Strategic Location Choice and Network Formation for Entry*

Keizo Mizuno†
School of Business Administration
Kwansei Gakuin University

Yasunori Okumura
School of Economics
Han-nan University

September 2011

Abstract

In our model, an entrant has two beneficial strategies: collaboration with incumbents and location choice. We analyze the effect of allowing an entrant the opportunity to collaborate with incumbents on its location choice. First, we show that when collaboration requires mutual consent, the entrant has an incentive to distort its location from a welfare viewpoint to collaborate with incumbents and facilitate its entry. Second, the existing collaboration among incumbents cannot always be a device that deters entry, depending on the magnitude of the collaboration effect.

Keywords: entry, location choice, network, pairwise stability.
JEL classification: D85, L13, L14, L41.

---

*We would like to thank Makoto Hanazono, Shingo Ishiguro, Shinsuke Kambe, and Noriaki Matsushima for their useful comments, which have helped to improve this paper. We also thank Kojun Hamada, Keisaku Higashida, Hiroaki Ino, Tatsuaki Kuroda, and seminar participants at Contract Theory Workshop, Kwansei Gakuin University, Nagoya University, and Niigata University for their valuable discussions. The financial support of the Japanese Ministry of Education, Culture, Sports, Science, and Technology (MEXT) (Grant No.21530243) is gratefully acknowledged.

†The corresponding author. 1-1-155 Uegahara, Nishinomiya, Hyogo 662-8501 Japan. Tel: +81-798-54-6181; fax: +81-798-51-0903; *e-mail* : kmizuno@kwansei.ac.jp.

1 Introduction

This paper examines a case in which an entrant has two beneficial strategies: collaboration with incumbents and location choice. In particular, we analyze the effect of allowing an entrant the opportunity to collaborate with incumbents on the choice of location.

It is not rare for entrants to use collaboration with incumbents as a strategic tool. When an entrant has a broad distribution channel or excellent technology for the production of an input that is used to produce a final good, the entrant may easily collaborate with incumbents in the final good market by sharing its knowledge with the incumbent. For example, Samsung, which already possessed a broad distribution channel and top-class technology in the semiconductor market, collaborated with Sony when it entered the LCD market and with Seagate when it entered the hard-disk market. Facilitating an entrant's collaboration with incumbents seems to be justified by the improvement in information technology, such as the prevalence of the Internet or cloud-computing systems. This is because the Internet increases the frequency of contacts, and cloud-computing systems induce the common use of network systems among firms.

To understand better the issues discussed in this paper, suppose two retail shops are located at the two extreme ends of a line city. The shops have the same retail technology, engage in price competition, and obtain some positive profits. Now, assume that an outsider plans to open a shop in the same city. The two incumbent retail shops may collaborate with each other to deter the entry of a new competitor. If the incumbents collaborate with each other, they lower their resale costs by sharing some facilities, such as production plants, storehouses, or distribution channels. However, assume that once the outsider opens a shop, it will be allowed to collaborate with the incumbent retail shops. In such a case, where would the outsider choose to locate a shop in the city? Would it be easy for the outsider to open its shop in the city?[1]

Using the standard setting of a Hotelling model, we propose a curious answer to these questions: Once the outsider enters the market, it does not always locate at the center of the line (or it does not always maximally horizontally differentiate its goods from those of the incumbents). That is, its location is distorted towards the location of one of the two incumbents. Moreover, collaboration between the two incumbents to reduce their production cost cannot always deter a competitor's entry. In fact, the outsider may be welcomed to the market, even when the incumbents have collaborated on an entry barrier.

Let us restate this answer in detail. In our model, we define a collaboration of firms as a *pairwise link* between firms with mutual consent. First, we show that when an entrant is allowed to form a link and the link formation requires mutual consent, the entrant has an incentive to distort its location to form a link with each of the incumbents. Second, we show that the existing link formed by the incumbents cannot be a device to deter entry when the incremental change of the cost-reducing effect generated by forming that link scarcely depends on the number of links.

These two results are produced by two driving forces: *the cost-reducing effect*, which is a direct effect generated by the network formation, and *the price-competition effect*, which

[1] Needless to say, the location choice in this example can be replaced by a choice of a product characteristic such as the color of the product.

is a mixed effect generated by the network formation and the entrant's location choice. Suppose that an entrant tries to form a link with an incumbent. The cost-reducing effect works positively for both the entrant and the incumbent, which implies that competition in the product market becomes fiercer with the link than without it. The entrant's location choice can also affect the fierceness of the product market competition. The entrant's location choice affects the relative magnitude between the cost-reducing effect and the price-competition effect, which in turn determines the success probability of forming a link. Hence, the entrant has an incentive to distort its location to obtain the incumbent's agreement to form the link. We will explain the details of the mechanism in the following analysis.

The crucial prerequisite for our results is that both the incumbents and the entrant have an opportunity to form a link once the entrant enters the market. In fact, we verify in the following analysis that when only the incumbents have an opportunity to form a link, the entrant undoubtedly locates at the center of the line (i.e., it maximally horizontally differentiates its product from those of the incumbents). In addition, we see in that case that the link between the incumbents can always be an effective tool for deterring entry.

The previous studies by Goyal and Moraga-González (2001), Goyal and Joshi (2003), and Deroian and Gannon (2006) relate most closely to this paper in the sense that they deal with the relationship between strategic network formation with bilateral agreements between rival firms and their market competition.[2] Goyal and Moraga-González employ the three-stage game in which firms choose a R&D effort for cost reduction after a network structure is established, and the market competition follows in the final stage. They examine the effects of market rivalry and spillovers on network structure. Goyal and Joshi also address the relationship between a firm's incentive to form a network and the nature of market competition. Deroian and Gannon restrict their attention more to the effect of spillovers on network structure by assuming that a R&D effort contributes to quality improvement.[3] In contrast to these three studies, our main issue is the relationship between a potential entrant's two beneficial strategies, i.e., collaboration with incumbents and location choice, and their effects on its capability for entry when the network formation with bilateral agreements affects the market competition among rival firms.

Using a repeated game framework, Friedman and Thisse (1993) examines the relationship between collaboration among firms and their location choices. They show that partial tacit collusion on price fosters minimum product differentiation (i.e., the firms' agglomeration). Our study differs from that of Friedman and Thisse in that we consider not only the threat of potential entry but also the possibility of collaborative networks that include entrants in a horizontal product differentiation model.[4]

[2] For the basic notions and applications of strategic network formation with pairwise links, see Goyal (2007) and Jackson (2008).

[3] Another interesting application of strategic network models to market competition is by Song and Vannetelbosch (2007), who analyze the effect of government policies on the stability and efficiency of networks of R&D collaboration among firms.

[4] Okumura (2011) deals with an endogenous formation of collaborative networks in a spacial model. Within the framework of a circular model, however, his main focus is the characterization of networks in equilibrium, given the number of firms and each firm's location. In contrast to his study, this paper deals with the relationship between endogenously formed networks and a firm's location choice.

The next section describes the framework of the model. In Section 3, we first characterize the pairwise stable networks, taking a potential entrant's entry and its location as given. We then analyze an entrant's location choice and the feasibility of entry under the derived pairwise stable networks. Concluding remarks are in Section 4.

2 The Model

We examine a linear Hotelling model with two incumbents, firm 1 and firm 2, and one potential entrant, firm e. Consumers are uniformly distributed on the interval $[0, 1]$ with a mass of 1. Firm 1 is located at the left end, 0, while firm 2 is at the right end, 1; the incumbents' goods are maximally horizontally differentiated. Once firm e decides to enter the market by incurring an entry sunk cost, F, it can choose its location, a.

All of the firms sell the good, whose reservation value for each consumer is v.[5] Each consumer buys a single unit of the good from one of the firms. Then, the utility of a consumer located at θ is represented by

$$u\left(p_i; \theta\right) = v - \left(\theta - s_i\right)^2 t - p_i,$$

where s_i is the location of firm i ($i = 1,\ 2,\ e$), t is the parameter of the quadratic transportation cost incurred by the consumer, and p_i is the price set by firm i.

Our model departs from the existing literature by allowing the firms to collaborate with other firms in the market. In particular, a firm has an opportunity to form a *pairwise* collaborative link with one of the other firms in the market. Moreover, we assume that firm i's constant marginal cost of production, c_i, is represented by a function of the number of the collaborative links. That is,

$$c_i = c\left(\eta_i\left(g\right)\right),$$

where $\eta_i\left(g\right)$ is the number of pairwise links that firm i has with the other firms in the *network* g. Here, a network represents a structure of the pairwise links in the market. For example, if we see a network, $\widetilde{g}$, where firm e forms a pairwise link with firm 1 after it enters the market and firm 1 also forms a pairwise link with firm 2, we state that $\eta_1\left(\widetilde{g}\right) = 2$, $\eta_2\left(\widetilde{g}\right) = 1$, and $\eta_e\left(\widetilde{g}\right) = 1$. We also assume that a pairwise link has an effect of lowering the marginal cost of production as follows:

$$c\left(0\right) > c\left(1\right) > c\left(2\right).$$

That is, a firm's marginal cost is strictly decreasing in the number of pairwise links. Note that the marginal cost depends only on the number of links, not on the distance between the two firms that form a link. This means that a firm's marginal cost is independent of its location choice.[6]

[5] We assume that v is sufficiently high so that all consumers buy the good from one of the firms.

[6] We follow Goyal and Joshi (2003) regarding the formulation of the cost-reducing effect through collaborative links between firms. Note that this formulation excludes the possibility of spillovers. That is, the marginal cost does not depend on the number of links that a *neighbor firm* (i.e., a firm that consists

To derive equilibrium networks, we use a concept of *pairwise stability* attributed to Jackson and Wolinsky (1996). In particular, a network, g, is said to be *pairwise stable* if any firm that is linked to another in the network has a strict incentive to maintain the link, and any two firms that are not linked have no strict incentive to form a link with each other. Pairwise stability captures the idea of mutual consent. In the next section, we introduce the formal definition of pairwise stability in our model. Moreover, to predict which networks are likely to emerge and refine the set of pairwise stable networks, we use a dynamic process for network formation, the details of which will be also explained in the next section.

In the following analysis, we restrict our attention to an interior solution in which all three firms can obtain a positive demand and a positive profit after firm e's entry, irrespective of the type of network that is established. To do so, we must make some assumptions, because we generically examine an oligopoly with firms that may have different marginal costs through a link formation. The following assumption is required to restrain the cost differentials among firms.

Assumption 1 *(i)* $\Delta_0 < (3/5)\,t$, *(ii)* $\Delta_1 < (3/4)\,t$.

In (i) and (ii) of the above assumption, Δ_0 and Δ_1 represent the (production) cost differentials among firms. In particular, $\Delta_0 \equiv c(0) - c(1)$ and $\Delta_1 \equiv c(1) - c(2)$. In other words, Δ_0 (resp. Δ_1) represents the cost-reducing effect of the first (resp. second) link formed. Assumption 1 requires that the cost-reducing effects of the first and second links are not large relative to each consumer's (unit) transportation cost. This assumption is necessary for the existence of the interior solution, because the monopolization or the duopolization can occur in our Hotelling model only when a cost differential among firms is large.[7] In fact, under Assumption 1, the interior solution is guaranteed as long as firm e chooses its location from the range of $[1-\overline{a}(t),\ \overline{a}(t)]$, where $\overline{a}(t)$ is defined as follows.[8,9]

$$\overline{a}(t) \equiv \min\left\{1 - (2\Delta_0/(3t-\Delta_0)),\ \left(1+\sqrt{1-(4\Delta_0/3t)}\right)/2,\ 1-(2\Delta_1/3t)\right\}.$$

If t approaches ∞, $\overline{a}(t)$ approaches 1. To focus on the interior solution, we assume that a is chosen from $1-\overline{a}(t)$ to $\overline{a}(t)$. In reality, this assumption seems to be justified, because an entrant cannot locate too close to an incumbent because of some factors, such as the existence of loyal consumers who favor the incumbent's goods, and so on. The analytical results derived in the next section are ensured as long as t is sufficiently large.

---

of a pair to a particular firm) has. See the concluding remarks on this point in Section 4.

[7] Matsumura and Matsushima (2009) and Meza and Tombak (2009) examine the effect of cost differentials among firms in a Hotelling model, which is called an *asymmetric duopoly* model.

[8] $\overline{a}(t)$ is easy to derive by the following procedure. First, given a network, g, and firm e's location, a, we obtain conditions under which all three firms have a positive demand and a positive profit in the third stage. Second, comparing the conditions derived under all networks and using Assumption 1, we derive the upper bound of firm e's location, $\overline{a}(t)$, that is a sufficient condition for the existence of the interior solution.

[9] If we allow firm e to choose $a \in (0, \overline{a}(t)) \cup (\overline{a}(t), 1)$, we cannot only guarantee the interior solution but also the existence of any type of location equilibrium in pure strategies. The proof of this point is available upon request.

The timing of the game is as follows. In the first stage, firm e decides whether or not to enter the market by incurring an entry sunk cost, F. Once it decides to enter, it also determines its location, a. In the second stage, all firms in the market determine their pairwise links, which means that each firm's marginal cost is determined in this stage. Then, we derive the equilibrium network structures among firms in the market by adopting the concept of pairwise stability with a dynamic process, the details of which are explained in the next section. In the third stage, the firms compete in price (i.e., Bertrand competition) and consumers buy goods from one of the firms in the market.

3 The Equilibrium

In this section, we derive the equilibrium by using a backward induction argument.

3.1 Bertrand competition

We examine the outcome of Bertrand competition in the third stage, given firm e's location, a, in a type of network structure.

Suppose firm e enters and chooses a. Restricting our attention to an interior solution in which all three firms can obtain a positive demand and a positive profit in equilibrium, we denote a consumer who is indifferent between buying from firm 1 and buying from firm e by $\theta_{1e} \in (0, a)$. Then, we have

$$\begin{aligned} v - \theta_{1e}^2 t - p_1 &= v - (a - \theta_{1e})^2 t - p_e \\ \text{or } \theta_{1e} &= \frac{p_e - p_1}{2at} + \frac{a}{2}. \end{aligned}$$

Similarly, denoting a consumer who is indifferent between buying from firm 2 and buying from firm e by $\theta_{e2} \in (a, 1)$, we have

$$\begin{aligned} v - (\theta_{e2} - a)^2 t - p_e &= v - (1 - \theta_{e2})^2 t - p_2 \\ \text{or } \theta_{e2} &= \frac{p_2 - p_e}{2(1-a)t} + \frac{1+a}{2}. \end{aligned}$$

Then, the firms' respective demands are

$$\begin{aligned} D_1(p_1, p_e) &= \theta_{1e} = \frac{p_e - p_1}{2at} + \frac{a}{2}, \\ D_e(p_1, p_e, p_2) &= \theta_{e2} - \theta_{1e} = \frac{p_2 - p_e}{2(1-a)t} - \frac{p_e - p_1}{2at} + \frac{1}{2}, \\ D_2(p_e, p_2) &= 1 - \theta_{e2} = \frac{p_e - p_2}{2(1-a)t} + \frac{1-a}{2}. \end{aligned}$$

Each firm maximizes its profit with respect to price. Deriving the first-order conditions and rearranging them, we have the equilibrium prices as follows:

$$\begin{aligned} p_1^* &= \frac{1}{6}(4-a)c_1 + \frac{1}{3}c_e + \frac{1}{6}ac_2 + \frac{1}{2}at, \\ p_e^* &= \frac{1}{3}(1-a)c_1 + \frac{2}{3}c_e + \frac{1}{3}ac_2 + a(1-a)t, \\ p_2^* &= \frac{1}{6}(1-a)c_1 + \frac{1}{3}c_e + \frac{1}{6}(3+a)c_2 + \frac{1}{2}(1-a)t. \end{aligned}$$

One remark deserves to be mentioned in the equilibrium prices. In the interior solution, firm e competes in price with two incumbents (i.e., firms 1 and 2), whereas each of the two incumbents competes directly only with firm e. We then implicitly assume that firm e sets a uniform price (i.e., no price discrimination) to compete with two different incumbents. However, the equilibrium price set by each of the incumbents depends not only on its own cost and firm e's cost but also on the other incumbent's. This reflects an indirect effect through firm e's price and location strategies. This point relates to the price-competition effect that works in the second stage of link formation.

