The Welfare Effects of Third-Degree Price Discrimination in a Differentiated Oligopoly[*]

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November 1, 2010

Abstract

This paper studies the relationship between horizontal product differentiation and the welfare effects of third-degree price discrimination in oligopoly. By deriving linear demands from a representative consumer's utility and focusing on symmetric equilibrium of a pricing game, we characterize conditions, relating to such demand properties as substitutability and complementarity, for price discrimination to improve social welfare. In particular, we show that price discrimination can improve social welfare (especially) if firms' brands are substitutes in the the market where the discriminatory price is higher and are complements in the market where it is lower, but it never improves vice versa. We conjecture, however, that consumer surplus never improves by price discrimination: welfare improvement by price discrimination is solely due to the increase in the firms' profits. This means that there is little or no chance that firms suffer from "prisoners' dilemma", that is, firms are mostly or always better off by switching from uniform pricing to price discrimination. It is also shown, contrary to the intuition, that competition due to strong substitutability does not play a positive role for price discrimination to improve social welfare because an effect of a mal-distribution only remains as in the case of monopoly.

Keywords: Third-Degree Price Discrimination; Oligopoly; Social Welfare; Horizontal Product Differentiation; Substitutability; Complementarity.

JEL classification: D43, D60, L11, L13.

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[*]We are grateful to seminar participants at Kwansei Gakuin University for helpful comments. Adachi also acknowledges a Grant-in-Aid for Young Scientists (B) from the Japanese Ministry of Education, Science, Sports, and Culture (21730184). Matsushima acknowledges a Grant-in-Aid for Scientific Research (B) from the Ministry (21730193). Any remaining errors are our own.

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1 Introduction

Product differentiation is one of the main reasons why firms can enjoy some market power: it enables them to sell products that are no longer perfect substitutes. For example, Coca Cola and PepsiCo sell similar types of soda, though it is arguably believed that they differentiate in taste, thus each firm is attracting some consumers over another. It is often the case that firms sell *complementary* products under imperfect competition. Examples abound, including soda and hot dogs, PC and its applications, fashion clothes and shoes, and so on.

If firms have some control over the price that consumers face, it is natural that they take advantage of it. *Third-degree price discrimination*, among others, is a marketing technique that is widely used in imperfectly competitive markets. In third-degree price discrimination, the seller uses identifiable signals (e.g., age, gender, location and time of use) to categorize buyers into different segments, or submarkets, and each segment is given a constant price per unit. Behind the recent rising trend of third-degree price discrimination are rapid progresses in information processing technology including, notably, the widespread use of the Internet in the last two decades.[1]

In this paper, we examine the welfare effects of *oligopolistic* third-degree price discrimination, explicitly taking into account product differentiation as a source of ***market power and strategic interaction***. An important question that awaits a careful study is whether third-degree price discrimination is a good or bad thing in the presence of product differentiation. Answering this question is important because it helps antitrust authorities to evaluate the pros and cons of price discrimination in markets that are of importance in many situations: those of *price discrimination with oligopoly and product differentiation*. Our focus is on *horizontal* product differentiation to consider *substitutability* as well as *complementarity*.[2] By deriving linear demands from a representative consumer's utility and focusing on symmetric equilibrium of a pricing game, we **characterize conditions,**

[1] See Shy (2008) on how the advances in the information technology have made pricing tactics more and more readily practicable for sellers.

[2] With *horizontal* product differentiation, some consumers prefer product A to B while others prefer B to A. On the other hand, *vertical* product differentiation captures the situation where all consumers agree on the ranking of products. See, e.g., Belleflamme and Peitz (2010, Ch.5) for more on the difference between horizontal and vertical product differentiation.

relating to such demand properties as substitutability and complementarity, for price discrimination to improve social welfare. In particular, we show that price discrimination can improve social welfare (especially) if firms' brands are substitutes in the market where the discriminatory price is higher and are complements in the market where it is lower, but it never improves vice versa. We conjecture, however, that consumer surplus never improves by price discrimination: welfare improvement by price discrimination is solely due to the increase in the firms' profits. **This means that there is little or no chance that firms suffer from "prisoners' dilemma", that is, fims are mostly or always better off by switching from uniform pricing to price discrimination.** It is also shown, contrary to the intuition, that competition due to strong substitutability does not play a positive role for price discrimination to improve social welfare because an effect of a maldistribution only remains as in the case of monopoly.

Since Pigou's (1920) seminal work, the central question that has been posed in the analysis of third-degree price discrimination is about its welfare effects: what are the effects of third-degree price discrimination on consumer surplus and Marshallian social welfare (the sum of the consumer surplus and firms' profit)? In the literature, however, little has been known about the welfare effects of *oligopolistic* third-degree price discrimination since the publication of a seminal paper by Holmes (1989), who analyzes the output effects of third-degree price discrimination in oligopoly, but does not study the welfare effects.[3] On the other hand, the welfare effects of *monopolistic* third-degree price discrimination is relatively well known. Since the work by Robinson (1933), it has been well known that when all submarkets are served under uniform pricing,[4] price discrimination must decrease social welfare unless aggregate output increases. It implies that an increase in aggregate output is a *necessary* condition for social welfare to improve by third-degree price

[3]See Armstrong (2006) and Stole (2007) for comprehensive surveys on price discrimination with imperfect competition. In contrast to Holmes' (1989) analysis focuses on *symmetric* Nash equilibrium (where *all firms behave identically*), another important work by Corts (1998) relax the symmetry to show that *asymmetry* in firms' best response functions is necessary to have unambiguous welfare effects (in the case where prices in all markets drop it results in unambiguous welfare improvement, and in the case where prices in all market jump it results in unambiguous welfare deterioration). **Our focus on symmetric equilibrium lies on the assumption that all firms agree in their ranking in pricing (see Stole (2008) for the details), and is motivated by our recognition that this situation is more natural than the asymmetric cases in many examples of third-price discrimination.**

[4]Under uniform pricing, firms may be better off by refusing supply to some of the submarkets. See, e.g., Hausman and MacKie-Mason (1988) for this issue.

discrimination.[5] In particular, price discrimination *necessarily decreases social welfare if demands are linear* because aggregate output remains constant.[6] The welfare consequences of *oligopolistic* third-degree price discrimination, however, remain largely unknown. It is, therefore, important to study oligopolistic third-degree price discrimination, because only a small number of goods are supplied by monopolists in the real world and more and more competing firms price discriminate their products and services.

This paper investigates the relationship between product differentiation and an associated change in social welfare with the regime change from uniform pricing to price discrimination when all submarkets are open under uniform pricing.[7] To model price competition with product differentiation, we adopt the Chamberlin-Robinson approach (named by Vives (1999, p.243)): a "representative" consumer (i.e., a virtual individual that is made of aggregation of infinitesimal and identical consumers) is assumed to value the variety of goods. In this paper, we consider the (fully parametrized) linear demand structure to obtain an explicit solution as well as an explicit condition for all submarkets to be open under uniform pricing. The benefit of this specification is that we do not have to **simply assume** [**ERASE**: put restrictions o**n**] **such endogenous events as market opening take place. In addition,** while Holmes (1989) assumes substitutability of products, our formulation **allows one to include complementarity as well in a welfare analysis.**

One important difference between monopoly and oligopoly is that in monopoly, the price elasticity of demand in each submarket has a one-to-one relationship with the optimal discriminatory price: the larger the price elasticity is, the lower the discriminatory price is.

[5] Aguirre, Cowan and Vickers (2010) offer a comprehensive analysis: they find sufficient conditions that relate the curvatures of direct and indirect demand functions in separate markets. While they allow nonlinear demand functions, they put, as many papers in the literature do, a restriction on an endogenous event: all markets are simply assumed to be open. Cowan (2007) also offers a similar analysis by assuming a restricted class of demand functions.

[6] See, e.g., Schamalensee (1981), Varian (1985), Schwartz (1990) and Bertoletti (2004). In contrast to these studies, Adachi (2002, 2005) shows that, when there are consumption externalities, price discrimination can increase social welfare even if aggregate output remains the same (see also Ikeda and Nariu (2009)). Ikeda and Toshimitsu (2010) shows that if the quality is endogenously chosen, price discrimination necessarily improves social welfare.

[7] In a closely related study, Dastidar (2006) also considers the welfare effects of third-degree price discrimination in oligopoly by focusing on, as this paper does, symmetric Nash equilibrium. In comparison to Dastidar (2006), our study explicitly takes into account **such demand properties as substitutability and complementarity to characterize conditions for price discrimination to improve social welfare.**

In oligopoly, however, it may not be the case. This is because *strategic interaction* affects the pricing decision of each firm. In particular, the price elasticity that a firm faces in a discriminatory market would be in general different from the elasticity that the firms *as a whole* (i.e., collusive oligopoly) face. In this paper, we show that *in equilibrium* this "firm-level" price elasticity has a simple expression in terms of product differentiation. More specifically, as a special case of Holmes' (1989) result, it is verified that *in equilibrium* the firm-level price elasticity decomposes into the "market-level" elasticity and the "strategic-related" elasticity (the precise meanings are given in the text), and that the latter elasticity simply coincides with the degree of product differentiation.[8] It is observed from numerical and graphical analysis that this "strategic-related" elasticity play an important role in the determination of discriminatory prices and social welfare. One benefit from using linear demands is that we can do these exercises of welfare evaluation without complications that are associated with demand concavity/convexity.

