Partial privatization in mixed oligopoly with foreign competitors

Ming Hsin LIN

Faculty of Economics, Osaka University of Economics*

November, 2007

Abstract

This paper investigates the relations between partial privatization and foreign competition. Introducing the aspect of partial privatization into Fjell and Pal (1996)'s model, we find that the effects of foreign firms' entry on a public firm's output, profits and on the home country's welfare are different from Fjell and Pal' results, if the public firm is a *partially privatized* one. In addition, socially optimal degree of privatization has strong relations with market opening as well as foreign acquisition of domestic private firms. These findings provide new policy implication for a government is considering privatizing its public firm, while facing the pressure of market opening.

Keywords: Mixed oligopoly, partial privatization, market opening, foreign acquisition

1. Introduction

Since one of the pioneer works by De Fraja and Delbono (1989), the study on mixed oligopoly has now been developed dramatically in recent years. According as the trend that many countries are privatizing their state-owned industries, one essential concern of

* Faculty of Economics, Osaka University of Economics, 2-2-8 Osumi, Higashiyodogawa-ku, Osaka, 533-8533, Japan. Tel.: +81-6-6328-2431; fax: +81-6-6327-1825.

E-mail address: linmh@osaka-ue.ac.jp (M.H. Lin).

the literature is on the issue of privatization.[1] At the same time, corresponding with the movement of economic globalization, there is also another concern on the issue of competition with foreign firms.[2] In line with these two concerns, a number of mixed oligopoly models have been developed following in the two directions: privatization and foreign competition.

For example, given the result of De Fraja and Delbono (1989) that suggests a public firm should be privatized and should maximize profits rather than welfare, Matsumura (1998) explicitly considers the possibility of partial privatization and shows that neither full privatization nor full nationalization is optimal. Lee and Hwang (2003) elaborate on the framework of Matsumura (1998) by allowing managerial inefficiency and show that partial privatization is a reasonable choice of government in a monopoly as well as a mixed duopoly. In each study, there are no foreign private firms in the market.

On the other hand, Fjell and Pal (1996) construct a mixed oligopoly model, in which a welfare-maximizing public firm competes with domestic private firms and foreign private firms, to examine the effects of an open door policy and the effects of foreign acquisition of domestic private firms. In their study, the issue of privatization is not discussed. Pal and White (1998) adopt Fjell and pal's model to investigate the consequence of full privatization and strategic trade policy taking the form of domestic

[1] Privatization, which is defined as the deliberate sale of a state-owned enterprise by a government to private sector, now appears as a legitimate toll of statecraft for governments of more than 100 countries in Asia and Europe (Lee and Hwang, 2003). As current examples in Middle East, Jordan decided start to sell out the share of Royal Jordanian, a fully (100%) government-owned airline, from the end of 2007. Egypt Central Bank announced plans to sell out 80% share of Cairo Band to private sector before the first half of 2008.

[2] In the mixed oligopoly literatures with foreign competition, two major policy concerns are on market opening policy and international trade policy.

production subsidy or import tariff. Fjell and Heywood (2002) extend Fjell and Pal' (Cournot-Nash) model into a Stackelberg one, to examine the effects of an open door policy, foreign acquisitions and full privatization. These two works discuss the case of complete privatization only, and the possibility of *partial* privatization is neglected.[3]

One simple question may arise, here. For a government is considering privatizing its public firm and is forced to open the corresponding domestic market at the same time, do policy suggestions based on the well known results of the literatures still work well when both the aspects of *partial* privatization and foreign competition are taken into account? In fact, it is anticipated that the speed of privatizing China's state-owned enterprises will accelerated and more foreign firms will be encouraged to enter China's huge markets (Chao and Yu, 2006). It is therefore important to investigate this question.

Given this importance, recent works involved both aspects of partial privatization and foreign competition appeared in the literature. Chang (2005) use a mixed duopoly model with cost asymmetry to examine optimal trade policy (import tariff) and full/partial privatization policy. Chao and Yu (2006) construct a mixed oligopoly model to examine how partial privatization or foreign competition affects the optimal import tariff. Chang (2007) also examine optimal trade policy (trade tax/subsidy), industrial policy (production tax/subsidy), and privatization policies under a mixed duopoly model with strategic managerial incentives. In each of these studies, the focus is on the strategic trade policy and domestic private firms are not included.[4] While, focusing on

[3] Without the consideration of privatization, Matsumura (2003) also investigates a Stackelberg model where the welfare-maximizing public firm competes with a single foreign private firm.

[4] Recently, as an extension of the traditional framework of international mixed oligopoly, there are mixed oligopoly model with two countries each with public and private firms competing in a single international market. See Barcena-Ruiz and Garzon (2005), Dadpay and Heywood (2006).

the market opening policy, Han and Ogawa (2007) clarify how market opening affects the degree of privatization of a public firm. In their study, the market opening policy takes the form of raising the restricted share of foreign investments, as well as allowing the entry of private firms which are partially owned by foreigners. Again, complete domestic private firms (without any foreign capital) are not included in their model, too.

This paper is attempt to introduce the framework of partial privatization by Matsumura (1998) into Fjell and Pal (1996)'s model to re-examine the effects of competition with foreign private firms, including market opening policy and foreign acquisition of domestic private firms. Though the direction of extension is straightforward in this present paper, the analytical results as comparisons with Fjell and Pal's ones are different and are perhaps interesting. In addition, this extension makes it possible to argue the relations between the socially optimal degree of privatization and foreign competition (by allowing foreign private firms' entry as well as foreign acquisition of domestic private firms). The policy implications based on the new results seems meaningful.