The equilibrium profits are described as follows:

$$\begin{aligned} \Pi_1 &= D_1(p_1^*, p_e^*)(p_1^* - c_1) = \frac{1}{2at}(p_1^* - c_1)^2, \\ \Pi_e &= D_e(p_1^*, p_e^*, p_2^*)(p_e^* - c_e) = \frac{1}{2a(1-a)t}(p_e^* - c_e)^2, \\ \Pi_2 &= D_1(p_1^*, p_e^*)(p_2^* - c_2) = \frac{1}{2(1-a)t}(p_2^* - c_2)^2. \end{aligned}$$

When firm e does not enter the market, the equilibrium prices and profits are easily derived in a similar way. Hence, we report them when we discuss the characteristics of firm e's entry decision and its location choice in Section 3.

3.2 The formation of pairwise stable networks

We now proceed to the analysis of network structure (i.e., the formation of pairwise links) in the second stage. In this subsection, we examine the case in which firm e enters the market and locates at a. Because our model has a symmetric structure, we can restrict our attention to the case in which $a \in \left[\frac{1}{2}, \overline{a}(t)\right]$. The case in which firm e does not enter the market will be mentioned in Subsection 3.3.

Suppose a network structure, g, is established. Then, given g, the equilibrium prices and profits are rewritten as

$$
\begin{aligned}
p_1^*(g,a) &= \frac{1}{6}(4-a)\,c(\eta_1(g)) + \frac{1}{3}c(\eta_e(g)) + \frac{1}{6}ac(\eta_2(g)) + \frac{1}{2}at, & (1)\\
p_e^*(g,a) &= \frac{1}{3}(1-a)\,c(\eta_1(g)) + \frac{2}{3}c(\eta_e(g)) + \frac{1}{3}ac(\eta_2(g)) + a(1-a)\,t, & (2)\\
p_2^*(g,a) &= \frac{1}{6}(1-a)\,c(\eta_1(g)) + \frac{1}{3}c(\eta_e(g)) + \frac{1}{6}(3+a)\,c(\eta_2(g)) + \frac{1}{2}(1-a)\,t. & (3)\\
\Pi_1(g,a) &= \frac{1}{2at}\left(p_1^*(g,a) - c(\eta_1(g))\right)^2, & (4)\\
\Pi_e(g,a) &= \frac{1}{2a(1-a)\,t}\left(p_e^*(g,a) - c(\eta_e(g))\right)^2, & (5)\\
\Pi_2(g,a) &= \frac{1}{2(1-a)\,t}\left(p_2^*(g,a) - c(\eta_2(g))\right)^2. & (6)
\end{aligned}
$$

We define firm i's *link-incentive function* as follows.

$$
f_i(g,a,ij) \equiv p_i^*(g+ij,a) - c(\eta_i(g+ij)) - \left(p_i^*(g,a) - c(\eta_i(g))\right), \qquad (7)
$$

where ij represents a direct pairwise link between firms i and j ($i,j = 1$, 2, e, and $i \neq j$). Then, $g+ij$ represents the network obtained by adding the link ij to network g. Hence, the link-incentive function $f_i(g,a,ij)$ defines the difference in firm i's price-cost margin under network $g+ij$ from that under network g, given any location a. Apparently, the sign of $f_i(g,a,ij)$ is the same as that of $\Pi_i(g+ij,a) - \Pi_i(g,a)$. Hence, hereafter, we use $f_i(g,a,ij)$ to analyze firm i's incentive to form pairwise links. In fact, we have:

1. When $f_i(g,a,ij) > 0$, firm i has an incentive to add ij to network g. Otherwise, it has no incentive to do so.

2. When $f_i(g-ij,a,ij) < 0$, firm i has an incentive to sever ij from network g. Otherwise, it has no incentive to do so.

Then, we obtain the following lemma. (All proofs of lemmas and propositions are relegated to the Appendix.)

Lemma 1 *Suppose* $\eta_i(g) = \eta_j(g)$ *where* $i,j = 1,2,e$ *and* $i \neq j$. *Then, for any* $a \in \left[\frac{1}{2}, \overline{a}(t)\right)$, *we have:*

For $ij \notin g$, $f_i(g,a,ij) > 0$ *and* $f_j(g,a,ij) > 0$.

For $ij \in g$, $f_i(g-ij,a,ij) > 0$ *and* $f_j(g-ij,a,ij) > 0$.

Lemma 1 states that as long as the number of links is the same between firm i and firm j in a given network, g, the link ij must be included in g (i.e., they both have an incentive to form a link between them), irrespective of the location of an entrant, a. We should remember that this statement holds to all the firms, including firm e.

In our analysis, we use a concept of pairwise stability attributed to Jackson and Wolinsky (1996). The pairwise stability in our model is defined as follows.

Definition 1 *A network* g *is* ***pairwise stable*** *if*

(i) for all $ij \in g$, $f_i(g-ij,a,ij) \geq 0$ *and* $f_j(g-ij,a,ij) \geq 0$.

(ii) for all $ij \notin g$, *if* $f_i(g,a,ij) > 0$, *then* $f_j(g,a,ij) < 0$.

The pairwise stability captures the idea of mutual consent. It supposes that a pair of players can communicate and must agree to form a link.

[Insert Figure 1 around here.]

Now, we attempt to characterize the pairwise stable networks in our model. All the possible networks are drawn in Figure 1. From the result of Lemma 1, it is easy to find that the networks g^0, g^4, g^5, and g^6 are not pairwise stable, because they contain firms i and j such that $\eta_i(g) = \eta_j(g)$, and the link between them is not included in the associated network.

The remaining networks, g^1, g^2, g^3, and g^7 are candidates for pairwise stable networks. In fact, the set of pairwise stable networks depends on the degree of the cost-reducing effect of a link on a firm's marginal cost, as shown in the following lemma. Before reporting the lemma, we define $\Delta \equiv \Delta_0/\Delta_1$. That is, Δ represents the ratio of the cost-reducing effect of the first link to that of the second link. We call the case where $\Delta > 1$ *the case of decreasing incremental change of the cost-reducing effect*. That is, when $\Delta > 1$, the cost-reducing effect of the first link is larger than that of the second link. Similarly, we call the case where $\Delta < 1$ *the case of increasing incremental change of the cost-reducing effect*. In that case, the cost-reducing effect of the first link is smaller than that of the second link.

Then, we report Lemma 2.

Lemma 2 *When* $a \in \left[\frac{1}{2}, \sqrt{3}-1\right)$, *we have:*
(i) If $\Delta > \frac{3+a}{1-a}$, g^1, g^2, g^3, *and* g^7 *are pairwise stable.*
(ii) If $\frac{3+a}{1-a} \geq \Delta > \frac{2+a}{a}$, g^1, g^3, *and* g^7 *are pairwise stable.*
(iii) If $\frac{2+a}{a} \geq \Delta > \frac{2+a}{2}$, g^3 *and* g^7 *are pairwise stable.*
(iv) If $\frac{2+a}{2} \geq \Delta \geq 1-a$, g^7 *is pairwise stable.*
(v) If $1-a > \Delta \geq \frac{a}{2+a}$, g^3 *and* g^7 *are pairwise stable.*
(vi) If $\frac{a}{2+a} > \Delta \geq \frac{1-a}{3-a}$, g^2, g^3 *and* g^7 *are pairwise stable.*
(vii) If $\frac{1-a}{3-a} > \Delta > 0$, g^1, g^2, g^3 *and* g^7 *are pairwise stable.*
When $a \in \left[\sqrt{3}-1, \overline{a}(t)\right)$, *the above (i), (ii), (iii), (vii) hold, and (iv) to (vi) are replaced by the followings:*
(iv') If $\frac{2+a}{2} \geq \Delta \geq \frac{a}{2+a}$, g^7 *is pairwise stable.*
(v') If $\frac{a}{2+a} > \Delta \geq 1-a$, g^2 *and* g^7 *are pairwise stable.*
(vi') If $1-a > \Delta \geq \frac{1-a}{3-a}$, g^2, g^3 *and* g^7 *are pairwise stable.*

In Lemma 2, the location of an entrant affects not only the threshold values for the range of the ratio of the cost-reducing effect of links, but also the set of pairwise stable networks [compare *(iv)* to *(vi)* and *(iv')* to *(vi')*]. The crucial point of Lemma 2 is that the set of pairwise stable networks varies, depending on the ratio of the cost-reducing effect of links.[10]

[10] Instead of the concept of pairwise stability, one can use the other equilibrium concepts in our model. For example, we ensure that if *the pairwise Nash stability* is used in our model, the pairwise stable sets are exactly the same as those of the pairwise Nash stability. Similarly, if we use *the strong stability* in the sense of Jackson and van den Nouweland (2005), the strong stable networks are g^1, g^2, and g^3. Thus, no strong stable network exists in the case in which $(2+a)/2 \geq \Delta \geq 1-a$.

