

Volume and Share Quotas in Oligopoly*

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Abstract

In this paper, we theoretically discuss volume and share quotas in Cournot oligopoly. We show the equivalence between specific taxes and volume quotas and share quotas with respect to equilibrium quantities. By using these results, we analyze comparative statics effects of volume and share quotas. Further, we apply the results to the examination of an international oligopoly model with tariffs, import volume quotas, and import share quotas. Finally, we extend the model to endogenize the set of firms and derive a non-equivalence result of volume quotas and specific taxes.

Keywords: Volume quota; Share quota; Specific tax; Cournot oligopoly; International oligopoly

JEL classification: D43; F12; F13; L51

1 Introduction

In this paper, we theoretically discuss an oligopolistic market with two ways of quantity control: a volume quota and a share quota. If there is a volume quota system in a market, a firm cannot set their quantity of production more than the volume quota level assigned to it. There are several real world examples of volume quotas. In a number of marine product markets, a fisherman is limited by the individual quota assigned to him. Moreover, hunters often face bag and possession limits; that is, they cannot harvest or possess animal species over certain limited numbers. Authorities impose such volume quotas in order to preserve natural resources.¹ Moreover, in a market of a country, a foreign firm may be restricted by the import volume quota assigned to it. Authorities impose import volume quotas in order to protect domestic producers by limiting the volume that the foreign firms can import.

If there are share quotas in a market, a firm cannot expand its market share over the share quota level assigned to it. In many regions, an anti-trust

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¹See, for example, Stavins (2011) for several policy problems on preserving natural resources and environmental quality.

authority can take measures to enhance competition in a market if the share of a particular firm is high in the market.² For example, Standard Oil was divided into 34 independent companies because its total share was approximately 90%. Since a firm is often reluctant to be divided, it may refrain from increasing its market share when it is already large. Therefore, firms may face some explicit or implicit share quota. Moreover, national governments occasionally assign some countries to a ratio of the domestic consumption in order to benefit domestic firms. See Kosteki (1991) with regard to the policy in the real world.

Here, we will consider a Cournot quantity competition model and compare the effects of the three policies—a volume quota, a share quota and a specific tax—on market outcomes. There is extensive literature on the effects of a change in the specific tax rate on Cournot equilibria. See for instance Vives (2001, ch.4) on some analysis of the effects. However, relatively few studies examine the effects of changes in quotas.³ In this paper, we analyze comparative statics effects of volume and share quotas on market outcomes. When an authority aims to protect natural resources, a specific tax is an alternative policy to a volume quota. Therefore, our study may be helpful in answering the quotas-versus-taxes policy question.⁴ Moreover, an import tariff is an alternative policy to an import volume quota and an import share quota. In fact, OECD (2005) states that the reduction or elimination of import tariffs has made non-tariff barriers, which includes import volume and share quotas, relatively more conspicuous. Our study may also be useful to discuss the optimal trade policy.

First, we will show the equivalence of volume quotas and specific taxes in terms of equilibrium quantities. That is, equilibrium quantities under any volume quota system are equal to those under some specific tax systems. Moreover, the equilibrium quantities under any specific tax system are equal to those under some volume quota systems. Second, by using this equivalence result, we analyze comparative statics effects of volume quotas. Third, we will show that the equilibrium quantities under any share quota system are equal to those under some volume quota systems and those under some specific tax systems. Moreover, if there are some firms not bound by the volume quotas assigned to them, the equilibrium quantities under the volume quota system are equal to those under some share quota systems. Fourth, by using the third result, we analyze comparative statics effects of share quotas.

Further, our results can be applied to analyses of international oligopoly. There is extensive literature on the equivalence or non-equivalence of tariffs and import volume quotas. See, for example, Bhagwati (1965, 1968) and Shibata (1968), Itoh and Ono (1982, 1984). Hwang and Mai (1988) and Fung (1989) show the equivalence in a Cournot model with one domestic firm and one foreign

²In several regions such as US, Europe, China, and Japan, if the market share of a particular firm is over 50%, the anti-trust authority can regard the market as monopolistic and take measures to enhance competition.

³Several works discuss import quotas in the context of international oligopoly. We will present some of these works later.

⁴A considerable number of studies have theoretically discussed the quotas-versus-taxes policy question in many different contexts. See, for example, Jensen (2008) for a review of the literature.

firm.⁵ We show the equivalence in a Cournot model with an arbitrary number of domestic and foreign firms. In addition, the level of the volume quota of a foreign firm can be different from that of another foreign firm. Moreover, firms do not need to be bound by the volume quotas assigned to them. There are several previous works on import share quotas. For example, Sweeney et al. (1977), Mai and Hwang (1989) and Denicolò and Garella (1999) consider Stakelberg oligopoly models where domestic firms are leaders and foreign firms are followers. In this paper, we model a Cournot oligopoly with share quotas. Moreover, in our model, the level of the share quota of a firm can differ from that of another firm and some share quotas do not need to be binding.

We will extend the model to endogenize the set of firms. In this case, the equilibrium quantities under any specific tax vectors are equal to those under some volume quota vectors. However, there are some volume quota vectors under which the equilibrium quantities are not equal to those under any specific tax vectors. That is, we derive a non-equivalence result of volume quotas and specific taxes when firms are endogenously determined.