Specifically, while Fjell and Pal showed that a foreign firm's entry exactly increases the public firm's output. We show that if the public firm is a partially privatized one (with a degree of privatization more than 33%), then the entry increases the privatized public firm's output if the number of domestic private firms is sufficiently large. Fjell and Pal also showed that the entry exactly reduces the profits of the public firm. We shows that, given an appropriate degree of privatization, the entry increases the partially privatized public firm's profit when the number of domestic private firms is sufficiently large and the number of foreign private firms is sufficiently small. Moreover, this paper shows that given a small level of privatization, the entry increases welfare if and only if

the number of domestic firms is small relative to the number of foreign private firms (corresponding to the result of Fjell and Pal). However, given a certain large level of privatization, welfare increases even though the number of domestic private firms is larger than that of foreign private firms.

Second, Fjell and Pal showed that foreign acquisition of a domestic private firm exactly decreases the profits of the state-owned public firm. In contrast, this paper with the consideration of partial privatization shows that the foreign acquisition increases the profits of the *drastically* privatized public firm. Finally, as new results related with the relations between the socially optimal degree of privatization and foreign competition, we finds that if a new foreign private firm entered the market, then the socially optimal degree of privatization rises (falls) when the number of domestic private firms is small (sufficiently large), given an appropriate number of foreign private firms. Furthermore, we also find that if a domestic private firm is acquired by foreign nationals, then the optimal degree of privatization falls.

The paper is organized as follows. In Section 2, the mixed oligopoly model with two stages game is presented. In Section 3, the Cournot-Nash outcomes in the second stage are derived and then the effects of open-door policy (allowing entry of foreign private firms) are examined. In Section 4, the socially optimal degree of privatization for the public firm is derived, and the relevance been this optimal degree and open-door policy are demonstrated. The issue of foreign acquisition of domestic firms is discussed in Section 5. Concluding remarks follow in Section 6.

2. The Model

We consider a mixed oligopoly model, originally constructed by Fjell and Pal (1996),

in which foreign private firms exist. In our model, there are one partially privatized public firm (firm 0), *m* domestic private firms, and *n* foreign private firms. Each firm produces a homogenous commodity and has identical technologies, which can be represented by the cost function as $C(q) = f + (1/2)q^2$. That is, the marginal cost is increasing in outputs.[5] Furthermore, the fixed cost f is assumed to be zero for simplicity.[6]

Following Fjell and Pal (1996), we assume that the inverse demand function is linear and is given as $p = a - \left(q_0 + \sum_{i=1}^{m} q_i + \sum_{j=1}^{n} q_j\right)$, where p is the market price, q_0 is the output of the privatized public firm, q_i is the output of the domestic private firm i $(i = 1, \dots, m)$, and q_j is the output of the foreign private firm j $(j = 1, \dots, n)$, $i \neq j$. Consequently, the consumer surplus is given by $CS = (1/2)\left(q_0 + \sum_{i=1}^{m} q_i + \sum_{j=1}^{n} q_j\right)^2$. Each domestic private firm i maximizes its own profit denoted by $\pi_i = pq_i - (1/2)q_i^2$, and each foreign private firm j maximizes its profits denoted by $\pi_j = pq_j - (1/2)q_j^2$.

The partially privatized public firm is jointly owned by both the government and private sector, and the government owns a share of $(1 - \theta) \in [0,1]$ of the firm. Following Matsumura (1998), we assume that partially privatized public firm maximize the weighted average of social welfare and its profits, which is defined as $V = \theta\pi_0 + (1 - \theta)W$, where $\pi_0 = pq_0 - (1/2)q_0^2$ denotes the profit of the privatized public

[5] The assumption of the increasing marginal cost is crucial when the privatized public firm and the private firms have identical technologies. The justifications for this assumption have been provided by Fjell and Pal (1996), Pal and White (1998), among others.

[6] Fjell and Pal (1996) also assumed $f = 0$, then indicated the results change for $f > 0$ in footnote 3 and 5 in their study. These indications apply to our corresponding results, which are not explicitly presented in the paper, as well. As we can see below, our main results holds for $f > 0$.

firm, and $W = CS + \pi_0 + \sum_{i=1}^{m} \pi_i$ denotes the social welfare of the home country.[7] Note that θ can be used to measure the degree of privatization. In the case where $\theta = 0$, firm 0 is a state-owned public firm who maximizes the social, while in the case where $\theta = 1$, firm 0 is a private firm who maximizes its own profits.

Finally, we consider a two-stage game. In the first stage, the government chooses the degree of privatization, which is represented by the parameter θ, to maximizes the social welfare of the home country (the defined function W). In the second stage, given θ, all private firms choose its quantity to maximize its own profits, while the privatized public firm choose its quantity to maximize its objective function (the defined function V) , in the standard Cournot-Nash fashion. We will proceed to derive the sub-game perfect equilibrium by backward induction.

