Network externalities between carriers or machines: how they work in the smartphone industry *

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Abstract

In this paper, we consider a duopoly model where two firms sell two differentiated products and there is a network externality between either carriers or machines. We derive the equilibria of these games and illustrate the effects of a change in quality and in strength of network externality on the equilibrium quantity of each good. Furthermore, I analyze the effect of the change in the production cost for high-quality goods on carriers' equlibrium profit.Then, I find that two kinds of network externalities work differently in equlibrium and should be distinct.

Keywords: Smartphone market, Multi-product firm, Duopoly, Cannibalization, Network externality

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1 Introduction

Over the last decade, mobile phones have spread rapidly in many developed countries. In the market for traditional mobile phones, there is just one network externality (network effect), as has been recognized since the seminal work of Katz and Shapiro (1985).[1]

In addition to these standard mobile phones, smartphones, for example, the *iPhone* from Apple, have recently increased their share and importance in our daily lives. Figure 1, for example, illustrates the market for smartphones in Japan.

Insert Figure 1 here.

One notable property of the smartphone market that differs from the market for standard mobile phones is that it contains the following two externalities.

First, there is a network externality between carriers that has been considered in the existing literature, such as Katz and Shapiro (1985) and Chen and Chen (2011). According to this externality, a consumer who purchases a product or service from a certain firm gains a network benefit when other consumers purchase the same or different product or service from the same firm. In Japan, for example, there are three major carriers, NTT DoCoMo, KDDI, and Softbank, all of which provide some special services that are mutually beneficial for their respective customers.

Second, we should recognize the existence of another important network externality between distinct types of smartphones supplied *to different carriers* by the same producer of smartphone devices.[2]In the real world, for instance, a customer of a carrier who has

[1]In Belleflamme and Peitz (2011, p.549), network effects has been formally defined as follows: "A product is said to exhibit network effects if each user's utility is increasing in the number of other users of that product or products compatible with it."

[2]In Kitamura (2013), I define this externality as follows: "A consumer who purchases a product from a certain firm gains a network benefit when other consumers purchase the same product from the same or different firm."

Apple's *iPhone* gains a network benefit when the number of *iPhone* users increases, even when these users are customers of other carriers. This network benefit takes the form of enhancement of reputation about the *iPhone*, or an increase in complementary goods, such as application software for the *iPhone*. Thus, even if consumers who use the *iPhone* do not use the same carrier, all consumers gain a network benefit from the increase in the number of *iPhone* users. To the best of our knowledge, this externality has received no attention in the previous studies that consider network externality.

In order to analyze such a market, one has to consider the idea of cannibalization. Cannibalization means that a company reduces the sales of one of its products by introducing a similar, competing product in the same market. Although Katz and Shapiro (1985) and Chen and Chen (2011) analyze the oligopolistic market in which each firm supplies a single product, considering the real economy, there are oligopolistic markets in which each firm produces and sells multiple products that are differentiated vertically in the same market. From each consumer's point of view, the quality of technology that each firm uses to produce its goods is different. Therefore, each consumer places different values on the high-quality goods of each firm. An example of this type of market is the "beer-like" beverage market that emerged in Japan in 1994. This market is composed of beer and *happoshu* or *low-malt beer. (*Happoshu) or low-malt beer is a tax category of Japanese liquor that most often refers to a beer-like beverage with less than 67% malt content. In the Japanese alcoholic-beverage tax system, lower tax is imposed on low-malt beer than "beer" with more than 67% malt content. Consequently, the market price of the former is lower than that of the latter. Therefore, leading makers such as Kirin, Asahi, and Sapporo Breweries sell beer and low-malt beer brands in the same beer-like beverage market. This market is not only horizontally but also vertically differentiated. Similarly, multi-product firms (abbreviated as "MPFs" hereafter) exist in the smartphone market. For example, in Japan, both KDDI and Softbank supply Apple's *iPhone* and Google's

Android smartphone. Although only Softbank supplied the *iPhone* initially, KDDI has also adopted it recently.

Haruvy and Prasad (1998), a study closely related to mine, analyzed a market in which a monopolist sells a high-end and low-end version of the same product. The authors find some conditions under which producing both goods is optimal in the market with network externality. However, although each firm produces two differentiated goods, the two goods are sold in different markets, each with different types of consumers. In our model, we assume that both goods are supplied to the *same* market.

Furthermore, the *iPhone* is made by only Apple (that is, vertical integration), but *Google's* smartphones are made by many different producers. That is, *Google* only supplies the *Android* platform, and when the platform is updated, each producer must fix the programming of their product to apply the new platform programming. So, *Android* smartphones have more bugs, as compared with Apple's *iPhone*. Therefore, even in the smartphone market, there may exist vertical differentiation in quality.[3] Thus, in the real world, there may be many MPFs that differentiate their goods not only horizontally but also in quality, in the same market.

The remainder of this paper is organized in the following manner. Section 2 presents the model. Sections 3 prove and discuss the main results. Section 4 provides the conclusion.

[3] Another example of vertical differentiation in this industry is confirmed by the following outcome of Geekbench (the first URL is for the iPhone and the second for Android smartphones). This shows that the iPhone and Android smartphones differ in quality.
URL: http://browser.primatelabs.com/geekbench2/1030202
URL: http://browser.primatelabs.com/android-benchmarks

2 The model

In this section, I analyze a intermediaries' business model in the smartphone service industry with two kinds of network externalities. In this study, I focus on a dealer intermediation; that is, a carrier acts as a dealer who buys the goods from the smartphone producers and sells them to the consumers. In particular, I consider here the case that each carrier has strong market power and it can choose not only from which smartphone producer it gets in from them, but the quantity and unit sales commission of each device. To pay attention to the externality between machines or devices, I omit carriers' phone services charges, because carriers in Japan charge their customers a fixed communication services fee, including an installment plan for the smartphone.[4]

Suppose there are two carriers, $(i = 1, 2)$ and two differentiated smartphone producers, each producing one kind of goods (good H and good L) that differ in terms of quality. Let V_H and V_L denote the quality level of the two goods. Then, the maximum amount consumers are willing to pay for each good is assumed to be $V_H > V_L > 0$. Further, we assume $V_H = (1 + \mu)V_L$, where μ represents the difference in quality between the two goods. For simplicity, we normalize the quality of the low-quality good as $V_L = 1$. Good $\alpha(= H, L)$ is assumed to be homogeneous for any consumer. Moreover, suppose that each firm has constant returns to scale and that $c_H > c_L$, where c_α is the marginal cost of good α. This implies that a high-quality good incurs a higher cost of production than a low-quality good. Without loss of generality, we also assume that $c_L = 0$. For simplicity, suppose that each firm has no production and fixed costs. Further, both carriers can decided unit sales commission P_α for smartphone producers and their revenue(profit) for one unit of product is only from it. Under these assumptions, each carriers' profit is

[4]Smartphone users of any carrier can practically use the call service free of charge by using the software, "Line."

defined as follows:

$$\Pi_i = P_{iH}x_{iH} + P_{iL}x_{iL} \qquad i = 1, 2, \tag{1}$$

where $x_{i\alpha}$ is the supply of product α sold by carrier i. Each carrier chooses $P_{i\alpha}$, its unit sales commission for product α, and $x_{i\alpha}$, its supply quantity of product α that maximizes this profit function, so that smartphone producers enter the smartphone market and supply their product with consumers through the carrier itself.. The profit of smartphone producer α is given by

$$\pi_\alpha = \sum_{i=1}^{2}(p_{i\alpha} - P_{i\alpha} - c_\alpha)x_{i\alpha} \qquad i = 1, 2 \;\; \alpha = H, L \tag{2}$$

where, $p_{i\alpha}$ is a retail price of good α sold by carrier i.

Now, we describe the consumers' behavior in our model.

Following the standard specification in the literature–for example, Katz and Shapiro (1985)–we assume that there is a continuum of consumers that is characterized by a taste parameter θ that is uniformly distributed between $-\infty$ and $r > 0$ with density 1. It is assumed that a consumer of type $\theta \in (-\infty, r], r > 0$ obtains a net surplus from one unit of good α of firm i at price $p_{i\alpha}$. Furthermore, we assume that there exists *a network externality between carriers* or *a network externality between machines*. The former implies that a consumer who purchases a product or service from a certain firm gains a network benefit when other consumers purchase the same or different product from the same firm. We define the latter externality as follows: A consumer who purchases a product from a certain firm gains a network benefit when other consumers purchase the same product, regardless of its carrier.

Then, the surplus of the consumer θ who buys good α $(= H, L)$ from carrier i $(= 1, 2)$

is given by[5]

$$U_{i\alpha}(\theta) = V_\alpha\theta + \nu V_\alpha g^e_{i\alpha} - p_{i\alpha}, \qquad i = 1, 2, \quad \alpha = H, L, \tag{3}$$

where ν represents the strength of the network externality. $g^e_{i\alpha}$ is the expectation of network benefit that a consumer obtains by purchasing one unit of good α from firm i. More precisely, we assume that the function $g^e_{i\alpha}(\centerdot)$ is linear and define $g^e_{i\alpha}$ as follows in the two cases of network externality:

- Network externality between carriers

$$\begin{aligned} g^e_{i\alpha} &\equiv g_{i\alpha}(x^e_{iH}, x^e_{iL}, x^e_{jH}, x^e_{jL}, \phi_c) \\ &= x^e_{iH} + x^e_{iL} + \phi_c(x^e_{jH} + x^e_{jL}) \\ &= X^e_i + \phi_c X^e_j, \quad i,\ j = 1, 2, i \neq j, \alpha = H, L. \end{aligned} \tag{4}$$

  Here, $X^e_i = x^e_{iH} + x^e_{iL}$ and ϕ_c is the degree of compatibility between carriers.

- Network externality between machines or devices

$$\begin{aligned} g^e_{i\alpha} &\equiv g_{i\alpha}(x^e_{iH}, x^e_{iL}, x^e_{jH}, x^e_{jL}, \phi_m) \\ &= x^e_{1\alpha} + x^e_{2\alpha} + \phi_m(x^e_{1\beta} + x^e_{2\beta}) \\ &= X^e_\alpha + \phi_m X^e_\beta, \quad \alpha,\ \beta = H, L, \alpha \neq \beta, i = 1, 2. \end{aligned} \tag{5}$$

  Here, $X^e_\alpha = x^e_{1\alpha} + x^e_{2\alpha}$ and ