Privatization Neutrality Theorem in Free Entry Markets

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Abstract

It is known that if the number of entering firms is endogenous (free entry markets), privatization is not necessarily welfare neutral in mixed oligopolies under a uniform production subsidy policy. We revisit this problem by considering another policy tool, the output floor regulation. We investigate three free entry models with different time structures, a Cournot and two Stackelberg models. We find that neutrality is restored in free entry markets under the optimal output floor regulation, regardless of the time structure.

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1 Introduction

Whether the public ownership in firms influences their activities is a longstanding, important research question. Some empirical studies suggested that public ownership reduces the production efficiency or profitability of the firms, and others suggested that it does not affect their operations significantly. Thus, an effect of such ownership is a controversy issue from an empirical viewpoint (Megginson and Netter, 2001). In the theoretical literature on mixed oligopolies, many studies showed that privatization of a public firm can either enhance or reduce welfare, depending on the competition structure, but they assumed that the government does not adopt direct regulations or tax-subsidy policies. White (1996) showed that the implications of privatization as described in this literature change drastically if such policies are explicitly considered. He introduced simple nondiscriminatory unit production subsidy into Cournot models and showed that this subsidy policy yields the first-best outcome in mixed as well as private oligopolies, and thus, privatization does not matter under the optimal subsidy policy (the privatization neutrality theorem).

Many studies suggested that this result is robust in various economic circumstances. Hashimzade et al. (2007) considered product differentiation, Tomaru (2006) adopted a partial privatization approach formulated by Matsumura (1998), and Kato and Tomaru (2007) considered non-profit-maximizing private firms. The privatization neutrality theorem holds in all these cases. Poyago-Theotoky (2001) and Tomaru and Saito (2010) considered Stackelberg duopolies where the public firm is the leader and the private firm is the follower, and showed that privatization does not affect welfare in either case.[1]

However, all of these models assumed that the number of firms are given exogenously. Cato and Matsumura (2013) showed that the privatization neutrality theorem depends on this assumption.

[1] However, Poyago-Theotoky (2001) and Tomaru and Saito (2010) assumed that all private firms move simultaneously after privatization. Fjell and Heywood (2004) showed that if the privatization does not change the time structure, the privatization neutrality theorem does not hold true in Stackelberg models.

The government must control both the number of entering firms and the output of each firm in free entry markets and the production subsidy policy is not suitable for this purpose.[2]

We revisit this problem by considering a different policy, an output floor regulation.[3] We show that the privatization neutrality theorem holds if we use output floor regulations. We consider three time structures in free entry markets, all firms produce simultaneously, the leaders produce before the entry of private firms (strongly persistent leadership model), and the leaders produce after such entry and before private firms produce (weakly persistent leadership model).[4] We show that in all three models, privatization does not affect welfare. In other words, the privatization neutrality theorem holds for free entry markets if we consider the optimal output floor regulation, regardless of the time structure.[5]

2 The Model

In this section, a set of firms is exogenously given by $N = \{1, 2, \cdots, n\}$. These firms produce a homogeneous good, and firm i chooses its output x_i. Let $\mathbf{x} = (x_1, x_2, \cdots, x_n)$, $X = x_1 + \cdots + x_n$,

[2] In the literature on mixed oligopolies, the analysis of free entry markets is rich and diverse. Anderson et al. (1997) and Matsumura et al. (2009) investigated monopolistic competition. Matsumura and Kanda (2005) investigated the optimal degree of privatization. Wang and Chen (2010) showed the importance of cost difference between public and private firms. Ino and Matsumura (2010) and Wang and Lee (2013) examined Stackelberg competition. Ghosh and Sen (2012) and Ghosh et al. (2013) demonstrated the closed relationship between trade and privatization policies. None of them, however, discussed the privatization neutrality theorem.

[3] In the real world, we can observe a output floor regulation in mixed oligopolies. For example, in Japan, we have to prepare ten or more guest rooms to run a hotel and there are several hotels (called Kokumin Shukusya) owned and managed by local and national governments. For examples and properties of output floors in more general context, see De Fraja and Iossa (1998) and Matsumura and Okumura (2013).

[4] For the concept and rationalization of weakly and strongly persistent leadership models, see Ino and Matsumura (2012).