3. Cournot-Nash outcomes and effects of open-door policy (2nd stage)

In the second stage, given the policy variable θ of the government, we first derive the Cournot-Nash outcomes, and then examine the effects of an open-door policy that allows foreign firms to enter the mixed oligopoly, by the comparative statics analysis. Solving the optimization problems defined in section 2, we have the Cournot-Nash outcomes as follows.

$$q_i^* = q_j^* = a \cdot (1 + \theta)/[(4 + m + 2n) + \theta(2 + m)]$$

$$q_0^* = a \cdot [2 + (1 - \theta)n]/[(4 + m + 2n) + \theta(2 + m)]$$

$$p^* = 2q_i^* = 2a \cdot (1 + \theta)/[(4 + m + 2n) + \theta(2 + m)]$$

$$\pi_i^* = \pi_j^* = (3/2)q_i^* = (3/2)a^2 \cdot [(1 + \theta)/\Delta]^2$$

[7] Following Fjell and Pal (1996), the profits of foreign private firms are not included in the social welfare of the home country.

Where $\Delta \equiv (4 + m + 2n) + \theta(2 + m)$

$\pi_0^* = a^2 \cdot [(2 + n) - \theta n][(2 - n) + \theta(4 + n)]/(2\Delta^2)$

$W^* = (a^2/2\Delta^2)(1 + \theta)$

$$\times [(8 + 7m + 8n + 4mn + m^2 + 3n^2) + \theta(3m - 4n + m^2 - n^2)]$$

The superscript " * " stands for the corresponding equilibrium values. Note that if $\theta = 0$ (i.e., the government has 100% ownership of firm 0), these outcomes get back Fjell and Pal's ones.[8]

Now given θ, the effects of entry of a foreign private firm can be investigated by differentiation of the corresponding equilibrium values with respect to the number of foreign private firms *n*. The results are as follows.

(1) $\partial q_0^*/\partial n = (1 + \theta)[m(1 - \theta) - 2\theta] \gtreqless 0$

(2) $\partial m\pi_i^*/\partial n = (a^2/\Delta^3)(-6m)(1 + \theta)^2 < 0$

(3) $\partial CS^*/\partial n = (a^2/\Delta^3)(1 + \theta)[(8 + 4m + 8n) + \theta(4m)] > 0$

(4) $\partial \pi_0^*/\partial n = (a^2/\Delta^3)(-)(1 + \theta) \times \big[(8 + 4n + mn) - \theta(2m + 6n + 2mn) +$

$\theta 22+m(2+n) >< 0$

(5) $\partial W^*/\partial n = (a^2/\Delta^3)(-)(1 + \theta) \times \big[(2m - 4n + mn) - \theta(6n + 2mn) + \theta^2(2 +$

$m(2+n) >< 0$

When a comparison is made with the results of Fjell and Pal (1996, proposition 2) results, we find that if the public firm is partially privatized, the effects of the entry of a foreign private firm on the privatized public firm's output, profits and the social welfare are different. The differences are perhaps interesting.

[8] Note that if the number of foreign private firms is too large ($n > [(2 + 4\theta)/(1 - \theta)]$), the public firm's profits turn into be negative. In this case, lump-sum transfer to the firm may be conducted.

Proposition 1. If a foreign private firm enters the mixed oligopoly market, then

(a) given an appropriate θ, the privatized public firm's output increases (decreases) when the number of the domestic firms is large (sufficiently small).

(b) given an appropriate θ, the privatized public firm's profit increases when the number of domestic private firms is sufficiently large and the number of foreign private firms is sufficiently small.

(c) given a small θ, welfare increases if and only if the number of domestic firms is small relative to the number of foreign private firms. While given a large θ, welfare increases even though the number of domestic firms is large relative to the number of foreign private firms.

####Figure 1, Figure 2####about here

First, while Fjell and Pal showed that, independent on the number of domestic private firms *m*, a foreign firm's entry increases the public firm's output. Our analysis result part (a) shows that if the public firm is partially privatized (more than 33%), then the privatized public firm's output increases if and only if *m* is sufficiently large (see Figure 1). The intuition can be explained by reviewing the reaction functions approach provided by Fjell and Pal. In their analysis where $\theta = 0$, the entry increases the public firm's output since adding a new foreign firm not only shifts the reaction function of 'all other firms' outward, but also shifts the public firm's reaction function outward. In our study with $\theta \in [0, 1]$, given an appropriate θ, a foreign firm's entry shifts the reaction function of the privatized public firm outward as well.[9] The output of the privatized

[9] Note that this shift of the privatized public firm's reaction function starts from different position: the larger the given θ is, the shift starts from inward much more. On the other hand, the shift of the

public firm increases, if and only if the entry shifts the reaction function of 'all other firms' outward slightly. This slight shift occurs when the number of domestic private firm is sufficiently large, which implies that the output of the private firm is sufficiently small.

Second, when we look at the effects on the public firm's profits, Fjell and Pal showed that the entry exactly reduces the profits of the public firm. In our study, the result of part (b) shows that given an appropriate θ, the entry may increase the partially privatized public firm's profits, depending on the numbers of the domestic firms *m* and that of foreign private firms *n*. Figure 2 is provided to show this result.[10] Intuitively, either the firm is a full public or full privatized one, an entry reduces the firm's profits. However, if the public firm is privatized partially, it is possible that its profits increase since the entry reduces the price but may increase it output. By the result of part (a) we know that entry increase its output if and only if *m* is sufficiently large. This turns out to be that case that the privatized public firm's profits increase.

####Figure 3 and Figure 4####about here

Third, in the analysis without the consideration of partially privatization, Fjell and Pal showed that a foreign firm's entry increase the social welfare if and only if *m* is small relative to *n*. In our study with the consideration of partially privatization, the result of part (c) shows that Fjell and Pal's result still holds if the degree of privatization is small. However, if the degree is sufficiently large, then the entry increases the social welfare even *m* is *large* relative to *n*. To demonstrate this result well, let us draw Figure 3 and 4

---

all other firms' reaction function is independent to θ.

