Competition among the Big and the Small with Different Product Substitution*

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Abstract

This paper employs a linear monopolistic competition model to revisit the impacts of the large firm's entry in the differentiated good market where large and small firms coexist. The large firm determines both the range of product varieties and the quantity of each variety while the small firm produces only one variety and freely enters the market. We find that the different substitutablitities between the products of large firms and those of

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small firms play a critical role in determining the impacts exerted by the entry of the large firm. The entry of the large firm may cause a rise or a fall of the incumbent large firms' output, price and profit, depending on the comparison of the substitutability within large firms and small firms and the substitutability across these two types of firms. In addition,the impact of the large firm's entry on cosumer welfare and social welfare is also ambiguous. Our welfare analysis implies that whether the government should enforce regulations against the entry of large firms depends on the product substitution level across large and small firms in the industry.

Keywords: big firms, small firms, product substitution, entry behavior, market impacts

JEL classification: D21, D43, L11, L13

1 Introduction

Many industries consist of a few large firms and a large number of small firms, such as retailing, manufacturing, and catering. The large firms are usually influential in the market, able to affect the market price of the products, while the small firms' impacts are negligible. It is questionable whether the standard imperfect competition theory still works to describe the market where the large and small firms coexist. As argued by Shimomura and Thisse (2012), neither oligopoly nor monopolistic competition can fully capture such a market. Thus, it is worth investigating firms' behavior and social welfare in this market structure.

Moreover, different industries with the coexistence of large and small firms see distinct changes in firms' behavior after the entry of a large firm. According to the empirical study by Igami (2011), in the supermarket industry in Japan, after the relaxation of the Big Retail Store Law which induced the entry of large supermarkets, large supermarkets were inclined to shrink (or even exit) but small supermarkets responded positively. Nevertheless, firms reacted differently in the Korean furniture market. After the entry of IKEA into Korea in 2014, the large national furniture makers, such as HANSSEM and LIVART, enjoyed an increase in their revenue. After the establishment of the IKEA store in Gwangmyeong, the revenue of Livart's Gwangmyeong branch increased 27%, while Hanssem's Gwangmyeong store saw a 10% rise in sales over the same period of the previous year. Small furniture makers, however, suffered from over 70% decrease in their revenue on average, and many were at the edge of shutdown.[1] These two contrast-

[1]Source: John Choi. "Korea's Large Furniture Makers Boost Revenues Thanks to IKEA." Korea Bizwire. March 27, 2015. http://koreabizwire.com/koreas-large-furniture-makers-boost-revenues-thanks-to-ikea/32438

ing cases invite us to wonder why the impacts exerted by a large firm's entry vary across different industries.

In reality, some governments implicitly or explicitly restrict the entry of large firms into local markets with laws and regulations, such as the Royer-Raffarin Law in France and zoning laws in UK, Poland, Korea and India, etc. However, it is worth examining whether the barriers to the large firms' entry set by the government have a sound theoretical ground.

The present paper studies the impacts of large firms' entry in the market with large and small firms by combining the Cournot model with the linear monopolistic competition model. Our model is characterized by the following three aspects. First, each large firm supplies a non-negligible range of product varieties, which is endogenously determined [2]. Second, each small firm produces only one variety with a negligible quantity, but can freely enter or exit from the market. Third, the substitutability of products may differ across large and small firms[3]. All firms move simultaneously. We find that the different product substitutabilitities between large and small firms play a critical role in determining the impacts exerted by the entry of the large firm. When the products of large firms and those of small firms have different levels of substitution, the entry of the large firm may cause a rise or a fall of the incumbent large firms' output, price and profit, depending on the comparison of the substitutability within large firms and small firms and the substitutability across these two types of firms. If the substitutability across these

[2]Bernard et al. (2006) show that multi-product firms are almost omnipresent in the U.S. manufacturing industry. According to the data between 1979 and 1992, multi-product firms account for 41% of the total number of firms but supply 91% of total output. In addition, 89% of multi-product firms adjust their product range every five years.

[3]Our analytical framework is based on Singh and Vives (1984), Ottaviano and Thisse (1999) and Ottaviano et. al. (2002), but is distinct from them in the above-mentioned three respects.

two types of firms is relatively larger than the substitutability within large firms and small firms, the squeezing effect due to the shrinkage of the competitive fringe outweighs the substitution effect among large firms, causing a rise in the market power of the large firms. Otherwise, the squeezing effect is not strong enough to compensate for the substitution effect among large firms, and consequently the large firms have to reduce their price and output.

This may explain the different impacts of a large firm's entry in the Japanese supermarket industry and the Korean furniture market. In the Japanese supermarket industry, as Igami (2011) observes, the small size and convenience provides a dimension of differentiation for small supermarkets. Therefore, large supermarkets are less differentiated than small ones so that the squeezing effect is not strong enough to offset the competition effect for the large incumbents in the Japanese supermarket industry. On the contrary, with unique designs, the large furniture makers are more differentiated than small makers so that the squeezing effect outweighs the competition effect for large makers in the Korean furniture market. We also find that the welfare effects are ambiguous.

This paper is closely related to the seminal work by Shimomura and Thisse (2012). To the best of our knowledge, Shimomura and Thisse (2012) is the first paper that connects large oligopolistic firms and small monopolistic competitors. In a general equilibrium framework with CES utility, they show that in this mixed market structure, the entry of large firms increases the incumbent large firms' profit and raises welfare. This paper employs their idea of perceiving large firms as oligopolies and small firms as monopolistic competitors. However, we are distinct from Shimomura and Thisse (2012) in the following aspects. First, we establish a partial equilibrium framework with a quasi-linear utility function with quadratic subutility, while Shimomura and Thisse (2012) build a general equilibrium framework with CES utility. Part of our results is the same as Shimomura and Thisse (2012) if the income effects are washed out in their model. Second, different from Shimomura and Thisse (2012), which assumes large firms produce one variety, we consider large firms as multiproduct firms with endogenous choices on the product range to provide a more generalized result. We also test the robustness by assuming large firms as single-product firms in the discussion and find that our results hold qualitatively. Most importantly, we relax the assumption of the same elasticity of substitution among all firms in Shimomura and Thisse (2012) by assigning different levels of substitution across large and small firms. As we will see, the difference of substitution across large and small firms is our key distinction from Shimomura and Thisse (2012). We find that the different substitutabilities across large and small firms play a critical role in determining whether entry is beneficial or harmful to large firms and social welfare.[4]

The present paper is also related to other studies on the issues concerning the coexistence of large and small firms. The first strand is the widely used dominant firm model, which models the dominant firm as the leader and the price maker, while assumes that small firms are the followers who face increasing marginal cost and behave like price takers. Representative works include Chen (2003) and Gowrisankaran and Holmes (2004). Unlike the dominant firm model, we do not assume that small firms have increasing marginal cost or are price takers. Another strand is to use the traditional Stackelberg model to deal with such issues, as represented by Etro (2004, 2006) and Ino and Matsumura (2012), etc. In

[4]Another related work is Parenti (2013), which adopts a framework similar to ours. However, he also assumes the level of substitution is the same across large and small firms and investigates a completely different issue.

their models, the firm is large in the sense that it is both the leader and the first mover. The small firms are followers but their individual behavior influences the market price. The small firms and large firms can supply homogenous goods in a Stackelberg game. This paper is different from the Stackelberg model in that i) we consider a differentiated good market, ii) we do not assume the commitment power of the large firms, and iii) small firms are negligible in the market. Besides, some studies differentiate between large and small firms from the perspective of firm heterogeneity in quality (Ishibashi and Matsushima 2009) or technology (Matsumura and Matsushima, 2010). All in all, the present paper studies issues that are different from the above literature.

The rest of the paper is organized as follows. We construct the model in Section 2. Results are shown in Section 3. Section 4 discusses the robustness of the established results.