[5] We can show that our result holds in the Bertrand and endogenous contract models with a differentiated product market discussed in Matsumura and Ogawa (2012) if we consider a price ceiling regulation (Bertrand) or a combination of price ceiling and output floor regulations (an endogenous price-quantity contract). The detailed results are available from the authors upon request.

and $X_{-i} = X - x_i$. The firms' cost functions are identical and are given by $C(\cdot)$, which satisfies $C' > 0$ and $C'' > 0$.[6] Let $P(X)$ be the inverse demand function satisfying $P'(X) < 0$. Firm i's profit is given by $\pi_i(\mathbf{x}) = P(X)x_i - C(x_i)$. We assume that the marginal revenue is decreasing; that is, $P'(X) + P''(X)x_i \leq 0$ for all $X \geq x_i \geq 0$. This is a weak and standard assumption. See for example Dixit (1986), Farrell and Shapiro (1990) and Vives (2001, ch.4).

Social welfare W is defined as the sum of consumer surplus and the firms' profits and is given as

$$W(\mathbf{x}) = \int_0^X P(z)dz - P(X)X + \sum_{i=1}^{n} \pi_i.$$

Each firm maximizes the following objective function with respect to its output x_i,

$$(1 - \alpha_i) W(\mathbf{x}) + \alpha_i \pi_i(\mathbf{x}),$$

where $\alpha_i \in [0, 1]$. If $\alpha_i = 1$, then firm i is a private firm that maximizes its profit. If $\alpha_i = 0$, then firm i is a public firm that maximizes social welfare. If $\alpha_i \in (0, 1)$, then firm i is a partially privatized firm that maximizes a weighted average of social welfare and its profit. That is, α_i represents the degree of privatization of firm i (see Matsumura,1998).

Let $b_i = b(X_{-i}; \alpha_i)$ be the best reply function of firm i, given the other firms' outputs. This best reply function is derived from the following first-order condition,

$$P(X) + \alpha_i P'(X)x_i - C'(x_i) = 0.$$

The second-order condition is satisfied under the assumptions made in this section. We have $\partial b(X_{-i}; \alpha_i) / \partial X_{-i} \in (-1, 0)$; that is, each firm's best reply function is downward sloping.

Let $\varnothing(X; \alpha_i)$ be a cumulative best reply function of i that is the optimal output of i which consistent with an aggregate output X. That is, $\varnothing(X; \alpha_i)$ is the unique solution of $x_i = b(X - x_i; \alpha_i)$. See Vives (2001, ch.4) for more detailed on this function. It should be noted that $\partial\varnothing(X; \alpha_i) / \partial X < 0$ under the assumptions mentioned above.

[6] Our main results are satisfied even if the cost function is linear; that is, $C'' = 0$.

We consider an output floor regulation where each firm cannot set its output below the floor level $\underline{x} \in \mathbb{R}^1_{++}$. The cumulative best response function of a firm i is

$$\hat{\varnothing}\left(X;\alpha_i,\underline{x}\right) = \max\left\{\varnothing\left(X;\alpha_i\right),\underline{x}\right\}.$$

Because $\partial\varnothing\left(X;\alpha_i\right)/\partial X < 0$,

$$\frac{\partial\hat{\varnothing}\left(X;\alpha_i,\underline{x}\right)}{\partial X} \leq 0 \text{ and } \frac{\partial\sum_{i=1}^{n}\hat{\varnothing}\left(X;\alpha_i,\underline{x}\right)}{\partial X} \leq 0.$$

Next, we derive the optimal outputs of the firms that maximize social welfare. Let $\left(x_1^F,\cdots,x_n^F\right)$ be the socially optimal outputs vector. By the first order condition, $x_1^F = \cdots = x_n^F = x^F$ satisfies

$$P(nx^F) - C'\left(x^F\right) = 0. \tag{1}$$

Finally, we provide a result as regards the equilibrium outputs of the firms where α, n and the time structure (m firms are Stackelberg leaders and $n-m$ firms are Stackelberg followers, where $m = 0,1,2,...,n$) are given. The following result is proved by Matsumura and Okumura (2013).

Result 1 *For any α, n and time structure, if $\underline{x} = x^F$, then the equilibrium output of any firm is equal to x^F.*

This result implies that when the set of firms is exogenously given, the optimal output floor level is not dependent on α and the time structure, and thus, the privatization neutrality theorem holds.

In the next section, we consider three free entry market models where n is endogenously determined.

3 Free entry models

In this section, we investigate three free entry models with different time structures. The first is a Cournot model where all firms choose their outputs simultaneously. The second and the third are Stackelberg models where incumbent firms choose their output and entrants then choose their outputs. First, we describe the common structure of the three models.