[10] In Figure 2, roughly given $\theta \in (0.5 \sim 0.85)$, a foreign firm's entry increases the privatized public firm's profits with the combination of the number of firms (m=35 to 45, n=1).

based on Eq. (5). In Figure 3, the firms' number *m* and *n* are shown on the vertical axis. The horizontal line with $n = 8$, and the critical curve separate the analytical area into four regions. In both region III (where $m < n$) and region II (where $m > n$), a foreign firm's entry exactly increases the home country's social welfare. Furthermore, Figure 4 (with various numbers of *n*) shows us that if *n* is smaller than 4 (8), then Fjell and Pal's result hold if $\theta < 0.15$ ($\theta < 0.35$).

The intuition behind the result of part (c) is as follows. The social welfare includes the consumer surplus, the profits of the privatized public firm and domestic private firms. In the full public firm case ($\theta = 0$), the entry increases the consumer surplus since the price falls down, while reduce the profits of the public firm and the profits of all the private firms. Welfare increases if and only if the loss in the private firms' profits is relatively small. This occurs when *m* is small relative to *n*. However, in the situation that the degree of privatization is sufficiently large, the profits of the privatized public firm increase if *m* is sufficiently large and *n* is small. In addition, the loss in the profits of the domestic private firms (shown in Eq.(2)) is relatively small, since it is decreasing in θ if *m* is sufficiently large and *n* is small.[11] Thus, result of part (c) holds.

4. Partial privatization and open door policy (1st stage)

In the first stage, the government chooses the degree of privatization θ to maximize the social welfare of its own country. Before derive the socially optimal degree, let us examine the effects of an additional θ on each part included in the social welfare. By differentiating the corresponding equilibrium values in the second stage with respect to

[11] This relationship can be easily confirmed by paying attention that $\partial|\partial m\pi_i^*/\partial n|/\partial\theta < 0$ has opposite sign of Eq. (A.4) in Appendix.

θ, we have the following results.

(6) $\partial m\pi_i^*/\partial\theta = (a^2/\Delta^3)2(1+n)(3m)(1+\theta) > 0$

(7) $\partial CS^*/\partial\theta = (a^2/\Delta^3)2(1+n)(-2)[(2+m+2n)+\theta(m)] < 0$

(8) $\partial\pi_0^*/\partial\theta = (a^2/\Delta^3)2(1+n)\times[(4+4n+mn+n^2)-\theta(4+2m+6n+mn+$

$n2 >< 0$

(9) $\partial W^*/\partial\theta = (a^2/\Delta^3)2(1+n)\{[m(1+n)+n^2]-\theta[m(1+n)+(4+6n+n^2)]\}$

Lemma 1. $\partial m\pi_i^*/\partial\theta > 0$. *If the government raises the degree of privatization in the first stage, then the total profits of domestic private firms increase in the second stage.*

This result is perhaps interesting. The government raises the degree of privatization means that the privatized public firm behaves with less concern on the consumer surplus and on the profits of the domestic private firms. However, as a result of the competition in the second stage, the profits of domestic private firms increase. This result holds because that if the government raises θ in the first stage, the privatized public firm reduces its output in the second stage. Then the best response for domestic private firms is to increase its output. Furthermore, raises in the first stage raises the price in the second stage.

Now, by setting Eq. (9) is equal to zero and solving the equation for θ, we have the socially optimal degree of the privatization as follows.[12]

$\theta^* = [m(1+n)+n^2]/[m(1+n)+(4+6n+n^2)]$

This optimal degree of privatization θ^* is a function of the number of domestic private firms *m* and that of foreign private firms *n*. Differentiating this optimal value with

[12] It can be confirmed that the second-order condition $\partial^2 W^*/\partial\theta^2 < 0$ is satisfied.

respect to *m* and *n*, respectively, we have the following results:

(10) $\partial\theta^*/\partial \mathrm{m} = (1+\mathrm{n})(4+6\mathrm{n})/[\mathrm{m}(1+\mathrm{n})+(4+6\mathrm{n}+\mathrm{n}^2)]^2 > 0$

(11) $\partial\theta^*/\partial \mathrm{n} = 2\,[\mathrm{n}(4+3\mathrm{n})-\mathrm{m}]/[\mathrm{m}(1+\mathrm{n})+(4+6\mathrm{n}+\mathrm{n}^2)]^2 \gtreqless 0$

Proposition 2.

(a) The optimal degree of privatization θ^* *is in* $(0,1)$. *This result holds not only for* $m \geq 1,\ n \geq 1$, *but also holds for either* $m = 0$ *or* $n = 0$.

(b) If the entry of domestic private firms is advancing, a welfare-maximizing government should raise the degree of privatization of its public firm.

(c) If the government is allowing the entry of foreign private firms, then it should raise (lower) the degree of privatization, when the number of its domestic private firms is small (sufficiently large), given an appropriate number of foreign private firms.