2 The model

2.1 Preference and demand

Consider a closed economy consisting of two sectors. Firms in sector 1 are perfectly competitive and produce the homogenous product under constant return to scale. Sector 2 provides the differentiated products that are produced by two types of firms. The first type of firms are large in size, and the number of these firms is exogenous. The second type of firms are infinitessimal and freely enter or exit from the market.

On the demand side, the large and small firms differ in three respects. First, each large firm imposes a non-negligible impact on the market and competes in an oligopolistic manner, while each small firm is negligible in the market and behaves as a monopolistic competitor. Here we follow the approach by Shimomura and Thisse (2012). Second, each large firm produces a range of varieties, and strategically chooses both the product range and the quantity of each variety, while each small firm only produces one variety of product. Third, the varieties are equally substitutable within the group of large firms and that of small firms, but the level of substitution across these two types of firms can be different.

The utility of the representative consumer U is described by a quasi-linear utility with a quadratic subutility:

$$\begin{aligned} U \;=\; & \alpha[\int_0^N q_S(i)di + \sum_{m=1}^{M} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega] \\ & -\frac{\beta}{2}\sum_{m=1}^{M} \int_{\omega\in\Omega_m} [q_L^m(\omega)]^2 d\omega - \frac{\beta}{2}\int_0^N [q_S(i)]^2 di \\ & -\frac{\gamma_1}{2}[\int_0^N q_S(i)di]^2 - \frac{\gamma_2}{2}[\sum_{m=1}^{M} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega]^2 \\ & -\gamma_3[\int_0^N q_S(i)di][\sum_{m=1}^{M} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega] + q_0, \end{aligned} \tag{1}$$

where $q_S(i)$ is the quantity of small firm i with $i \in [0, N]$. The output of each small firm is of zero measure, and the total mass of small firms is N, describing the competitive fringe. The set of varieties produced by the large firm m ($m = 1, ..., M$) is represented by Ω_m, and the quantity for variety $\omega \in \Omega_m$ is $q_L^m(\omega)$. The total number of the incumbent large firms is M, with $M \geq 2$. Here we treat M and $|\Omega_m|$ as continuous variables. The output of sector 1 is q_0, which is treated as the numeraire. Consumer preferences are characterized by five parameters, which are α, β, and γ_i $(i = 1, 2, 3)$. The intensity of preferences for the differentiated product is measured by $\alpha > 0$, which determines the size of the differentiated good market, whereas $\beta > 0$ implies the consumer's preference for a diversified consumption of products. The substitutability between varieties is characterized by γ_i $(i = 1, 2, 3)$. Specifically, the substitutability among the varieties produced by small firms and that among the varieties of large firms are expressed by γ_1 and γ_2, respectively, and the cross substitutability between the varieties of large firms and those of small firms is expressed by γ_3. The products are closer substitutes when γ_i $(i = 1, 2, 3)$ is higher. The products of the small and large firms have the same level of substitution when $\gamma_1 = \gamma_2 = \gamma_3$ and have different substitutabilities otherwise. Finally, to ensure the concavity of the quadratic subutility, we have $\beta/N + \gamma_1 > 0$, $\beta/(M\left|\Omega\right|) + \gamma_2 > 0$, and $(\beta/N + \gamma_1)[\beta/(M\left|\Omega\right|) + \gamma_2] > \gamma_3^2$.[5] (See **Appendix A-1**.)

The representative consumer's budget constraint is:

$$\int_0^N p_S(i)q_S(i)di + \sum_{m=1}^{M} \int_{\omega\in\Omega_m} p_L^m(\omega)q_L^m(\omega)d\omega + q_0 = I,$$

where $p_S(i)$ and $p_L^m(\omega)$ are the prices of small firm i and large firm m's variety ω, respectively. The representative consumer's income is I, which is exogenously given. The inverse demand functions facing small firms and large firms are determined

[5] Here γ_i $(i = 1, 2, 3)$ can be negative as long as the conditions for the concavity of the utitility function hold. Hence the products can be complementary among the firms. We focus on the case when the products are substitutes in the rest of our analysis.

by the maximization of the consumer's utility subject to the budget constraint:

$$p_S(i) = \alpha - \beta q_S(i) - \gamma_1 Q_S - \gamma_3 Q_L, \tag{2}$$

$$p_L^m(\omega) = \alpha - \beta q_L^m(\omega) - \gamma_3 Q_S - \gamma_2 Q_L. \tag{3}$$

where $Q_S \equiv \int_0^N q_S(i)di$ and $Q_L \equiv \sum_{m=1}^{M} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega$ are the total output of the small firms and that of the large firms, respectively.

2.2 Firms

Both large and small firms incur variable costs and fixed costs. All firms incur a common and constant marginal cost, which is normalized to zero, whereas the fixed cost may differ across the two types of firms.

2.2.1 Small firms

The profit function of the small firms is:

$$\Pi_S(i) = p_S(i)q_S(i) - (f^e + f^p),$$

where $\Pi_S(i)$ is the profit of small firm i, and f^e and f^p are the entry cost and fixed production cost of the small firm, respectively. To simplify our denotation and explanation, we denote $f \equiv f^e + f^p$ as the total fixed cost of a small firm.

Plugging $p_S(i)$ of equation (2) into the above profit function, $\Pi_S(i)$ can be rewritten as:

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - f, \tag{4}$$

Each small firm maximizes its profit with respect to its quantity $q_S(i)$.

The free entry and exit of small firms pins down the equilibrium profit of the small firm to zero:

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - f = 0. \tag{5}$$

2.2.2 Large firms

The profit of the large firm is:

$$\Pi_L^m(\Omega_m, q_L^m(\omega)) = \int\limits_{\omega\in\Omega_m} (p_L^m(\omega)q_L^m(\omega) - F)d\omega,$$

where $\Pi_L^m(\Omega_m, q_L^m(\omega))$ is the profit of large firm m, and F is the fixed production cost for the large firm to produce one variety. [6]

Substituting $p_L^m(\omega)$ of equation (3) into the above profit function, $\Pi_L^m(\Omega_m)$ can be rewritten as:

$$\begin{aligned}\Pi_L^m(\Omega_m, q_L^m(\omega)) &= \{\alpha - \gamma_3 Q_S - \gamma_2[\sum_{k\neq m}\int\limits_{\omega\in\Omega_k} q_L^k(\omega)d\omega]\}[\int\limits_{\omega\in\Omega_m} q_L^m(\omega)d\omega] \\ &\quad -\beta\int\limits_{\omega\in\Omega_m}[q_L^m(\omega)]^2 d\omega - \gamma_2[\int\limits_{\omega\in\Omega_m} q_L^m(\omega)d\omega]^2 - F\left|\Omega_m\right|.\end{aligned} \tag{6}$$

The large firm maximizes its profit with respect to both its product range Ω_m and the quantity of each variety $q_L^m(\omega)$. Note that the varieties do not overlap with each other.

All firms behave simultaneously. The equilibrium is mainly characterized by

[6] Here we do not consider the free entry and exit of large firms. Therefore, the entry cost for the incumbent large firms is normalized to zero.

the mass of small firms N^*, the output of the small firm $q_S^*(i)$, the product range of the large firm $|\Omega_m^*|$, and the output of each variety for the large firm $q_L^{m*}(\omega)$. These variables are determined by the profit maximization of the small firm with respect to $q_S(i)$, the free entry condition of small firms, and the profit maximization of the large firm with respect to $|\Omega_m|$ and q_L^m.

2.3 Social Welfare

The social welfare comprises consumer surplus and producer surplus. Consumer surplus is measured by:

$$CS = U - I,$$

Hence, the change of consumer surplus is the same as that of consumer's utility.