According to Lemma 2, there can be multiple pairwise stable networks. To refine the pairwise stable networks, we use a dynamic process for network formation, which is explained as follows.[11] At the initial period, called period 0, the two incumbents (i.e., firms 1 and 2) decides whether they will form a link or not. Then, after period 0, a link ij $(i, j = 1, 2, e$, and $i \neq j)$, irrespective of whether it is in the existing network or not, is randomly identified to be updated with uniform probability. If the link ij is already in the existing network, then either firm i or j can decide to sever the link. If the link is not in the network, and at least one of the two firms involved would benefit from adding it and the other would be at least as well off given the existing network, the link is added. Then, if the process reaches a fixed configuration, it must be a stable network. In our setting, the presumption that the opportunity to form a collaboration between incumbents comes first is natural, because their collaboration can have an entry-deterrence effect when there exists a threat of potential entry.[12]

Then, we obtain the following lemma.

Lemma 3 *When $\Delta > \frac{2+a}{2}$ or $\Delta < 1 - a$, the network formation process converges to g^3. When $1 - a \leq \Delta \leq \frac{2+a}{2}$, it converges to g^7.*

Lemma 3 states that when the ratio of the cost-reducing effect of links is large, irrespective of whether the incremental change of the cost-reducing effect is increasing or not, the existing link between the two incumbents remains and the entrant cannot form a pairwise link with any incumbent. On the other hand, when the ratio of the cost-reducing effect is small, all the firms, including the entrant, can form two pairwise links with the other two firms (the *complete* network).

The intuition of Lemma 3 is explained by two effects generated by the formation of pairwise links. One is *the cost-reducing effect*, which is a direct effect of link formation, whereas the other is *the price-competition effect*, which is a mixed effect generated by the network formation and the entrant's location choice. Let us explain it in more detail.

First, consider the case where $\Delta > \frac{2+a}{2}$. Suppose firms 1 and e are chosen at the first period, given the initial state $g_{(0)} = g^3$, and consider firm 1's incentive for link formation.[13] The fact that $\Delta > \frac{2+a}{2}$ means that the cost-reducing effect of the second link is much smaller than that of the first link. Then, when firm 1 does not form a link with firm e, it can enjoy a higher price and obtain a larger market share than those with the link, as long as firm e does not have $e2$. This is because firm e has a cost disadvantage over firms 1 and 2. On the other hand, if firm 1 forms $1e$, the price it can set should be lower than the price it can set without the link. This is because firm e becomes more efficient with

[11] Our dynamic process is similar to the process introduced by Watts (2001) with the exception of the move at the initial round. Okumura (2007) applies Watts' dynamic process to the consideration of an oligopoly model with network formation. Introducing another type of dynamic process, called "learning dynamics", Bala and Goyal (2001) also insist on the effectiveness of the dynamic process as a refinement tool of the equilibria of a static game in network formation. See p.1187 of Bala and Goyal (2001).

[12] Instead, if two of all three firms are randomly chosen from the initial period, multiple kinds of networks still remains in equilibrium, depending on the value of Δ. For example, there can be an equilibrium in which each of g^2, g^3 and g^7 appear with probability 1/3 in some range of Δ. However, in other ranges of Δ, the conversion to each of g^3 and g^7 holds, as shown in Lemma 3.

[13] Note that at period 0, firms 1 and 2 certainly form their link for any level of Δ.

a link than without it, although it still has a cost disadvantage over firm 1. Indeed, we verify this fact by

$$p_1^*\left(g^3+1e,a\right)-p_1^*\left(g^3,a\right)=-\frac{1}{6}\left(4-a\right)\Delta_1-\frac{1}{3}\Delta_0<0.$$

Because the cost-reducing effect generated by $1e$ (i.e., the second link for firm 1) is small, the price-competition effect given firm e's location, a, can be larger than the cost-reducing effect for firm 1. Hence, firm 1 has no incentive to form $1e$. By the same reasoning, we can verify that firm 2 has no incentive to form $e2$ if it is chosen at the first period.

Second, consider the case where $\Delta<1-a$. In this case, the cost-reducing effect of the second link is larger than that of the first link. This makes firm e cost-disadvantageous over the two incumbents if it forms a link with one of them. Again, suppose firms 1 and e are chosen at the first period. Then we have

$$p_e^*\left(g^3+12,a\right)-p_e^*\left(g^3,a\right)=-\frac{1}{3}\left(1-a\right)\Delta_1-\frac{2}{3}\Delta_0<0,$$

and ensure that, for firm e, the cost-reducing effect of $1e$ is much smaller than the price-competition effect. Hence, firm e has no incentive to form $1e$.

This argument can be applied to the case in which firms e and 2 are chosen at the first period. In sum, for $\Delta>\frac{2+a}{2}$ or $\Delta<1-a$, the network g^3 survives in the dynamic process because, as for a firm that tries to form a link, the cost-reducing effect is smaller than the price-competition effect. On the contrary, for $1-a\leq\Delta\leq\frac{2+a}{2}$, the cost-reducing effect is larger than the price-competition effect, so that each of the three firms has an incentive to form a link with each other.

3.3 Entry and location choice

Now, we turn to the first stage in which firm e makes its entry decision, and we address two important questions in our analysis: First, which location would firm e choose once it enters the market? Second, is it easy for firm e to enter the market when it is allowed to form links with the incumbents?

In this stage, if firm e decides to enter, it also determines its location, a. Using Assumption 1, we obtain the primary finding of this paper.

Proposition 1 *In equilibrium, we have:*

(i) For $\Delta\in(0,\ 1/2]$, if $\left(3t-4\Delta_0\right)^2\leq 36t^2\Delta\left(1-\Delta\right)$, firm e chooses $a^*=1-\Delta\,(>1/2)$ and enters the market if and only if $F\leq F^{*1}\equiv(1/2)\,t\Delta\left(1-\Delta\right)$. Otherwise, $a^*=\frac{1}{2}$ and firm e enters the market if and only if $F\leq F^{*2}\equiv(1/72t)\left(3t-4\Delta_0\right)^2$.

(ii) For $\Delta\in[1/2,\ 5/4]$, firm e chooses $a^*=1/2$ and enters the market if and only if $F\leq F^{*3}\equiv(1/8)\,t$.

(iii) For $\Delta\in(5/4,\ 3/2)$, if $\left(3t-4\Delta_0\right)^2\leq 72t^2\left(\Delta-1\right)\left(3-2\Delta\right)$, firm e chooses $a^*=2\left(\Delta-1\right)(>1/2)$ and enters the market if and only if $F\leq F^{*4}\equiv t\left(\Delta-1\right)\left(3-2\Delta\right)$. Otherwise, $a^*=1/2$ and firm e enters the market if and only if $F\leq F^{*2}\equiv(1/72t)\left(3t-4\Delta_0\right)^2$.

(iv) For $\Delta\geq 3/2$, firm e chooses $a^*=1/2$ and enters the market if and only if $F\leq F^{*2}\equiv(1/72t)\left(3t-4\Delta_0\right)^2$.

The result of Proposition 1 is explained by a firm's incentive to form a link with another firm. We describe the details of this point below.

Consider the case where $0 < \Delta \leq 1/2$. In this case, according to Lemma 3, if firm e chooses a between $1-\Delta$ and $\overline{a}(t)$, the complete network g^7 is formed. This result occurs because the cost-reducing effect is larger than the price-competition effect for any two firms that try to form a link. Let us explain this point in detail.