2 Model and Equilibrium with Specific Taxes

We consider quantity competition with homogeneous products. In Sections 2 to 4, we assume that the set of firms $N = \{1, \dots, n\}$ are exogenously given. Each firm i simultaneously decides its quantity q_i . Let a quantity vector be $q = (q_1, q_2, \dots, q_n)$ and $Q = \sum q_i$ where $n > 2$.

Let $P(Q)$ be an inverse demand function satisfying $P' < 0$ and $P' + QP'' \leq 0$. The cost function of each firm i is given by $C_i(q_i)$. We assume that $C'_i > 0$ and $C''_i - P' > 0$ for all i . Moreover, we focus on interior solutions; that is, $q_i > 0$ for all i in equilibrium.

First, we will consider a specific tax system. The specific tax rate of a firm can be different from that of another firm. The tax rate assigned to i is given by t_i . Let $t = (t_1, \dots, t_n) \in \mathbb{R}_+^n$ be a tax rate vector and $q_i^*(t)$ be the equilibrium output where the cost function of i is $C_i(q_i) + t_i q_i$ for all i . Moreover, $q^*(t) = (q_1^*(t), \dots, q_n^*(t))$ and $Q^*(t) = \sum q_i^*(t)$. The profit of firm i under t_i is $\pi_i(q; t_i) = P(Q)q_i - C_i(q_i) - t_i q_i$. We focus on an interior solution $q_i^*(t) > 0$ for all i . By the first order condition, the best-reply function of i denoted by $br_i(Q_{-i}, t_i)$ satisfies

$$P'(Q_{-i} + br_i)br_i + P(Q_{-i} + br_i) - C'_i(br_i) - t_i = 0. \quad (1)$$

Note that there is a unique equilibrium quantity vector. By the first order

⁵More recent papers such as Matschke (2003) and Chen and Hwang (2006) examine the effects of volume quotas and tariffs in oligopoly models with uncertainty.

condition, for i and $j(\neq i)$,

$$\frac{\partial q_i^*(t)}{\partial t_i} = \frac{nP' + P''(Q^* - q_i^*) - C''}{(P' - C'')((n+1)P' + Q^*P'' - C'')} < 0, \quad (2)$$

$$\frac{\partial q_i^*(t)}{\partial t_j} = -\frac{P' + q_i^*P''}{(P' - C'')((n+1)P' + Q^*P'' - C'')} > 0, \quad (3)$$

$$\frac{\partial Q^*(t)}{\partial t_i} = \frac{1}{(n+1)P' - C'' + Q^*P''} < 0. \quad (4)$$

We will use these results for analyzing comparative statics effects of volume quotas and share quotas.

3 Volume Quotas

In this section, we consider a volume quota system. Let $q^V = (q_1^V, \dots, q_n^V)$ be a volume quota vector. Firm i cannot have an output over q_i^V . When we consider a volume quota system, $t = 0$. Let $\widehat{br}_i(Q_{-i}, q_i^V)$ be the best-reply function of i under q^V . Then,

$$\widehat{br}_i(Q_{-i}, q_i^V) = \min \{br_i(Q_{-i}, 0), q_i^V\}. \quad (5)$$

Let an equilibrium quantity vector be $q^{**}(q^V) = (q_1^{**}(q^V), \dots, q_n^{**}(q^V))$ under q^V . Then, we obtain the following result.

Lemma 1 *There exists a unique equilibrium quantity vector for each q^V .*

The proof for this is presented in the Appendix..

We will have the equivalence result of volume quotas and specific taxes. Let $N^{**}(q^V) = \{j \in N \mid q_j^{**}(q^V) = q_j^V\}$ be the set of the firms bound by the volume quota level assigned to them.

Theorem 1 *First, for each $q^V \geq 0$, there exist some $t \geq 0$ satisfying $q^{**}(q^V) = q^*(t)$. Second, for each $t \geq 0$, there is some q^V satisfying $q^*(t) = q^{**}(q^V)$ and $j \in N^{**}(q^V)$ for all j such that $t_j > 0$.*

The proof is presented in the Appendix.

This result implies the equivalence of volume quotas and specific taxes in terms of the equilibrium quantities. Obviously, the equivalence is also satisfied with respect to the market price. However, if $q^{**}(q^V) = q^*(t)$, the profits of firms under q^V and t may be different. That is, we obtain the following result.

Remark 1 *Consider t and q^V satisfying $q^*(t) = q^{**}(q^V)$. For any j such that $t_j > 0$, the profit under q^V is higher than that under t . For any i such that $t_i = 0$, the profit under q^V is equal to that under t .*

This result is important when we consider the effect of q^V and t on social welfare. Let social welfare be given by

$$SW = \delta_C CS(Q) + \sum_{i \in N} \delta_i \pi_i(q; t_i) + \delta_G \sum_{i \in N} t_i q_i, \quad (6)$$

where CS represents consumers' surplus that depends only on Q , and $\delta_j \geq 0$ for all $j = 1, \dots, n, C, G$. If $\delta_1 = \dots = \delta_n = \delta_G$, then t is equivalent to q^V in terms of social welfare. However, if otherwise, then the equivalence result in terms of social welfare can break down. An important example is the case of international oligopoly. We will discuss that in Section 5.