The rest of the paper is organized as follows. The next section present a model and give preliminary results. In Section 3, welfare analysis is presented. Section 4 concludes the paper.

2 The Model

In this section, we first set up the model and then provide preliminary results that are necessary for welfare analysis in the next section.

2.1 Setup

Firms produce (horizontally) differentiated products and compete in price to sell their products (directly) to consumers. A firm sells only one type of its own product, hence it can also be interpreted as a *brand*. Markets are partitioned according to identifiable signals (e.g., age, gender, location and time of use).[9] The qualifier "horizontally" denotes that firms differentiate by targeting consumer heterogeneity in tastes, rather than by

[8] Our analysis below shows that Holmes' (1989) decomposition also holds for the case of complementarity with linear demands.

[9] There are no inter-dependencies between separate markets. Layson (1998) and Adachi (2002) study the welfare effects of monopolistic third-degree price discrimination in the presence of inter-dependencies.

sorting in terms of the quality. For simplicity, we assume that all firms have the same constant marginal cost, $c \geq 0$. Resale among consumers must be impossible; otherwise some consumers will be better off buying the good at a lower price from other consumers (arbitrage).

Following Robinson (1933) and most subsequent papers in the literature, we suppose that the whole market are divided into two subgroups: the "*strong*" markets and the "*weak*" markets. Loosely put, a strong (weak) market is a "larger" ("smaller") market.[10] Consumers' preference in market $m \in \{s, w\}$ (s denotes (the set of) the strong markets and w the weak markets) is represented by the following quasi-linear utility function:

$$U_m(q_m^A, q_m^B) \equiv \alpha_m \cdot (q_m^A + q_m^B) - \frac{1}{2}\left(\beta_m [q_m^A]^2 + 2\gamma_m q_m^A q_m^B + \beta_m [q_m^B]^2\right),$$

where $|\gamma_m| < \beta_m$ denotes the degree of horizontal product differentiation in market m, q_m^j is the amount of consumption/output produced by firm j for market m ($j \in \{A, B\}$), and $\beta_m > 0$.[11] The goods in market m are *substitutes* if $\gamma_m > 0$, *complements* if $\gamma_m < 0$, or *independent* if $\gamma_m = 0$: the less the value of γ_m, the *more* differentiated firms' products are.[12] Notice that the direction in the sign associates with the usual definitions of complementarity/substitutability: when the firms' goods are substitutes (complements), the marginal utility from consuming an additional unit of the good purchased from one firm is lower (higher) when he or she is consuming a more amount of the good purchased from the other firms. The ratio $\gamma_m/\beta_m \in (-1, 1)$ is interpreted as the *(normalized) measure of horizontal product differentiation* in market m (see Belleflamme and Peitz (2010, p.65)). As we see in Section 3, γ_m/β_m plays an important role in interpreting the equilibrium prices under price discrimination.

Utility maximization by the representative consumer yields the inverse demand func-

[10] More precisely, we define, following the literature, a strong (weak) market as the market where the price increases (decreases) by price discrimination. Notice that this is the definition based on an "equilibrium"result from optimizing behavior (either in monopoly or oligopolistic pricing). Appendices A1 and A2 show the parametric restrictions for a market to be strong or weak in the model presented below.

[11] More precisely, we assume that the utility function has a quasi-linear form of $U_m(q_m^A, q_m^B) + q_0$, where q_0 is the "composite" good (produced by the competitive sector) whose (competitive) price is normalized to be one. Thus, there are no income effects on the determination of demands in the markets that are focused, validating partial equilibrium analysis. This quadratic utility function is a standard one to justify linear demands (see, e.g., Vives (1999, p.145)). Here, the symmetry between firms is additionally imposed.

[12] In the case of independency in market m ($\gamma_m = 0$), each firm behaves as a monopolist of its own brand in market m. Hence, the results from the studies of monopolistic third-degree price discrimination with linear demands apply.

tion for firm j in each market m, $p_m^j(q_m^j, q_m^{-j}) = \alpha_m - \beta_m q_m^j - \gamma_m q_m^{-j}$. The demand functions in market m are thus given by

$$\begin{cases} q_m^A(p_m^A, p_m^B) = \dfrac{\alpha_m}{\beta_m + \gamma_m} - \dfrac{\beta_m}{\beta_m^2 - \gamma_m^2} p_m^A + \dfrac{\gamma_m}{\beta_m^2 - \gamma_m^2} p_m^B \\ q_m^B(p_m^A, p_m^B) = \dfrac{\alpha_m}{\beta_m + \gamma_m} + \dfrac{\gamma_m}{\beta_m^2 - \gamma_m^2} p_m^A - \dfrac{\beta_m}{\beta_m^2 - \gamma_m^2} p_m^B. \end{cases} \tag{1}$$

Notice here that the symmetry in firms' demands, $q_m^A(p', p'') = q_m^B(p'', p')$. As implied above, we follow Holmes (1989) and many others to focus on symmetric Nash equilibrium where all firms set the same price in one market.[13] With little abuse of notation, let $q_m(p) = q_m^A(p, p)$. For a simpler exposition, the number of firms and the number of discriminatory markets are both two. These numbers can be arbitrary and the results presented below hold as long as we focus on symmetric Nash equilibrium.

Social welfare in market m is defined by

$$SW_m(q_m^A, q_m^B) \equiv U_m(q_m^A, q_m^B) - c \cdot (q_m^A + q_m^B)$$

and thus the aggregate social welfare is given by

$$SW(\{q_m^A, q_m^B\}_m) \equiv \sum_m SW_m(q_m^A, q_m^B).$$

We measure social efficiency by this aggregate social welfare. We can also define aggregate consumer surplus by

$$CS(\{p_m^A, p_m^B\}_m) \equiv \sum_m CS_m(p_m^A, p_m^B)$$

where consumer surplus in market m is

$$CS_m(p_m^A, p_m^B) \equiv U_m[q_m^A(p_m^A, p_m^B), q_m^B(p_m^A, p_m^B)] - p_m^A q_m^A(p_m^A, p_m^B) - p_m^B q_m^B(p_m^A, p_m^B),$$

as well as aggregate corporate surplus (profit) by

$$\Pi(\{p_m^A, p_m^B\}_m) \equiv \sum_m \sum_j (p_m^j - c) q_m^j(p_m^A, p_m^B)$$

[13] See Corts (1998) for interesting issues that arise from the asymmetric equilibrium.

so that the aggregate social welfare is divided in the following way:

$$SW(\{q_m^A(p_m^A, p_m^B), q_m^B(p_m^A, p_m^B)\}_m) = CS(\{p_m^A, p_m^B\}_m) + \Pi(\{p_m^A, p_m^B\}_m).$$

We consider two regimes, *uniform pricing* ($r = U$) and *price discrimination* ($r = D$): under uniform pricing, firms set a common unit price for all separate markets, and under price discrimination, they can set a different price in each market. Throughout this paper, we restrict our attention to the case where all markets are served under either regime.

Because the profit of firm $j \in \{A, B\}$ is given by

$$\pi^j(p_s^A, p_s^B, p_w^A, p_w^B) \equiv \sum_m (p_m^j - c) q_m^j(p_m^A, p_m^B),$$

we know that under symmetry, the symmetric equilibrium price under uniform pricing, p^*, is given by

$$[q_s(p^*) + q_w(p^*)] + (p^* - c)\left[\frac{\partial q_s^A(p^*, p^*)}{\partial p_m^A} + \frac{\partial q_w^A(p^*, p^*)}{\partial p_m^A}\right] = 0, \quad (2)$$

while the equilibrium prices in market m under price discrimination, p_m^*, are determined by the following first-order condition:

$$q_m(p_m^*) + (p_m^* - c)\frac{\partial q_m^A(p_m^*, p_m^*)}{\partial p_m^A} = 0. \quad (3)$$

Again, one caveat here is the well-known problem in the literature on third-degree price discrimination: ***under uniform pricing, when a market is sufficiently small, it may not be served by either firm.*** While many papers in the literature simply assume that all market open under uniform pricing, we provide a more specified structure in the next subsection to guarantee this is the case and to proceed the analysis further on.

Note also the differences between Corts (1998) and this paper. Let $BR_m^j(p_m^k) \equiv \arg\max_{p^j}(p^j - c)q_m^j(p^j, p_m^{-j})$ be firm j's best response function in market m under price discrimination, given firm k's price in market m, p_m^k. Corts (1998) makes four assumptions as to the profit functions and the best response functions. In our settings, Assumptions 1-3 in Corts (1998) are all satisfied.[14] However, Assumption 4 in Corts (1998), which requires in this paper's notation

$$BR_m^j(p^{-j}) > BR_{-m}^j(p^{-j})$$

[14] Assumption 1 in Corts (1998) ensures the uniqueness of the best response, Assumption 2 strategic complementarity, and Assumption 3 the stability.

does not necessarily hold. In this sense, our model specification puts less restrictions on the economic primitives than Corts (1998) does.