Part (a) describes that neither full nationalization nor full privatization is optimal in mixed oligopoly. This result holds not only for the market where a public firm competes with both domestic and foreign private firms, but also for the market where a public firm competes with domestic or foreign private firms only. The optimal degree of privatization exists because accelerating the privatization has marginal costs and marginal benefits on welfare. Specifically, an addition of θ raises the price and thus reduces the consumer surplus (the marginal cost on welfare), while it increases the profits of both domestic private firms and the privatized public firms (two ways of the marginal benefits on welfare).[13] Furthermore, the marginal cost and the combined

[13] It can be conformed that the profit function of the privatized public firm is concave in θ. Note that in the first stage, the government does not choose a pretty large θ that causes the firm's profit

marginal benefits are decreasing in θ.[14]

Now, the economic intuition for part (b) and (c) can be explained as follows. Adding a new domestic private firm shifts the curve of the marginal cost downward (see Eq. (A.2) in Appendix), it also shifts the curve of the combined marginal benefits inward or outward, too (Eqs. (A.1) and (A.3)). In the case of the outward-shift of the marginal benefits curve, the optimal degree of privatization exactly increases. In the other case of the inward-shift of the marginal benefit curve, the optimal degree also increases since the downward-shift of the marginal cost curve is relatively large. Then, the result of part (b) holds. On the other hand, adding a new foreign private firm basically shifts the curve of the marginal cost upward (Eq. (A.5)), it also shifts the curve of the combined marginal benefits inward or outward (Eqs. (A.4) and (A.6)). If the number of domestic private firms *m* is sufficiently large, then the upward-shift of the marginal cost curve is relative large. This leads the optimal degree decrease and the result of part (c) holds.

5. Partial privatization and foreign acquisition of domestic firms

In this section, let us first examine the economic effects of foreign acquisition of a domestic private firm, and then discuss the issue of partial privatization and the foreign acquisition. Given the degree of privatization θ, we compare the equilibrium outcomes before and after a domestic firm is acquired by foreign nationals. By these comparisons we find that the effects on outputs, all private firms' profits, consumer surplus and social welfare are exactly the same as Fjell and Pal (1996, proposition 3), except the effect on

---

turn into decreasing.

[14] Note that if *n* is sufficiently large to *m*, then the marginal benefit from the increases of domestic private firms' profits only, is increasing in θ.

the profit of the privatized public firm.[15] Here we have the following lemma:

Lemma 2. Given appropriate numbers of domestic and foreign private firms, foreign acquisition of a domestic private firm increases the privatized public firm's profits if the degree of privatization is sufficiently large.

Proof:

Denote the profit of the privatized public firm after the acquisition as π_0^A, then we have

(12) $$\pi_0^A - \pi_0^* = [(1-\theta)(1+\theta)a^2]\left[\alpha + \beta\theta + \gamma\theta^2\right]/\{2\Delta^2[\Delta + (1-\theta)]^2\},$$

$$\alpha = (-)(52 + 16m + 64n + 20mn + m^2 + 27n^2 + 6mn^2 + 2m^2n + 4n^3)$$

$$\beta = (+)(32 + 20m + 68n + 20mn + 4m^2 + 32n^2 + 4mn^2 + 4n^3)$$

$$\gamma = (+)(20 + 20m + 20n + 16mn + 5m^2 + 3n^2 + 2mn^2 + 2m^2n)$$

While this comparison result cannot be unambiguously signed, simulation makes the pattern clear. The simulation results are shown in Table 1. Q.E.D.

####Table 1#### about here

Fjell and Pal (1996) has shown that foreign acquisition of a domestic private firm exactly decreases the profits of the state-owned public firm, which coincides with our study when $\theta = 0$. However, in our study with the consideration of partially privatization, the foreign acquisition increases the profits of the drastically privatized public firm. The economic intuition is as follows. When $\theta = 0$, reducing a domestic private firm increases the public firm's profit, while adding a new foreign private firm reduces its profits. The foreign acquisition reduces the public firm's profit since the

[15] All the comparisons have been done by the author. These presentations are not included in this paper for brief.

decrease is larger than the increase in the profit. However, when $\theta > 0$, reducing a domestic private firm exactly increases the privatized public firm's profit. Furthermore, as shown in proposition 1-(b), adding a new foreign private firm may increase its profits too.[16] Thus, the result of lemma 2 emerges.

Now, let as discuss the relationship between the optimal degree of privatization and the foreign acquisition. Let θ^A denotes the optimal degree of privatization after the foreign acquisition, then it can be shown easily that $\theta^A < \theta^*$. Here we have the following proposition:

Proposition 3. If a welfare-maximizing government is allowing foreign acquisition of domestic private firms, then it should lower the degree of privatization of its public firm.

The economic intuition can be provided based on the explanation for part (b) and (c) of proposition 2. Recall, that an addition of θ has marginal costs and marginal benefits (each of them depends on the two numbers of private firms *m* and *n*) on welfare. Due to the foreign acquisition, one of domestic private firm disappears, while one new foreign private firm appears. These two changes shift the curve of the combined marginal benefits inward or outward, however each of the change exactly shifts the curve of the marginal cost upward largely. Then, proposition 3 holds.

6. Conclusions

[16] Note that in Table 1, the larger m is (and the smaller n is), the specified range of θ where foreign acquisition increases the privatized public firm's profit becomes wider. This corresponds with the result of proposition 1-(b).

This paper introduced the analytical framework of partial privatization by Matsumura (1998) into the mixed oligopoly model by Fjell and Pal (1996) where foreign private firms exists, to re-examine the allocation of production as well as the effects of open-door policy and the effects of foreign acquisition of domestic firms. By taking the aspect of partial privatization of a stated-owned firm into account, the analytical results obtained in this paper, as comparison with corresponding results provided by Fjell and Pal, are different and are perhaps interesting. In addition, relying on this straightforward extension, the relations between partial privatization and foreign competition, which has not been investigated in the typical mixed oligopoly studies, has been clarified in this paper. These findings may provide some new policy implication on the literature.