The firms are divided into two: incumbents and entrants. A large number of potential entrants exists and they decide whether to enter the market. Let $I = \{1, 2, \cdots, m\}$ and $E = \{m+1, m+2, \cdots, n\}$ be the sets of incumbents and entrants in the market, respectively. That is, m incumbents are already present in the market. We assume $\alpha_i = 1$ for all $i \in E$; that is, all entrants are private firms. In other words, public firms are incumbents, if such firms exist. Note that we allow that some private firms are also incumbents. We assume that the number of entrants are positive, that is, $n > m$ in any equilibrium. The sufficient condition of this assumption is that m is sufficiently small. Let $K > 0$ be the entry cost that each entrant incurs. We ignore the integer constraint for the number of entrants.

We consider social welfare. Let $x^F(n)$ be the first-best output of each firm for a given n. The social welfare function $W(\mathbf{x}, n)$ is given by

$$W(\mathbf{x}, n) = \int_0^X P(z)dz - P(X)X + \sum_{i=1}^{n} \pi_i - (n-m)K.$$

Let n^{**} be a socially optimal number of firms and $\mathbf{x}^{**} \in \mathbb{R}_+^{n^{**}}$ be a socially optimal output vector. We can show that n^{**} satisfies

$$P\left(n^{**}x^F(n^{**})\right)x^F(n^{**}) - C\left(x^F(n^{**})\right) - K = 0$$

and $\mathbf{x}^{**} = \left(x^F(n^{**}), \cdots, x^F(n^{**})\right)$ where $x^F(n)$ satisfies (1) for a given n. In order to simplify the notation, let $x^{**} = x^F(n^{**})$ and hence $\mathbf{x}^{**} = (x^{**}, \cdots, x^{**})$. Note that the socially optimal outcome (or the first-best outcome) is equal to the perfectively competitive market equilibrium outcome, and the equilibrium conditions for perfectively competitive market is zero profit condition and zero margin condition (that is, price is equal to the marginal cost), and those coincide with the above two conditions.

3.1 Cournot model

In this subsection, we consider a free entry Cournot model with an output floor regulation. First, the potential entrants simultaneously determine whether to enter the market. Second, after observing the number of entrants, the incumbents and the entrants simultaneously decide their outputs.

Let the Cournot equilibrium output vector of firms be $\mathbf{x}^N(n,\alpha,\underline{x}) = \left(x_1^N(n,\alpha,\underline{x}),\cdots,x_n^N(n,\alpha,\underline{x})\right)$ and $X^N(n,\alpha,\underline{x}) = \sum x_i^N(n,\alpha,\underline{x})$ when the number of the firms is given by n. Then, $X^N(n,\alpha,\underline{x})$ is the unique solution of $\sum_{i=1}^n \hat{\varnothing}\left(X;\alpha_i,\underline{x}\right) = X$ and $x_i^N(n,\alpha,\underline{x}) = \hat{\varnothing}\left(X^N(n,\alpha,\underline{x});\alpha_i,\underline{x}\right)$ for all i. Therefore, we have $\partial X^N(n,\alpha,\underline{x})/\partial n > 0$ and $\partial x_i^N(n,\alpha,\underline{x})/\partial n \leq 0$.

Because an entrant's fixed entry cost is K, an entrant obtains $\pi_i(x^N(n,\alpha,\underline{x})) - K$. If a potential entrant does not enter the market, its profit is zero. Because each potential entrant is a private firm, the equilibrium number of firms $n^*\left(\alpha,\underline{x}\right)$ satisfies $\pi_j(\mathbf{x}^N(n^*\left(\alpha,\underline{x}\right),\alpha,\underline{x})) - K = 0$ for all $j \in E = \{m+1,\cdots,n^*\left(\alpha,\underline{x}\right)\}$. The free entry equilibrium output of i is $x_i^*\left(\alpha,\underline{x}\right) = x_i^N(n^*\left(\alpha,\underline{x}\right),\alpha,\underline{x})$, and the aggregate output is $X^*\left(\alpha,\underline{x}\right) = \sum_{i=1}^{n^*(\alpha,\underline{x})} x_i^*\left(\alpha,\underline{x}\right)$.

We have the following result. Theorem 1 states that the privatization neutrality theorem holds for the free entry market under the optimal output floor regulation.