Next, we will compare the equilibrium quantities under two volume quota levels q^V and $q^{V'}$ satisfying that $q_l^{V'} < q_l^V$ and $q_{-l}^{V'} = q_{-l}^V$ and l is bound by $q_l^{V'}$. In order to compare the equilibrium quantities, let $t : \mathbb{R}_+^n \rightarrow \mathbb{R}_+^n$ be such that

$$\begin{aligned} & \text{for all } i \notin N^{**}(q^V), t_i(q^V) = 0, \\ & \text{for all } j \in N^{**}(q^V) \text{ and } q_j^V > 0, q_j^*(t(q^V)) = q_j^V, \\ & \text{for all } k \in N^{**}(q^V) \text{ and } q_k^V = 0, t_k(q^V) = P(0). \end{aligned}$$

Then, $t(q^V)$ is a tax rate vector satisfying $q^*(t(q^V)) = q^{**}(q^V)$. If an increase in q^V does not change $N^{**}(q^V)$, we can easily analyze comparative statics effects of q^V . However, in general, $N^{**}(q^V)$ is dependent on q^V .

Proposition 1 *Suppose that q^V and $q^{V'}$ satisfies that $q_l^{V'} < q_l^V$ and $q_{-l}^{V'} = q_{-l}^V$. If $q_l^{**}(q^V) \leq q_l^{V'}$, then $q^{**}(q^V) = q^{**}(q^{V'})$. Otherwise; i.e., if $q_l^{**}(q^V) > q_l^{V'}$, then*

1. $Q^{**}(q^V) > Q^{**}(q^{V'})$,
2. $q_l^{**}(q^{V'}) = q_l^{V'}$,
3. for all $k (\neq l) \in N^{**}(q^V)$, $q_k^{**}(q^{V'}) = q_k^{**}(q^V) = q_k^V$,
4. for all $i (\neq l) \notin N^{**}(q^V)$, $q_i^{**}(q^{V'}) > q_i^{**}(q^V)$.

The proof for this is presented in the Appendix.

By Proposition 1, if $q_l^{**}(q^V) > q_l^{V'}$, then $q_l^{**}(q^V) > q_l^{**}(q^{V'})$ and $q_k^{**}(q^V) \leq q_k^{**}(q^{V'})$ for all $k \neq l$. That is, the quantity of the firm whose quota level is decreased is reduced, but the quantities of the other firms are not reduced. However, the overall quantity reduces due to the decrease in the quota level.

4 Share Quotas

In this section, we focus on a market share quota system. In a share quota system, $t = 0$. Let a share quota vector be $s = (s_1, s_2, \dots, s_n)$ satisfying $\sum s_i \geq 1$ and $s_i \in [0, 1]$ for each i . The profit of a firm is

$$\begin{aligned} \pi_i(q_i, Q_{-i}; 0) & \text{ if } q_i/Q \leq s_i \text{ or } Q < \epsilon, \\ & -Z \text{ if } q_i/Q > s_i \text{ and } Q \geq \epsilon, \end{aligned}$$

where ϵ is a small positive integer and $Z > 0$. Note that $\epsilon > 0$ implies that no share quotas are binding if the total production is sufficiently small. We will explain why the assumption is adopted.

Let $q_i^*(s)$ be the equilibrium quantity of i under s and $Q^*(s) = \sum q_i^*(s)$. Let $N^*(s) = \{i \in N \mid q_i^*(s)/Q^*(s) = s_i\}$ be the set of firms bound by the share quota assigned to them.

Lemma 2 *If $\sum s_i > 1$, then there exists a unique equilibrium quantity vector under s .*

The proof for this is presented in the Appendix..

If $\epsilon = 0$, then $q_1 = q_2 = \dots = q_n = 0$ is always an equilibrium quantity vector. We assume $\epsilon > 0$ in order to avoid the uninteresting equilibrium. If $\sum s_i = 1$, then there are multiple equilibrium quantity vectors. That is, $q_i^* = (s_1\delta, \dots, s_n\delta)$ is an equilibrium quantity vector for a sufficiently small $\delta (> \epsilon)$. Hereafter, we restrict our attention to the case of $\sum s_i > 1$, that is, any share quota systems that control the shares of all firms are not considered.

Now, we relate the three systems: a share quota system, volume quota system, and a specific tax system. Let

$$q_i^V(s) = s_i Q^*(s) \text{ and } s_i(q^V) = q_i^V / Q^{**}(q^V).$$

We have the following result.

Theorem 2 *First, for each s such that $\sum s_i > 1$, there exist some $q^V \geq 0$ and $t \geq 0$ satisfying $q^*(s) = q^{**}(q^V) = q^*(t)$. Second, if there are some firms not bound by q^V , there is some s such that $q^*(s) = q^{**}(q^V)$. Third, if $t_i = 0$ for some i , there is some s such that $q^*(s) = q^*(t)$.*

The proof for this is presented in the Appendix.

By Theorems 1 and 2, if there are some firms not bound by q^V or if $t_i = 0$ for some i , the three systems are equivalent in terms of the equilibrium quantity. Moreover, s and q^V are equivalent in terms of social welfare defined as (6).