Since small firms earn zero profit, producer surplus is given by the sum of all large firms' profits:

$$PS = \sum_{m=1}^{M} \Pi_L^m,$$

Then, social welfare SW is the sum of consumer surplus and producer surplus:

$$SW = U - I + \sum_{m=1}^{M} \Pi_L^m. \tag{7}$$

3 Results

In this section, we derive the equilibrium results and conduct the comparative static analysis to investigate the impacts of the entry of a large firm on the other firms' behavior and social welfare.

Small Firms

A small firm only accounts for the impact of the market's total production because its own impact on the market is negligible. The small firm maximizes its profit given by equation (4) with respect to its output $q_S(i)$, yielding the optimal quantity of the small firm for an expected total output of large firms Q_L and mass of small firms N:

$$q_S^*(Q_L, N) = \frac{\alpha - \gamma_3 Q_L}{2\beta + \gamma_1 N}. \tag{8}$$

Using equation (2), the price of the small firm can be expressed by:

$$p_S^*(Q_L, N) = \beta \frac{\alpha - \gamma_3 Q_L}{2\beta + \gamma_1 N}. \tag{9}$$

Accordingly, the equilibrium price of the small firm decreases with the mass of small firms and the total output of large firms.

Entry and exit are free for small firms. Using equation (5) after plugging in (8) and (9), the equilibrium mass of small firms with a given total output of large firms Q_L is:

$$N^*(Q_L) = \frac{1}{\gamma_1}[\sqrt{\frac{\beta}{f}}(\alpha - \gamma_3 Q_L) - 2\beta]. \tag{10}$$

which decreases with the total output of large firms.

Substituting (10) into (8), the optimal quantity of each small firm is:

$$q_S^* = \sqrt{\frac{f}{\beta}}.$$

Owing to free entry and exit, the quantity produced by the small firm is independent of the behavior of large firms. In other words, the aggregate behavior of small firms responds to the change in the market condition only by adjusting the competitive fringe.

Plugging q_S^* into (9) yields the equilibrium price of small firms:

$$p_S^* = \sqrt{\beta f}.$$

Large Firms

Unlike small firms, large firms impose non-negligible impacts on the market. Large firm m maximizes its profit given by equation (6) with respect to its output $q_L^m(\omega)$, yielding the optimal quantity of each variety, given the total output of small firms Q_S, the total output of other large firms $Q_{-L} = \sum_{j \neq m} \int_{\omega \in \Omega_j} q_L^j(\omega) d\omega$, and its own product range $|\Omega_m|$:

$$q_L^{m*}(Q_S, Q_{-L}, |\Omega_m|) = \frac{\alpha - \gamma_3 Q_S - \gamma_2 Q_{-L}}{2(\beta + \gamma_2 |\Omega_m|)}. \tag{11}$$

Everything else being equal, an increase in firm m's product range (larger $|\Omega_m|$) result in a reduction in the quantity of each variety, implying cannibalization.

The product range of large firm m, $|\Omega_m^*|$, that maximizes (6) after substituting (11) satisfies:

$$2(\beta + \gamma_2 |\Omega_m^*|) = \sqrt{\frac{\beta}{F}}[\alpha - \gamma_3 Q_S - \gamma_2 Q_{-L}]. \tag{12}$$

We obtain the optimal output per variety for the large firm from equations (11) and (12):

$$q_L^{m*} = \sqrt{\frac{F}{\beta}}.$$

which is determined only by the fixed cost of large firms and the demand parameters, but independent of its product range or other firms' behavior. This implies that the large firm reacts to the change in the market condition only by adjusting its product range $|\Omega_m^*|$.

Substituting q_L^{m*} into equation (12), we obtain the equilibrium product range $|\Omega_m^*|$ given the expected aggregate output of small firms Q_S:

$$|\Omega_m^*|\,(Q_S) = \frac{\sqrt{\beta/F}(\alpha - \gamma_3 Q_S) - 2\beta}{\gamma_2(M+1)}. \tag{13}$$

In equilibrium, the total output of big firms can be expressed by $Q_L^* = M\,|\Omega_m^*|\,q_L^{m*}$, and the aggregate output of small firms is $Q_S^* = N^* q_S^*$. Plugging these two expressions into (10) and (13), the mass of small firms and the product range of each big firm are:

$$\begin{aligned}
N^* &= \sqrt{\frac{\beta}{f}}\frac{\alpha[\gamma_2(M+1) - \gamma_3 M] - 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{f} - \gamma_3 M\sqrt{F}]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M},\\
|\Omega^*| &= \sqrt{\frac{\beta}{F}}\frac{\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}.
\end{aligned}$$

Substituting N^*, $|\Omega^*|$, q_S^* and q_L^{m*} into equation (3), the price of the large firm in equilibrium is:

$$p_L^* = \sqrt{\beta F^p} + \frac{\gamma_2[\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}.$$

Substituting the equilibrium range of varieties $|\Omega^*|$, the output of each variety q_L^{m*} and the equilibrium price of large firms p_L^* into equation (6), we obtain the equilibrium profit of the large firm:

$$\Pi_L^* = \frac{\gamma_2[\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})]^2}{[\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M]^2}.$$

And the total output is:

$$\begin{aligned} Q^* &= \frac{1}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}\{\alpha M(\gamma_1 + \gamma_2 - 2\gamma_3) + \gamma_2(\alpha - 2\sqrt{\beta f}) \\ &\quad -2M\sqrt{\beta}[(\gamma_2 - \gamma_3)\sqrt{f} + (\gamma_1 - \gamma_3)\sqrt{F}]\}. \end{aligned}$$

We focus on the market with the coexistence of large and small firms. To ensure the market is mixed and stable in equilibrium, all the endogenous variables should be positive. The following proposition establishes the conditions.

Proposition 1 *There exists a unique mixed market equilibrium if the following three conditions hold:*

(i) $\alpha(\gamma_1 - \gamma_3) > 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})$;

(ii) $\alpha[\gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{f} - \gamma_3 M\sqrt{F}]$;

(iii) $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

Conditions (i) and (ii) ensure the existence of large firms and small firms, respectively. Condition (iii) is the sufficient condition to guarantee the stability of the established model (see **Appendix A-2**). Conditions (i) and (ii) require that the market size should be sufficiently large, and conditions (ii) and (iii) also require the number of large firms should not be too large.

These three conditions can be illustrated by the aggregate reaction functions of the large and small firms. The aggregate reaction of large firms to the competitive fringe is:

$$Q_L(Q_S) = Mq_L^* \left|\Omega_m\right| (Q_S^*) = \frac{M}{\gamma_2(M+1)}(\alpha - 2\sqrt{\beta F} - \gamma_3 Q_S),$$

The aggregate reaction of the competitive fringe to the large firms is:

$$Q_S(Q_L) = \frac{1}{\gamma_1}(\alpha - 2\sqrt{\beta f} - \gamma_3 Q_L).$$

The coexistence of large and small firms in equilibrium requires that the two aggregate reaction functions intersect and the intersection be a stable equilibrium. Figure 1 depicts these two aggregate reaction functions. The stability of the intersection requires that the slope of $Q_L(Q_S)$ be flatter than the slope of $Q_S(Q_L)$, i.e. $\gamma_3 M/\gamma_2(M+1) < \gamma_1/\gamma_3$. This condition is equivalent to condition (iii). If condition (iii) does not hold, the mixed market equilibrium is not stable, resulting in the equilibrium with only small firms or the equilibrium with large firms only. To ensure that the two aggregate reaction functions intersect, two more conditions are necessary. On the horizontal axis, the intercept of $Q_S(Q_L)$ should be smaller than the intercept of $Q_L(Q_S)$, i.e. $(\alpha - 2\sqrt{\beta f})/\gamma_1 < (\alpha - 2\sqrt{\beta F^p})/\gamma_3$, which is equivalent to condition (i). On the vertical axis, the intercept of $Q_S(Q_L)$ should be larger than the intercept of $Q_L(Q_S)$, i.e. $(\alpha - 2\sqrt{\beta f})/\gamma_3 > (\alpha - 2\sqrt{\beta F^p})M/\gamma_2(M+1)$, which is equivalent to condition (ii).