Theorem 1 *If $\underline{x} = x^{**}$, then $n^*(\alpha,\underline{x}) = n^{**}$ and $x_i^*(\alpha,\underline{x}) = x^{**}$ for all $i = 1,\cdots,n^*(\alpha,\underline{x})$ and $(\alpha_1,\cdots,\alpha_m) \in [0,1]^m$.*

Proof. By Result 1, if $\underline{x} = x^{**}$, then $x^N(n^{**},\alpha,\underline{x}) = x^{**}$ and

$$P\left(X^N(n^{**},\alpha,\underline{x})\right)x^N(n^{**},\alpha,\underline{x}) - C\left(x^N(n^{**},\alpha,\underline{x})\right) = P\left(n^{**}x^{**}\right)x^{**} - C\left(x^{**}\right) = 0.$$

Next, we show $\pi_i(\mathbf{x}^N(n,\alpha,\underline{x})) - K < 0$ for all n such that $n > n^{**}$ for all $i \in E$. Because $\partial X^N(n,\alpha,\underline{x})/\partial n > 0$ and $\partial\varnothing\left(X^N;\alpha_i\right)/\partial X^N < 0$, $\varnothing\left(X^N(n^{**},\alpha,\underline{x});\alpha_i\right) > \varnothing\left(X^N(n,\alpha,\underline{x});\alpha_i\right)$ for all i. Thus, since $\hat{\varnothing}\left(X^N(n^{**},\alpha,\underline{x});\alpha_i,\underline{x}\right) = x_i^N(n^{**},\alpha,\underline{x}) = \underline{x} = x^{**}$, $x_i^N(n,\alpha,\underline{x}) = \underline{x} = x^{**}$ for all i. We have

$$\pi\left(x_i^N(n,\alpha,\underline{x})\right) = P(nx^{**})x^{**} - C\left(x^{**}\right) < P(n^{**}x^{**})x^{**} - C\left(x^{**}\right) = K.$$

Finally, we show $\pi_i(\mathbf{x}^N(n,\alpha,\underline{x}))-K>0$ for all n such that $n<n^{**}$ for all $i\in E$. First, suppose $x_i^N(n,\alpha,\underline{x})=\underline{x}=x^{**}$ for $i\in E$. Because $\partial X^N(n,\alpha,\underline{x})/\partial n>0$, $X^N(n,\alpha,\underline{x})<X^N(n^{**},\alpha,\underline{x})=n^{**}x^{**}$. Therefore, we have

$$\pi\left(\mathbf{x}^N(n,\alpha,\underline{x})\right)=P(X^N(n,\alpha,\underline{x}))x^{**}-C\left(x^{**}\right)>P(n^{**}x^{**})x^{**}-C\left(x^{**}\right)=K.$$

Second, suppose $x_i^N(n,\alpha,\underline{x})>\underline{x}=x^{**}$. Then, $\hat{\varnothing}\left(X^N(n^{**},\alpha,\underline{x});\alpha_i,\underline{x}\right)=\varnothing\left(X^N(n^{**},\alpha,\underline{x});\alpha_i\right)>x^{**}$. Thus,

$$\begin{aligned}\pi\left(\mathbf{x}^N(n,\alpha,\underline{x})\right)&=P(X^N(n,\alpha,\underline{x}))x_i^N(n,\alpha,\underline{x})-C\left(x_i^N(n,\alpha,\underline{x})\right)\\&>P(X_{-i}^N(n,\alpha,\underline{x})+x^{**})x^{**}-C\left(x^{**}\right)\end{aligned}$$

where $X_{-i}^N(n,\alpha,\underline{x})=X^N(n,\alpha,\underline{x})-x_i^N(n,\alpha,\underline{x})$. Because $X^N(n,\alpha,\underline{x})<X^N(n^{**},\alpha,\underline{x})=n^{**}x^{**}$ and $\underline{x}=x^{**}$,

$$P(X_{-i}^N(n,\alpha,\underline{x})+x^{**})x^{**}-C\left(x^{**}\right)>P(n^{**}x^{**})x^{**}-C\left(x^{**}\right)=K.$$

Hence we have Theorem 1. **Q.E.D.**

Without regulation, the output of each entrant is too small from the social welfare viewpoint, and this leads to excessive entries (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). Under the optimal output floor regulation, the output of each entrant is optimal from the social welfare viewpoint, and this also yields the optimal number of entries, resulting in the first-best outcome regardless of α for incumbents.

3.2 Weakly persistent leadership model

In this subsection, we consider a weakly persistent model with an output floor regulation. This model is based on that of Ino and Matsumura (2012). First, the potential entrants simultaneously determine whether to enter the market. Second, after observing the number of the entrants, the incumbents simultaneously decide their outputs. Finally, after observing the number of the entrants and the outputs of the incumbents, the entrants that enters the market simultaneously decide their outputs. That is, the incumbents are the Stackelberg leaders and the entrants are the followers.

First, we assume that $E = \{m+1, m+2, \cdots, n\}$ and an aggregate output of the leaders X^L are given. The aggregate equilibrium output of the firms $F\left(X^L, n, \underline{x}\right)$ is the unique solution of $(n-m)\,\hat{\varnothing}\left(X; 1, \underline{x}\right) = X - X^L$. Therefore, $\partial F\left(X^L, n, \underline{x}\right)/\partial X^L \in (0,1]$, $\partial F\left(X^L, n, \underline{x}\right)/\partial n > 0$, and $\partial F\left(X^L, n, \underline{x}\right)/\partial \underline{x} \geq 0$. Moreover, for a given $\left(X^L, n, \underline{x}\right)$, $x^E\left(X^L, n, \underline{x}\right) = \hat{\varnothing}\left(F\left(X^L, n, \underline{x}\right); 1, \underline{x}\right)$ represents an entrant's equilibrium output. Note that $\partial x^E\left(X^L, n, \underline{x}\right)/\partial X^L < 0$.

Next, we consider the output of a leader. Let $\hat{\varnothing}^W\left(X; \alpha_i, \underline{x}\right)$ be the cumulative best response function of $i \in I$. Then, $\hat{\varnothing}^W\left(X; \alpha_i, \underline{x}\right) = \max\left\{\underline{x}, \varnothing^W\left(X; \alpha_i\right)\right\}$ where $\varnothing^W\left(X; \alpha_i\right)$ satisfies

$$\begin{aligned} P(X) - C'\left(\varnothing^W\left(X; \alpha_i\right)\right) + (1-\alpha_i)(n-m)\,C'\left(x^E\right)\frac{\partial x^E\left(X^L, n, \underline{x}\right)}{\partial X^L} \\ +\alpha_i \frac{\partial F\left(X^L, n, \underline{x}\right)}{\partial X^L} P'(X)\varnothing^W\left(X; \alpha_i\right) = 0. \end{aligned} \tag{2}$$

Since $\partial F\left(X^L, n, \underline{x}\right)/\partial X^L > 0$ and $\partial x^E\left(X^L, n, \underline{x}\right)/\partial X^L < 0$, $P(X) - C'\left(\varnothing^W\left(X; \alpha_i\right)\right) \geq 0$ for any X and α_i. The equilibrium aggregate output $X^W\left(n, \alpha, \underline{x}\right)$ is a unique solution of

$$F\left(\sum_{i=1}^{m} \hat{\varnothing}^W\left(X; \alpha_i, \underline{x}\right), n, \underline{x}\right) = X.$$

In addition, let $x_i^W\left(n, \alpha, \underline{x}\right) = \hat{\varnothing}^W\left(X^W\left(n, \alpha, \underline{x}\right); \alpha_i, \underline{x}\right)$ for $i \in I$, $x_j^W\left(n, \alpha, \underline{x}\right) = \hat{\varnothing}\left(X^W\left(n, \alpha, \underline{x}\right); 1, \underline{x}\right)$ for $j \in E$.

Finally, let $n^W\left(\underline{x}, \alpha\right)$ be such that

$$\pi_i(\mathbf{x}^W\left(n^W\left(\alpha, \underline{x}\right), \alpha, \underline{x}\right)) - K = 0,$$

where $\mathbf{x}^W\left(n, \alpha, \underline{x}\right)) = \left(x_1^W\left(n, \alpha, \underline{x}\right), \cdots, x_n^W\left(n, \alpha, \underline{x}\right)\right)$.

Moreover, $x_i^W\left(n, \alpha, \underline{x}\right) = \hat{\varnothing}^W\left(X^W\left(n^W\left(\alpha, \underline{x}\right), \alpha, \underline{x}\right); \alpha_i, \underline{x}\right)$ for $i \in I$,
$x_j^W\left(\alpha, \underline{x}\right) = \hat{\varnothing}\left(X^W\left(n^W\left(\alpha, \underline{x}\right), \alpha, \underline{x}\right); 1, \underline{x}\right)$ for $j = m+1, \cdots, n^W\left(\alpha, \underline{x}\right)$.

Thus, we have the following result. Theorem 2 again states that the privatization neutrality theorem holds under the optimal output floor regulation at this free entry market, too.