Suppose that firms 1 and e are chosen at the first period after firm e chooses a between $1-\Delta$ and $\overline{a}(t)$ and firms 1 and 2 form the link 12 at period 0 (i.e. $g_{(0)} = g^3$). In the case where $0 < \Delta \leq 1/2$, it is apparent that firm 1 definitely has an incentive to form the link $1e$, because it can enjoy a large cost-reducing effect through $1e$. On the other hand, firm e's incentive to form $1e$ depends on the relative magnitude of the cost-reducing effect through the formation of the link $1e$ and the price-competition effect which is a mixed effect generated by the link $1e$ and firm e's location. Here, from (1) and (2), we can verify that, as a (i.e., the distance between firms 1 and e) becomes larger, firm 1 sets a higher price $p_1^*(g,a)$ associated with a lower demand $D_1(p_1^*, p_e^*)$ for its goods, given any network g. Then, firm 1's incentive for a high price with a low demand induces firm e to be far away from firm 1. In fact, the range of $[1-\Delta,\ \overline{a}(t)]$ is the range of firm e's location at which firm e prefers the network g^4 (i.e., accepting the link $1e$) to the network g^3 (i.e., refusing it). This is because in g^4 firm e can compete with an equally efficient rival (i.e., firm 2) on the opposite side, whereas in g^3 firm e faces two more efficient firms on both sides. Thus, firm e chooses $a^* = 1-\Delta$ in g^4 to avoid fierce price competition in the retail market. This is ensured by

$$\frac{\partial \Pi_e(g^7, a)}{\partial a} < 0 \text{ for any } a \in [1-\Delta, \overline{a}(t)).$$

Subsequently, the formation of the link $1e$ induces firms e and 2 to form the link $e2$. Thus, the network g^7 is finally established. Hence, we have $a^* = 1-\Delta$ with the network g^7.

On the contrary, if firm e chooses location a between $1/2$ and $1-\Delta$, g^3 still stands in the second stage. This result occurs because, for firm e, the cost-reducing effect of the link $1e$ is smaller than the price-competition effect in the range between $1/2$ and $1-\Delta$. Then, to avoid fierce price competition in the retail market, firm e chooses $a^* = 1/2$ in g^3.[14] Thus, we have $a^* = 1/2$ with the network g^3.

In sum, in the case where $0 < \Delta \leq 1/2$, we have two equilibrium locations; $a^* = 1-\Delta$ with g^7 and $a^* = 1/2$ with g^3. Needless to say, firm e chooses the location that gives a higher profit. Then, we can verify that if $(3t - 4\Delta_0)^2 \leq (>) 36t^2\Delta(1-\Delta)$, $a^* = 1-\Delta$ ($a^* = 1/2$) gives a higher profit to firm e (the condition in (i)).

Similarly, in the case where $1/2 < \Delta \leq 5/4$, we ensure that the network g^7 is formed in the second stage, irrespective of firm e's location $a \left(\in \left[\frac{1}{2}, \overline{a}(t)\right]\right)$, by the result of Lemma 3 and the condition that $1/2 < \Delta$ (see the proof of Lemma 3 in the Appendix). Then, firm e chooses $a^* = 1/2$.

In the case where $5/4 < \Delta < 3/2$, the incumbent network g^3 survives if firm e chooses location a between $1/2$ and $2(\Delta - 1)$, whereas the complete network g^7 is formed if firm e chooses a in the range between $2(\Delta-1)$ and $\overline{a}(t)$. Hence, firm e chooses $a^* = 1/2$ in

[14] Indeed, $\partial \Pi_e(g^3, a)/\partial a < 0$ for any $a \in \left[\frac{1}{2}, 1-\Delta\right)$.

g^3, whereas it chooses $a^* = 2(\Delta - 1)$ in g^7. The condition that indicates firm e's location choice is given by the statement in Proposition 1.

Last, in the case where $\Delta \geq 3/2$, we ensure that network g^3 survives in the second stage, irrespective of firm e's location $a \left(\in \left[\frac{1}{2}, \overline{a}(t)\right)\right)$, by the result of Lemma 3 and the condition that $2(\Delta - 1) > 1$. Then, firm e chooses $a^* = 1/2$.

In Proposition 1, it is difficult to interpret the conditions that indicate firm e's location choice when two candidates exist (i.e., the case where $0 < \Delta \leq 1/2$ or $5/4 < \Delta < 3/2$). Let us give an interpretation for these conditions. Consider the case in which $0 < \Delta \leq 1/2$. When does firm e choose $a^* = 1/2$ rather than $a^* = 1 - \Delta$? Note that, given Assumption 1(i) (i.e., $\Delta_0 < (3/5)\,t$), the larger Δ_0, the smaller firm e's profit in g^3 (i.e., $\Pi_e^*(g^3, 1/2)$). This result occurs because the two incumbents who have one link with each other become more efficient as Δ_0 becomes larger. On the other hand, firm e's profit in g^7 (i.e., $\Pi_e^*(g^7, 1-\Delta)$) becomes larger as Δ_0 becomes larger, when $\Delta \leq 1/2$. Hence, as the cost-reducing effect of the first link becomes large, firm e has more incentive to choose $a^* = 1 - \Delta$, given that other parameters remain constant.

Next, consider the effect of the parameter Δ_1. It is easy to check that $\partial\Pi_e^*(g^7, 1-\Delta)/\partial\Delta_1 < 0$ when $\Delta \leq 1/2$. When the marginal cost is the same among all firms, the profit-maximizing location for firm e is $a = 1/2$. Then, because $a^* = 1 - \Delta$ gets farther from $1/2$ as Δ_1 becomes larger, firm e's profit under g^7 decreases. Hence, it is more likely for firm e to take $a^* = 1/2$ rather than $a^* = 1 - \Delta$ when Δ_1 is large.

Similarly, we can give an interpretation of the condition for the case in which $5/4 < \Delta < 3/2$. In fact, because $\partial\Pi_e^*(g^7, 2(\Delta-1))/\partial\Delta_1 > 0$ when $\Delta > 1$, it is more likely for firm e to take $a^* = 1/2$ than $a^* = 2(\Delta - 1)$ when Δ_1 becomes large.[15]

We summarize these findings as a corollary.

Corollary 1 *Consider the case in which* $0 < \Delta \leq 1/2$. *Then, when* Δ_0 *is large or* Δ_1 *is small, firm e chooses* $a^* = 1 - \Delta$. *For the case in which* $5/4 < \Delta < 3/2$, *firm e chooses* $a^* = 2(\Delta - 1)$ *when* Δ_1 *is large.*

The results of Proposition 1 and Corollary 1 imply that when firms in the market have an opportunity to form links (i.e., collaborations) with each other, an entrant's location can be distorted from a welfare viewpoint. To clarify the driving force that generates the distortion of an entrant's location, we prepare two benchmarks and compare the results of the benchmarks with those of Proposition 1 and Corollary 1. The first benchmark is the case in which no firm in the market has an opportunity to form links, whereas the second is the case in which only the incumbents can form a link and the entrant is not allowed to do so.

In the first benchmark, it is easy to verify that firm e has an incentive to choose $a^{B*} = 1/2$, once it decides to enter the market. Similarly, firm e chooses $\widetilde{a}^{B*} = 1/2$ even in the second benchmark, because it would like to avoid fierce price competition with more efficient incumbents. Hence, we ensure that allowing firm e to form a link generates firm e's incentive to distort its location. We can state that when there exists a factor that facilitates collaboration with an entrant, it creates a distortion in the entrant's location.

[15] In this case, the effect of Δ_0 is ambiguous, because both $\Pi_e^*\left(g^3, 1/2\right)$ and $\Pi_e^*\left(g^7, 2(\Delta - 1)\right)$ are decreasing in Δ_0.