First, without the consideration of privatization of a public firm, Fjell and Pal (1996) found that a new entry of foreign private firms exactly increases the public firm's output but reduce its profits, and the entry increases the home country's welfare if and only if the number of its domestic private firms is smaller than that of foreign private firms. In contrast, this present paper found that the entry may reduce the partially privatized public firm's output depending on the number of domestic private firms only, and the entry may increases the partially privatized public firm's profits depending on both the numbers of domestic and foreign private firms.

Second, we found that Fjell and Pal' result related with the effect of foreign firms' entry on welfare still holds, if the degree of privatization is relatively small. However, given a certain large degree of privatization, the entry may increases the home country's welfare even the number of its domestic private firms is larger than that of foreign private firms. Moreover, the results related with the effects of foreign acquisition in this paper are the same as Fjell and Pal's ones, except that the effect on the profits of the

public firm. That is, foreign acquisition of a domestic private firm exactly reduces the profits of a public firm fully owned by the state. However, the acquisition may increase the profits of a privatized public firm with high degree of privatization.

The finding on the socially optimal degree of privatization and the findings on the relations between the optimal degree and foreign competition may provide policy implications as follows. For a government is considering privatizing its public firm, “full privatization” is not optimal, when its public firm competes with either one single/plural or domestic/foreign private firms in the market. For a government is considering privatizing its public firm and also opening the market to both domestic and foreign privatize firm, if entry of its domestic firms is advancing, then the government should accelerate the privatization. However, if the government is allowing entry of foreign private firms, then it should accelerate (restrict) the privatization when the number of its domestic firm is small (sufficiently large), relative to a certain number of foreign private firms.

Finally, we discuss the limitations for this paper and suggest some directions for future research. First, similar with many previous literatures, the linear inverse demand function and identical cost functions are assumed for tractability. On the other hand, there is discussion on the cost asymmetry between the public firm and private firms on the literature (for example, Lee and Hwang, 2003, Matsumura and Matsushima 2004). The extension for the specification on demand function as well the symmetry on cost function remains for future research. Second, in the present paper firms were exogenously assumed to be Cournot-Nash behaviors in the second stage. The examination of a desirable role (either Stackelberg leader or follower) of a partially privatized public firm, furthermore the investigation of endogenous order of moves,

would be interesting directions for future study.

Appendix

In this Appendix, we present the results of the corresponding differentiations, those are useful for understanding the explanations for Proposition 1-(c), and Proposition 2-(b), (c).

(A.1) $\partial^2 \mathrm{m}\pi_i^* / \partial\theta\,\partial \mathrm{m} = (\mathrm{a}^2/\Delta^4)12(1+\mathrm{n})(1+\theta)[2+\mathrm{n}+\theta-\mathrm{m}(1+\theta)]$

Eq. (A.1) is negative, when *n* is small and *m* is sufficiently large.

(A.2) $\partial\left|\frac{\partial CS^*}{\partial\theta}\right| / \partial \mathrm{m} = (\mathrm{a}^2/\Delta^4)(-8)(1+\mathrm{n})(1+\theta)[(\mathrm{m}+2\mathrm{n}+\mathrm{m}\theta)+(1-\theta)] < 0$

(A.3) $\partial^2 \pi_0^* / \partial\theta\,\partial \mathrm{m} = (\mathrm{a}^2/\Delta^4)2(1+\mathrm{n}) \times \big[(-)(12+8\mathrm{n}+2\mathrm{mn}+\mathrm{n}^2) - \theta\big(8-4\mathrm{m}+$

$2\mathrm{n}2)+\theta 2(8+4\mathrm{m}+16\mathrm{n}+2\mathrm{mn}+3\mathrm{n}2 >< 0$

(A.4) $\partial^2 \mathrm{m}\pi_i^* / \partial\theta\,\partial \mathrm{n} = (\mathrm{a}^2/\Delta^4)(6\mathrm{m})(1+\theta)[(-2-4\mathrm{n})+\mathrm{m}+\theta(2+\mathrm{m})]$

Eq. (A.4) is positive, when *n* is small and *m* is sufficiently large.

(A.5) $\partial\left|\frac{\partial CS^*}{\partial\theta}\right| / \partial \mathrm{n} = (4\mathrm{a}^2/\Delta^4)\big[(4+2\mathrm{m}+\mathrm{m}^2-4\mathrm{n}^2)+\theta(8+4\mathrm{m}+8\mathrm{n}+2\mathrm{m}^2)+$

$\theta 2\mathrm{m}(2+\mathrm{m})$

Eq. (A.5) is positive, except that *n* is extremely large.

(A.6) $\partial^2 \pi_0^* / \partial\theta\,\partial \mathrm{n} = (2\,\mathrm{a}^2/\Delta^4)\big[(8+12\mathrm{m}+8\mathrm{n}+14\mathrm{mn}+\mathrm{m}^2+2\mathrm{n}^2+2\mathrm{m}^2\mathrm{n}+$

$\mathrm{mn}2+\theta 4\mathrm{n}+4\mathrm{mn}-2\mathrm{m}2+8\mathrm{n}2+2\mathrm{mn}2)-\theta 2(20+16\mathrm{m}+28\mathrm{n}+$

$18\mathrm{mn}+3\mathrm{m}2+6\mathrm{n}2+2\mathrm{m}2\mathrm{n}+3\mathrm{mn}2 >< 0.$

References:

Barcena-Ruiz, J.C., Garzon, M.B., 2005. International trade and strategic privatization. Review of Development Economics 9, 502-513.