Theorem 2 *If $\underline{x} = x^{**}$, then $n^W(\alpha, \underline{x}) = n^{**}$ and $x_i^W(\alpha, \underline{x}) = x^{**}$ for all $i = 1, \cdots, n^W(\alpha, \underline{x})$ and $(\alpha_1, \cdots, \alpha_m) \in [0,1]^m$.*

Proof. By Result 1, if $\underline{x} = x^{**}$, then $x_i^W(n^{**}, \alpha, \underline{x}) = x^{**}$ and

$$P\left(X^W(n^{**}, \underline{x})\right) x_i^W(n^{**}, \alpha, \underline{x}) - C\left(x_i^W(n^{**}, \alpha, \underline{x})\right) = P\left(n^{**}x^{**}\right) x^{**} - C\left(x^{**}\right) = 0$$

for all i.

Next, we show $\pi_j(\mathbf{x}^W(n, \alpha, \underline{x})) - K < 0$ for all n such that $n > n^{**}$ and $j \in E$. Because $x_i^W(n, \alpha, \underline{x}) \geq \underline{x}$ for all i, $X^W(n, \alpha, \underline{x}) > X^W(n^{**}, \alpha, \underline{x})$. Thus, we have $x_i^W(n, \alpha, \underline{x}) = x^{**}$ and

$$\pi\left(\mathbf{x}^W(n, \alpha, \underline{x})\right) = P(nx^{**})x^{**} - C\left(x^{**}\right) < P(n^{**}x^{**})x^{**} - C\left(x^{**}\right) = K.$$

Finally, we show $\pi_j(\mathbf{x}^W(n, \alpha, \underline{x})) - K > 0$ for all n such that $n < n^{**}$ and $j \in E$. First, we show $X^W(n, \alpha, \underline{x}) \leq nx^F(n)$ for all $n < n^{**}$. Suppose not; that is, $X^W(n, \alpha, \underline{x}) > nx^F(n)$. There exists at least one firm i such that $x_i^W(n^{**}, \alpha, \underline{x}) > x^F(n)$. Because $x^F(n) > x^F(n^{**}) = x^{**} = \underline{x}$ and $x^F(n) > \varnothing\left(nx^F(n); 1\right) > \varnothing\left(X^W(n, \underline{x}); 1\right)$, $x^F(n) > x_j^W(n^{**}, \alpha, \underline{x})$ for all $j \in E$. Then, $x_i^W(n^{**}, \alpha, \underline{x}) > x^F(n)$ for some $i \in I$. Because $P(nx^F(n)) - C'(x^F(n)) = 0$ and $x^F(n) > \underline{x}$,

$$P(X^W(n, \alpha, \underline{x})) - C'(x_i^W(n^{**}, \alpha, \underline{x})) < 0.$$

However, this contradicts the first order condition (2) of firm i. Thus, $X^W(n, \alpha, \underline{x}) \leq nx^F(n) < n^{**}x^{**}$. Furthermore, since $\hat{\varnothing}\left(X^W(n, \alpha, \underline{x}); 1, \underline{x}\right) \geq \underline{x} = x^{**}$ we have

$$\begin{aligned} P(X^W(n, \alpha, \underline{x}))\hat{\varnothing}\left(X^W(n, \alpha, \underline{x}); 1, \underline{x}\right) - C(\hat{\varnothing}\left(X^W(n, \alpha, \underline{x}); 1, \underline{x}\right)) &\geq P(X_{-i}^W(n, \alpha, \underline{x}) + x^{**})x^{**} - C(x^{**}) \\ \geq P(X^W(n, \alpha, \underline{x}))x^{**} - C(x^{**}) > P(n^{**}x^{**})x^{**} - C\left(x^{**}\right) &= K, \end{aligned}$$

where $X_{-i}^W(n, \alpha, \underline{x}) = X^W(n, \alpha, \underline{x}) - \hat{\varnothing}\left(X^W(n, \alpha, \underline{x}); 1, \underline{x}\right)$. **Q.E.D.**

In order to explain Theorem 2, we firstly consider the case where no regulation exists. Then, since the incumbents are leaders, each entrant's output are negatively dependent on the sum of incumbents' outputs. Thus, an incumbent with $\alpha > 0$ has an incentive to choose a larger output than each entrant for reducing the entrants' outputs in order to increase its profit. Then, the difference in the outputs between an incumbent and an entrant causes welfare distortion. Under the optimal output floor regulation, each entrant's output must be independent of the incumbents' outputs, and the incumbent loses the incentive to expand its output for this strategic purpose. Moreover, an incumbent with $\alpha < 1$ also has an incentive to choose a larger output than each entrant for improving social welfare. The incumbent also loses the incentive to increase its output for this purpose, because the output of each entrant is equal to x^{**}. Thus, the output floor regulation eliminates such distortion.

