)^3}{3}\right]$$

$$=\; 1 - \frac{\widehat{c}}{y}\left[\frac{a}{2}\,(\,\widehat{c}\,) + \frac{[\,1-2\,a\,]\,(\,\widehat{c}\,)^2}{3}\right] \;=\; 1 - \frac{\widehat{c}}{y}\left[\frac{a}{2}\,(\,\widehat{c}\,) + \frac{2}{3}\,(\,y - a\,\widehat{c}\,)\right]$$

$$= 1 - \frac{2\,\widehat{c}}{3} - \frac{a}{2\,y}\,(\,\widehat{c})^2 + \frac{2\,a}{3\,y}\,(\,\widehat{c})^2 \;\; = 1 - \frac{2\,\widehat{c}}{3} + \frac{a}{6\,y}\,(\,\widehat{c})^2$$

$$= 1 - \frac{2\,\widehat{c}}{3} + \frac{a}{6\,y}\left[\frac{2\,(y - a\,\widehat{c})}{1 - 2\,a}\right] \;=\; 1 - \frac{2\,\widehat{c}}{3} + \frac{a}{3\,y}\left[\frac{y - a\,\widehat{c}}{1 - 2\,a}\right]$$

$$= 1 - \frac{2\,\widehat{c}}{3} + \frac{1}{3}\left[\frac{a}{1 - 2\,a}\right] - \frac{a^2\,\widehat{c}}{3\,y\,[\,1 - 2\,a\,]}$$

$$= 1 + \frac{1}{3}\left[\frac{a}{1 - 2\,a}\right] - \frac{\widehat{c}}{3}\left[2 + \frac{a^2}{y\,(\,1 - 2\,a\,)}\right]$$

$$= 1 + \frac{a}{3\,[\,1 - 2\,a\,]} - \frac{\widehat{c}}{3\,y\,[\,1 - 2\,a\,]}\left[\,a^2 + 2\,y\,(1 - 2\,a)\,\right]. \tag{63}$$

Furthermore, (10), (26), and (62) imply that when $a = \frac{1}{2}$:

$$\frac{A_w}{T} \;=\; c^e - \frac{\int_0^{\widehat{c}} c\,dG(c)}{G\,(\widehat{c})} \;=\; 1 - \frac{\int_0^{\widehat{c}} \frac{c}{2}\,dc}{y} \;=\; 1 - \frac{1}{y}\left[\frac{c^2}{4}\right]_0^{\widehat{c}} \;=\; 1 - \frac{1}{4\,y}\,(\,\widehat{c})^2$$

$$= 1 - \frac{1}{4\,y}\,[\,2\,y\,]^2 \;=\; 1 - y\,.$$

(63) implies that when $a \neq \frac{1}{2}$:

$$\widehat{c} + \frac{A_w}{T} \;=\; \widehat{c} + 1 + \frac{a}{3\,[\,1 - 2\,a\,]} - \frac{\widehat{c}}{3\,y\,[\,1 - 2\,a\,]}\left[\,a^2 + 2\,y\,(1 - 2\,a)\,\right]$$

$$= 1 + \frac{a}{3\,[\,1 - 2\,a\,]} + \widehat{c}\left[1 - \frac{a^2 + 2\,y\,(1 - 2a)}{3\,y\,(1 - 2a)}\right]$$

$$= 1 + \frac{a}{3\,[\,1 - 2\,a\,]} + \widehat{c}\left[\frac{3\,y\,(1 - 2\,a) - a^2 - 2\,y\,(1 - 2\,a)}{3\,y\,(1 - 2\,a)}\right]$$

$$= 1 + \frac{a}{3\,[\,1 - 2\,a\,]} + \widehat{c}\left[\frac{y\,(1 - 2\,a) - a^2}{3\,y\,(1 - 2\,a)}\right] \;=\; \beta_2 + \alpha_2\,\widehat{c},$$

$$\text{where} \quad \beta_2 \;\equiv\; 1 + \frac{a}{3\,[\,1 - 2\,a\,]} \quad \text{and} \quad \alpha_2 \;\equiv\; \frac{y\,[\,1 - 2\,a\,] - a^2}{3\,y\,[\,1 - 2\,a\,]}\,. \tag{64}$$

(63) also implies that when $a \neq \frac{1}{2}$:

$$\widehat{c} - \frac{A_w}{N - T} \;=\; \widehat{c} - \frac{A_w}{T}\left[\frac{T}{N - T}\right] \;=\; \widehat{c} - \frac{A_w}{T}\left[\frac{y}{1 - y}\right]$$

$$= \widehat{c} - \frac{y}{1 - y}\left[1 + \frac{a}{3\,(1 - 2\,a)} - \frac{\widehat{c}}{3\,y\,(1 - 2\,a)}\left[\,a^2 + 2y\,(1 - 2\,a)\,\right]\right]$$

$$= -\frac{y}{1-y}\left[1+\frac{a}{3\,(1-2\,a)}\right]+\widehat{c}\left[1+\frac{a^2+2\,y\,(1-2\,a)}{3\,(1-y)\,(1-2\,a)}\right]$$

$$= -\left[\frac{y}{1-y}\right]\beta_2+\widehat{c}\left[\frac{3\,(1-y)\,(1-2\,a)+a^2+2\,y\,(1-2a)}{3\,(1-y)\,(1-2\,a)}\right]$$

$$= -\left[\frac{y}{1-y}\right]\beta_2+\widehat{c}\left[\frac{(1-2\,a)\,(3-3\,y+2\,y)+a^2}{3\,(1-y)\,(1-2\,a)}\right]$$

$$= -\left[\frac{y}{1-y}\right]\beta_2+\widehat{c}\left[\frac{(1-2\,a)\,(3-y)+a^2}{3\,(1-y)\,(1-2\,a)}\right] = -\left[\frac{y}{1-y}\right]\beta_2+\alpha_1\,\widehat{c},$$

$$\text{where}\quad \alpha_1 \equiv \frac{[\,1-2\,a\,]\,(3-y)+a^2}{3\,[\,1-y\,]\,[\,1-2\,a\,]}\,. \tag{65}$$