Now, we compare the equilibrium quantities under two share quotas s and s' satisfying $s'_l < s_l$ and $s'_{-l} = s_{-l}$. The following result is important for the comparison.

Lemma 3 *For each i , $q_i^V(s)$ and $s_i(q^V)$ are increasing in s_i and q_i^V , respectively.*

The proof for this is presented in the Appendix.

Then, we have the following result.

Proposition 2 *Suppose that s and s' satisfy that $s'_l < s_l$ and $s'_{-l} = s_{-l}$. If $q_l^*(s)/Q^*(s) \leq s'_l$, then $q^*(s) = q^*(s')$. Otherwise; i.e., if $q_l^*(s)/Q^*(s) > s'_l$, then*

1. $Q^*(s) > Q^*(s')$,
2. $q_l^*(s')/Q^*(s') = s'_l$,

3. for all $j \in N^*(s)$, $q_j^*(s') < q_j^*(s)$,
4. for all i such that $i \notin (N^*(s) \cup N^*(s'))$, $q_i^*(s') > q_i^*(s)$.

The proof for this is presented in the Appendix.

By Propositions 1 and 2, the comparative statics effects of volume quotas are similar to that of share quotas. However, an increase in the share quota that assigned to a firm may decrease the quantity of some other firm. By the first result of Proposition 2, an increase in the quota decreases the total quantity and thus increases the shares of the other firms. Therefore, all firms bound by the share quota must decrease their quantity.

5 Import Quotas

In this section, we consider import quotas and tariffs. In a market of a country, there are some domestic firms and/or some foreign firms. An import quota is imposed on a foreign firm. Note that the level of the import quota of a foreign country can be different from that of another foreign country. In fact, if country A forms an FTA with country B but does not with country C, then a firm in country B does not face any import quotas of country A but a firm in country C may face an import quota of country A.

First, let q^I represent a volume quota vector such that q_d^I is sufficiently large for all domestic firm d . We call q^I an import volume quota vector. Second, let s^I be a share quota vector such that $s_d^I = 1$ for all domestic firm d . We call s^I an import share quota vector. Under q^I and s^I , $d \notin N^{**}(q^I)$ and $d \notin N^*(s^I)$ for all domestic firms d , respectively. Moreover, let $\tau \geq 0$ be a specific tax vector such that $\tau_d = 0$ for each domestic firm d . We call τ a tariff vector.

Here we discuss national welfare of a country. Let N_D be the set of the domestic firms. National welfare is given by (6) satisfying $\delta_i > 0$ for $i = C, G$, $\delta_d > 0$ for all $d \in N_D$ and $\delta_f = 0$ for all $f \notin N_D$.

According to the results provided in previous sections, we obtain the following results.

- Corollary 1**
1. The equilibrium quantities under q^I (τ) are equal to those under some τ (q^I).
 2. If there is some domestic firm, then the equilibrium quantities under s^I (q^I) are equal to those under some q^I (s^I).
 3. If q^I binds some foreign firm, national welfare under q^I is lower than that under some τ . In addition, if s^I binds some foreign firm, national welfare under s^I is lower than that under some τ .
 4. Suppose that q^I and $q^{I'}$ satisfy that $q_f^{I'} < q_f^I$ and $q_{-f}^{I'} = q_{-f}^I$ for a foreign firm f . If $q_f^{**}(q^I) > q_f^{I'}$, then the total quantity of the domestic firms is increased and that of the foreign firms is decreased.

5. Suppose that s^I and $s^{I'}$ satisfy that $s_f^{I'} < s_f^I$ and $s_{-f}^{I'} = s_{-f}^I$ for a foreign firm f . If $q_f^*(s^I)/Q^*(s^I) > s_f^{I'}$, then the total quantity of the domestic firms is increased and that of the foreign firms is decreased.

The first result of Corollary 1 is a generalization of the results of Hwang and Mai (1988, Proposition 1) and Fung (1989, Proposition 1). They consider duopoly models with a domestic and foreign firm and an import quota that binds the foreign firm.

If $N_D \neq \emptyset$, $s_d^I = 1$ for $d \in N_D$ and $\sum s_i^I > 1$. Therefore, by Theorem 2, we have the second result of Corollary 1. The first and second results imply the equivalence of the three systems if there is at least one domestic firm.

By the first result, for any q^I , there is τ such that $q^{**}(q^I) = q^*(\tau)$. Since $\sum_{f \notin N_D} \tau_f q^*(\tau) > 0$, national welfare under τ is higher than that under q^I . In addition, by using the second result, we obtain the third result. Note that the first part of the third result is a generalization of the result of Chen and Hwang (2006, Proposition 3) who consider a duopoly model with one domestic firm and one foreign firm.

The fourth result of Corollary 1 implies that if the import volume quota of a firm is decreased, then the amount of import is decreased and the domestic production is increased. Therefore, if a country forms an FTA with a country, then the amount of import is increased and domestic production is decreased. Moreover, a reduction in an import share quota also decreases the amount of import and increases domestic production. Ono (1990) discusses an international oligopoly model with a foreign firm and domestic firms. He shows that there is a volume quota level where a reduction in the volume quota level always increases national welfare. By the fourth result of Corollary 1, this result continues to hold if the import volume quota of a country is different from that of another country. That is, there is some q^I such that a decrease in q_f^I increases national welfare. Moreover, there also be some s^I , such that a decrease in s_f^I increases national welfare.