In fact, when an entrant has a broad distribution channel or an excellent technology for the production of an input that is used to produce a final good, the entrant may easily collaborate with incumbents in the final good market by sharing its knowledge with the incumbent. For example, Samsung collaborated with Sony when it entered the LCD market and with Seagate when it entered the hard-disk market. In addition, one of the factors that facilitate a coalition among firms is the improvement of information technology, such as the prevalence of Internet or cloud-computing systems. The prevalence of Internet or cloud-computing systems aids communication among firms, even if they are distant from each other. Indeed, a newcomer finds it difficult to communicate with incumbents without the improvement of information technology, because it is not easy for the newcomer to access knowledge on business customs, the distribution of consumers, and so on.

Furthermore, we can obtain the analytical result regarding the combined effect of an entrant's collaboration with incumbents and its associated location choice on the incumbent's entry-deterrence strategy by providing the threshold level of the entry sunk cost for the two benchmarks. In the first benchmark in which there exists no opportunity for all of the firms to form pairwise links, firm e's profit is

$$\Pi_e^{B*}\left(\varnothing, a^{B*}\right)\left(=\Pi_e^*\left(g^0, a^{B*}\right)\right)=\frac{1}{8}t,$$

because it chooses $a^{B*}=1/2$. Hence, the threshold level of the entry sunk cost in the first benchmark is $F^{B*}=(1/8)\,t$. Note that $a^{B*}=1/2$ and $\Pi_e^*=(1/8)\,t$ hold as long as the marginal cost is the same among the three firms, including the entrant. Hence, a^{B*} and F^{B*} are the first-best solution in the sense that they are realized when the marginal costs of all the firms (firms 1, 2, and e) are $c\,(2)$.

Next, we derive the threshold level of the entry sunk cost in the second benchmark in which only the incumbents can form a pairwise link and the entrant is not allowed to do so. In this second benchmark, firm e chooses $\widetilde{a}^{B*}=1/2$, once it decides to enter the market. Thus, firm e's profit is given by

$$\widetilde{\Pi}_e^{B*}\left(g^3, \widetilde{a}^{B*}\right)=\frac{1}{72t}\left(3t-4\Delta_0\right)^2.$$

Hence, the threshold level of the entry sunk cost when there exists a link between the two incumbents is $\widetilde{F}^{B*}=\left(3t-4\Delta_0\right)^2/72t$. Comparing $\widetilde{F}^{B*}$ with F^{B*}, we ensure that the collaboration between the two incumbents can unquestionably have an entry-deterrence effect, because $\widetilde{F}^{B*}<F^{B*}$.

Comparing the threshold levels of the entry sunk cost in equilibrium (i.e., F^{*1} to F^{*4}) in Proposition 1 with F^{B*} or $\widetilde{F}^{B*}$, we immediately obtain the following finding.

Corollary 2 *In equilibrium, if the cost-reducing effect of a link is almost constant (i.e., the incremental change in the cost-reducing effect scarcely depends on the number of links), the link between incumbents never has an entry-deterrence effect. Otherwise, the incumbent link has an entry-deterrence effect, whereas its effect can be diluted with an entrant's location choice.*

According to Proposition 1, for $\Delta \in [1/2,\ 5/4]$, the threshold level of the sunk cost is exactly the same as that of the first benchmark; $F^{*3} = F^{B*} = (1/8)\,t$. That is, when the incremental change of the cost-reducing effect is near 1 (i.e., it scarcely depends on the number of links), the incumbent link has no entry-deterrence effect because the incumbents are also willing to form a link with an entrant. Indeed, in that case, the cost-reducing effect becomes larger than the price-competition effect for all firms, which, in turn, benefits *not only an entrant but also the two incumbents.* Then, the entrant is likely to be welcome so that the complete network g^7 is formed. Hence, the incumbent collaboration does not have an entry-deterrence effect in that case.

Except for that case, the incumbent link certainly has an entry-deterrence effect, even though the entrant also has an opportunity to form a link with an incumbent. However, when $0 < \Delta \leq 1/2$ and Δ_0 is large or Δ_1 is small, the entrant can dilute the entry-deterrence effect of the incumbent link with its location choice $a^* = 1 - \Delta$. Similarly, when $5/4 < \Delta < 3/2$ and Δ_1 is large, the entrant chooses $a^* = 2\,(\Delta - 1)$ to make the entry-deterrence effect of the incumbent link weak.

The result of Corollary 2 provides a crucial implication for the competition policy. According to Corollary 2, the entry-deterrence effect of the collaboration among incumbents depends on the magnitude of the collaboration effect (i.e., the cost-reducing effect generated by the collaboration) when there is an opportunity for the incumbents to collaborate with the entrant. The collaboration among firms may enhance social welfare. Therefore, in that situation, a competition authority should care about the information regarding the collaboration effect to evaluate the incumbents' alleged entry-deterrence strategies.

4 Concluding Remarks

In this paper, we have examined the case in which an entrant has two beneficial strategies; collaboration with incumbents and location choice. In particular, we have analyzed the effect of allowing the entrant to have an opportunity to collaborate with incumbents on its location choice. As stated in the text, it is not rare for entrants to use a strategic tool of collaboration with incumbents. For example, when an entrant has a broad distribution channel or an excellent technology for the production of an input that is used to produce a final good, the entrant may easily collaborate with incumbents in the final good market by sharing its knowledge with the incumbents. In addition, facilitating an entrant's collaboration with incumbents seems to be justified by the improvement of information technology such as the prevalence of Internet or cloud-computing systems.

Using the standard setting of a Hotelling model with a pairwise-link formation, we have demonstrated that when an entrant is allowed to form a link and the link formation requires mutual consent, the entrant has an incentive to distort its location to form a link with each of the incumbents. Then, we have demonstrated that the existing link formed by the incumbents cannot be a device to deter entry when the incremental change of the cost-reducing effect generated by forming a link scarcely depends on the number of links.

The model presented in this paper is quite simple, so that several extensions deserve to be mentioned for future research. Here, we will mention three of them. First, introducing multiple entrants or more than two incumbents may affect the analytical results derived above, because the relative magnitude between the cost-reducing effect and the price-competition effect changes according to the number of firms. Second, if the cost-reducing effect generated by a pairwise link formation depends not only on the number of links but also on the degree of spillovers (i.e., how many links a neighbor that has a link with a particular firm has with other firms), our analytical results become quite complicated. Third, the change of spatial structure from a linear city to a circular city seems to provide a dramatic qualitative change in the above results, as it emerges when examining other research issues.[16]

Appendix

A. Proof of Lemma 1

Consider a network, g, in which $\eta_1 = \eta_e = \eta$ and $\eta_2 = \widetilde{\eta}\,(\neq \eta)$. Suppose the link $1e \notin g$. Then, from (1) and (2), we have:

$$\begin{aligned} f_1(g, a, 1e) &\equiv p_1^*(g + 1e, a) - c(\eta + 1) - (p_1^*(g, a) - c(\eta)) \\ &= \frac{1}{6} a (c(\eta) - c(\eta + 1)) > 0. \\ f_e(g, a, 1e) &\equiv p_e^*(g + 1e, a) - c(\eta + 1) - (p_e^*(g, a) - c(\eta)) \\ &= \frac{1}{3} a (c(\eta) - c(\eta + 1)) > 0. \end{aligned}$$

Next, suppose $1e \in g$. Again, from (1) and (2), we have:

$$\begin{aligned} f_1(g - 1e, a, 1e) &\equiv p_1^*(g, a) - c(\eta) - (p_1^*(g - 1e, a) - c(\eta - 1)) \\ &= \frac{1}{6} a (c(\eta - 1) - c(\eta)) > 0. \\ f_e(g - 1e, a, 1e) &\equiv p_e^*(g, a) - c(\eta) - (p_e^*(g - 1e, a) - c(\eta - 1)) \\ &= \frac{1}{3} a (c(\eta - 1) - c(\eta)) > 0. \end{aligned}$$

Notice that all the above inequalities hold for any $a \in \left[\frac{1}{2}, \overline{a}(t)\right)$.

Likewise, we can verify the claim for any other pairs by using (1) to (3). ■

B. Proof of Lemma 2

Using (1) to (3) and the definition of pairwise stability, we need to check when each of the networks g^1, g^2, g^3, and g^7 can be pairwise stable.