2.2 Solutions and Preliminary Results

As an innocuous normalization, we set the constant marginal cost to zero, $c = 0$.[15] Given that

$$\frac{\partial q_m^A(p_m^A, p_m^B)}{\partial p_m^A} = -\frac{\beta_m}{\beta_m^2 - \gamma_m^2}$$

from (1) and that $q_m(p) = (\alpha_m - p)/(\beta_m + \gamma_m)$ is the symmetric demand function, the equilibrium discriminatory prices are (from (3))

$$p_m^* = \frac{\alpha_m(\beta_m - \gamma_m)}{2\beta_m - \gamma_m}$$

and from (2) the equilibrium uniform price is

$$p^* = \frac{(\beta_w - \gamma_w)(\beta_s - \gamma_s)[(\beta_w + \gamma_w)\alpha_s + (\beta_s + \gamma_s)\alpha_w]}{(2\beta_w - \gamma_w)(\beta_s^2 - \gamma_s^2) + (2\beta_s - \gamma_s)(\beta_w^2 - \gamma_w^2)} \ (\equiv p^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}))$$

under the regime of uniform pricing (where $\boldsymbol{\gamma} \equiv (\gamma_s, \gamma_w)$, $\boldsymbol{\alpha} \equiv (\alpha_s, \alpha_w)$ and $\boldsymbol{\beta} \equiv (\alpha_s, \alpha_w)$) *if both markets are open.* Appendix A2 shows that the size of the strong market should be sufficiently small for neither firm to have an incentive to deviate to closing the weak market, *and* that it should be also sufficiently large for the weak market to be open under uniform pricing. Thus, we put the restriction, $\alpha_s/\alpha_w \in (\underline{\alpha_s/\alpha_w}, \overline{\alpha_s/\alpha_w})$. The upper and lower bounds are functions of $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$, and the actual forms are given in Appendix A2.[16]

Notice that $\partial p_m^*/\partial \gamma_m = -\alpha_m\beta_m/(2\beta_m - \gamma_m)^2 < 0$, which implies that as γ_m becomes larger the discriminatory prices decreases. In addition, the uniform price and the discriminatory prices converge to the marginal cost because $\lim_{\gamma_m \uparrow \beta_m} p_m^* = 0 = \lim_{\gamma_m \uparrow \min(\beta_m)} p^*$ for all m. In contrast to the case of monopoly with linear demands, the difference in equilibrium aggregate output by regime change is *not necessarily zero* (see Appendix A1).

[15] Notice the innocuousness of the zero marginal cost assumption: it is equivalent to assuming a constant marginal cost if prices and consumers' willingness to pay are interpreted as net of the cost (by interpreting as $\alpha_m - c$ as α_m).

[16] The "weak" market is smaller than the "strong" market in the sense that the marginal utility $\partial U_m(q_m^A, q_m^B)/\partial q_m^j$ at $(q_m^A, q_m^B) = (0, 0)$ is greater in the strong market.

3 Welfare Analysis

This section consists of two subsections. The first subsection presents analytical properties that are useful for welfare analysis. We then investigate the welfare effects of price discrimination in the second subsection.

3.1 Analytical Properties

In symmetric equilibrium, social welfare under regime $r \in \{D, U\}$ is written by

$$SW^r = 2(\alpha_s q_s^r + \alpha_w q_w^r) - (\beta_s + \gamma_s)[q_s^r]^2 - (\beta_w + \gamma_w)[q_w^r]^2$$

where $q_m^D = q_m(p_m^*)$ and $q_m^U = q_m(p^*)$ are the equilibrium quantities in market m under the regimes of price discrimination and of uniform pricing, respectively (see Appendix A1 for the actual functional forms). Let ΔSW^* be defined by the equilibrium difference $SW^D - SW^U$. It is then given by

$$\begin{aligned}
\Delta SW^* &= \Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}) \\
&\equiv 2\left[\alpha_s(q_s^D - q_s^U) + \alpha_w(q_w^D - q_w^U)\right] \\
&\quad -(\beta_s + \gamma_s)\left(q_s^D - q_s^U\right)\left(q_s^D + q_s^U\right) - (\beta_w + \gamma_w)\left(q_w^D - q_w^U\right)\left(q_w^D + q_w^U\right) \\
&= \Delta q_s^*[2\alpha_s - (\beta_s + \gamma_s)(q_s^D + q_s^U)] + \Delta q_w^*[2\alpha_w - (\beta_w + \gamma_w)(q_w^D + q_w^U)],
\end{aligned}$$

where $\Delta q_m^* \equiv q_m^D - q_m^U$. It is further shortened, and thus we have the following proposition (see the proof in Appendix A3):

Proposition 1. *The equilibrium difference $\Delta SW^* = \Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by*

$$\Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = -\sum_{m \in \{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m} \cdot (p_m^* + p^*),$$

where $\Delta p_m^* \equiv p_m^* - p^*$.

This expression has the following graphical interpretation. Figure 1 shows the relationship between Δp_m^* and Δq_m^*. As Appendix A1 verifies, we have $\Delta p_m^* = -(\beta_m + \gamma_m)\Delta q_m^*$. This relationship can be interpreted as the situation where *in **symmetric** equilibrium* any firm faces the "virtual" inverse demand function, $p_m = \alpha_m - (\beta_m + \gamma_m)q_m$, in

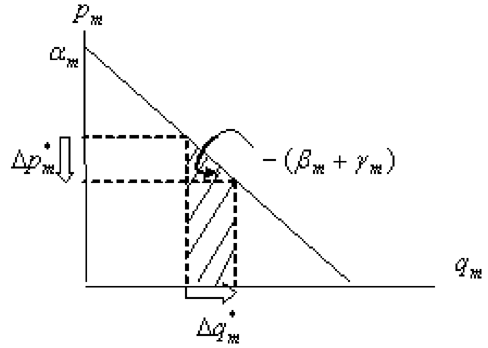

Figure 1: Equilibrium Changes in Quantity and Price Market m (for any firm)

market m (notice the difference from the original inverse demand function, $p_m^j(q_m^j, q_m^{-j}) = \alpha_m - \beta_m q_m^j - \gamma_m q_m^{-j}$). The welfare change in market m is depicted as the shaded trapezoid in Figure 1 (in this example, it is a welfare gain). Thus, its size is calculated by the sum of the upper and bottom segments ($p_m^* + p^*$) multiplied by height ($\Delta q_m^* = -\Delta p_m^*/(\beta_m + \gamma_m)$), devided by two. Noting that two identical firms exist in market m, we have $-\Delta p_m^*(p_m^* + p^*)/(\beta_m + \gamma_m)$ as a welfare change in market m.

If it is positive (when $\Delta p_m^* < 0$), then it is a welfare gain. Similarly, if it is negative (when $\Delta p_m^* > 0$), then it is a welfare loss. With other things being equal, the greater the value of γ_m, the gentler (and hence the elastic) the equilibrium inverse demand curve becomes. Complementarity between the brands makes the equilibrium inverse demand curve steep, and substitutability makes it gentle.

It seems that complementarity makes the equilibrum price elasticity less elastic, and substitutability makes it more elastic. However, in equilibrium, the opposite is true. More formally, we have the following property of the price elasticity. A simple calculation leads to the following lemma:

Lemma 1. *Let the price elasticity of demand in market m in equilibrium defined by*

$$\varepsilon_m(p_m^*) \equiv \left| -\frac{dq_m(p_m^*)}{dp_m^*} \frac{p_m^*}{q_m^D} \right|,$$

where $q_m(p_m^*) = (\alpha_m - p_m^*)/(\beta_m + \gamma_m)$. Then, the equilibrium price elasticity of demand *is expressed by*

$$\varepsilon_m(p_m^*) = \underbrace{1}_{\textit{market elasticity}} + \underbrace{\left(-\frac{\gamma_m}{\beta_m}\right)}_{\textit{cross-price elasticity}}. \tag{4}$$

Notice that $\varepsilon_m(p_m^*)$ is a constant, and it does not depend on q_m^D or even on the intercept, α_m, either. This decomposition is a special result of Holmes' (1989, p.246) general result: the *firm-level* elasticity is the sum of the *market* elasticity and the *cross-price* elasticity.[17]

The *market* elasticity of demand is a unit-free measure of responsiveness for the firms *as a whole*. However, *strategic interaction* makes it different from the elasticity that each firm bases on in its decision making: the *cross-price* elasticity measures of how much each firm "damages" the other firm ***in equilibrium***. In our model, strategic interaction is created by the very fact that firms (horizontally) differentiate their products or services. In our case of linear demands and the zero marginal cost, the market elasticity is exactly one as in the case of one-good monopoly with a linear demand curve (remember that price elasticity of demand is one when the marginal revenue curve crosses the constant marginal cost curve (i.e., the horizontal axis)).