3.3 Strongly persistent leadership model

In this subsection, we consider a strongly persistent model with an output floor regulation. This model is also based on that of Ino and Matsumura (2012). First, all incumbents simultaneously decides their output. Let X^L be their aggregate output. Second, after observing X^L, each potential entrant decides whether to enter the market. Thus, the number of entrants $n - m$ is determined at this stage. Third, the entrants in the market simultaneously choose their outputs.

First, we derive the entrant's equilibrium output for a given $\left(X^L, n, \underline{x}\right)$. Then, $F\left(X^L, n, \underline{x}\right)$ that is defined in the previous section. Suppose $X^L < X^*(\alpha, \underline{x})$. Because $\partial F\left(X^L, n, \underline{x}\right)/\partial n > 0$, the equilibrium number of firms $n^S\left(X^L, \alpha, \underline{x}\right)$ is a solution of

$$P\left(X^L + (n-m)\, x^E\left(X^L, n, \underline{x}\right)\right) x^E\left(X^L, n, \underline{x}\right) - C(x^E\left(X^L, n, \underline{x}\right)) - K = 0.$$

This implies that the equilibrium aggregate output $X^S\left(X^L, \alpha, \underline{x}\right)$ is a solution of

$$P(X)\, \hat{\varnothing}(X; 1, \underline{x}) - C(\hat{\varnothing}(X; 1, \underline{x})) - K = 0.$$

Hence if $X^L < X^*(\alpha, \underline{x})$, then $X^S\left(X^L, \alpha, \underline{x}\right) = X^*(\alpha, \underline{x})$ and $n^S\left(X^L, \alpha, \underline{x}\right) = m + \left(X^*(\alpha, \underline{x}) - X^L\right)/x^*(\alpha, \underline{x})$. On the other hand, if $X^L > X^*(\alpha, \underline{x})$, then

$$P\left(F\left(X^L, n, \underline{x}\right)\right) \hat{\varnothing}\left(F\left(X^L, n, \underline{x}\right); 1, \underline{x}\right) - C(\hat{\varnothing}\left(F\left(X^L, n, \underline{x}\right); 1, \underline{x}\right)) - K < 0$$

for all $n > m$. This implies that $X^S\left(X^L, \alpha, \underline{x}\right) = \min\left\{X^*\left(\alpha, \underline{x}\right), X^L\right\}$ and

$$n^S\left(X^L, \alpha, \underline{x}\right) = \left\{m + \frac{X^*\left(\alpha, \underline{x}\right) - X^L}{x^*\left(\alpha, \underline{x}\right)}, m\right\}.$$

That is, if the total output of the leaders is less than the free entry Cournot equilibrium, the equilibrium total output is equivalent to the free entry Cournot equilibrium total output level. This result show the robustness of a result of Ino and Matsumura (2012, Lemma 1) that consider the model where all firms are private and no regulation exists. Since we assume that the equilibrium number of entrants is positive, $X^S\left(X^L, \underline{x}\right) = X^L$ and $n^S\left(X^L, \underline{x}\right) = m$ are satisfied only in an off-equilibrium path.

Now, we consider the incumbents' outputs. Let $\hat{\varnothing}^S\left(X; \alpha_i, \underline{x}\right) = \max\left\{\underline{x}, \varnothing^S\left(X; \alpha_i\right)\right\}$ be the best reply function for $\left(X; \alpha_i, \underline{x}\right)$ where $\varnothing^S\left(X; \alpha_i\right)$ is the solution of

$$-\left(1-\alpha_i\right)\left(C'\left(x_i\right) + \left(C\left(x^*\left(\alpha, \underline{x}\right)\right) + K\right)\frac{\partial n^S\left(X^L, \underline{x}\right)}{\partial X^L}\right) + \alpha_i\left(P(X) - C'\left(x_i\right)\right) = 0. \quad (3)$$

The equilibrium aggregate output $X^S\left(\alpha, \underline{x}\right)$ is a solution of

$$F\left(X^L, n^S\left(X^L, \alpha, \underline{x}\right), \underline{x}\right) = X \text{ and } X^L = \sum_{i=1}^{m} \hat{\varnothing}^S\left(X; \alpha_i, \underline{x}\right).$$

In addition, let $x_i^S\left(\alpha, \underline{x}\right) = \hat{\varnothing}^S\left(X^S\left(\alpha, \underline{x}\right); \alpha_i, \underline{x}\right)$ for $i \in I$, $x_j^S\left(\alpha, \underline{x}\right) = \hat{\varnothing}_j\left(X^S\left(\alpha, \underline{x}\right); 1, \underline{x}\right)$ for $j \in E$.