First, consider g^1. Link $1e$ satisfies the condition of pairwise stability from Lemma 1. By checking the definition of pairwise stability, we can verify that link 12 satisfies its

[16] Complications would arise in such a model, mainly because all the incumbents are symmetric in a circular city model in the sense that they are located within equal distance, and they compete on both sides with rivals, even though we allow an entrant to choose its location between two incumbents.

condition when $\Delta > \frac{2+a}{a}$ or $\Delta < \frac{1-a}{3-a}$. Similarly, link $e2$ satisfies its condition when $\Delta > \frac{1}{a}$ or $\Delta < \frac{2}{3-a}$. Therefore, g^1 is pairwise stable when $\Delta > \frac{2+a}{a}$ or $\Delta < \frac{1-a}{3-a}$.

The same procedure for the check of pairwise stability conditions can be applied to g^2, g^3, and g^7. In fact, we ensure that: (i) g^2 is pairwise stable when $\Delta > \frac{3+a}{1-a}$ or $\Delta < \frac{a}{2+a}$, (ii) g^3 is pairwise stable when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$, and (iii) g^7 is pairwise stable for any $\Delta > 0$.

Furthermore, we know that $1-a \geq (<) \frac{a}{2+a}$ if and only if $\frac{1}{2} \leq a \leq \sqrt{3}-1$ $\left(\sqrt{3}-1 < a < \overline{a}(t)\right)$. Then, the results derived above are summarized as in the text. ■

C. Proof of Lemma 3

We must provide the proofs for two cases; the case in which $a \in \left[\frac{1}{2}, \sqrt{3}-1\right)$ and the case in which $a \in \left[\sqrt{3}-1, \overline{a}(t)\right)$. Hereafter, we denote the network at t-th period by $g_{(t)}$.

(i) The case where $a \in \left[\frac{1}{2}, \sqrt{3}-1\right)$

First, we consider the case where $a \in \left[\frac{1}{2}, \sqrt{3}-1\right)$. It is easy to verify that at the initial period, link 12 is certainly formed, because it is pairwise stable. Then, from Lemma 2, we know that, given $g_{(0)} = g^3$, g^3 and g^7 are the candidates for pairwise stable networks to which the dynamic process converges when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$. On the other hand, when $1-a \leq \Delta \leq \frac{2+a}{2}$, only g^7 is its candidate, whereas there is a possibility that no network survives this process.

Consider the case in which $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$. When firms 1 and e are chosen with probability $\frac{1}{3}$ at the 1st period, firm 1 has no incentive to form $1e$ because $f_1(g^3, a, 1e) < 0$ for $\Delta > \frac{2+a}{2}$. Hence, $g_{(1)} \left(= g_{(0)}\right) = g^3$. Next, suppose firms 1 and 2 are chosen with probability $\frac{1}{3}$. Because $12 \in g^3$, we have $g_{(1)} = g_{(0)} = g^3$ from Lemma 1. Lastly, suppose firms e and 2 are chosen with probability $\frac{1}{3}$. Because $\Delta < 1-a < a$ for $a \geq \frac{1}{2}$, $f_e(g^3, a, e2) < 0$ which implies that firm e has no incentive to form $e2$. Hence, $g_{(1)} = g_{(0)} = g^3$. Therefore, when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$, g^3 remains forever, i.e., the dynamic process converges to g^3.

Next, consider the case in which $1-a \leq \Delta \leq \frac{2+a}{2}$. Again, at the initial period, we have $g_{(0)} = g^3$. Suppose firms 1 and e are chosen with probability $\frac{1}{3}$ at the 1st period. Then, because $f_1(g^3, a, 1e) \geq 0$ and $f_e(g^3, a, 1e) \geq 0$ when $1-a \leq \Delta \leq \frac{2+a}{2}$, the two firms have an incentive to form link $1e$. Hence, $g_{(1)} = g_{(0)} + 1e = g^4$. Then, from Lemma 1, we know that firms e and 2 have an incentive to form link $e2$, because $\eta_e(g^4) = \eta_2(g^4) = 1$ and $e2 \notin g^4$. This means that, once firms e and 2 are chosen with probability $\frac{1}{3}$, g^4 converges to g^7. Next, suppose firms 1 and 2 are chosen at the 1st period. Because $12 \in g^3$, we have $g_{(1)} = g_{(0)} = g^3$ from Lemma 1. Notice that, once firms 1 and e are chosen after this period, g^3 goes to g^4. Again, g^4 converges to g^7 as long as firms e and 2 are chosen.

Suppose firms e and 2 are chosen at the 2nd period after $g_{(1)} = g_{(0)} = g^3$. We need to divide the case where $1-a \leq \Delta \leq \frac{2+a}{2}$ into three cases: the case in which $\frac{3-a}{2} < \Delta \leq \frac{2+a}{2}$, the case in which $a < \Delta \leq \frac{3-a}{2}$, and the case in which $1-a \leq \Delta \leq a$. When $\frac{3-a}{2} < \Delta \leq \frac{2+a}{2}$, g^3 converges to g^7 through g^4 as long as firms 1 and e are chosen. When $a < \Delta \leq \frac{3-a}{2}$, firms e and 2 have an incentive to form the link $e2$ under g^3, which means g^3 goes to g^6. Also, once firms 1 and e are chosen after this period, firms 1 and e have an incentive to form link $1e$, which means g^6 goes to g^7. The process at the case in which $1-a \leq \Delta \leq a$ is the same as the case in which $\frac{3-a}{2} < \Delta \leq \frac{2+a}{2}$. Hence, we can summarize that when firms 1 and 2 are chosen at the 1st period, the dynamic process converges to g^7.

Lastly, suppose firms e and 2 are chosen with probability $\frac{1}{3}$ at the 1st period. The analysis is the same as the case in which firms e and 2 are chosen at the 2nd period after $g_{(1)} = g_{(0)} = g^3$. Therefore, the process converges to g^7.

In sum, we ensure that when $1-a \leq \Delta \leq \frac{2+a}{2}$, the dynamic process with the initial state g^3 converges to g^7.

(ii) The case in which $a \in \left[\sqrt{3}-1, 1\right)$

From Lemma 2, we know that, given $g_{(0)} = g^3$, g^3 and g^7 are the candidates for pairwise stable networks to which the dynamic process converges when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$. On the other hand, when $1-a \leq \Delta \leq \frac{2+a}{2}$, only g^7 is its candidate. Hence, the procedure for checking the convergence of the dynamic process is the same as in (i), except that when $1-a \leq \Delta \leq \frac{2+a}{2}$, we need to divide the case into two cases: the case in which $\frac{a}{2+a} \leq \Delta \leq \frac{2+a}{2}$ and the case in which $1-a \leq \Delta < \frac{2+a}{2}$. However, the same reasoning as in the case in which $1-a \leq \Delta \leq \frac{2+a}{2}$ in (i) applies to both cases. Therefore, we derive the same result as in (i). ■

D. Proof of Proposition 1

Let us consider firm e's problem in the first stage. To do so, we first rewrite the conditions under which the network formation process converges to g^3 and g^7, respectively. In fact, the condition that $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$ is rewritten as

$$a < 2(\Delta - 1) \text{ or } a < 1 - \Delta.$$

We know that $2(\Delta-1) \geq (<) 1-\Delta$ if and only if $\Delta \geq (<) 1$. Hence, when $\Delta \geq 1$, we need to check only the condition that $a < 2(\Delta-1)$. Otherwise, we check the condition that $a < 1-\Delta$. Similarly, the condition that $1-a \leq \Delta \leq \frac{2+a}{2}$ is rewritten as

$$a \geq 1-\Delta \text{ and } a \geq 2(\Delta-1).$$

Again, we obtain that when $\Delta \geq 1$, we need to check only the condition that $a \geq 2(\Delta-1)$. Otherwise, we check the condition that $a \geq 1-\Delta$.

Then, using these conditions and the fact that $a \in \left[\frac{1}{2}, \overline{a}(t)\right)$, we have five cases to be considered: (i) $0 < \Delta \leq \frac{1}{2}$, (ii) $\frac{1}{2} < \Delta \leq 1$, (iii) $1 < \Delta \leq \frac{5}{4}$, (iv) $\frac{5}{4} < \Delta \leq \frac{3}{2}$, and (v) $\Delta > \frac{3}{2}$. Let us examine each of the cases separately.

(i) The case in which $0 < \Delta \leq \frac{1}{2}$.