$\widehat{c} \leq 1$ because $y \leq \frac{1}{2}$, by assumption. Therefore:

$$\widehat{c}-\frac{A_w}{N-T} \leq 1\,. \tag{66}$$

(64) implies that for $a \neq \frac{1}{2}$:

$$\widehat{c}+\frac{A_w}{T} \geq 1 \quad\Leftrightarrow\quad 1+\frac{1}{3}\left[\frac{a}{1-2\,a}\right]+\widehat{c}\left[\frac{y\,(1-2\,a)-a^2}{3\,y\,(1-2a)}\right] \geq 1$$

$$\Leftrightarrow\quad \widehat{c}\left[\frac{y\,(1-2\,a)-a^2}{3\,y\,(1-2a)}\right] \geq -\frac{1}{3}\left[\frac{a}{1-2\,a}\right] \quad\Leftrightarrow\quad \widehat{c}\left[\frac{y\,(1-2\,a)-a^2}{y}\right] \geq -a$$

$$\Leftrightarrow\quad \widehat{c}\left[\,y\,(1-2\,a)-a^2\,\right] \geq -a\,y \quad\Leftrightarrow\quad \widehat{c}\,y\,[\,1-2\,a\,]-\widehat{c}\,a^2+a\,y \geq 0$$

$$\Leftrightarrow\quad \widehat{c}\,y\,[\,1-2\,a\,]+a\,y-a^2\,\widehat{c} \geq 0\,. \tag{67}$$

Because $a \leq \frac{1}{2}$, the inequality in (67) holds if:

$$a\,y-a^2\,\widehat{c} \geq 0\,.$$

(61) implies:

$$y-a\,\widehat{c} = \frac{[\,1-2\,a\,]\,(\widehat{c})^2}{2} \quad\Rightarrow\quad a\,y-a^2\,\widehat{c} = \frac{a\,[\,1-2\,a\,]\,(\,\widehat{c}\,)^2}{2} \geq 0\,.$$

Therefore, the inequality in (67) holds, so:

$$\widehat{c}+\frac{A_w}{T} \geq 1\,. \tag{68}$$

Because $\widehat{c} + \frac{A_w}{T} \geq 1$ from (68), (60) and (64) imply:

$$G\left(\widehat{c} + \frac{A_w}{T}\right) = G\left(\beta_2 + \alpha_2\,\widehat{c}\right)$$
$$= \frac{1}{2} + a\left[\beta_2 + \alpha_2\,\widehat{c} - 1\right] + \left[\frac{1-2\,a}{2}\right]\left[1 - \left(2 - \beta_2 - \alpha_2\,\widehat{c}\right)^2\right]. \tag{69}$$

Because $\widehat{c} - \frac{A_w}{N-T} \leq 1$ from (66), (60) and (65) imply:

$$G\left(\widehat{c} - \frac{A_w}{N-T}\right) = G\left(-\left[\frac{y}{1-y}\right]\beta_2 + \alpha_1\,\widehat{c}\right)$$
$$= a\left[-\frac{y\,\beta_2}{1-y} + \alpha_1\,\widehat{c}\right] + \left[\frac{1-2\,a}{2}\right]\left[-\frac{y\,\beta_2}{1-y} + \alpha_1\,\widehat{c}\right]^2. \tag{70}$$

(69) and (70) imply:

$$G\left(\widehat{c} + \frac{A_w}{T}\right) - G\left(\widehat{c} - \frac{A_w}{N-T}\right)$$
$$= \frac{1}{2} + a\left[\beta_2 + \alpha_2\,\widehat{c} - 1\right] + \frac{1}{2} - a - \left[\frac{1-2\,a}{2}\right]\left[2 - \beta_2 - \alpha_2\,\widehat{c}\right]^2$$
$$- a\left[-\frac{y\,\beta_2}{1-y} + \alpha_1\,\widehat{c}\right] - \left[\frac{1-2\,a}{2}\right]\left[-\frac{y\,\beta_2}{1-y} + \alpha_1\,\widehat{c}\right]^2$$
$$= 1 + a\left[\beta_2 + \alpha_2\,\widehat{c} - 2 - \alpha_1\,\widehat{c} + \frac{y\,\beta_2}{1-y}\right]$$
$$- \left[\frac{1-2\,a}{2}\right]\left[\left(2 - \beta_2 - \alpha_2\,\widehat{c}\right)^2 + \left(-\frac{y\,\beta_2}{1-y} + \alpha_1\,\widehat{c}\right)^2\right]$$
$$= 1 - 2\,a + a\left[\alpha_2 - \alpha_1\right]\widehat{c} + a\,\beta_2\left[1 + \frac{y}{1-y}\right]$$
$$- \left[\frac{1-2\,a}{2}\right]\left[\left(2 - \beta_2 - \alpha_2\,\widehat{c}\right)^2 + \left(-\frac{y\,\beta_2}{1-y} + \alpha_1\,\widehat{c}\right)^2\right]$$
$$= 1 - 2\,a + a\left[\alpha_2 - \alpha_1\right]\widehat{c} + \frac{a\,\beta_2}{1-y}$$
$$- \left[\frac{1-2\,a}{2}\right]\left[\left(2 - \beta_2 - \alpha_2\,\widehat{c}\right)^2 + \left(-\frac{y\,\beta_2}{1-y} + \alpha_1\,\widehat{c}\right)^2\right] \equiv Z_1. \tag{71}$$