We present our last result. Theorem 3 states that the privatization neutrality theorem holds under the optimal output floor regulation at this free entry market, too.

Theorem 3 *If $\underline{x} = x^{**}$, then $n^S\left(\alpha, \underline{x}\right) = n^{**}$ and $x_i^S(\alpha, \underline{x}) = x^{**}$ for all $i = 1, \cdots, n^S\left(\alpha, \underline{x}\right)$ and for any $\left(\alpha_1, \cdots, \alpha_m\right) \in [0,1]^m$.*

Proof. Suppose $\underline{x} = x^{**}$. By Result 1, $X^S\left(X^L, \alpha, \underline{x}\right) = \min\left\{X^{**}, X^L\right\}$. Since we assume that the number of entrants is positive in any equilibrium, $X^S\left(X^L, \alpha, \underline{x}\right) = X^{**} > X^L$, $x^E\left(X^L, n, \underline{x}\right) = \underline{x} = x^{**}$ and $n^S\left(X^L, \underline{x}\right) = m + \left(X^{**} - X^L\right)/x^{**}$. Moreover, in this case, (3) is

$$-\left(1-\alpha_i\right)\left(C'\left(x_i\right) - \frac{C\left(x^{**}\right) + K}{x^{**}}\right) + \alpha_i\left(P(X^{**}) - C'\left(x_i\right)\right) = 0.$$

By the definition of x^{**}, $C'(x^{**}) = (C(x^{**}) + K)/x^{**}$. Therefore,

$$-(1-\alpha_i)\left(C'(x^{**}) - \frac{C(x^{**})+K}{x^{**}}\right) + \alpha_i \left(P(X^{**}) - C'(x^{**})\right) = 0.$$

Moreover, for any $x < (>) x^{**}$,

$$-(1-\alpha_i)\left(C'(x) - \frac{C(x^{**})+K}{x^{**}}\right) + \alpha_i \left(P(X^{**}) - C'(x)\right) > (<) 0.$$

This implies that $n^S(\alpha, \underline{x}) = n^{**}$ *and* $x_i^S(\alpha, \underline{x}) = x^{**}$ for all $i = 1, \cdots, n^S(\alpha, \underline{x})$. **Q.E.D.**

Without regulation, the number of entrants in the market depends on each incumbent's output. Thus, an incumbent with $\alpha < 1$ chooses a larger output than each entrant for reducing the number of entrants in order to moderate the excessive entry. Further, an incumbent with $\alpha > 0$ chooses a larger output than each entrant for increasing its profit. The difference in outputs between each incumbent and each entrant causes distortion. The optimal output floor regulation, however, reduces the number of entrants without strategic behaviors by the incumbents. Under the regulation the equilibrium price is equal to the marginal cost of each incumbent at the first-best production level. Therefore, each incumbent loses the incentive to expand its output beyond the first-best production level for the strategic purpose. This eliminates the welfare distortion.

4 Concluding Remarks

In this study, we investigated three free entry models with three different time structures, Cournot, and Stackelberg with weakly and strongly persistent leaderships. We showed that in all models, the privatization neutrality theorem holds under the output floor regulation. This result is in sharp contrast to that of Cato and Matsumura (2013) who showed that the privatization neutrality theorem does not hold at free entry markets under the optimal production subsidy policy. This result also in sharp contrast to that of Fjell and Heywood (2004) who showed that the privatization neutrality theorem does not hold in a Stackelberg model under the optimal production subsidy policy as long as the privatization does not affect the time structure. Our result suggested the importance and robustness of the output floor regulation in this context.

In this study, we assumed that all firms are domestically owned. In the literature on mixed oligopolies, foreign ownership in private firms often matters.[7] We can show that the privatization neutrality theorem does not hold with foreign penetration even under the uniform output floor regulation. Investigating another policy tool that yields the first-best outcome under such circumstances remains a topic for future research.

In this study, we consider complete information games with two-tier time structures. Shinkai (2000) showed that in his incomplete information game, three-tier structure changes the result drastically. In our context, we presume that three-tier time structure does not affect our results as long as we consider the complete information game. However, when there is incomplete information, our results can be changed. Investigating this problem also remains a topic for future research.

[7] See Corneo and Jeanne (1994) and Barcena-Ruiz and Garzon (2005a, b). See also Lin and Matsumura (2012) for the foreign penetration in both public and private firms.

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