In this case, because $\Delta < 1$, the condition that $a \geq (<) 1-\Delta$ is a relevant constraint for firm e's problem. Then, if $\frac{1}{2} \leq a < 1-\Delta$, g^3 is formed with probability 1 in the second stage. On the other hand, if $1-\Delta \leq a < \overline{a}(t)$, g^7 is formed with probability 1 in the second stage. Hence, we need to solve each of the two subproblems and then compare firm e's maximized profits to obtain firm e's profit-maximizing location.

The first subproblem is formulated as follows:

$$\begin{aligned}\max_{a}\Pi_e\left(g^3,a\right) &= \frac{1}{2a\left(1-a\right)t}\left(p_e^*\left(g^3,a\right)-c\left(\eta_e\left(g^3\right)\right)\right)^2 \\ s.t.\ \frac{1}{2} &\leq a<1-\Delta.\end{aligned}$$

Then, we obtain the solution $a^{*(i),1}=\frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^*\left(g^3,\frac{1}{2}\right)=\frac{1}{72}\left(3t-4\Delta_0\right)^2.$$

Similarly, the second subproblem is formulated by:

$$\begin{aligned}\max_{a}\Pi_e\left(g^7,a\right) &= \frac{1}{2a\left(1-a\right)t}\left(p_e^*\left(g^7,a\right)-c\left(\eta_e\left(g^7\right)\right)\right)^2 \\ s.t.\ 1-\Delta &\leq a<\overline{a}\left(t\right).\end{aligned}$$

We obtain the solution $a^{*(i),2}=1-\Delta$, and the associated profit is as follows:

$$\Pi_e^*\left(g^7,1-\Delta\right)=\frac{1}{2}t\Delta\left(1-\Delta\right).$$

We can ensure that both $a^{*(i),1}$ and $a^{*(i),2}$ can be a solution when $0<\Delta\leq\frac{1}{2}$, depending on the level of parameters. Then, comparing $\Pi_e^*\left(g^3,1/2\right)$ with $\Pi_e^*\left(g^7,1-\Delta\right)$ gives the condition in the text. Needless to say, the associated profits correspond to the threshold levels of entry sunk cost F^{*1} and F^{*2}.

(ii) The case in which $\frac{1}{2}<\Delta\leq 1$

Because $\Delta\leq 1$, the condition that $a\geq(<)\,1-\Delta$ is a relevant constraint for firm e's problem. Furthermore, because $1-\Delta<\frac{1}{2}$, firm e's problem is formulated as

$$\max_{a}\Pi_e\left(g^7,a\right)\quad s.t.\ \frac{1}{2}\leq a<\overline{a}\left(t\right).$$

We obtain the solution $a^{*(ii)}=\frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^*\left(g^7,\frac{1}{2}\right)=\frac{1}{8}t,$$

which corresponds to F^{*3}.

(iii) The case in which $1<\Delta\leq\frac{5}{4}$

Because $\Delta>1$, the condition that $a\geq(<)\,2\left(\Delta-1\right)$ is a relevant constraint for firm e's problem. Furthermore, because $2\left(\Delta-1\right)\leq\frac{1}{2}$, firm e's problem is formulated as

$$\max_{a}\Pi_e\left(g^7,a\right)\quad s.t.\ \frac{1}{2}\leq a<\overline{a}\left(t\right).$$

We obtain the solution $a^{*(iii)} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^* \left(g^7, \frac{1}{2} \right) = \frac{1}{8} t.$$

(iv) The case in which $\frac{5}{4} < \Delta \leq \frac{3}{2}$

Because $\Delta > 1$, the condition that $a \geq (<) \, 2 \, (\Delta - 1)$ is relevant for firm e's problem. Then, if $\frac{1}{2} \leq a < 2 \, (\Delta - 1)$, g^3 is formed with probability 1 in the second stage. On the other hand, if $2 \, (\Delta - 1) \leq a < 1$, g^7 is formed with probability 1 in the second stage. Again, we need to solve each of two subproblems and then compare firm e's maximized profits to obtain firm e's profit-maximizing location.

The first subproblem is formulated as follows:

$$\max_a \Pi_e \left(g^3, a \right) \quad s.t. \ \frac{1}{2} \leq a < 2 \, (\Delta - 1) \, .$$

Then, we obtain the solution $a^{*(iv),1} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^* \left(g^3, \frac{1}{2} \right) = \frac{1}{72} \left(3t - 4\Delta_0 \right)^2 ,$$

The second subproblem is formulated by:

$$\max_a \Pi_e \left(g^7, a \right) \quad s.t. \ 2 \, (\Delta - 1) \leq a < \overline{a} \, (t) \, .$$

We obtain the solution $a^{*(iv),2} = 2 \, (\Delta - 1)$, and the associated profit is as follows:

$$\Pi_e^* \left(g^7, 2 \, (\Delta - 1) \right) = t \, (\Delta - 1) \, (3 - 2\Delta) \, ,$$

which corresponds to F^{*4}. We can ensure that both $a^{*(iv),1}$ and $a^{*(iv),2}$ can be a solution when $\frac{5}{4} < \Delta \leq \frac{3}{2}$, depending on the level of parameters. Then, comparing $\Pi_e^* \left(g^3, 1/2 \right)$ with $\Pi_e^* \left(g^7, 2 \, (\Delta - 1) \right)$ gives the claim in the text.

(v) The case in which $\Delta > \frac{3}{2}$

Because $\Delta > 1$, the condition that $a \geq (<) \, 2 \, (\Delta - 1)$ is relevant for firm e's problem. Furthermore, because $2 \, (\Delta - 1) \geq 1$, firm e's problem is formulated as

$$\max_a \Pi_e \left(g^3, a \right) \quad s.t. \ \frac{1}{2} \leq a < \overline{a} \, (t) \, .$$

We obtain the solution $a^{*(v)} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^* \left(g^3, \frac{1}{2} \right) = \frac{1}{72} \left(3t - 4\Delta_0 \right)^2 .$$

Summarizing all of the above results gives the claim in the text. ■

References

[1] Bala, V. and Goyal, S. (2001), "A Noncooperative Model of Network Formation", *Econometrica* 68, 1181-1229.

[2] Deroian, F. and Gannon, F. (2006), "Quality-Improving Alliances in Differentiated Oligopoly", *International Journal of Industrial Organization* 24, 629-637.

[3] Friedman, J. W. and Thisse, J. (1993), "Partial Collusion Fosters Minimum Product Differentiation", *Rand Journal of Economics* 24, 631-645.

[4] Goyal, S. (2007), *Connections*, New Jersey: Princeton University Press.

[5] Goyal, S. and Joshi, S. (2003), "Networks of Collaboration in Oligopoly", *Games and Economic Behavior* 43, 57-85.

[6] Goyal, S. and Moraga-González, J. L. (2001), "R&D Networks", *Rand Journal of Economics* 32, 686-707.

[7] Jackson, M. O. (2008), *Social and Economic Networks*, New Jersey: Princeton University Press.

[8] Jackson, M. O. and van den Nouweland, A. (2005), "Strongly Stable Networks", *Games and Economic Behavior* 51, 420-444.

[9] Jackson, M. O. and Wolinsky, A. (1996), "A Strategic Model of Social and Economic Networks", *Journal of Economic Theory* 71, 44-74.

[10] Matsumura, T. and Matsushima, N. (2009), "Cost Differentials and Mixed Strategy Equilibria in a Hotelling Model", *Annals of Regional Science* 43, 215-234.

[11] Meza, S. and Tombak, M. (2009), "Endogenous Location Leadership", *International Journal of Industrial Organization* 27, 687-707.

[12] Okumura, Y. (2007), "A Network Formation Process Converges to the Complete Collaboration Network". *Mathematical Social Sciences* 53(2), 134-139.

[13] Okumura, Y. (2011), "Spatial Competition and Collaboration Networks", forthcoming in *International Journal of Game Theory.*

[14] Song, H. and Vannetelbosch, V. (2007), "International R&D Collaboration Networks", *The Manchester School* 75, 742-766.

[15] Watts, A. (2001) "A Dynamic Model of Network Formation", *Games and Economic Behavior* 34, 331-341.

Figure 1 All Networks