As we mention in Section 2, the ratio $\gamma_m/\beta_m \in (-1, 1)$ is interpreted as the normalized measure of horizontal product differentiation in market m. The negative of the ratio also measures the cross-price elasticity in Holmes (1989). From (4), we have the relationship, $\varepsilon_m(p_m^*) \lesseqgtr 1$ if and only if $\gamma_m \gtreqless 0$. That is, if the brands are complements ($\gamma_m < 0$), then the firm-level elasticity in equilibrium is *greater* than one, meaning that a one percent price cut by one firm makes more than a one percent increase in demand for the firm, thus an *increase* in revenue (hence in profit). The result is opposite if the brands are substitutes ($\gamma_m > 0$).

As to changes in equilibrium aggregate output, ΔQ^* (see Appendix A1 for the derivation), it is shown that if the aggregate output does not increase by price discrimination, then social welfare deteriorates, as verified by Betoletti (2004) in the case of monopoly

[17] Holmes (1989) shows the decomposition under the assumption of symmetric demands between firms: it also holds off equilibrium. The term "market elasticity" is borrowed from Stole (2007) (Holmes (1989) originally called it the "industry-demand elasticity").

with linear and nonlinear demands.[18]

Proposition 2. $\Delta Q^* \leq 0 \Rightarrow \Delta SW^* < 0$.

Given that market s is actually strong ($\alpha_s/\alpha_w > \underline{\alpha_s/\alpha_w}$), we have the following relationship:

$$\Delta Q^* \gtreqless 0 \Leftrightarrow \frac{\gamma_s}{\beta_s} \gtreqless \frac{\gamma_w}{\beta_w},$$

which is also a special case of Holmes' (1989) result that includes nonlinear demands: Holmes (1989, p.247) shows that the change in the aggregate output by price discrimination is positive if and only if the sum of the two terms, "the adjusted-concavity condition" and "elasticity-ratio condition", is positive. As its name implies, the first term is related to the demand curvature, and in our case of linear demands, it is zero. The second term is written by

$$\frac{\text{cross-price elasticity in market } s}{\text{market elasticity in market } s} - \frac{\text{cross-price elasticity in market } w}{\text{market elasticity in market } w},$$

which is equivalent to $\gamma_s/\beta_s - \gamma_w/\beta_w$ from Lemma 1. **The result that the output change, ΔQ^*, can be positive in duopoly is in shart contrast to the the case of monopoly where the output change is always zero with linear demands. In next subsection, we explore the possibility of $\Delta SW^* > 0$ in the differentiated oligopoly.**

3.2 Welfare-Improving Price Discrimination

We now explore the possibility of $\Delta SW^*(\boldsymbol{\gamma}, \boldsymbol{\alpha}, \boldsymbol{\beta}) > 0$ by reducing the number of the parameters. More specifically, we assume that $\alpha_s = 1 > \alpha_w > 0$. This is because price discrimination never improves welfare if $\alpha_s = \alpha_w$ (the formal proof is available upon request). Thus, $\alpha_s/\alpha_w > 1$ is necessary for social welfare to improve.

In the following analysis, we first consider the case of symmetry in product differentiation in the strong and the weak markets ($\gamma_s/\beta_s = \gamma_w/\beta_w$). We then allow asymmetric product differentiation. To do so, we first make an intuitive argument on what makes price discrimination improve social welfare. Given the equilibrium discriminatory price is

[18] Bertoletti's (2004) result is a generalization of the well-known result of Varian (1985) and Schawartz (1990) who state that $\Delta Q^* < 0 \Rightarrow \Delta SW^* < 0$.

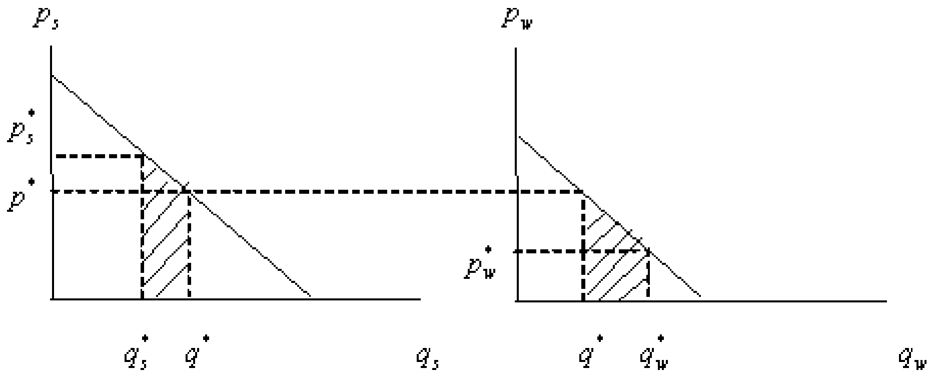

Figure 2: Asymmetry between the Strong and the Weak Markets

higher (lower) than the uniform price in the strong (weak) market, we know that (note that, $\Delta q_m^* = -\Delta p_m^*/(\beta_m + \gamma_m)$)

$$\Delta SW^* > 0 \Leftrightarrow \Delta q_w^* \cdot (p^* + p_w^*) > \Delta q_s^* \cdot (p^* + p_s^*).$$

For the latter inequality to hold, (1) Δq_w^* or $(p^* + p_w^*)$ is sufficiently *large*, and/or (2) $|\Delta q_s^*|$ or $(p^* + p_s^*)$ is sufficiently *small*. Figure 2 shows the asymmetry between the strong and the weak markets. Notice that the upper segment of the trapezoid of the welfare loss in the strong market and the bottom segment of the trapezoid of the welfare gain in the weak market have the same length (p^*). Thus, the larger $|\Delta q_s^*|$ is, the larger $(p^* + p_s^*)$ is. On the other hand, the larger Δq_w^* is, the smaller $(p^* + p_w^*)$ is. Hence, *the smaller* $|\Delta q_s^*|$, *the better for welfare improvement, while* Δq_w^* *should not be too small or too large.*

3.2.1 The Case of Symmetric Product Differentiation

Let the situation be called *symmetric* product differentiation if the measures of horizontal product differentiation coincide in the two markets (i.e., $\gamma_s/\beta_s = \gamma_w/\beta_w$). In this case, the two markets are homethetic in the sense that the only difference in the two markets is in the intercepts of the inverse demand curves. It is shown that if $\gamma_s = \gamma_w$ and $\beta_s = \beta_w$, then $\Delta Q^* = 0$ (see Appendix A1). This means that $|\Delta q_w^*| = |\Delta q_s^*|$. Because p_s^* is greater than p_w^* (which comes from the assumption $\alpha_s > \alpha_w$), the loss in the strong market is always larger than the gain in the weak market. We thus have the following proposition:

Proposition 3. *In the case of symmetric product differentiation, social welfare never improve by price discrimination (i.e., $\Delta SW^* < 0$ for all exogenous parameters).*

We therefore need to consider the case of $\gamma_s\beta_w \neq \gamma_w\beta_s$, which is called *asymmetric* product differentiation, to study the possibility of $\Delta SW^* > 0$.

3.2.2 The Case of Asymmetric Product Differentiation

To simplify the analysis, we consider the following two cases separately: (1) $\gamma_s = \gamma_w$; (2) $\beta_s = \beta_w$.

γ_m is common Let $\gamma \equiv \gamma_s = \gamma_w$. We allow β_s and β_w to be different and provide numerical analysis to contrast substitutability with complementarity for a fixed value of $(\alpha_w, \beta_s, \beta_w)$, and graphical arguments on the domains (β_s, β_w) for $\Delta SW^* > 0$, with the value of (γ, α_w) kept fixed.

Table 1 shows the result for the case of $\alpha_w = 0.85$.[19] The first and the second column corresponds to the case of substitutability ($\gamma = 0.3$), while the third and the fourth to the case of complementarity ($\gamma = -0.3$). The difference between the first and the second (the third and the fourth in the case of complementarity) columns is whether the own slope of the inverse demand curve in the strong market is greater than in the weak market (i.e., $\beta_s > \beta_w$). Notice that price discrimination improves social welfare only in the second case ($(\gamma, \beta_s, \beta_w) = (0.3, 0.75, 1.0)$). In this case, $|\Delta q_s^*|/q_s^*(p^*)$ is particularly small (2%), while $\Delta q_w^*/q_w^*(p^*)$, is not also too large (3%), in comparison to the other three cases.

[19] It is verified that all of the model parameters in the analysis below satisfy the restriction conditions provided in Appendix A2.