6 Endogenous Entry

In this section, we assume that the set of firms is endogenously determined. First, let a set of firms be E and its cardinality be e . For given E , the equilibrium quantities under t , q^V , and s are given by $q^*(t, E)$, $q^{**}(q^V, E)$ and $q^*(s, E)$, respectively.

We assume that the fixed entry cost $C_i(0) > 0$ for all i . Firm i enters this market if and only if its profit will be positive. Therefore, an equilibrium set of firms under t be $E^*(t)$ such that

$$\begin{aligned} \pi_j(q^*(t, E^*(t)); t_j) &\geq 0 \text{ for all } j \in E^*(t) \text{ and} \\ \pi_i(q^*(t, (E^*(t) \cup \{i\})); t_i) &< 0 \text{ for all } i \notin E^*(t). \end{aligned}$$

Similarly, $E^{**}(q^V)$ and $E^*(s)$ represent equilibrium sets of firms under q^V and s , respectively. Note that there may be multiple equilibrium sets of firms.

We consider q^V and t satisfying $q^*(t, E) = q^{**}(q^V, E)$. By Remark 1, if $\pi_i(q^*(t, E); t_i) \geq 0$, then $\pi_i(q^{**}(q^V, E); 0) \geq 0$. This implies, if firm i has no incentive to exit under t , then i also has no incentive to exit under q^V . Moreover, since $C_i(0) > 0$, if q_i^V is sufficiently small, then $\pi_i(q^{**}(q^V, E); 0) < 0$ for each E . Therefore, we obtain the following result.

Proposition 3 *Suppose that the set of firms is endogenously determined. For any $t \geq 0$ and $E^*(t)$, there is some q^V such that $q^*(t, E^*(t)) = q^{**}(q^V, E^{**}(q^V))$ and $E^*(t) = E^{**}(q^V)$. Moreover, if there is some i such that $t_i = 0$, then $q^*(t, E^*(t)) = q^*(s, E^*(s))$ and $E^*(t) = E^*(s)$.*

The proof for this is presented in the Appendix..

On the other hand, we can make some example where for some q^V , there is no t such that $q^*(t, E^*(t)) = q^{**}(q^V, E^{**}(q^V))$ and $E^*(t) = E^{**}(q^V)$. We will show this fact by examining the following example.

Suppose that there are two firms 1 and 2. $P(Q) = 15 - Q$, $C_i(q_i) = 14$ for all $q_i > 0$ and $i = 1, 2$. Let $q^V = (15, 3)$. Then $q_1^{**}(q^V, E^{**}(q^V)) = 6$, $q_2^{**}(q^V, E^{**}(q^V)) = 3$ and $E^{**}(q^V) = \{1, 2\}$. If $t = (0, 3)$, $q^*(t, \{1, 2\}) = q^{**}(q^V, \{1, 2\}) = (6, 3)$. However, $\pi_2(q^*(t, \{1, 2\}); t_2) = -5$ and thus $E^*(t) = \{1\} \neq E^{**}(q^V)$. Therefore, there is no t such that $q^*(t, E^*(t)) = (6, 3)$. It should be noted that if $s = (1, 1/3)$, then $q^*(s, E^*(s)) = (6, 3)$ and $E^*(s) = \{1, 2\}$. Thus, if the set of firms is endogenously determined, then the equivalence of volume quotas and specific taxes and that of share quotas and specific taxes break down.

7 Conclusion

We summarize the policy implications of this paper. First, we consider a short-run market competition; that is, the case in which the set of firms is exogenously given. In this case, specific tax and volume quota systems are equivalent with respect to the equilibrium quantities. An important difference between specific tax and volume quota systems is that the profit of some firm under the volume quota system is higher than that under the specific tax system; however, the governmental revenue under the specific tax system is higher under the volume quota system. Therefore, in the case in which an authority prioritizes the governmental revenue over the profits of firms, for example, in the case of the international market, then it should choose a specific tax system. Next, we consider a long-run market competition; i.e., the case where the set of active firms is endogenously determined. Then, market outcomes under any specific tax systems are the same as those under volume quotas. However, in some cases, market outcomes under a volume quota system are not equal to those under any specific tax systems. Therefore, when we consider a long-run market competition, a volume quota system is more flexible. Comparing volume and share quota systems, market outcomes under any share quota system are equal to those under some volume quota systems. On the other hand, market outcomes under some volume quota system are not equal to those under any share

quota systems. Therefore, we conclude that a volume quota system is better than a share quota system.

Next, we discuss the robustness of our results. Some of the results are robust even if we consider a Cournot oligopoly model with differentiated products. We assume that the inverse demand function for firm i 's products depends on its own quantity and the aggregate quantities of the rivals. If the best response functions of the firms have negative slopes larger than -1 , then there is a unique equilibrium. See Vives (2001, Ch.6) with regard to this point. In our model, we obtain the same equivalence results as Theorems 1 and 2.