(28) implies that $A_w > A_m$ if $Z_1 > \frac{1}{2}$. *Mathematica* reveals that this is the case for all

$y \in \left(0, \frac{1}{2}\right]$ and $a \in \left[0, \frac{1}{2}\right)$. Therefore, $A_w > A_m$ for any finite $N \geq 2T$ and $a \in \left[0, \frac{1}{2}\right)$.

Case B. $N < 2T$.

$\widehat{c} \geq 1$ because: (i) $G(\widehat{c}) = y$ from Lemma 1; (ii) $y \equiv \frac{T}{N} > \frac{1}{2}$ by assumption; and (iii) $G(1) = \frac{1}{2}$ due to the symmetry in (10). Therefore, (82) implies:

$$\frac{1}{2} + a[\widehat{c} - 1] + \frac{1-2a}{2}\left[1 - (2-\widehat{c})^2\right] - y = 0$$

$$\Rightarrow \quad [1-2a]\left[1-(2-\widehat{c})^2\right] + 2a[\widehat{c}-1] + 1 - 2y = 0$$

$$\Rightarrow \quad -[1-2a][2-\widehat{c}]^2 + 2a[\widehat{c}-1] + 2 - 2a - 2y = 0$$

$$\Rightarrow \quad -[1-2a](\widehat{c})^2 + 2a[\widehat{c}-1] + 4\widehat{c}[1-2a] - 4[1-2a] + 2 - 2a - 2y = 0$$

$$\Rightarrow \quad -[1-2a](\widehat{c})^2 + \widehat{c}[4-6a] + 4a - 2 - 2y = 0$$

$$\Rightarrow \quad \frac{1}{2}[2a-1](\widehat{c})^2 + [2-3a]\widehat{c} + 2a - 1 - y = 0. \tag{72}$$

Observe that:

$$[2-3a]^2 - 4\left[\frac{1}{2}\right][2a-1][2a-1-y] = 4 - 12a + 9a^2 + [2-4a][2a-1-y]$$

$$= 4 - 12a + 9a^2 + 4a - 2 - 2y - 8a^2 + 4a + 4ay = a^2 + 4ay - 4a - 2y + 2.$$

Therefore, because $a \neq \frac{1}{2}$, (72) implies:

$$\widehat{c} = \frac{-2 + 3a + \sqrt{a^2 + 4ay - 4a - 2y + 2}}{2a - 1}. \tag{73}$$

(10) and (26) imply:

$$\frac{A_w}{T} = c^e - \frac{\int_0^{\widehat{c}} c\, dG(c)}{G(\widehat{c})} = 1 - \frac{\int_0^1 c[a + (1-2a)c]\, dc + \int_1^{\widehat{c}} c[a + (1-2a)(2-c)]\, dc}{y}$$

$$= 1 - \frac{1}{y}\left[\frac{1}{2}ac^2\Big|_0^1 + \frac{1}{3}[1-2a]c^3\Big|_0^1 + \frac{1}{2}(a + 2[1-2a])c^2\Big|_1^{\widehat{c}} - \frac{1}{3}[1-2a]c^3\Big|_1^{\widehat{c}}\right]$$

$$= 1 - \frac{1}{y}\left[\frac{1}{2}a + \frac{1}{3}(1-2a) + \frac{1}{2}(a + 2[1-2a])(\widehat{c})^2 - \frac{1}{2}(a + 2[1-2a]) - \frac{1}{3}(1-2a)(\widehat{c})^3 + \frac{1}{3}(1-2a)\right]$$

$$= 1 - \frac{1}{y}\left[-\frac{1}{3}(1-2a) + (\widehat{c})^2(1 - \frac{3a}{2}) - \frac{1}{3}(1-2a)(\widehat{c})^3\right]. \tag{74}$$

(27) and (74) imply:

$$c_2 \;=\; \widehat{c} + \frac{A_w}{T} \;=\; \widehat{c} + 1 + \frac{1}{y}\left[\frac{1}{3}\,(\,1 - 2\,a\,) - (\widehat{c})^2\left(1 - \frac{3\,a}{2}\right) + \frac{1}{3}\,(\,1 - 2\,a\,)\,(\widehat{c})^3\right]$$

$$=\; \widehat{c} + 1 + \frac{1}{6\,y}\left[2\,(\,1 - 2\,a\,) - 6\,(\widehat{c})^2\left(1 - \frac{3\,a}{2}\right) + 2\,(\,1 - 2\,a\,)\,(\widehat{c})^3\right]$$

$$=\; \frac{1}{6\,y}\left[\,2 - 4\,a\, + 6\,y\,(1 + \widehat{c}) - 6\,(\widehat{c})^2 + 9\,a\,(\widehat{c})^2 + 2\,(\,1 - 2\,a\,)\,(\widehat{c})^3\,\right]$$