[Figure 1 around here]

These three conditions imply that a unique mixed market equilibrium does not exist if $\gamma_1 < \gamma_3$. If $\gamma_1 < \gamma_3$, the substitutability from the large firms' variety to the small firm's is larger than the substitutability among the small firms' varieties, thus the large firm can expand its production so that all small firms are squeezed out of the market. In addition, if $\gamma_1 = \gamma_2 = \gamma_3$, condition (i) implies that $f > F$. That is, if the varieties are equally substitutable among all firms, the existence of large firms requires that the total fixed cost of a small firm should be larger than the large firm's fixed production cost of each variety. When the large and small firms share the same fixed production cost, the small firm's entry cost should be positive so that the large firm enjoys economies of scope (Parenti, 2013). Even when the small firm's entry cost is close to zero, the large firm may also exist if it is more efficient in producing each variety.

Finally, an increase in the number of large firms M generates a clockwise rotation of $Q_L(Q_S)$ around its intercept on the horizontal axis, resulting in a rise in the total output of large firms and a fall of the aggregate output of small firms.

In the rest of our analysis, we focus on the market where both large and small firms exist.

Now we investigate the impacts of a large firm's entry. Proposition 2 establishes the results.

Proposition 2 *The entry of a large firm will exert the following impacts on firms' behavior:*

(i) The output and price level of the small firm do not change;

(ii) The output of each variety of the large firm do not change;

(iii) The competitive fringe shrinks;

(iv) The product range, price, and profit of each large firm rise (fall) if $\gamma_1\gamma_2 < (>)\gamma_3^2$*, and remain to be the same if* $\gamma_1\gamma_2 = \gamma_3^2$*;*

(v) The total output increases if $\gamma_1 > \gamma_3$ *and remains to be the same if* $\gamma_1 = \gamma_3$*.*

Proof. See **Appendix A-3**. ■

The first outcome is in line with the traditional monopolistic competition model. As shown by Figure 2, the free entry and exit of small firms shifts the demand curve such that there is only one equilibrium quantity, at which the average cost (AC) is tangent to the average revenue (AR) and marginal revenue (MR) intersects with marginal cost.

[Figure 2 around here]

The second result can be briefly explained as follows. The profit maximization of large firm m with respect to the output of each variety q_L^m yields $p_L^m - \beta q_L^m - \gamma_2 |\Omega_m| q_L^m = 0$, where the last term on the LHS is the internalization by the large firm. Applying the envelope theorem, the profit maximization of large firm m with respect to the product range $|\Omega_m|$ yields $p_L^m q_L^m - \gamma_2 |\Omega_m| (q_L^m)^2 = F$, where the second term on the LHS is the cannibalization effect. With linear demand and symmetric technology across varieties within the large firm, the cannibalization and internalization effects completely offset each other, and consequently the optimal output of each variety q_L^m is independent of the product scope $|\Omega_m|$. This implies that the large firm reacts to changes in the market condition by varying its product scope only.

The third result shows that the entry of a large firm may raise or reduce the prices and profits of the incumbent large firms when the substitutability across the products of large firms and those of small firms is different from the substitutability within the groups of large and small firms. To illustrate the mechanism, we establish the following two expressions:

$$p_S^* = \alpha - \beta q_S^* - \gamma_1 Q_S^* - \gamma_3 Q_L^*, \tag{14}$$

$$p_L^* = \alpha - \beta q_L^* - \gamma_2 Q_L^* - \gamma_3 Q_S^*. \tag{15}$$

The equilibrium conditions describing the demands for large and small firms, the profit maximization of large and small firms, and the free entry of small firms boil down to expressions (14) and (15). Here $Q_S^* = N^* q_S^*$ is the total output of small firms, and $Q_L^* = M\left|\Omega^*\right| q_L^*$ is the total output of large firms.

As shown by Figure 1, the entry of a large firm raises the equilibrium total output of large firms Q_L^*. Denote this increase in Q_L^* by ΔQ_L^*. Two opposing effects are generated by the entry of a large firm. First, according to equation (15), ΔQ_L^* generates a direct negative substitution effect on p_L^* by $-\gamma_2 \Delta Q_L^*$. Meanwhile, ΔQ_L^* also leads to the shrinkage of the competitive fringe, which has a positive effect on the large firms. As shown by the first argument in Proposition 2, p_S^* and q_S^* are not affected by a large firm's entry. According to equation (14), an increase in the total output of large firms ΔQ_L^* squeezes out the aggregate output of small firms by $\Delta Q_S^* = -(\gamma_3/\gamma_1)\Delta Q_L^*$. Then the substitution effect of the small firms on the large firms is weakened by the shrinkage of the competitive fringe, according to equation (15). Precisely, the indirect squeezing effect is measured by $(-\gamma_3)(-\gamma_3/\gamma_1)\Delta Q_L^* = (\gamma_3^2/\gamma_1)\Delta Q_L^*$. Therefore, whether the entry of a large firm raises or reduces the price of large firms depends on the comparison between the direct substitution effect and the indirect squeezing effect. If $\gamma_1\gamma_2 > \gamma_3^2$, which implies $-\gamma_2\Delta Q_L^* + (\gamma_3^2/\gamma_1)\Delta Q_L^* < 0$, then the negative substitution effect dominates the positive squeezing effect, and large firms have to reduce their price. Because $d\left|\Omega_m^*\right|/dM = (\sqrt{\beta/F}/\gamma_2)dp_L^*/dM$, in addition, the equilibrium product range of the large firm also shrinks, and consequently the equilibrium profit of each large firm decreases. If $\gamma_1\gamma_2 < \gamma_3^2$, on the other hand, then the positive squeezing effect dominates the negative substitution effect, and the price, product range and profit of each large firm rise. Finally, if $\gamma_1\gamma_2 = \gamma_3^2$, the positive squeezing effect exactly offsets the negative substitution effect, and consequently the large firms do not change their behavior. The last result is consistent with Shimomura and Thisse (2012) with the elimination of income effect and Parenti (2013).

Let us consider how the entry of large firm influences consumer welfare, producer surplus and social welfare. Proposition 3 establishes the results.

Proposition 3 *The entry of a large firm generates the following impacts on welfare:*

(i) Consumer welfare rises (falls) if $2E(\gamma_3^2-\gamma_1\gamma_2)M+D\sqrt{\beta}(\gamma_3\sqrt{f}-\gamma_1\sqrt{F}) < (>)0$*;*

(ii) Producer surplus rises (falls) if $\gamma_3^2 M-\gamma_1\gamma_2(M-1) > (<)0$*;*

(iii) Social welfare rises (falls) if $2\gamma_1\gamma_2 E + D\sqrt{\beta}(\gamma_1\sqrt{F}-\gamma_3\sqrt{f}) > (<)0$*.*

where $D=\gamma_1\gamma_2+(\gamma_1\gamma_2-\gamma_3^2)M > 0$*, and* $E=\alpha(\gamma_1-\gamma_3)-2\sqrt{\beta}(\gamma_1\sqrt{F}-\gamma_3\sqrt{f}) > 0$ *according to the conditions in Proposition 1.*

Proof. See **Appendix A-4**. ■

Proposition 3 shows that the entry of the large firm will only conditionally raise consumer surplus, producer surplus and social welfare. Because the conditions are complicated, we decompose the impacts of a large firm's entry as follows.

The impact of a large firm's entry on consumer welfare can be expressed by:

$$\frac{dCS}{dM}=\frac{q_S^*}{2}\frac{dQ_S^*}{dM}+\frac{q_L^*}{2}\frac{dQ_L^*}{dM}-Q_L^*\frac{dp_L^*}{dM}.$$

The entry of a large firm generates three effects on consumer welfare. The first term represents the effect of the competitive fringe, which is negative. The second term represents the effect of the total output of large firms, which is positive. The third term represents the effect of large firms' price, which is ambiguous, depending on the relative levels of substitution across large and small firms.