	$(\gamma, \beta_s, \beta_w) =$			
	$(0.3, 1.0, 0.75)$	$(0.3, 0.75, 1.0)$	$(-0.3, 1.0, 0.75)$	$(-0.3, 0.75, 1.0)$
p^*	0.3582	0.3644	0.5235	0.5423
p_s^* $(\Delta p_s^*/p^*)$	0.4118 (15%)	0.3750 (3%)	0.5652 (8%)	0.5833 (8%)
p_w^* $(\Delta p_w^*/p^*)$	0.3188 (−11%)	0.3500 (−4%)	0.4958 (−5%)	0.4804 (−11%)
Δq_s^* $(\Delta q_s^*/q_s^*(p^*))$	−0.9412 (−8%)	−0.0101 (−2%)	−0.0596 (−9%)	−0.0912 (−9%)
Δq_w^* $(\Delta q_w^*/q_w^*(p^*))$	0.0375 (8%)	0.0111 (3%)	0.0615 (8%)	0.0884 (20%)
ΔSW^*	−0.0063	0.0005	−0.0022	−0.0123
ΔCS_s^*	−0.0507	−0.0127	−0.0543	−0.0797
ΔCS_w^*	0.0384	0.0109	0.0419	0.0598
$\Delta\Pi^*$	0.0060	0.0023	0.0102	0.0076
ΔQ^*	−0.0037	0.0009	0.0019	−0.0028

Table 1: Substitutability versus Complementarity with $\beta_s \neq \beta_w$ ($\alpha_w = 0.85$)

First, consider the case of substitutable goods ($\gamma > 0$). Notice that when $\beta_s > \beta_w$, the strong market has a higher value of price elasticity than the weak market does (see equation (4)). The equilibrium price in the strong market p_s^*, however, is at a *higher* level than in the case of $\beta_s < \beta_w$ (0.4118 vs. 0.3750). **This seemingly paradoxical result is due to the strategic effects: the firms want to "cooperate" because they are afraid of being retaliated when the market is more price elastic. Now, if the market is "integrated" (i.e., uniform pricing is forced), then the market price in the strong market is expected to drop in a larger extent than in the case of $\beta_s < \beta_w$, because the strong market has a higher value of price elasticity (more competitive) than the weak market has when $\beta_s > \beta_w$.** In Table 1, we see the price in the strong market drop from 0.4118 to 0.3582 (−6%) when $(\gamma, \beta_s, \beta_w) = (0.3, 1, 0.75)$, while p_s drops from 0.3750 to 0.3644 (−3%) when $(\gamma, \beta_s, \beta_w) = (0.3, 0.75, 1.0)$. To sum up, *when the strong market is relatively **less** price elastic, the regime of uniform pricing does not lower the price in the strong market enough.* As a result, uniform pricing may harm social welfare. In other words, *price discrimination **may** improve welfare.*

Even though the products are complements, a similar logic can apply to the property of price discrimination. When the products are complements, the price changes and the associated production changes are large due to the greater elasticity created by complementarity. In fact, welfare loss is larger in the fourth case **(where the strong market has a higher value of price elasticity than the weak market does)** than in the third case ($|-0.0123] > |-0.0022|$). As to the changes in equilibrium aggregate output, it is positive in our second case, It is also positive in the third case while in other two cases it is negative. These results **are consistent with Proposition 2**: ***an increase in the aggregate output is necessary for welfare to improve by price discrimination*, as in the case of monopoly.**

The difference between substitutability and complementarity is further investigated graphically. Figures 3 and 4 depict the region of $\Delta SW^* > 0$ for the cases of substitutability ($\gamma = 0.3$) and of complementarity ($\gamma = -0.3$), respectively (with $\alpha_w = 0.85$). Notice that $(\beta_s, \beta_w) = (0.75, 1.0)$ in Table 1 is contained in the shaded region of Figure 3. The result for the case of substitutability is an expected one from the argument above. For the case of complementarity, the combination of "high β_s and low β_w" works for welfare improvement, an opposite result to the case of substitutability. Notice that complementarity makes the demand in each market more price elastic. With elasticity being high enough, a higher value of β_s makes the uniform price higher, and thus the price change introduced by price discrimination becomes smaller due to the high value of β_s, making less inefficiency of price discrimination in the strong market.

In Figure 3, the white area around the north-west corner violates the condition that $\alpha_s/\alpha_w > \underline{\alpha_s/\alpha_w}$. The violation means that the discriminatory price at the strong market with α_s is *lower* than that at the weak market with α_w (note that $\alpha_s > \alpha_w$). In other words, **the discriminatory price at the market with a higher intercept (α_s) is *lower* than that at the market with a lower intercept** (α_w). Following the definition of "strong" market in Section 2, we now redefine the former market "weak market" and the latter one "strong' market." On this white area **where $\beta_s < \beta_w$ holds**, the **redefined** "weak" market with **a higher intercept** is ***more*** elastic than the **redefined** "strong" market **with a lower intercept** is. As mentioned earlier, when the "weak" market is elastic, the increase in quantity in the weak market is **not high enough to offset the loss from** the decrease in quantity in the strong market, that is, $\Delta Q < 0$. In fact, on this white area, price discrimination deteriorates the total social surplus.

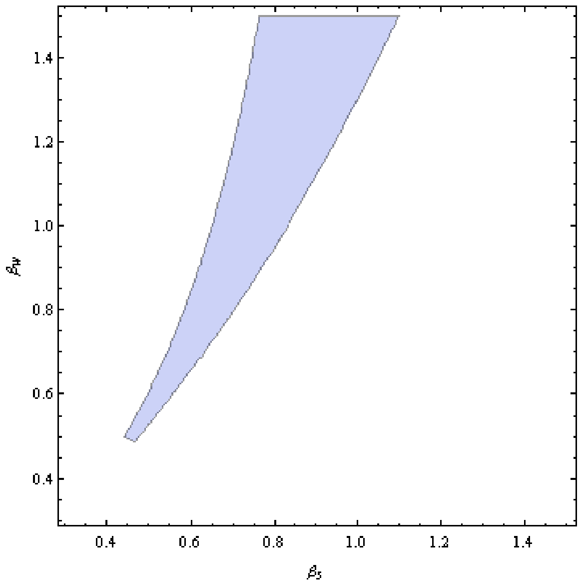

Figure 3: Substitutability ($\gamma = 0.3$) in the Case of $\alpha_w = 0.85$

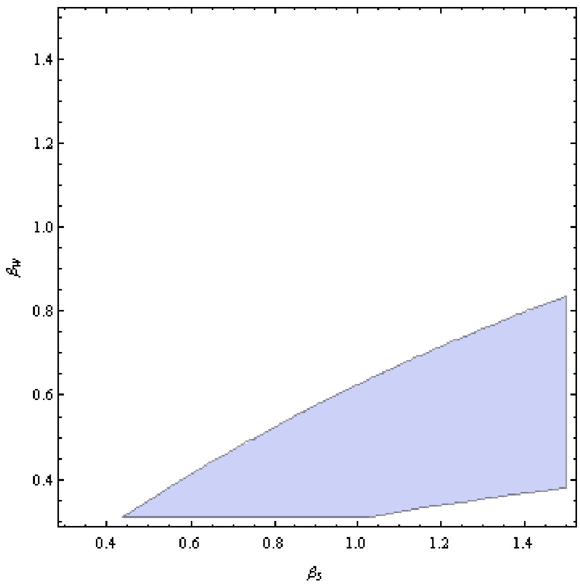

Figure 4: Complementarity ($\gamma = -0.3$) in the Case of $\alpha_w = 0.85$

Lastly, it is verified that consumer surplus never improves by price discrimination for the cases of $(\gamma, \alpha_w) = (0.3, 0.85)$ and of $(\gamma, \alpha_w) = (-0.3, 0.85)$. Thus, this and other numeral results suggest that welfare improvement by price discrimination is solely due to the increase in the firms' profits. **In particular, we conjecture that there is little or no chance that firms suffer from "prisoners' dilemma", that is, fims are mostly or always better off by switching from uniform pricing to price discrimination.**

β_m is common Now we allow γ_s and γ_w to be different, letting $\beta \equiv \beta_s = \beta_w$ to avoid unnecessary complications. We make numerical and graphical arguments on the domains (γ_s, γ_w) that make $\Delta SW^* > 0$ for fixed values of $(\alpha_w, \beta_s, \beta_w)$.

Figure 5 depicts the region of $\Delta SW^* > 0$ with $\alpha_w = 0.85$ and $\beta = 1.0$. We can apply the same logic in the case where γ_i is common to this case. Figure 5 contains a similar property to that in Figure 3. It is necessary to improve the total social surplus by price discrimination that $\gamma_s/\beta_s > \gamma_w/\beta_w$, that is, $\gamma_s > \gamma_w$. Figure 5 shows that $\Delta SW^* < 0$ if $\gamma_s < \gamma_w$. Notice that price discrimination never improves social welfare in the second quadrant ($\gamma_s < 0 < \gamma_w$). That is, ***if the two brands are complementary in the strong market while the firms sell substitutable goods in the weak market, then price discrimination necessarily deteriorates social welfare.*** This result seems to hold for other parameter values because $\Delta SW^* \leq 0$ if $\beta_s = \beta_w$ and $\gamma_s = \gamma_w$: in the northwestern region separated by $\gamma_s = \gamma_w$, social welfare would be negative. The intuitive reason is that complementarity in the strong market makes the price change by price discrimination more responsive, which creates more inefficiency, while substitutability in the weak market makes the price change less responsive. The latter positive effect is not sufficiently large to cover the former negative effect.