$$=\; \frac{1}{6\,y}\left[\,2\,(\,1 - 2\,a\,)\,(\widehat{c})^3 + 3\,(3\,a - 2)\,(\widehat{c})^2 + 6\,y\,\widehat{c} + 2\,(1 - 2\,a + 3\,y)\,\right] \tag{75}$$

and

$$c_1 \;=\; \widehat{c} - \frac{A_w}{N - T} \;=\; \widehat{c} - \frac{A_w}{T}\left[\frac{T}{N - T}\right] \;=\; \widehat{c} - \left[\frac{y}{1 - y}\right]\frac{A_w}{T}$$

$$=\; \widehat{c} - \frac{y}{1 - y} + \frac{1}{1 - y}\left[-\frac{1}{3}\,(\,1 - 2\,a\,) + (\widehat{c})^2\,(1 - \frac{3\,a}{2}) - \frac{1}{3}\,(\,1 - 2\,a\,)\,(\widehat{c})^3\right]$$

$$=\; \frac{1}{1 - y}\left[-\frac{1}{3}\,(\,1 - 2\,a\,) + (\widehat{c})^2\,(1 - \frac{3\,a}{2}) - \frac{1}{3}\,(\,1 - 2\,a\,)\,(\widehat{c})^3 + (1 - y)\,\widehat{c} - y\right]$$

$$=\; \frac{1}{6\,[\,y - 1\,]}\left[2\,(\,1 - 2\,a\,) - 6\,(\widehat{c})^2\,(1 - \frac{3\,a}{2}) + 2\,(\,1 - 2\,a\,)\,(\widehat{c})^3 - 6\,(1 - y)\,\widehat{c} + 6\,y\right]$$

$$=\; \frac{1}{6\,[\,y - 1\,]}\left[\,2 - 4\,a - 6\,(\widehat{c})^2 + 9\,a\,(\widehat{c})^2 + 2\,(\,1 - 2\,a\,)\,(\widehat{c})^3 - 6\,(1 - y)\,\widehat{c} + 6\,y\,\right]$$

$$=\; \frac{1}{6\,[\,y - 1\,]}\left[\,2\,(\,1 - 2\,a\,)\,(\widehat{c})^3 - 3\,(2 - 3\,a)\,(\widehat{c})^2 - 6\,(1 - y)\,\widehat{c} + 2\,(1 - 2\,a + 3\,y)\,\right]. \tag{76}$$

From (60):

$$G\left(\widehat{c} + \frac{A_w}{T}\right) - G\left(\widehat{c} - \frac{A_w}{N - T}\right) - \frac{1}{2} \;=\; Z_2\,,$$

where

$$Z_2 \;\equiv\; a\,[\,c_2 - 1\,] + \frac{1 - 2\,a}{2}\left[\,1 - (2 - c_2)^2\,\right] - a\,c_1 - \left[\frac{1 - 2\,a}{2}\right](c_1)^2\,. \tag{77}$$

and where c_1 and c_2 are defined in (75) and (76).

Mathematica reveals that $Z_2 > \frac{1}{2}$ (so $A_m < A_w$ from (28)) for all $y \in (\frac{1}{2}, 1)$. ■

Proposition 5A. $A_m \to A_w$ as $\alpha \to \frac{1}{2}$ when $g(c)$ is as specified in (10).

Proof. (61) implies that for $a \neq \frac{1}{2}$:

$$2\,y - 2\,a\,\widehat{c} \;=\; [\,1 - 2\,a\,]\,(\,\widehat{c})^2 \quad \Rightarrow \quad \widehat{c} \;\to\; 2\,y \quad \text{as} \;\; a \;\to\; \frac{1}{2}\,. \tag{78}$$

(63) implies:

$$\begin{aligned}
\frac{A_w}{T} \;&=\; 1 + \frac{a}{3\,[\,1 - 2\,a\,]} - \frac{a^2\,\widehat{c}}{3\,[\,1 - 2\,a\,]\,y} - \frac{2\,\widehat{c}}{3} \\
&=\; 1 - \frac{2\,\widehat{c}}{3} + \frac{a}{3\,[\,1 - 2\,a\,]}\left[\,1 - \frac{a\,(\,\widehat{c})^2}{y}\,\right] \\
&=\; 1 - \frac{2\,\widehat{c}}{3} + \frac{a}{6\,y}\left[\,\frac{2\,y - 2\,a\,\widehat{c}}{1 - 2\,a}\,\right] \;=\; 1 - \frac{2\,\widehat{c}}{3} + \frac{a}{6\,y}\,(\,\widehat{c})^2\,.
\end{aligned} \tag{79}$$

The equality in (79) reflects (78). (78) and (79) imply that as $a \to \frac{1}{2}$:

$$\frac{A_w}{T} \;\to\; 1 - \frac{2}{3}\,[\,2\,y\,] + \frac{1}{12\,y}\,[\,2\,y\,]^2 \;=\; 1 - \frac{4\,y}{3} + \frac{y}{3} \;=\; 1 - y\,. \tag{80}$$

(78) and (80) imply that as $a \to \frac{1}{2}$:

$$\begin{aligned}
&\widehat{c} + \frac{A_w}{T} \;\to\; 2\,y + 1 - y \;=\; 1 + y\,, \;\; \text{and} \\
&\widehat{c} - \left[\,\frac{y}{1 - y}\,\right]\frac{A_w}{T} \;\to\; 2\,y - \left[\,\frac{y}{1 - y}\,\right][\,1 - y\,] \;=\; y\,.
\end{aligned} \tag{81}$$