The impact on producer surplus simply depends on the comparison between the profit of the large entrant and the change in the profits of the large incumbents. Producer surplus deteriorates only if the entry of the large firm leads to the reduction in the profit of large incumbents that outweighs the profit made by the entrant.

The impact on social welfare depends on the comparison between the negative effect of the competitive fringe and the positive effect of large firms' total output.

In particular, the sufficient condition for consumer surplus and social welfare to rise is $\gamma_3^2 < \gamma_1\gamma_2$ and $\gamma_3\sqrt{f} < \gamma_1\sqrt{F}$. Intuitively, $\gamma_3^2 < \gamma_1\gamma_2$ indicates that the substitutability across large and small firms should be relatively smaller than the substitutability within these two types of firms. In this case, the squeezing effect on the competitive fringe is dominated by the competition effect from the entry of a large firm, and consequently consumers benefit from the intensified competition among large firms. In addition, $\gamma_3\sqrt{f} < \gamma_1\sqrt{F}$, which is equivalent to $\sqrt{f/\beta} < (\gamma_1/\gamma_3)\sqrt{F/\beta}$, implies that switching from consuming the product of the small firm to the product of the large firm is beneficial to the consumer because the small firm's good is more substitutable than the large firm's good. Finally, the condition for an increase in producer surplus that is aligned to the sufficient conditions for consumer surplus and social welfare to rise is $\gamma_3^2 < \gamma_1\gamma_2$ and $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$. Because the entry of a large firm results in a fall in the profit of each large firm when $\gamma_3^2 < \gamma_1\gamma_2$, according to Proposition 2, these two conditions imply that the producer surplus increases only when the profit earned by the entrant large firm outweighs the profit loss of the incumbent large firms. Therefore, the sufficient condition that increases consumer surplus, producer surplus, and social welfare is $\gamma_3\sqrt{f} < \gamma_1\sqrt{F}$, $\gamma_3^2 < \gamma_1\gamma_2$, and $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

When $\gamma_3^2 > \gamma_1\gamma_2$, on the other hand, the entry of a large firm always increases producer surplus because the profit of each large firm is higher. As explained earlier, the increase in large firms' profits originates from weakened competition among large firms due to the squeezing effect on the competitive fringe. The government may be cautious of this case because consumer surplus and social welfare fall when $\gamma_3^2 > \gamma_1\gamma_2$, and $\gamma_3\sqrt{f} > \gamma_1\sqrt{F}$. In other words, the increase in producer surplus may be due to the mitigated competition in the market, which can be harmful to consumers and social welfare.

4 Discussion

In this section, we test the robustness of our results.

Single-product large firm When the varieties of large firms are exogenously given, say, $|\Omega_m| = 1$, both large and small firms are single-product firms. In this case, our results are robust, and the change in each large firm's output is qualitatively the same as the change in the large firm's variety choice in our original model. Specifically, the impact of a large firm's entry generates the same impacts on firms' behavior as in Proposition 2. The welfare effects are also ambiguous, with slight changes in the conditions. The conditions for the unique mixed market equilibrium are also modified. The following proposition establishes the results. (See **Appendix A-5**)

Proposition 4 *When both large and small firms are single-product firms,*

(i) There exists a unique mixed market equilibrium if the following three conditions hold:

(i-1) $\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$*;*

(i-2) $\alpha[2\beta + \gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta f}[2\beta + \gamma_2(M+1)]$*;*

(i-3) $\alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta F} > [\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M]\sqrt{F/(\beta + \gamma_2)}$*.*

(ii) The impacts of a large firm's entry on firms' behavior are the same as Proposition 2.

(iii) The entry of a large firm generates the following impacts on social welfare:

(iii-1) Consumer welfare rises (falls) if $A\gamma_1(2\beta+\gamma_2)+B(\gamma_1\gamma_2-\gamma_3^2)M > (<)0$*;*

(iii-2) Producer surplus rises (falls) if $I^2(\beta+\gamma_2)[\gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M]/H^3 > (<)F$*;*

(iii-3) Social welfare rises (falls) if $B\gamma_1(2\beta + \gamma_2) + A(\gamma_1\gamma_2 - \gamma_3^2)M > (<)0$*.*

where $A = \alpha\beta(\gamma_1-\gamma_3)-\gamma_2\gamma_3\sqrt{\beta f}$, $B = \alpha(\gamma_1-\gamma_3)(3\beta+2\gamma_2)+\gamma_3\sqrt{\beta f}(4\beta+3\gamma_2)$, $H = \gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M > 0$*, and* $I = \alpha(\gamma_1 - \gamma_3)+2\gamma_3\sqrt{\beta f} > 0$ *according to the conditions in (i).*

This reduced model relates to Shimomura and Thisse (2012), who assume that large and small firms are single-product firms. Shimomura and Thisse (2012) show that the entry of a large firm shrinks the competitive fringe and thus generates the market expansion effect on large firms. This market expansion effect is amplified by the income effect, raising the profits of large firms and leading to a welfare-improving result. The key to their result is the income effect that amplifies the market expansion effect on the large firms. We distinguish our model from theirs by excluding the income effect and explicitly introducing the different substitutability between large and small firms. We conclude that the entry of the large firm may result in an increase or a decrease of each large firm's output, and may be harmful or beneficial to consumer surplus and social welfare, depending on the different levels of substitution across large and small firms.

Income effect As mentioned in Proposition 2, large firms do not change their behavior with the entry of a new large firm if $\gamma_1 = \gamma_2 = \gamma_3$, which corresponds to Shimomura and Thisse (2012) with the income effects washed out. Here we would like to elaborate more on the elimination of income effects in the CES framework.

The utility in Shimomura and Thisse (2012) is expressed by a nested Cobb-Douglas function with CES subutility of the differentiated good market:

$$U = \mathbf{Q}^{\alpha} X^{1-\alpha}.$$

where $\mathbf{Q} = [\int_0^N (q_S(i))^{\rho} di + \sum_{j=1}^{M} (q_L^j)^{\rho}]^{1/\rho}$ is the CES composite good, and $0 < \rho < 1$ is an inverse measure of the degree of differentiation across varieties. The consumption of the homogeneous good is represented by X, and α represents the substitution between the composite good and the homogeneous good, satisfying $0 < \alpha < 1$. As α falls, the consumption on the composite good also goes down, and it is readily shown that the income effect diminishes. With α approaching zero, the income effect becomes negligible, and the large firms' total profits play a negligible role in the consumer's expenditure on the composite good.

Another way to eliminate the income effect, as also mentioned by Shimomura and Thisse (2012), is to redistribute the profit to the absentee shareholders. In this case, the profits earned by large firms are not enjoyed and spent by the representative consumer, and consequently the income is exogenously given.

The third way to eliminate the income effect is to nest the CES composite good in a quasi-linear utility function:

$$U = \mathbf{Q} + q_0.$$

where $\mathbf{Q}$ is the composite good as before, and q_0 is the numeriare good. This utility function is in the spirit of existing monopolistic competition literature, such as Krugman (1979, 1980), Feenstra and Ma (2007), etc. It is readily shown that the free entry and exit of small firm fixes $\mathbf{Q}$, which is independent of the number of large firms. As a consequence, the behavior of the large firm does not change with the entry of a large firm.

Other discussion Finally, we also find that the entry of a large firm will qualitatively exert the same impacts achieved by Propositions 1, 2, and 3 if we consider the following cases.

(i) Large firms and small firms are vertically differentiated. In this case, α is replaced by α_L for the large firm and by α_S for the small firm. If $\alpha_L > (<)\alpha_S$, the products of the large firms have a higher (lower) quality than the small firms.