Chang, W.W., 2005. Optimal trade and privatization policies in an international duopoly

with cost asymmetry. Journal of International trade and Economic Development 14, 19-42.

Chang, W.W., 2007. Optimal trade, industrial, and privatization policies in a mixed duopoly with strategic managerial incentives. Journal of International trade and Economic Development 16 (1), 31-.

Chao, C-C., Yu, E.S.H., 2006. Partial privatization, foreign competition, and optimum tariff. Review of International Economics 14(1), 87-92.

Dadpay, A., Heywood, J.S., 2006. Mixed oligopoly in a single international market. Australian Economic Papers 45(4), 269-280.

De Fraja G., Delbono, F., 1989. Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41, 302-311.

Fjell, K., Heywood, J.S., 2002. Public Stackelberg leadership in a mixed oligopoly with foreign firms. Australian Economic Papers 41, 267-281.

Fjell, K., Pal, D., 1996. A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 29, 737-743.

Han, L.H., Ogawa, H., 2007. Partial privatization and market opening policies: A mixed oligopoly approach. Economic Research Center (Nagoya University), Discussion paper, No.E07-3.

Lee, S-H., Hwang, H-S, 2003. Partial ownership for the public firm and competition. The Japanese Economic Review 54(3), 324-335.

Matsumura, T., 2003. Stackelberg mixed duopoly with a foreign competitor. Bulletin of Economic Research 55, 275-287.

Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 70, 473-483.

Matsumura, T., Matsushima, N., 2004. Endogenous cost differentials between public and private enterprises: A mixed duopoly approach. Economica 71, 671-688.

Pal, D., While, M.D., 1998. Mixed oligopoly, privatization, and strategic trade policy. Southern Economic Journal 65(2), 264-281.

Figure 1.

Region for a foreign entry decreases the partially privatized public firm's output

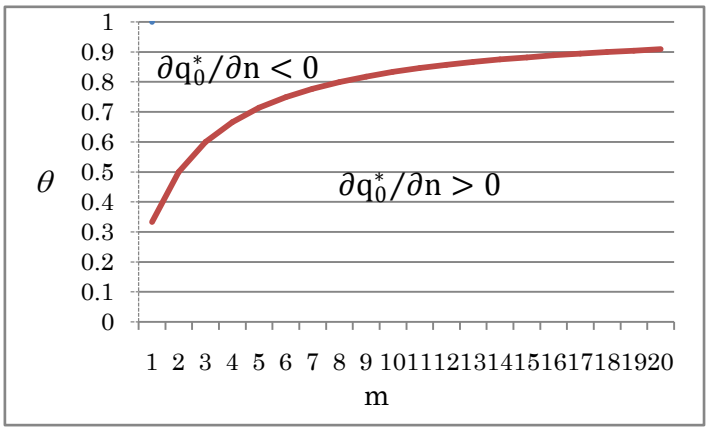

Figure 2.

Regions for a foreign entry increases the partially privatized public firm's profits

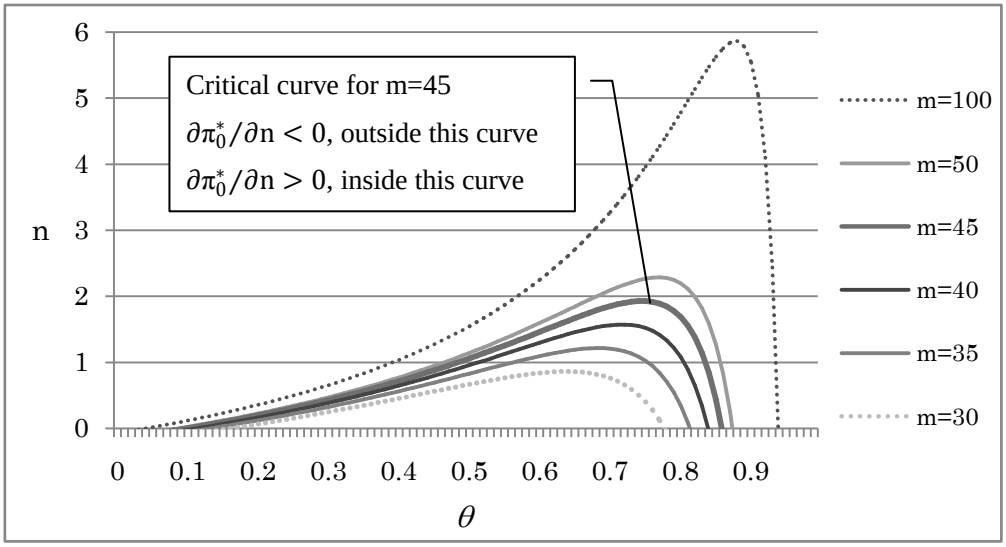

Figure 3.