On the other hand, it is possible that price discrimination improves social welfare ***if the firms' brands are substitutes in the strong market (*** $\gamma_s > 0$ ***) and are complements in the weak market (*** $\gamma_w < 0$ ***).*** Figure 5 also shows that the combination of strong complementarity in the weak market and weak complementarity in the weak market (i.e., $|\gamma_w|$ larger than $|\gamma_s|$) is suited to welfare gain. This result is as expected: strong complementarity in the weak market keeps the discriminatory price low enough to

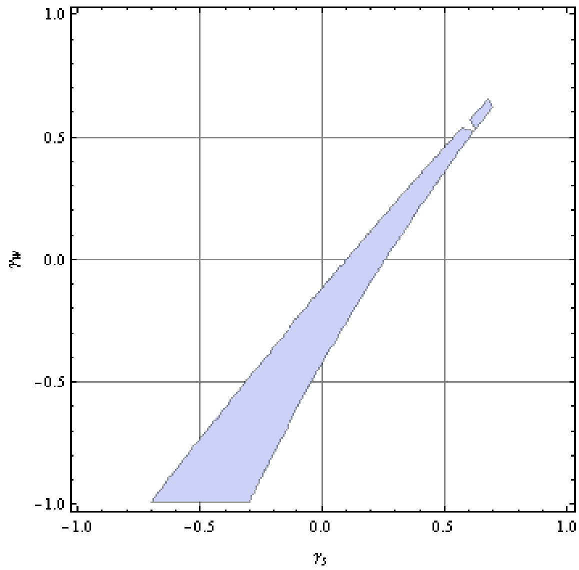

Figure 5: The Region of $\Delta SW^* > 0$ for the Case of $\alpha_w = 0.85$ and $\beta_s = \beta_w = 1.0$

offset the loss from the price increase in the strong market. However, it is also verified that consumer surplus ***never*** improves by price discrimination.

Notice also that around the northeastern corner in Figure 5 (around $(\gamma_s, \gamma_w) = (1.0, 1.0)$) and the southwestern corner in Figure 3, the brands are close to ***perfect substitutes***. We might expect that fiercer competition under price discrimination due to the substitutability would definitely play a positive role in increasing social welfare. In fact, price differentials between the strong and the weak markets are small because discriminatory prices in both markets are very close to the marginal cost. Thus, the distortions caused by the existence of strategic interactions (due to horizontal product differentiation) that are related to the effects on social welfare are thus negligible. *Paradoxically, because of this reason, price discrimination under fierce competition due to strong substitutability cannot improve social welfare.* In fact, the harmful effects of third-degree price discrimination that are similar to the case of monopoly only remain: consumers whose willingness-to-pay is the same pay different prices just because they belong to different submarkets.

Lastly, we focus on one case of asymmetric product differentiation. Table 2 shows the result for the case of $\alpha_w = 0.85$ and $\beta_s = \beta_w = 1.0$.[20] The first case, where the two brands are substitutes in the strong market while they are complementary goods in the week market, has smaller changes both in prices and in quantities than the second case has. Social welfare improves by price discrimination in the first case. The price differentials in the latter case are greater: what happens after the regime change from uniform pricing to price discrimination is that while competition in the weak market becomes fiercer due to the substitutability, complementarity softens the competition to get the discriminatory price in the strong market higher.

	$(\gamma_s, \gamma_w) =$ $(0.1, -0.1)$	$(-0.1, 0.1)$
p^*	0.4588	0.4663
p_s^* $(\Delta p_s^*/p^*)$	0.4737 (3%)	0.5238 (12%)
p_w^* $(\Delta p_w^*/p^*)$	0.4452 (−3%)	0.4026 (−14%)
Δq_s^* $(\Delta q_s^*/q_s^*(p^*))$	−0.0615 (−3%)	−0.0640 (−11%)
Δq_w^* $(\Delta q_w^*/q_w^*(p^*))$	0.0150 (3%)	0.0578 (17%)
ΔSW^*	0.0009	−0.0131
ΔCS_s^*	−0.0145	−0.0646
ΔCS_w^*	0.0120	0.0481
$\Delta \Pi^*$	0.0034	0.0034
ΔQ^*	0.0014	−0.0061

Table 2: Asymmetric Product Differentiation ($\alpha_w = 0.85$ and $\beta = 1.0$)

4 Concluding Remarks

In this paper, we study the relationship between horizontal product differentiation and the welfare effects of third-degree price discrimination in oligopoly with linear demands. By deriving linear demands from a representative consumer's utility and focusing on symmetric equilibrium of a pricing game, we **characterize conditions, relating to such demand properties as substitutability and complementarity, for price discrimination to improve social welfare. In particular,** we show that price discrimination

[20] It is verified that all of the model parameters satisfy the restriction conditions provided in Appendix A2.

can improve social welfare (especially) if firms' brands are substitutes in the market where the discriminatory price is higher and are complements in the market where it is lower, but it never improves vice versa. We conjecture, however, that consumer surplus never improves by price discrimination: welfare improvement by price discrimination is solely due to the increase in the firms' profits. **Accordingly, we also conjecture that there is little or no chance that firms suffer from "prisoners' dilemma", that is, fims are mostly or always better off by switching from uniform pricing to price discrimination.** It is also shown, contrary to the intuition, that competition due to strong substitutability does not play a positive role for price discrimination to improve social welfare because an effect of a maldistribution only remains as in the case of monopoly.

In the present paper, we focus only on symmetric equilibrium of the pricing game to obtain analytical insight. This limitation would be particularly unappealing if one wishes to consider firm heterogeneity to use equilibrium predictions from strategic models.[21] This and other interesting issues await future research.

Appendices

A1. Changes in Equilibrium Prices and Quantities by Price Discrimination

Equilibrium quantities produced by each firm under price discrimination in market m are

$$q_m(p_m^*) = \frac{\alpha_m \beta_m}{(2\beta_m - \gamma_m)(\beta_m + \gamma_m)},$$

where the denominator is positive because $|\gamma_m| < \beta_m$.

Under uniform pricing, *if both markets are open* (see Appendix A2 for the verification of market opening), then tedious calculation shows that the equilibrium uniform price is

$$p^* = \frac{(\beta_m - \gamma_m)(\beta_{m'} - \gamma_{m'})[\alpha_m(\beta_{m'} + \gamma_{m'}) + \alpha_{m'}(\beta_m + \gamma_m)]}{\Phi^U}, \tag{A1}$$

where $m \neq m'$ $(m, m' \in \{s, w\})$ and $\Phi^U \equiv \sum_{m \neq m'}(\beta_m^2 - \gamma_m^2)(2\beta_{m'} - \gamma_{m'})$. The denominator and the numerator are also found positive because $|\gamma_m| < \beta_m$. One can verify that

[21] Galera and Zaratieguia (2006) consider duopolistic third-degree price discrimination with heterogeneity in constant marginal cost and show that price discrimination can improve social welfare even if the total output does not change: it favors the low-cost firm to cut its prices significantly and this cost saving may overcome the welfare losses from price discrimination.

the equilibrium quantities under uniform pricing in market $m \neq m'$ are then given by

$$q_m(p^*) = \frac{\alpha_m[\beta_m(\beta_{m'}^2 - \gamma_{m'}^2) + \beta_{m'}(\beta_m^2 - \gamma_m^2)] + (\alpha_m - \alpha_{m'})(\beta_m^2 - \gamma_m^2)(\beta_{m'} - \gamma_{m'})}{(\beta_m + \gamma_m)\Phi^U}. \tag{A2}$$

Now, let

$$\Delta p_m^* \equiv p_m^* - p^* = \frac{(\beta_m^2 - \gamma_m^2)[\alpha_m(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'}) - \alpha_{m'}(\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)]}{(2\beta_m - \gamma_m)\Phi^U}$$

be defined as the changes in the equilibrium prices from uniform pricing to price discrimination in each market. Thus, if we define the *strong* (*weak*) market as the market where the equilibrium price increases (decreases) by price discrimination, then market m is strong if and only if

$$\alpha_m > \frac{(\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)}{(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'})}\alpha_{m'}.$$

This implies that, in contrast to the case of monopoly with inter-market dependencies (see Adachi (2002)), the condition on the intercepts, $\alpha_m > \alpha_{m'}$, is not exactly the necessary and sufficient condition for market m to be strong: *if $\gamma_{m'}\beta_m$ is much larger than $\gamma_m\beta_{m'}$* (note that either or both can be negative), *then market m with $\alpha_m > \alpha_{m'}$ can be weak.* Of course, if $\beta_m = \beta_{m'}$ and $\gamma_m = \gamma_{m'}$, then $\alpha_m > \alpha_{m'}$ is the necessary and sufficient condition for market m to be strong.