(ii) Large firms and small firms have the same or different marginal costs. In the constant marginal cost case, the variable costs of the big and small firms are respectively $c_L q_L$ and $c_S q_S$. If firms incur increasing marginal cost, the variable costs of the big and small firms can be represented by $c_L q_L^2/2$ and $c_S q_S^2/2$respectively.

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Appendices

Appendix A-1: Proof of the necessary and sufficient conditions for the concavity of the quadratic subutility function

To ensure the concavity of the quadratic subutility function, the second-order condition should be negative definite. Although we have infinite varieties of big and small firms, we take the grid points to approximate the utility.

Consider $x_S(i)$, $i \in [0, N]$, and $x_L(j)$, $j \in [0, |\Omega_m|]$. Suppose the number of small firms is n_S, and the number of varieties of large firm m is n_L^m, $m = 1, ..., M$.

Take the grid points for the varieties of small firms and large firm m as $(N/n_S)i$, $i = 1, ..., n_S$ and $(|\Omega_m| / n_L^m)j$, $j = 1, ..., n_L^m$, respectively.

As long as $x_s(i)$, $i \in [0, N]$ and $x_L(j)$, $j \in [0, |\Omega_m|]$ are integrable, the utility function can be approximated by:

$$
\begin{aligned}
U \quad = \quad & \alpha[\sum_{i=1}^{n_S} x_s(\frac{N}{n_S}i)\frac{N}{n_S} + \sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)\frac{|\Omega_m|}{n_L^m}] \\
& -\frac{\beta}{2}[\sum_{i=1}^{n_S} x_s(\frac{N}{n_S}i)^2\frac{N}{n_S} + \sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)^2\frac{|\Omega_m|}{n_L^m}] \\
& -\frac{\gamma_1}{2}[\sum_{i=1}^{n_S} x_s(\frac{N}{n_S}i)\frac{N}{n_S}]^2 - \frac{\gamma_2}{2}[\sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)\frac{|\Omega_m|}{n_L^m}]^2 \\
& -\gamma_3[\sum_{i=1}^{n_S} x_s(\frac{N}{n_S}i)\frac{N}{n_S}][\sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)\frac{|\Omega_m|_m}{n_L^m}].
\end{aligned}
$$

which limits to the original utility function as $n_S \to \infty$ and $n_L^m \to \infty$.

The second-order derivative of the quadratic subutility with respect to $x_S(i)$ ($i \in [0, N]$) and $q_L^m(j)$ ($j \in \Omega_m$) should be negative definite, i.e. for any $x \neq \mathbf{0}$, $-x^T Hx > 0$, where:

$$\begin{aligned}
-x^T Hx \;&=\; \beta[\sum_{i=1}^{n_S} x_S(\frac{N}{n_S}i)^2 \frac{N}{n_S} + \sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)^2 \frac{|\Omega_m|}{n_L^m}] + \gamma_1[\sum_{i=1}^{n_S} x_S(\frac{N}{n_S}i)\frac{N}{n_S}]^2 \\
&\quad + \gamma_2[\sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)\frac{|\Omega_m|}{n_L^m}]^2 + 2\gamma_3[\sum_{i=1}^{n_S} x_S(\frac{N}{n_S}i)\frac{N}{n_S}][\sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)\frac{|\Omega_m|}{n_L^m}].
\end{aligned}$$

We identify the necessary and sufficient condition for H to be negative definite in the following two steps. First, we find the minimized value $-x^T Hx$ in terms of β, γ_1, γ_2, γ_3 and x. Second, we identify the sufficient condition for the minimized value of $-x^T Hx$ to be positive.

Step 1:

Suppose $a = \sum_{i=1}^{n_S} x_S(\frac{N}{n_S}i)\frac{N}{n_S}$, and $b = \sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)$. Then:

$$-x^T Hx = \beta[\sum_{i=1}^{n_S} x_S(\frac{N}{n_S}i)^2 \frac{N}{n_S} + \sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)^2 \frac{|\Omega_m|}{n_L^m}] + \gamma_1 a^2 + \gamma_2 b^2 + 2\gamma_3 ab.$$

By Jensen's inequality, we have $\sum_{i=1}^{n_S} x_S(\frac{N}{n_S}i)^2 \frac{N}{n_S} + \sum_{m=1}^{M}\sum_{j=1}^{n_L^m} x_L^m(\frac{|\Omega_m|}{n_L^m}j)^2 \frac{|\Omega_m|}{n_L^m} \geq a^2/N + b^2/(M\,|\Omega_m|)$.

We normalize x such that $a^2+b^2 = 1$. The minimization of the value of $-x^T Hx$ is then expressed as:

$$\min_{a,b}\; \beta(\frac{a^2}{N} + \frac{b^2}{M\,|\Omega_m|}) + \gamma_1 a^2 + \gamma_2 b^2 + 2\gamma_3 ab$$

$$\text{subject to } a^2 + b^2 = 1.$$

The Lagrangian function is

$$L = \beta(\frac{a^2}{N} + \frac{b^2}{M\,|\Omega_m|}) + \gamma_1 a^2 + \gamma_2 b^2 + 2\gamma_3 ab + \lambda(a^2 + b^2 - 1).$$

Here λ is the Lagrangian multiplier. The first order conditions of L with respect to a, b and λ yield

$$\begin{aligned} 2a(\frac{\beta}{N} + \gamma_1 + \lambda) + 2\gamma_3 b &= 0, \\ 2b(\frac{\beta}{M\,|\Omega_m|} + \gamma_2 + \lambda) + 2\gamma_3 a &= 0, \\ a^2 + b^2 &= 1. \end{aligned} \tag{16}$$

To ensure the objective function is minimized, the Hessian matrix should be positive definite:

$$\begin{pmatrix} 2(\beta/N + \gamma_1 + \lambda) & \gamma_3 \\ \gamma_3 & 2(\beta/(M\,|\Omega_m|) + \gamma_2 + \lambda) \end{pmatrix}.$$

which requires $\beta/N + \gamma_1 + \lambda > 0$ and $\beta/M\,|\Omega_m| + \gamma_2 + \lambda > 0$. Hence:

$$\lambda = [-(\beta/N + \beta/(M\,|\Omega_m|) + \gamma_1 + \gamma_2) + \sqrt{(\beta/N + \gamma_1 - \beta/(M\,|\Omega_m|) - \gamma_2)^2 + 4\gamma_3^2}]/2.$$

Denote:

$$\Psi = \frac{-(p-q) + \sqrt{(p-q)^2 + 4\gamma_3^2}}{2}.$$

where $p = \beta/N + \gamma_1$, and $q = \beta/(M\,|\Omega_m|) + \gamma_2$.

Substituting λ into equation (16), we have $a = -\Psi b/\gamma_3$.

Let $\Theta = -x^T Hx$. Then,

$$\Theta = b^2(\frac{p}{\gamma_3^2}\Psi^2 - 2\Psi + q).$$

Step 2:

Now we identify the conditions on which Θ is positive. Observe that Θ is a quadratic function of Ψ. There are four combinations of p and q that determine the shape of Θ in terms of Ψ. We find the necessary and sufficient conditions for Θ to be positive in the following four cases.

1. $p > 0$ and $q > 0$.

In this case, Θ is a convex function of Ψ. A sufficient condition for Θ to be positive is $p/\gamma_3^2 > 0$ and $4 - 4pq/\gamma_3^2 > 0$. In other words, Θ is always positive if $p > 0$, $q > 0$, and $pq > \gamma_3^2$.