Finally, we provide directions for future research. First, when we discuss volume quotas, our model is a Cournot oligopoly game where strategy spaces of firms are restricted. Bhaskar and To (1999, 2003) and Kaas and Madden (2008) analyze the effects of minimum wages in a wage competition model. Their models are oligopsonistic wage competition models in which strategy spaces of firms are restricted. Therefore, the structure of the models is similar to ours. A similar result to our equivalence result between volume quotas and specific taxes is satisfied in an oligopsonistic wage competition model with a minimum wage system. Moreover, a similar result is also satisfied in a Bertrand competition model with a price ceiling system.⁶ That is, the equivalence of price ceilings and specific taxes in terms of equilibrium prices is satisfied in a Bertrand model. We plan to discuss the effects of price ceilings and minimum wages. Second, in our model, firms are exogenously assigned their quota. Individual transferable quota systems have recently attracted attention from economists and policymakers.⁷ When we model a transferable quota system, the quota levels are endogenously determined in a market. The application of our results to models of a transferable quota system remains an interesting topic for future research.

Appendix

(Proof of Lemma 1) The proof of Lemma 1 is similar to that of existence and uniqueness of the Cournot equilibrium without any restrictions. See, for instance, Vives (2001, Ch.4, p.97-98) with regard to the proof.

Let $\tilde{q}_i(Q, q_i^V)$ be the optimal output of i which is consistent with Q under q_i^V . Since $\partial br_i(Q_{-i})/\partial Q_{-i} \leq 0$, $\partial \tilde{q}_i(Q, q_i^V)/\partial Q \leq 0$. Therefore, $\sum \tilde{q}_i(Q, q_i^V)$ is nonincreasing in Q and will intersect only once the 45° line. Therefore, we have Lemma 1. (Q.E.D.)

(Proof of Theorem 1) Fix a volume quota vector q^V . Since $j \in N^{**}(q^V)$ has

⁶Earle et al. (2007) discuss a Cournot oligopoly model with price ceilings. Our principle cannot be used to analyze their model, but can be applied to a Bertrand oligopoly with price ceilings.

⁷Newell et al. (2005) state that individual transferable quota systems have been used in some major fisheries in seventeen countries such as Australia, Canada, Iceland and New Zealand. See, for example, Jensen (2008) for a review of the theoretical studies.

no incentive to decrease its output,

$$P(Q^{**}(q^V)) + q_j^V P'(Q^{**}(q^V)) - C'_j(q_j^V) \geq 0.$$

Thus, there is $t_j \geq 0$ satisfying

$$P(Q^{**}(q^V)) + q_j^V P'(Q^{**}(q^V)) - C'_j(q_j^V) - t_j = 0. \quad (7)$$

For $i \notin N^{**}(q^V)$, since i has no incentive to increase or decrease its output,

$$P(Q^{**}(q^V)) + q_i^{**}(q^V) P'(Q^{**}(q^V)) - C'_i(q_i^{**}(q^V)) = 0.$$

Therefore, if t satisfies (7) for all $j \in N^{**}(q^V)$ and $t_i = 0$ for all $i \notin N^{**}(q^V)$, then $q^{**}(q^V) = q^*(t)$.

Next, we will show the second fact. Fix a specific tax rate vector t . By the first order condition of firm j , if $t_j > 0$,

$$\begin{aligned} P(Q^*(t)) + q_i^*(t) P'(Q^*(t)) - C'_j(q_i^*(t)) - t_j &= 0 \implies \\ P(Q^*(t)) + q_i^*(t) P'(Q^*(t)) - C'_j(q_i^*(t)) &> 0. \end{aligned}$$

Therefore, if $q_j^V = q_j^*(t)$ for all $t_j > 0$ and q_i^V is sufficiently large for all $t_i = 0$, then $q^{**}(q^V) = q^*(t)$. (Q.E.D.)

(Proof of Proposition 1) Each firm is in either

$$\begin{aligned} A &= \{\alpha \mid \alpha \in N^{**}(q^V) \text{ and } \alpha \in N^{**}(q^{V'})\}, \text{ or} \\ B &= \{\beta \mid \beta \notin N^{**}(q^V) \text{ and } \beta \notin N^{**}(q^{V'})\}, \text{ or} \\ C &= \{\gamma \mid \gamma \in N^{**}(q^V) \text{ and } \gamma \notin N^{**}(q^{V'})\}, \text{ or} \\ D &= \{\delta \mid \delta \notin N^{**}(q^V) \text{ and } \delta \in N^{**}(q^{V'})\}. \end{aligned}$$

For $\alpha (\neq l) \in A$, $q_\alpha^{**}(q^V) = q_\alpha^{**}(q^{V'}) = q_\alpha^V$. By Theorem 1, for $\beta (\neq l) \in B$, $t_\beta = t_\beta(q^V) = t_\beta(q^{V'}) = 0$. Moreover, for $\gamma (\neq l) \in C$, $q_\gamma^{**}(q^V) = q_\gamma^V > q_\gamma^{**}(q^{V'})$ and $t_\gamma(q^{V'}) = 0 \leq t_\gamma(q^V)$. Further, if $\delta (\neq l) \in D$, then $q_\delta^{**}(q^V) < q_\delta^V = q_\delta^{**}(q^{V'})$ and $t_\delta(q^V) = 0 \leq t_\delta(q^{V'})$. Therefore,

$$\begin{aligned} X &= q_l^{**}(q^V) + \sum_{\alpha \in A \setminus \{l\}} q_\alpha^{**}(q^V) + \sum_{\gamma \in C \setminus \{l\}} q_\gamma^{**}(q^V) \\ &> q_l^{**}(q^{V'}) + \sum_{\alpha \in A \setminus \{l\}} q_\alpha^{**}(q^{V'}) + \sum_{\gamma \in C \setminus \{l\}} q_\gamma^{**}(q^{V'}). \end{aligned}$$