Turning attention to output, we have

$$\begin{aligned} \Delta q_m^* &\equiv q_m(p_m^*) - q_m(p^*) &\text{(A3)} \\ &= -\frac{(\beta_m - \gamma_m)[\alpha_m(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'}) - \alpha_{m'}(\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)]}{(2\beta_m - \gamma_m)\Phi^U}. &(1) \end{aligned}$$

as the equilibrium changes in output from uniform pricing to price discrimination for each firm in strong and weak markets, respectively. It is then verified that Δp_m^* and Δq_m^* are related in the following way:

$$\Delta p_m^* = -(\beta_m + \gamma_m)\Delta q_m^*, \tag{A4}$$

so that we have $q_m(p_m^*) > q_m(p^*)$ if and only if $p_m^* < p^*$. One can also derive the change in equilibrium aggregate output:

$$\Delta Q^* \equiv \Delta q_s^* + \Delta q_w^* = \frac{(\beta_w\gamma_s - \beta_s\gamma_w)[\alpha_s(\beta_s - \gamma_s)(2\beta_w - \gamma_w) - \alpha_w(\beta_w - \gamma_w)(2\beta_s - \gamma_s)]}{(2\beta_s - \gamma_s)(2\beta_w - \gamma_w)\Phi^U},$$

which does not necessarily coincides with zero as opposed to the case of monopoly with linear demands.

Now, although market m is strong even if $\alpha_m = \alpha_{m'}$ as long as $(\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'}) > (\beta_{m'} - \gamma_{m'})(2\beta_m - \gamma_m)$, we assume that $\alpha_m \neq \alpha_{m'}$. This is because if $\alpha_m = \alpha_{m'}$, then we have

$$\Delta Q^* = -\frac{\alpha_m(\beta_{m'}\gamma_m - \beta_m\gamma_{m'})^2}{(2\beta_m - \gamma_m)(2\beta_{m'} - \gamma_{m'})\Phi^U} \leq 0,$$
$$\Delta p_m^* = \frac{\alpha_m(\beta_m^2 - \gamma_m^2)(\beta_m\gamma_{m'} - \beta_{m'}\gamma_m)}{(2\beta_m - \gamma_m)\Phi^U}$$

and most importantly, ΔSW^*, the difference in social welfare under price discrimination and under uniform pricing (introduced in Section 3), can never positive (the formal proof is upon request). Thus, *unequal values of intercepts of the two markets are necessary for price discrimination to improve social welfare.* Hence, for markets s and w to be actually strong and weak, respectively, it is necessary to have

$$\frac{\alpha_s}{\alpha_w} > \max\left[\frac{(\beta_w - \gamma_w)(2\beta_s - \gamma_s)}{(\beta_s - \gamma_s)(2\beta_w - \gamma_w)}, 1\right].$$

The reason why it is not sufficient is that we must verify the parameter restriction for market w to be open under uniform pricing, and we verify it in Appendix A2.

For later use (Appendix A3), we also calculate the sum of a firm's output under uniform pricing and the under price discrimination in each market $m \in \{s, w\}$:

$$q_m(p_m^*) + q_m(p^*) = \frac{\alpha_m(3\beta_m - \gamma_m)}{(\beta_m + \gamma_m)(2\beta_m - \gamma_m)} - \frac{p^*}{\beta_m + \gamma_m}. \tag{A5}$$

A2. Market Opening under Uniform Pricing

Remember that the symmetric equilibrium under uniform pricing in the main text and Appendix A1 is obtained, *given that* both markets are supplied by either firm under uniform pricing ($q_s(p^*) > 0$ and $q_w(p^*) > 0$). In this appendix, we obtain a (sufficient) condition that guarantees that in equilibrium each firm supplies to the weak market under uniform pricing. To do so, we consider one firm's incentive not to deviate from the equilibrium by stopping its supply to the weak market.

Suppose firm j supplies only to the strong market, given the rival firm supplying to both markets with the equilibrium price, p^* (see Appendix A1). Let firm j's price when deviating from the equilibrium price under the regime of uniform pricing be denoted by p'. Then, when firm j closes the weak market, its profit is written by[22]

$$\widetilde{\pi}(p', p^*) = p' \cdot q_s^j(p', p^*)$$

where

$$q_s^j(p', p^*) = \frac{\alpha_s}{\beta_s + \gamma_s} - \frac{\beta_s}{\beta_s^2 - \gamma_s^2} p' + \frac{\gamma_s}{\beta_s^2 - \gamma_s^2} p^*.$$

Now, it is verified that

$$\arg\max_{p' \neq p^*} \widetilde{\pi}(p^*) = \frac{\alpha_s(\beta_s - \gamma_s)}{2} + \frac{\gamma_s}{2} p^* \ (\equiv p'').$$

Note that firm j's profit function when it deviates to any price other than the equilibrium price would not be necessarily (globally) concave because it would be kinked at the threshold price where the weak market closes, as depicted in Figure 6.

If p'' attains the local maximum as in Panels (1) and (2) in Figure 6, then one needs to solve for the restriction on the set of parameters that guarantees that the equilibrium profit when both markets are open

$$p^* \left(\frac{\alpha_s - p^*}{\beta_s + \gamma_s} + \frac{\alpha_w - p^*}{\beta_w + \gamma_w} \right)$$

is no smaller than the maximized profit when firm j deviates to close the weak market

$$\max_{p' \neq p^*} \widetilde{\pi}(p^*).$$

One would, however, find it too complicated to obtain the set of parameters from this inequality. Thus, we instead focus on the case that corresponds to Panel (3) in Figure 6. This gives a sufficient condition for the weak market to open. Notice that by definition,

[22] Given p^*, the upper bound of p' such that $q_s^j(p'^*) \geq 0$ is larger than that such that $q_w^j(p'^*) \geq 0$ if and only if $\alpha_s > (\beta_w - \gamma_w)(2\beta_s - \gamma_s)\alpha_w / ((\beta_s - \gamma_s)(2\beta_w - \gamma_w))$. That is, for any p' such that $q_w^j(p'^*) \geq 0$, $q_s^j(p'^*) \geq 0$. In other words, given p^*, the strong market opens if the weak market opens. The upper bound of p' such that $q_s^j(p'^*) \geq 0$ is $(\beta_s - \gamma_s)\alpha_s + \gamma_s p^*$. The upper bound of p' such that $q_w^j(p'^*) \geq 0$ is $(\beta_w - \gamma_w)\alpha_w + \gamma_w p^*$. The former minus the latter is

$$\frac{[(\beta_s - \gamma_s)(2\beta_w - \gamma_w)\alpha_s - (\beta_w - \gamma_w)(2\beta_s - \gamma_s)\alpha_w](\beta_s(\beta_w^2 - \gamma_w^2) + \beta_w(\beta_s^2 - \gamma_s^2))}{\beta_s\beta_w((\beta_s^2 - \gamma_s^2)(2\beta_w - \gamma_w) + (\beta_w^2 - \gamma_w^2)(2\beta_s - \gamma_s))} > 0.$$

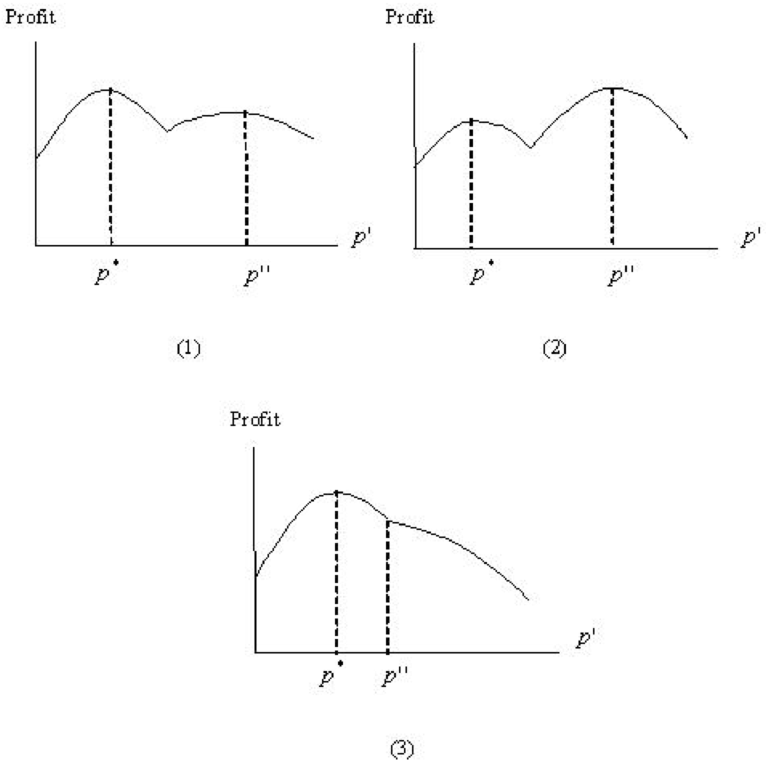

Figure 6: Profit when Deviating from the Equilibrium Price under Uniform Pricing

p'' must satisfy $q_w^j(p'', p^*) \leq 0$, given p^*. If this violates, then it is the case of Panel (3) in Figure 6. It shortens to

$$\alpha_w > \frac{(\beta_s - \gamma_s)[2(\beta_w - \gamma_w)(\beta_w^2 - \gamma_w^2)\beta_s + (2\beta_w - \gamma_w)(\beta_s^2 - \gamma_s^2)\beta_w]}{(\beta_w - \gamma_w)[2(2\beta_s - \gamma_s)(\beta_w^2 - \gamma_w^2)\beta_s + (4\beta_s - \gamma_s)(\beta_s^2 - \gamma_s^2)\beta_w]}\alpha_s.$$