If $pq < \gamma_3^2$, $\Psi = [-(p-q)+\sqrt{(p-q)^2 + 4\gamma_3^2}]/2 > [-(p-q)+\sqrt{(p-q)^2 + 4pq}]/2 = q$. In addition, $(\gamma_3^2 - pq)^2 = \gamma_3^2(\gamma_3^2 - pq) - pq(\gamma_3^2 - pq) < \gamma_3^2(\gamma_3^2 - pq)$, implying that $\gamma_3^2 - |\gamma_3|\sqrt{\gamma_3^2 - pq} < pq$. Hence $\Psi > q > (\gamma_3^2 - |\gamma_3|\sqrt{\gamma_3^2 - pq})/p$. On the other hand, $pq < \gamma_3^2$ implies $q < \gamma_3^2/p$. Hence $\Psi = [q - p + \sqrt{(q-p)^2 + 4\gamma_3^2}]/2 < [\gamma_3^2/p - p + \sqrt{(\gamma_3^2/p - p)^2 + 4\gamma_3^2}]/2 = \gamma_3^2/p < (\gamma_3^2 + |\gamma_3|\sqrt{\gamma_3^2 - pq})/p$. Therefore, $(\gamma_3^2 - |\gamma_3|\sqrt{\gamma_3^2 - pq})/p < \Psi < (\gamma_3^2 + |\gamma_3|\sqrt{\gamma_3^2 - pq})\gamma_3/p$, and Θ is consequently negative. Thus, $pq > \gamma_3^2$ is a necessary and sufficient condition to ensure a positive Θ in this case.

2. $p > 0$ and $q < 0$.

In this case, Θ is a convex function of Ψ. Since $pq < \gamma_3^2$ always holds, $(\gamma_3^2 - |\gamma_3|\sqrt{\gamma_3^2 - pq})/p < 0 < \Psi < (\gamma_3^2 + |\gamma_3|\sqrt{\gamma_3^2 - pq})/p$, and Θ is consequently negative.

3. $p < 0$ and $q > 0$.

In this case, Θ is a concave function of Ψ, and $pq < 0 < \gamma_3^2$ always holds. Then $(\gamma_3^2 - pq)^2 = \gamma_3^2(\gamma_3^2 - pq) - pq(\gamma_3^2 - pq) > \gamma_3^2(\gamma_3^2 - pq)$, implying that $\gamma_3^2 - |\gamma_3| \sqrt{\gamma_3^2 - pq} > pq$ and $[\gamma_3^2 - |\gamma_3| \sqrt{\gamma_3^2 - pq}]/p < q$. As shown earlier, $pq < \gamma_3^2$ implies that $\Psi > q$. Therefore, $\Psi > (\gamma_3^2 - |\gamma_3| \sqrt{\gamma_3^2 - pq})/p$, and Θ is consequently negative.

4. $p < 0$ and $q < 0$.

In this case, Θ is a concave function of Ψ. If $pq > \gamma_3^2$, Θ is always negative. If $pq < \gamma_3^2$, $\Psi > 0 > (\gamma_3^2 - |\gamma_3| \sqrt{\gamma_3^2 - pq})/p$, and Θ is consequently negative.

Summing up the above four cases, therefore, the subutility function is concave when $\beta/N + \gamma_1 > 0$, $\beta/(M|\Omega_m|) + \gamma_2 > 0$, and $(\beta/N + \gamma_1)(\beta/(M|\Omega_m|) + \gamma_2) > \gamma_3^2$.[7]

Appendix A-2: Proof of Proposition 1.

Given the equilibrium values of $q_S^* = \sqrt{f/\beta}$ and $q_L^* = \sqrt{F/\beta}$, the free entry condition of small firm and the profit maximization of large firm yield the following two expressions of dynamic adjustment process:

$$\dot{N}(N, |\Omega|) = d_1[\alpha q_S^* - \beta q_S^{*2} - (\gamma_1 N q_S^* + \gamma_3 M |\Omega| q_L^*) q_S^* - f],$$

$$\dot{|\Omega|}(N, |\Omega|) = d_2\{[\alpha - \beta q_L^* - \gamma_3 N q_S^* - \gamma_2 (M+1) |\Omega| q_L^*] q_L^* - F\}.$$

where $\dot{N} = dN/dt$, $\dot{|\Omega|} = d|\Omega|/dt$, $d_1 > 0$ and $d_2 > 0$ are the speed of dynamic adjustment. Without loss of generality, set $d_1 = d_2 = 1$. To ensure the local stability of the established model, the Jacobian matrix derived from the above two

[7]The proof can be more general if we replace $M|\Omega_m|$ with $\sum_{m=1}^{M} |\Omega_m|$.

expressions is required to be negative definite:

$$J = \begin{pmatrix} \partial \dot{N}/\partial N & \partial \dot{N}/\partial |\Omega| \\ \partial |\dot{\Omega}|/\partial N & \partial |\dot{\Omega}|/\partial |\Omega| \end{pmatrix} = \begin{pmatrix} -\gamma_1 q_S^{*2} & -\gamma_3 M q_S^* q_L^* \\ -\gamma_3 q_S^* q_L^* & -\gamma_2 (M+1) q_L^{*2} \end{pmatrix}.$$

$J^1 = -\gamma_1 q_S^* < 0$, and $J^2 = [\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M] q_S^{*2} q_L^{*2} > 0$. Hence $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

Appendix A-3: Proof of Proposition 2.

Let $D = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M$, and $E = \alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})$. By Proposition 1, $D > 0$ and $E > 0$. From the obtained results, we have:

$dq_S^*/dM = 0$, $dp_S^*/dM = 0$, $dq_L^*/dM = 0$, and $dN^*/dM = -\gamma_2\gamma_3 E\sqrt{\beta/f}/D^2 < 0$. $dp_L^*/dM = \gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E/D^2$, $d\,|\Omega^*|\,/dM = (\gamma_3^2 - \gamma_1\gamma_2)E\sqrt{\beta/F}/D^2$, $d\Pi_L^*/dM = 2\gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E^2/D^3$, which are positive if $\gamma_1\gamma_2 < \gamma_3^2$ and negative if $\gamma_1\gamma_2 > \gamma_3^2$, and $dQ^*/dM = \gamma_2(\gamma_1 - \gamma_3)E/D^2 \geqq 0$.

Appendix A-4: Proof of Proposition 3.

The consumer surplus, producer surplus and social welfare can be expressed as:

$$\begin{aligned} CS^* &= \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\,|\Omega^*|\, q_L^{*2}) - \frac{\gamma_1}{2} Q_S^{*2} \\ &\quad - \frac{\gamma_2}{2} Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^*, \\ PS^* &= \frac{\gamma_2 M E^2}{D^2}, \\ SW^* &= \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\,|\Omega^*|\, q_L^{*2}) - \frac{\gamma_1}{2} Q_S^{*2} \\ &\quad - \frac{\gamma_2}{2} Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^* + \frac{\gamma_2 E^2}{D^2}. \end{aligned}$$

The impact of a marginal increase of M on consumer surplus is:

$$\frac{dCS^*}{dM} = -\frac{\gamma_2 E}{D^2}[\frac{EM(\gamma_3^2 - \gamma_1\gamma_2)}{D} + \frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{f} - \gamma_1\sqrt{F})].$$

which is positive if $2E(\gamma_3^2 - \gamma_1\gamma_2)M + D\sqrt{\beta}(\gamma_3\sqrt{f} - \gamma_1\sqrt{F}) < 0$ and negative if $2E(\gamma_3^2 - \gamma_1\gamma_2)M + D\sqrt{\beta}(\gamma_3\sqrt{f} - \gamma_1\sqrt{F}) > 0$.

The impact of a marginal increase of M on producer surplus is:

$$\frac{dPS^*}{dM} = \frac{\gamma_2 E^2}{D^3}[\gamma_3^2 M - \gamma_1\gamma_2(M-1)].$$

which is positive (negative) if $\gamma_3^2 M - \gamma_1\gamma_2(M-1) > (<)0$.