(60) and (81) imply that as $a \to \frac{1}{2}$:

$$\begin{aligned}
&G\left(\widehat{c} + \frac{A_w}{T}\right) \;\to\; G\,(1 + y) \;=\; \frac{1}{2} + \frac{1}{2}\,[\,1 + y - 1\,] \;=\; \frac{1 + y}{2}\,, \;\; \text{and} \\
&G\left(\widehat{c} - \frac{A_w}{N - T}\right) \;\to\; G\,(y) \;=\; \frac{y}{2} \\
&\Rightarrow \;\; G\left(\widehat{c} + \frac{A_w}{T}\right) - G\left(\widehat{c} - \frac{A_w}{N - T}\right) \;\to\; \frac{1 + y}{2} - \frac{y}{2} \;=\; \frac{1}{2}\,. \quad \blacksquare
\end{aligned}$$

Appendix B. Additional Findings

The analysis in the text considers settings where the most extreme jury service costs (c) are relatively unlikely. Although these settings may be the most likely to arise in practice, a setting could conceivably arise in which the most extreme c's are relatively likely. To consider this possibility, suppose that $g(c)$ is a piecewise linear, symmetric, V-shaped density (with $c^e = c^d$) on the normalized support $[0, 2]$.[43] Formally, suppose that for $a \in [0, \frac{1}{2})$:

$$g(c) = \begin{cases} 1 - a + [2a - 1]c & \text{if } 0 \leq c \leq 1 \\ 3a - 1 + [1 - 2a]c & \text{if } 1 \leq c \leq 2. \end{cases} \tag{82}$$

This density declines at the constant rate $1 - 2a$ on $[0, 1]$ and increases at the corresponding rate on $[1, 2]$. The two segments of the symmetric density become more steeply sloped as a declines to 0.

Proposition 8 indicates that majority rule often introduces a bias in favor of OJS when $g(c)$ is a symmetric V-shaped density with moderate slope (i.e., for $a \in [0.042, 0.5)$). However, a bias against OJS can arise for symmetric V-shaped densities with more pronounced slope if $N/T > 1$ is relatively small or if N/T is finite and sufficiently large.

Proposition 8. *If $a \in [0.042, 0.5)$ for the density specified in (82), then $A_m > A_w$ for all finite $N/T > 1$. If $a = 0$ for this density, then: (i) $A_m > A_w$ if $N/T \in (1.7, 2.414)$; whereas (ii) $A_m < A_w$ if $N/T \in (1, 1.7)$ or if finite $N/T \geq 2.44$.*[44]

Proof. For the density in (82):

$$G(c) = \int_0^c (1 - a + [2a - 1]\eta)\, d\eta = [1 - a]c + \frac{c^2}{2}[2a - 1] \quad \text{for } c \in [0, 1]. \tag{83}$$

$$G(c) = \int_0^1 (1 - a + [2a - 1]c)\, dc + \int_1^c (3a - 1 + [1 - 2a]\eta)\, d\eta$$

$$= \frac{1}{2} + [3a - 1][c - 1] + \frac{1}{2}[1 - 2a]\left[c^2 - 1\right] \quad \text{for } c \in [1, 2]. \tag{84}$$

Case A. $N \geq 2T$.

Define $y \equiv \frac{T}{N}$. $\widehat{c} \leq 1$ because: (i) $G(\widehat{c}) = y$ from Lemma 1; (ii) $y \leq \frac{1}{2}$ by assumption; and (iii) $G(1) = \frac{1}{2}$ due to the symmetry in (82). Therefore, from (83):

$$G(\widehat{c}) = y \quad \Rightarrow \quad [1 - a]\widehat{c} + \frac{1}{2}[2a - 1](\widehat{c})^2 = y$$

$$\Rightarrow \frac{2[1 - a]}{2a - 1}\widehat{c} + (\widehat{c})^2 = \frac{2y}{2a - 1} \quad \Rightarrow \quad (\widehat{c})^2 = \frac{2y}{2a - 1} - \frac{2[1 - a]}{2a - 1}\widehat{c}. \tag{85}$$

[43] This focus on the $[0, 2]$ support is again without loss of generality.

[44] If $a \in (0, 0.042)$, $A_m - A_w$ can be either positive or negative, depending on the value of N/T. To illustrate, when $a = 0.02$, $A_m < A_w$ when $\frac{N}{T} \in (1.098, 1.6) \cup (2.665, 11.161)$, and $A_m > A_w$ otherwise.

From (26):

$$\frac{A_w}{T} = c^e - \frac{\int_0^{\widehat{c}} c\, g(c)\, dc}{G(\widehat{c})}$$

$$\Rightarrow \quad \widehat{c} + \frac{A_w}{T} = \widehat{c} + c^e - \frac{\int_0^{\widehat{c}} c\, g(c)\, dc}{G(\widehat{c})} = c^e + \widehat{c} - \frac{\int_0^{\widehat{c}} c\, g(c)\, dc}{G(\widehat{c})} \tag{86}$$

$$= c^e + \frac{1}{G(\widehat{c})}\left[\widehat{c}\, G(\widehat{c}) - \int_0^{\widehat{c}} c\, g(c)\, dc\right] \geq c^e = 1\,. \tag{87}$$

Because $\widehat{c} \leq 1$, (82) implies:

$$\int_0^{\widehat{c}} c\, g(c)\, dc = \int_0^{\widehat{c}} c\,[\,1 - a + (\,2\,a - 1\,)\,c\,]\, dc$$

$$= \frac{1}{2}[\,1 - a\,](\widehat{c})^2 + \frac{1}{3}[\,2\,a - 1\,](\widehat{c})^3\,. \tag{88}$$