Together with the argument in Appendix A1, we, throughout the paper, assume the relative size of the intercept of the strong market satisfies $\alpha_s/\alpha_w \in (\underline{\alpha_s/\alpha_w}, \overline{\alpha_s/\alpha_w})$, where

$$\begin{aligned} \underline{\frac{\alpha_s}{\alpha_w}} &= \underline{\frac{\alpha_s}{\alpha_w}}(\boldsymbol{\gamma}, \boldsymbol{\beta}) \\ &\equiv \max\left[\frac{(\beta_w - \gamma_w)(2\beta_s - \gamma_s)}{(\beta_s - \gamma_s)(2\beta_w - \gamma_w)}, 1\right] \end{aligned}$$

and

$$\begin{aligned} \overline{\frac{\alpha_s}{\alpha_w}} &= \overline{\frac{\alpha_s}{\alpha_w}}(\boldsymbol{\gamma}, \boldsymbol{\beta}) \\ &\equiv \frac{(\beta_w - \gamma_w)[2(2\beta_s - \gamma_s)(\beta_w^2 - \gamma_w^2)\beta_s + (4\beta_s - \gamma_s)(\beta_s^2 - \gamma_s^2)\beta_w]}{(\beta_s - \gamma_s)[2(\beta_w - \gamma_w)(\beta_w^2 - \gamma_w^2)\beta_s + (2\beta_w - \gamma_w)(\beta_s^2 - \gamma_s^2)\beta_w]}, \end{aligned}$$

and this restriction is sufficient for markets s and w to be actually strong and weak.

A3. Proof of Proposition 1 (Calculating ΔSW^* as a Function of p^*, p_s^* and p_w^*)

By using equations (A1-A5), we can calculate

$$\begin{aligned} \Delta SW^* &= \sum_{m \in \{s,w\}} [2\alpha_m(q_m(p_m^*) - q_m(p^*)) - (\beta_m + \gamma_m)[q_m(p_m^*) - q_m(p^*)][q_m(p_m^*) + q_m(p^*)]] \\ &= \sum_{m \in \{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}[-2\alpha_m + (\beta_m + \gamma_m)(q_m(p_m^*) + q_m(p^*))] \\ &= \sum_{m \in \{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}\left[-2\alpha_m + (\beta_m + \gamma_m)\left[\frac{\alpha_m(3\beta_m - \gamma_m)}{(\beta_m + \gamma_m)(2\beta_m - \gamma_m)} - \frac{p^*}{\beta_m + \gamma_m}\right]\right] \\ &= \sum_{m \in \{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}\left[-\frac{\alpha_m(\beta_m - \gamma_m)}{2\beta_m - \gamma_m} - p^*\right] \\ &= -\sum_{m \in \{s,w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m}(p_m^* + p^*). \end{aligned}$$

A4. Proof of Proposition 2

Using the explicit forms for Δp_m^* and for ΔQ^*(derived in Appendix A1), we have

$$\Delta Q^* = -2\left(\frac{\Delta p_s^*}{\beta_s + \gamma_s} + \frac{\Delta p_w^*}{\beta_w + \gamma_w}\right),$$

which implies that $\Delta Q^* \leq 0$ if and only if

$$\frac{\Delta p_s^*}{\beta_s + \gamma_s} \geq -\frac{\Delta p_w^*}{\beta_w + \gamma_w}.$$

Now, suppose that $\Delta Q^* < 0$. Then, we have

$$\begin{aligned}
\Delta SW^* &= -\frac{\Delta p_s^*}{\beta_s + \gamma_s}(p_s^* + p^*) - \frac{\Delta p_w^*}{\beta_w + \gamma_w}(p_w^* + p^*) \\
&\leq -\frac{\Delta p_s^*}{\beta_s + \gamma_s}(p_s^* + p^*) + \frac{\Delta p_s^*}{\beta_s + \gamma_s}(p_w^* + p^*) \\
&= -\frac{\Delta p_s^*(p_s^* - p_s^*)}{\beta_s + \gamma_s}(p_s^* + p^*) < 0.
\end{aligned}$$

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Appendices for Referees

Derivation of $\Delta SW^* \leq 0$ for $\alpha_m = \alpha_{m'}$

We can proceed:

$$
\begin{aligned}
\Delta SW^* &= \frac{(\beta_w+\gamma_w)(p^{*2}-p_s^{*2})+(\beta_s+\gamma_s)(p^{*2}-p_w^{*\,2})}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)} \\
&= -\frac{(\beta_w+\gamma_w)(p^*+p_s^*)\Delta p_s^*+(\beta_s+\gamma_s)(p^*+p_w^*)\Delta p_w^*}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)} \\
&= -(p^*+p_s^*)\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)(\beta_s^2-\gamma_s^2)}{(\beta_s+\gamma_s)(2\beta_s-\gamma_s)\Phi^U} - (p^*+p_w^*)\frac{\alpha_m(\beta_w\gamma_s-\beta_s\gamma_w)(\beta_w^2-\gamma_w^2)}{(\beta_w+\gamma_w)(2\beta_w-\gamma_w)\Phi^U} \\
&= -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
&\quad \times \big[(p^*+p_w^*+p_s^*-p_w^*)(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w) \\
&\qquad\qquad -(p^*+p_w^*)(\beta_w^2-\gamma_w^2)(\beta_s+\gamma_s)(2\beta_s-\gamma_s)\big] \\
&= -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
&\quad \times \big[(p^*+p_w^*)\{(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w)-(\beta_w^2-\gamma_w^2)(\beta_s+\gamma_s)(2\beta_s-\gamma_s)\} \\
&\qquad +(p_s^*-p_w^*)(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w)\big] \\
&= -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
&\quad \times [(p^*+p_w^*)(\beta_s+\gamma_s)(\beta_w+\gamma_w)(\beta_s\gamma_w-\beta_w\gamma_s) \\
&\qquad +(p_s^*-p_w^*)(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)(2\beta_w-\gamma_w)] \\
&= -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
&\quad \times [(p^*+p_w^*)(\beta_s+\gamma_s)(\beta_w+\gamma_w)(\beta_s\gamma_w-\beta_w\gamma_s) \\
&\quad +\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)((\beta_s^2-\gamma_s^2)(2\beta_w-\gamma_w)+(\beta_w^2-\gamma_w^2)(2\beta_s-\gamma_s))}{(2\beta_s-\gamma_s)\Phi^U}(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)\Big] \\
&= -\frac{\alpha_m(\beta_s\gamma_w-\beta_w\gamma_s)^2}{(\beta_s+\gamma_s)(\beta_w+\gamma_w)(2\beta_s-\gamma_s)(2\beta_w-\gamma_w)\Phi^U} \\
&\quad \times [(p^*+p_w^*)(\beta_s+\gamma_s)(\beta_w+\gamma_w) \\
&\quad +\frac{\alpha_m(\beta_s^2-\gamma_s^2)(\beta_w+\gamma_w)((\beta_s^2-\gamma_s^2)(2\beta_w-\gamma_w)+(\beta_w^2-\gamma_w^2)(2\beta_s-\gamma_s))}{(2\beta_s-\gamma_s)\Phi^U}\Big] \leq 0.
\end{aligned}
$$

It is easy to see that the equality holds if and only if $\beta_s\gamma_w-\beta_w\gamma_s=0$.

Numerical Calculation

We use Mathematica to obtain numerical results. For the case of symmetric product differentiation, the following results are used to obtain numerical values in Table 1.

	$(\gamma, \beta_s, \beta_w) =$ (0.3, 1, 0.75)	(0.3, 0.75, 1.0)	(−0.3, 1.0, 0.75)	(−0.3, 0.75, 1.0)
$q_s^*(p^*)$	0.4937	0.6053	0.6807	1.0171
$q_w^*(p^*)$	0.4684	0.3735	0.7255	0.4396

Table A1: $q_s^*(p^*)$, $q_w^*(p^*)$ and ΔQ^* with $\beta_s \neq \beta_w$ ($\alpha_w = 0.85$)

The following table is for Table 2.

	$(\gamma_s, \gamma_w) =$ (0.1, −0.1)	(−0.1, 0.1)
$q_s^*(p^*)$	0.4920	0.5931
$q_w^*(p^*)$	0.4347	0.3489

Table A2: $q_s^*(p^*)$, $q_w^*(p^*)$ and ΔQ^* with $\beta_s = \beta_w = 1.0$ ($\alpha_w = 0.85$)