Since $P(Q^{**}(q^V)) + q_k^{**}(q^V) P'(Q^{**}(q^V)) - C'_k(q_k^{**}(q^V)) - t_k(q^V) = 0$ for $k (\neq l) \in B \cup D$,

$$P'(Q^{**}(q^V)) (Q^{**}(q^V) - X) + |B \cup D| P(Q^{**}(q^V)) - \sum_{k \in B \cup D} C'_k(q_k^{**}(q^V)) - t_k(q^V) = 0.$$

Then, $\partial Q^{**}(\mathbf{q}^V)/\partial X > 0$ and $\partial Q^{**}(\mathbf{q}^V)/\partial t_k < 0$. Thus, we have $Q^{**}(\mathbf{q}^{V'}) < Q^{**}(\mathbf{q}^V)$.

We will show the second fact. By Theorem 1, $t_l(\mathbf{q}^V) \geq 0$ and

$$P(Q^{**}(\mathbf{q}^V)) + q_l^{**}(\mathbf{q}^V)P'(Q^{**}(\mathbf{q}^V)) - C'_l(q_l^{**}(\mathbf{q}^V)) - t_l(\mathbf{q}^V) = 0.$$

Since $Q^{**}(\mathbf{q}^{V'}) < Q^{**}(\mathbf{q}^V)$ and $q^{V'} < q_l^{**}(\mathbf{q}^V)$,

$$\begin{aligned} P(Q^{**}(\mathbf{q}^{V'})) + q_l^{**}(\mathbf{q}^V)P'(Q^{**}(\mathbf{q}^{V'})) - C'_l(q_l^{**}(\mathbf{q}^V)) &> 0 \Rightarrow \\ P(Q^{**}(\mathbf{q}^{V'})) + q^{V'}P'(Q^{**}(\mathbf{q}^{V'})) - C'_l(q_l^{**}(\mathbf{q}^V)) &> 0. \end{aligned}$$

Therefore, $q_l^{**}(\mathbf{q}^{V'}) = q_l^{V'}$.

Next, we will show the third fact. For each $k(\neq l) \in N^{**}(\mathbf{q}^{V'})$, $P(Q^{**}(\mathbf{q}^{V'})) + q_k^V P'(Q^{**}(\mathbf{q}^{V'})) - C'_k(q_k^{V'}) \leq 0$. Since $Q^{**}(\mathbf{q}^{V'}) < Q^{**}(\mathbf{q}^V)$, $P(Q^{**}(\mathbf{q}^V)) + q_k^V P'(Q^{**}(\mathbf{q}^V)) - C'_k(q_k^V) < 0$. Therefore, $k \in N^{**}(\mathbf{q}^V)$.

Finally, we will show the fourth fact. First, for $\delta \in D$, $q_\delta^{**}(\mathbf{q}^V) < q_\delta^V = q_\delta^{**}(\mathbf{q}^{V'})$. Second, since $Q^{**}(\mathbf{q}^{V'}) < Q^{**}(\mathbf{q}^V)$, $q_\beta^{**}(\mathbf{q}^V) < q_\beta^{**}(\mathbf{q}^{V'})$ for all $\beta \in B$. Therefore, for all $k \notin N^{**}(\mathbf{q}^V)$, $q_k^{**}(\mathbf{q}^{V'}) > q_k^{**}(\mathbf{q}^V)$. (Q.E.D.)

(Proof of Lemma 2) Let $\tilde{\vartheta}_i(Q, s_i)$ be the optimal output of i which is consistent with Q under s . Since ϵ is small, there is no equilibrium satisfying $Q^*(s) < \epsilon$. Thus, suppose $Q \geq \epsilon$. Then,

$$\tilde{\vartheta}_i(Q, s_i) = \min \{\vartheta_i(Q), s_i Q\}.$$

Since $\vartheta_i(Q)$ is decreasing in Q , $\tilde{\vartheta}_i(Q, s_i)$ is a single peaked function of Q . We will show that there is a unique Q satisfying

$$\sum \tilde{\vartheta}_i(Q, s_i) = Q. \quad (8)$$

Since $\vartheta_i(Q)$ is decreasing in Q and $\sum s_i > 1$, $\sum \tilde{\vartheta}_i(Q, s_i) > Q$ if Q is sufficiently small. Moreover, $\tilde{\vartheta}_i(Q, s_i)$ is decreasing in Q for sufficiently large Q . Since $\sum \tilde{\vartheta}_i(Q, s_i)$ is continuous, there exists some Q that satisfies (8). Let \underline{Q} be the smallest integer that satisfies (8). Note that $\sum \tilde{\vartheta}_i(Q, s_i) < Q$ for all $Q < \underline{Q}$. If $\sum_j s_j \geq 1$ for j such that $\tilde{\vartheta}_j(Q, s_j) = s_j Q$, then $\sum \tilde{\vartheta}_i(Q, s_i) > Q$. Therefore, $\sum_j s_j < 1$ for j such that $\tilde{\vartheta}_j(\underline{Q}, s_j) = s_j \underline{Q}$. For any $Q \geq \underline{Q}$, the slope of $\sum \tilde{\vartheta}_i(Q, s_i)$ is less than 1. Hence there is no other Q that satisfies (8) and $\underline{Q} = Q^*(s)$. (Q.E.D.)