The impact of a marginal increase of M on social welfare is:

$$\frac{dSW^*}{dM} = \frac{\gamma_2 E}{2D^3}[2\gamma_1\gamma_2 E - D\sqrt{\beta}(\gamma_3\sqrt{f} - \gamma_1\sqrt{F})].$$

which is positive if $2\gamma_1\gamma_2 E - D\sqrt{\beta}(\gamma_3\sqrt{f} - \gamma_1\sqrt{F}) > 0$ and negative if $2\gamma_1\gamma_2 E - D\sqrt{\beta}(\gamma_3\sqrt{f} - \gamma_1\sqrt{F}) < 0$.

Appendix A-5: Proof of Proposition 4.

(4-i)

Given the equilibrium value of $q_S^* = \sqrt{f/\beta}$, the free entry condition of small firm and the profit maximization of large firm yield the following two expressions of dynamic adjustment process:

$$\dot{N}(N, q_L) = d_1[\alpha q_S^* - \beta q_S^{*2} - (\gamma_1 N q_S^* + \gamma_3 M q_L)q_S^* - f],$$

$$\dot{q_L}(N, q_L) = d_2\{[\alpha - 2\beta q_L^* - \gamma_3 N q_S^* - \gamma_2(M+1)q_L]\}.$$

where $\dot{N} = dN/dt$, $\dot{q_L} = dq_L/dt$, $d_1 > 0$ and $d_2 > 0$. To ensure the local stability of the established model, the Jacobian matrix derived from the above two expressions is required to be negative definite:

$$J = \begin{pmatrix} \partial\dot{N}/\partial N & \partial\dot{N}/\partial q_L \\ \partial\dot{q_L}/\partial N & \partial\dot{q_L}/\partial q_L \end{pmatrix} = \begin{pmatrix} -\gamma_1 q_S^{*2} & -\gamma_3 M q_S^* \\ -\gamma_3 q_S^* & -2\beta - \gamma_2(M+1) \end{pmatrix}.$$

$J^1 = -\gamma_1 q_S^{*2} < 0$, and $J^2 = [\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M]q_S^{*2} > 0$. Hence $\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

(4-ii)

Let $H = \gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M$, and $I = \alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f}$. By (4-i), $H > 0$ and $I > 0$. From the obtained results, we have:

$dq_S^*/dM = 0$, $dp_S^*/dM = 0$, and $dN^*/dM = -(\beta + \gamma_2)\gamma_3 I\sqrt{\beta/f}/H^2 < 0$. $dq_L^*/dM = (\gamma_3^2 - \gamma_1\gamma_2)I/H^2$, $dp_L^*/dM = (\beta+\gamma_2)(\gamma_3^2 - \gamma_1\gamma_2)I/H^2$, $d\Pi_L^*/dM = 2(\beta + \gamma_2)(\gamma_3^2 - \gamma_1\gamma_2)E^2/D^3$, which are positive if $\gamma_1\gamma_2 < \gamma_3^2$ and negative if $\gamma_1\gamma_2 > \gamma_3^2$, and $dQ^*/dM = (2\beta + \gamma_2)(\gamma_1 - \gamma_3)I/H^2$, which is positive (negative) if $\gamma_1 > (<)\gamma_3$.

(4-iii)

The consumer surplus, producer surplus and social welfare can be expressed as:

$$
\begin{aligned}
CS^* &= \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\left|\Omega^*\right| q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} \\
&\quad -\frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^*, \\
PS^* &= \frac{\gamma_2 M I^2}{H^2}, \\
SW^* &= \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\left|\Omega^*\right| q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} \\
&\quad -\frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^* + \frac{\gamma_2 I^2}{H^2}.
\end{aligned}
$$

The impact of a marginal increase of M on consumer surplus is:

$$
\frac{dCS^*}{dM} = \frac{I}{2H^2}[A\gamma_1(2\beta+\gamma_2) + B(\gamma_1\gamma_2 - \gamma_3^2)M].
$$

where $A = \alpha\beta(\gamma_1-\gamma_3) - \gamma_2\gamma_3\sqrt{\beta f}$, and $B = \alpha(\gamma_1-\gamma_3)(3\beta+2\gamma_2) + \gamma_3\sqrt{\beta f}(4\beta+3\gamma_2)$. dCS^*/dM is positive if $A\gamma_1(2\beta+\gamma_2) + B(\gamma_1\gamma_2-\gamma_3^2)M > 0$ and negative if $A\gamma_1(2\beta+\gamma_2) + B(\gamma_1\gamma_2-\gamma_3^2)M < 0$.

The impact of a marginal increase of M on producer surplus is:

$$
\frac{dPS^*}{dM} = \frac{(\beta+\gamma_2)E^2}{D^3}[\gamma_1(2\beta+\gamma_2) - (\gamma_1\gamma_2-\gamma_3^2)M] - F.
$$

which is positive if $(\beta+\gamma_2)I^2[\gamma_1(2\beta+\gamma_2) - (\gamma_1\gamma_2-\gamma_3^2)M] > H^3F$ and negative if $(\beta+\gamma_2)I^2[\gamma_1(2\beta+\gamma_2) - (\gamma_1\gamma_2-\gamma_3^2)M] < H^3F$.

The impact of a marginal increase of M on social welfare is:

$$
\frac{dSW^*}{dM} = \frac{I}{2H^2}[B\gamma_1(2\beta+\gamma_2) + A(\gamma_1\gamma_2 - \gamma_3^2)M].
$$

which is positive if $B\gamma_1(2\beta + \gamma_2) + A(\gamma_1\gamma_2 - \gamma_3^2)M > 0$ and negative if $B\gamma_1(2\beta + \gamma_2) + A(\gamma_1\gamma_2 - \gamma_3^2)M < 0$.

Figures

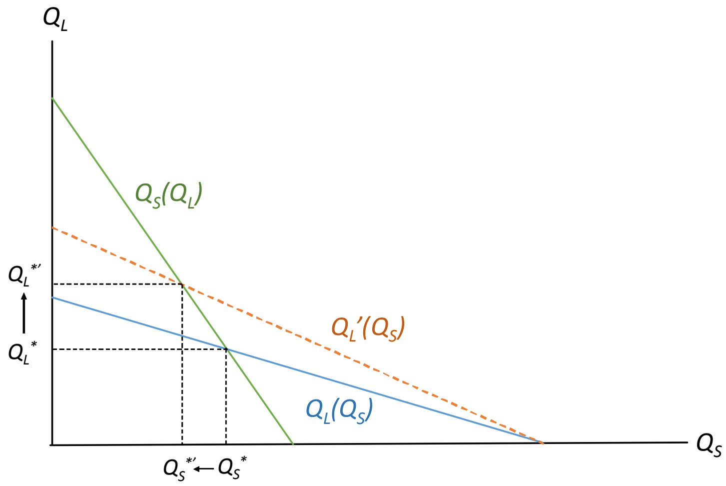

Figure 1

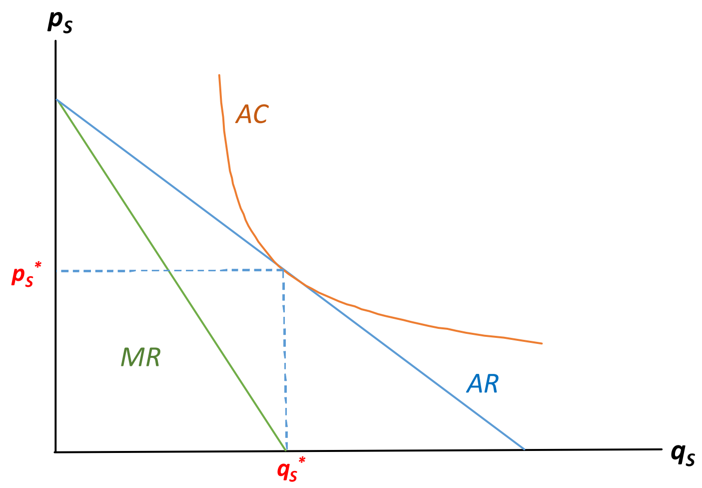

Figure 2