(85), (86), and (88) imply:

$$c_2 \equiv \widehat{c} + \frac{A_w}{T} = 1 + \widehat{c} - \frac{1}{y}\left[\frac{1}{2}(\,1 - a\,)(\widehat{c})^2 + \frac{1}{3}(\,2\,a - 1\,)(\widehat{c})^3\right]$$

$$= \widehat{c} + 1 - \frac{(\widehat{c})^2}{y}\left[\frac{1}{2}(\,1 - a\,) + \frac{1}{3}(\,2\,a - 1\,)\,\widehat{c}\right]$$

$$= \widehat{c} + 1 - \frac{1}{y}\left[\frac{1 - a}{2} + \left(\frac{2\,a - 1}{3}\right)\widehat{c}\right]\left[\frac{2\,y}{2\,a - 1} - \frac{2\,(\,1 - a\,)}{2\,a - 1}\,\widehat{c}\right]$$

$$= \widehat{c} + 1 - \frac{1}{y}\left[\frac{1 - a}{2}\right]\left[\frac{2\,y}{2\,a - 1}\right] - \frac{1}{y}\left[\frac{2\,a - 1}{3}\right]\widehat{c}\left[\frac{2\,y}{2\,a - 1}\right]$$

$$+ \frac{1}{y}\left[\frac{1 - a}{2}\right]\left[\frac{2\,(1 - a)}{2\,a - 1}\right]\widehat{c} + \frac{1}{y}\left[\frac{2\,a - 1}{3}\right]\left[\frac{2\,(1 - a)}{2\,a - 1}\right](\widehat{c})^2$$

$$= \widehat{c} + 1 - \frac{1 - a}{2\,a - 1} - \frac{2}{3}\,\widehat{c} + \frac{[\,1 - a\,]^2}{y\,[\,2\,a - 1\,]}\,\widehat{c} + \frac{2\,[\,1 - a\,]}{3\,y}\,(\widehat{c})^2$$

$$= 1 - \frac{1 - a}{2\,a - 1} + \widehat{c}\left[\frac{1}{3} + \frac{(\,1 - a\,)^2}{y\,(\,2\,a - 1\,)}\right] + \frac{2\,[\,1 - a\,]}{3\,y}\left[\frac{2\,y}{2\,a - 1} - \frac{2\,(\,1 - a\,)}{2\,a - 1}\,\widehat{c}\right]$$

$$= 1 - \frac{1 - a}{2\,a - 1} + \frac{4}{3}\left[\frac{1 - a}{2\,a - 1}\right] + \widehat{c}\left[\frac{1}{3} + \frac{(\,1 - a\,)^2}{y\,(\,2\,a - 1\,)} - \frac{4\,(\,1 - a\,)^2}{3\,y\,(\,2\,a - 1\,)}\right]$$

$$= 1 + \frac{1}{3}\left[\frac{1 - a}{2\,a - 1}\right] + \widehat{c}\left[\frac{1}{3} - \frac{(\,1 - a\,)^2}{3\,y\,(\,2\,a - 1\,)}\right]$$

$$= \; 1 + \frac{1}{3}\left[\frac{1-a}{2\,a-1}\right] + \frac{\widehat{c}}{3}\left[1 - \frac{(\,1-a\,)^2}{y\,(\,2\,a-1\,)}\right] \;=\; A_2 + B_2\,\widehat{c} \tag{89}$$

$$\text{where} \quad A_2 \;\equiv\; 1 + \frac{1}{3}\left[\frac{1-a}{2\,a-1}\right] \quad \text{and} \quad B_2 \;\equiv\; \frac{1}{3}\left[1 - \frac{(\,1-a\,)^2}{y\,(\,2\,a-1\,)}\right].$$

(89) implies:

$$c_1 \;\equiv\; \widehat{c} - \frac{A_w}{N-T} \;=\; \widehat{c} - \frac{A_w}{T}\left[\frac{T}{N-T}\right] \;=\; \widehat{c} - \frac{A_w}{T}\left[\frac{\frac{T}{N}}{1-\frac{T}{N}}\right]$$

$$= \; \widehat{c} - \frac{A_w}{T}\left[\frac{y}{1-y}\right] \;=\; \widehat{c} - \left[\frac{y}{1-y}\right][\,A_2 + B_2\,\widehat{c} - \widehat{c}\,] \tag{90}$$

$$= \; \left[1 + (1-B_2)\left(\frac{y}{1-y}\right)\right]\widehat{c} - \left[\frac{y}{1-y}\right] A_2 \;=\; A_1 + B_1\,\widehat{c} \tag{91}$$

$$\text{where} \;\; A_1 \;\equiv\; -\left[\frac{y}{1-y}\right] A_2 \;\; \text{and} \;\; B_1 \;\equiv\; 1 + [\,1-B_2\,]\left[\frac{y}{1-y}\right].$$

(84) and (89) imply:

$$G\,(c_2) \;=\; \frac{1}{2} + [\,3\,a-1\,]\,[\,A_2 + B_2\,\widehat{c} - 1\,] + \frac{1}{2}\,[\,1-2\,a\,]\,\left[\,(A_2 + B_2\,\widehat{c})^2 - 1\,\right]. \tag{92}$$

(83) and (91) imply:

$$G\,(c_1) \;=\; [\,1-a\,]\,[\,A_1 + B_1\,\widehat{c}\,] + \frac{1}{2}\,[\,2\,a-1\,]\,[\,A_1 + B_1\,\widehat{c}\,]^2. \tag{93}$$