(Proof of Theorem 2) Since $q^*(s) = q^{**}(\mathbf{q}^V(s))$, the first sentence is obvious. We provide the proof of the second sentence. If there is a firm i satisfying $q_i^V > q_i^{**}(\mathbf{q}^V)$, then $\sum s_k(\mathbf{q}^V) > 1$. Suppose $\mathbf{q} = \mathbf{q}^{**}(\mathbf{q}^V)$ under $s(\mathbf{q}^V)$. Then, since

$$P(Q^{**}(\mathbf{q}^V)) + q_j^V P'(Q^{**}(\mathbf{q}^V)) - C'_j(q_j^V) \geq 0 \text{ for } j \in N^{**}(\mathbf{q}^V),$$

j has no incentive to decrease its quantity under $s(\mathbf{q}^V)$. Moreover, since $s_j(\mathbf{q}^V) = q_j^V / Q^{**}(\mathbf{q}^V)$, j has no incentive to increase its quantity under $s(\mathbf{q}^V)$.

Further, for all $i \notin N^{**}(q^V)$, $q_i^{**}(q^V)$ is the best-reply to $q_{-i}^{**}(q^V)$. Therefore, $q^*(s(q^V)) = q^{**}(q^V)$. The third result is obvious from the second result and Theorem 1. (Q.E.D.)

(Proof of Lemma 3) Since $\tilde{\vartheta}_i(Q, s'_i) \leq \tilde{\vartheta}_i(Q, s_i)$ for $s'_i < s_i$ and all Q , we have $Q^*(s') \leq Q^*(s)$. Thus, $q_i^V(s)$ is increasing in s_i . Next, we will consider q_i^V and $q_i^{V'} (< q_i^V)$. By Proposition 1, if $q_i^{**}(q^V) \leq q_i^{V'}$, then $Q^{**}(q^V) = Q^{**}(q^{V'})$. On the other hand, if $q_i^{**}(q^V) > q_i^{V'}$, then $q_i^{**}(q^{V'}) = q_i^{V'}$ and $Q_{-i}^{**}(q^V) < Q_{-i}^{**}(q^{V'})$. Therefore, $s_i(q^V)$ is increasing in q_i^V . (Q.E.D.)

(Proof of Proposition 2) Suppose $q_l^*(s)/Q^*(s) > s'_l$. First, since $q^V(s) > q^V(s')$, $Q^*(s) \geq Q^*(s')$ because of Theorem 2 and Proposition 1. Since $q_l^*(s)/Q^*(s) \geq q_l^*(s)/Q^*(s) > s'_l$, $q_l^{**}(q^V(s)) > s'_l Q^*(s') = q_l^V(s')$. By Proposition 1, $Q^*(s) > Q^*(s')$. Second, since $q_l^*(s) > s'_l Q^*(s) > s'_l Q^*(s') = q_l^V(s')$,

$$\begin{aligned} P(Q^*(s)) + q_l^*(s)P'(Q^*(s)) - C'_j(q_l^*(s)) &\geq 0 \Rightarrow \\ P(Q^*(s')) + q_l^V(s')P'(Q^*(s')) - C'_j(q_l^*(s')) &> 0. \end{aligned}$$

Hence $q_l^*(s')/Q^*(s') = s'_l$. Third, since $Q^*(s) > Q^*(s')$, $q_k^V(s) > q_k^V(s')$ for all k and thus $q_j^*(s) > q_j^*(s')$ for all $j \in N^*(s)$. Finally, since $Q^*(s) > Q^*(s')$, $q_i^*(s') > q_i^*(s)$ for all i such that $i \notin (N^*(s) \cup N^*(s'))$. (Q.E.D.)

(Proof of Proposition 3) For given t and $E^*(t)$, consider q^V such that $q_j^*(t, E^*(t)) = q_j^{**}(q^V, E^*(t))$ for all $j \in E^*(t)$ and q_i^V is a sufficiently small for all $i \notin E^*(t)$. Then,

$$\pi_i(q^{**}(q^V, (E^*(q^V) \cup \{i\})); 0) < 0$$

for all $i \notin E^*(t)$. Moreover, by Remark 1, if $\pi_i(q^*(t, E^*(t)); t_i) \geq 0$, then $\pi_i(q^{**}(q^V, E^*(t)); 0) \geq 0$. Therefore, $E^*(t) = E^{**}(q^V)$. Likewise, we can show that if there is some i such that $t_i = 0$, then $q^*(t, E^*(t)) = q^*(s, E^*(s))$ and $E^*(t) = E^*(s)$. (Q.E.D.)

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