Regions for a foreign entry increase the home country's social welfare

(example for n=8)

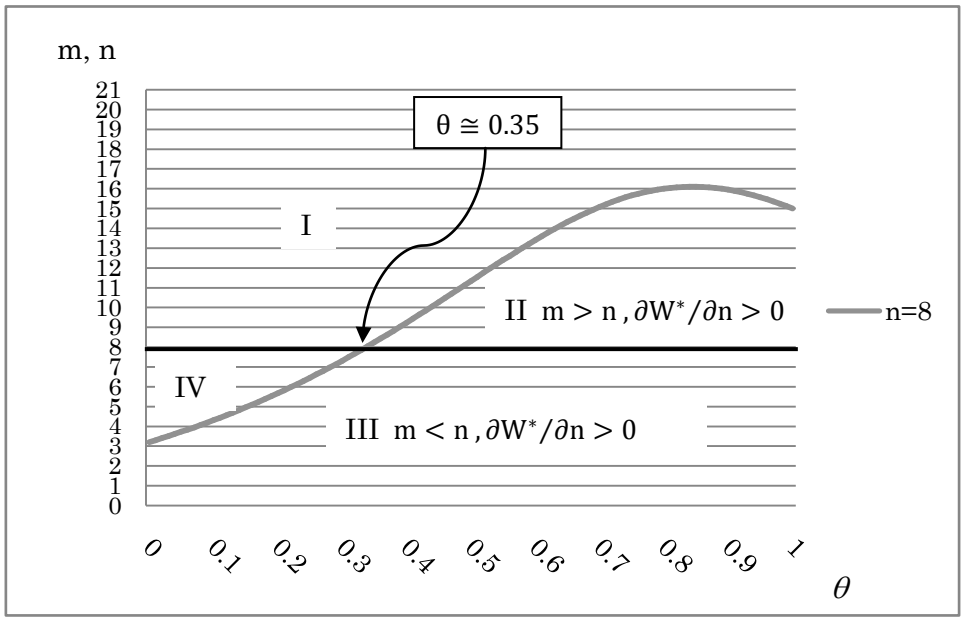

Figure 4.

Regions for a foreign entry increase the home country's social welfare.

(example for various *n*)

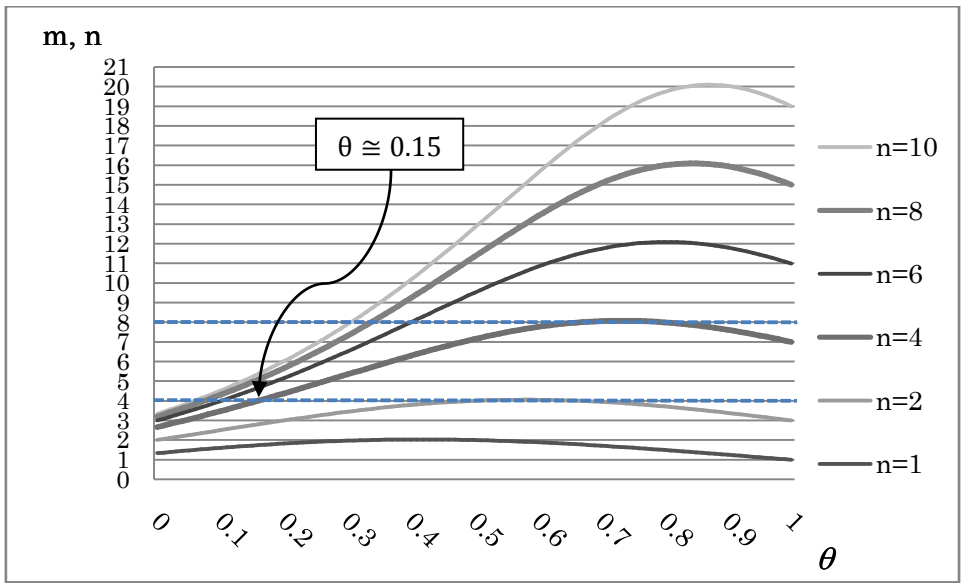

Table 1. The effect of a foreign acquisition on the privatized public firm's profit

M/N	1	2	3	4	5	6	7
1	+ for θ>0.76	+ for θ>0.78	+ for θ>0.80	+ for θ>0.82	+ for θ>0.83	+ for θ>0.84	+ for θ>0.85
2	+ for θ>0.71	+ for θ>0.75	+ for θ>0.78	+ for θ>0.80	+ for θ>0.82	+ for θ>0.84	+ for θ>0.85
3	+ for θ>0.67	+ for θ>0.73	+ for θ>0.76	+ for θ>0.79	+ for θ>0.81	+ for θ>0.83	+ for θ>0.85
4	+ for θ>0.64	+ for θ>0.71	+ for θ>0.75	+ for θ>0.78	+ for θ>0.80	+ for θ>0.82	+ for θ>0.84
5	+ for θ>0.61	+ for θ>0.69	+ for θ>0.74	+ for θ>0.78	+ for θ>0.80	+ for θ>0.82	+ for θ>0.84
6	+ for θ>0.59	+ for θ>0.68	+ for θ>0.73	+ for θ>0.77	+ for θ>0.80	+ for θ>0.82	+ for θ>0.83
7	+ for θ>0.58	+ for θ>0.67	+ for θ>0.73	+ for θ>0.76	+ for θ>0.79	+ for θ>0.81	+ for θ>0.83
8	+ for θ>0.57	+ for θ>0.67	+ for θ>0.72	+ for θ>0.76	+ for θ>0.79	+ for θ>0.81	+ for θ>0.83
9	+ for θ>0.55	+ for θ>0.65	+ for θ>0.71	+ for θ>0.76	+ for θ>0.79	+ for θ>0.80	+ for θ>0.83
10	+ for θ>0.55	+ for θ>0.65	+ for θ>0.71	+ for θ>0.75	+ for θ>0.78	+ for θ>0.81	+ for θ>0.83

Note: + foreign acquisition increases the privatized public firm's profit within the ranges specified.