(92) and (93) imply:

$$G\,(c_2) - G\,(c_1) \;=\; \frac{1}{2} + [\,3\,a-1\,]\,[\,A_2 + B_2\,\widehat{c} - 1\,] + \left[\frac{1-2\,a}{2}\right]\left[\,(A_2 + B_2\,\widehat{c})^2 - 1\,\right]$$

$$-\; [\,1-a\,]\,[\,A_1 + B_1\,\widehat{c}\,] - \left[\frac{2\,a-1}{2}\right][\,A_1 + B_1\,\widehat{c}\,]^2$$

$$= \; \frac{1}{2} - (3\,a-1) - \left(\frac{1-2\,a}{2}\right) + [\,3\,a-1\,]\,[\,A_2 + B_2\,\widehat{c}\,] + \left[\frac{1-2\,a}{2}\right][\,A_2 + B_2\,\widehat{c}\,]^2$$

$$-\,[\,1-a\,]\,[\,A_1 + B_1\,\widehat{c}\,] + \left[\frac{1-2\,a}{2}\right][\,A_1 + B_1\,\widehat{c}\,]^2 \;\equiv\; \Psi_1(a)\,,$$

where $\widehat{c}$ is the solution to (85), so:

$$\widehat{c} \;=\; \frac{1}{2}\left[-\frac{2\,[\,1-a\,]}{2\,a-1} + \sqrt{\frac{4\,[\,1-a\,]^2}{[\,2\,a-1\,]^2} + \frac{8\,y}{2\,a-1}}\;\right]$$

$$= \frac{1}{2}\left[-\frac{2\,[\,1-a\,]}{2\,a-1} + \frac{1}{2\,a-1}\sqrt{4\,[\,1-a\,]^2 + 8\,y\,[\,2\,a-1\,]}\,\right]$$

$$= \frac{-\,(1-a) + \sqrt{[\,1-a\,]^2 + 2\,y\,[\,2\,a-1\,]}}{2\,a-1}\,. \tag{94}$$

Mathematica reveals that for all $a \in [\,0.042,\, 0.5)$, $\Psi_1(a) < \frac{1}{2}$ (so $A_m > A_w$ from (28)) for all $y \in (0, \frac{1}{2})$. *Mathematica* also reveals that if $a = 0$, then: (i) $\Psi_1(a) > \frac{1}{2}$ for all $y \in (0, 0.41)$, i.e., for all finite $\frac{N}{T} \geq 2.44$; and (ii) $\Psi_1(a) < \frac{1}{2}$ for all $y \in [0.41,\, 0.5)$, i.e., for $\frac{N}{T} \in (1,\, 2.44)$.

Case B. $N < 2\,T$.

$\widehat{c} \geq 1$ because: (i) $G\,(\,\widehat{c}\,) = y$ from Lemma 1; (ii) $y \equiv \frac{T}{N} > \frac{1}{2}$ by assumption; and (iii) $G(1) = \frac{1}{2}$ due to the symmetry in (82). Therefore, (82) implies:

$$\int_0^{\widehat{c}} c\,g(c)\,dc \;=\; \int_0^1 c\,[\,1-a+(\,2\,a-1\,)\,c\,]\,dc + \int_1^{\widehat{c}} c\,[\,3\,a-1+(\,1-2\,a\,)\,c\,]\,dc$$

$$= [\,1-a\,]\left[\frac{c^2}{2}\right]_0^1 + [\,2\,a-1\,]\left[\frac{c^3}{3}\right]_0^1 + [\,3\,a-1\,]\left[\frac{c^2}{2}\right]_1^{\widehat{c}} + [\,1-2\,a\,]\left[\frac{c^3}{3}\right]_1^{\widehat{c}}$$

$$= \frac{1-a}{2} + \frac{2\,a-1}{3} - \frac{3\,a-1}{2} - \frac{1-2\,a}{3} + \frac{3\,a-1}{2}\,(\,\widehat{c}\,)^2 + \frac{1-2\,a}{3}\,(\,\widehat{c}\,)^3$$

$$= \frac{3-3\,a+4\,a-2-9\,a+3-2+4\,a}{6} + \frac{3a-1}{2}\,(\,\widehat{c}\,)^2 + \frac{1-2\,a}{3}\,(\,\widehat{c}\,)^3$$

$$= \frac{1-2\,a}{3} + \frac{3\,a-1}{2}\,(\,\widehat{c}\,)^2 + \frac{1-2\,a}{3}\,(\,\widehat{c}\,)^3\,. \tag{95}$$

(86) and (95) imply:

$$c_2 \;\equiv\; \widehat{c} + \frac{A_w}{T} \;=\; \widehat{c} + 1 - \frac{1}{y}\left[\frac{1-2\,a}{3} + \frac{3\,a-1}{2}\,(\,\widehat{c}\,)^2 + \frac{1-2\,a}{3}\,(\,\widehat{c}\,)^3\right], \tag{96}$$

where $\widehat{c}$ is determined by:

$$G(\,\widehat{c}\,) \;=\; \frac{1}{2} + [\,3\,a-1\,]\,[\,\widehat{c}-1\,] + \left[\frac{1-2\,a}{2}\right]\left[\,(\,\widehat{c}\,)^2 - 1\,\right] \;=\; y$$

$$\Rightarrow\;\; \frac{1}{2} + [\,3\,a-1\,]\,\widehat{c} - (3\,a-1) - \frac{1-2\,a}{2} + \left[\frac{1-2\,a}{2}\right](\,\widehat{c}\,)^2 \;=\; y$$

$$\Rightarrow\;\; \left[\frac{1-2\,a}{2}\right](\,\widehat{c}\,)^2 + [\,3\,a-1\,]\,\widehat{c} + 1 - 2\,a - y \;=\; 0$$

$$\Rightarrow\;\; (\,\widehat{c}\,)^2 + \frac{2\,[\,3\,a-1\,]}{1-2\,a}\,\widehat{c} + \frac{2\,[\,1-2\,a-y\,]}{1-2\,a} \;=\; 0$$

$$\Rightarrow \ \widehat{c} \ = \ \frac{1}{2}\left[-\frac{2\,[\,3\,a-1\,]}{1-2\,a} + \sqrt{\frac{4\,[\,3\,a-1\,]^2}{[\,1-2\,a\,]^2} - \frac{8\,[\,1-2\,a-y\,]}{1-2\,a}}\,\right]$$

$$= \ \frac{1}{2}\left[-\frac{2\,[\,3\,a-1\,]}{1-2\,a} + \frac{2}{1-2\,a}\sqrt{[\,3\,a-1\,]^2 - 2\,[\,1-2\,a-y\,]\,[\,1-2\,a\,]}\,\right]$$

$$= \ \frac{-(3\,a-1)+\sqrt{a^2+2\,a+2\,y-4\,a\,y-1}}{1-2\,a}\,. \tag{97}$$

(90) and (96) imply:

$$c_1 \ \equiv \ \widehat{c} - \frac{A_w}{T}\left[\frac{y}{1-y}\right] \ = \ \widehat{c} - \frac{y}{1-y}\,[\,c_2-\widehat{c}\,]$$

$$= \ \widehat{c}\left[1+\frac{y}{1-y}\right] - \frac{y\,c_2}{1-y} \ = \ \frac{\widehat{c}}{1-y} - \frac{y\,c_2}{1-y}\,. \tag{98}$$

(84) and (96) imply:

$$G\,(c_2) \ = \ \frac{1}{2} + [\,3\,a-1\,]\,[\,c_2-1\,] + \left[\frac{1-2\,a}{2}\right]\left[\,(c_2)^2-1\,\right]. \tag{99}$$

(83) and (98) imply:

$$G\,(c_1) \ = \ [\,1-a\,]\,c_1 + \left[\frac{2\,a-1}{2}\right](c_1)^2\,. \tag{100}$$

(99) and (100) imply:

$$G\,(c_2) - G\,(c_1) \ = \ \frac{1}{2} + [\,3\,a-1\,]\,[\,c_2-1\,] + \left[\frac{1-2\,a}{2}\right]\left[\,(c_2)^2-1\,\right] - [\,1-a\,]\,c_1 - \left[\frac{2\,a-1}{2}\right](c_1)^2$$

$$= \ \frac{1}{2} - (3\,a-1) - \left(\frac{1-2\,a}{2}\right) + [\,3\,a-1\,]\,c_2 + \left[\frac{1-2\,a}{2}\right](c_2)^2 - [\,1-a\,]\,c_1 - \left[\frac{2\,a-1}{2}\right](c_1)^2$$

$$= \ 1-2\,a + [\,3\,a-1\,]\,c_2 - [\,1-a\,]\,c_1 + \left[\frac{1-2\,a}{2}\right]\left[\,(c_2)^2+(c_1)^2\,\right] \ \equiv \ \Psi_2(a)\,.$$

Mathematica reveals that for all $a \in [\,0.042,\, 0.5)$, $\Psi_2(a) < \frac{1}{2}$ (so $A_w < A_m$ from (28)) for all $y \in \left[\,\frac{1}{2}, 1\right)$. *Mathematica* also reveals that if $a = 0$, then: (i) $\Psi_2(a) > \frac{1}{2}$ for all $y \geq 0.586$, i.e., for $\frac{N}{T} \in (1,\, 1.7\,]$; and (ii) $\Psi_2(a) < \frac{1}{2}$ for all $y \in (0.5,\, 0.586)$, i.e., for $\frac{N}{T} \in (1,\, 1.7)$. ■

Now consider the modified OJS policy analyzed in Section 5, where an individual's request for exemption from jury service is approved with probability $p \in (0, \overline{p}]$, where $\overline{p} < 1$. Proposition 9 reports that such limits on exemptions from jury service can reduce the extent to which majority rule entails a bias against OJS.

Proposition 9. *$A_m \lesseqgtr A_w$ as $\overline{p} \gtreqless 0.75$ under the optimal modified OJS policy if $g(c)$ is as specified in equation (82) with $a = 0$.*

Recall from Proposition 8 that when $N/T > 1$ is sufficiently small or finite and sufficiently large, majority rule introduces a bias against OJS when $g(c)$ is as specified in equation (82) with $a = 0$. Proposition 9 reports that in the corresponding setting when $\overline{p} < 0.75$, majority rule entails a bias in favor of OJS for all finite N/T. This conclusion reflects the fact that as $\overline{p}$ declines, the welfare gains and losses from OJS become less sensitive to c (because the individuals with the highest c's who request exemption from jury service are not ensured of exemption). Consequently, the extent to which majority rule fails to fully reflect the intensity of preferences for OJS is diminished, which eliminates the bias against OJS under the identified conditions.[45]

[45] The identified bias in favor of OJS also reflects the more limited potential gains from OJS as $\overline{p}$ declines.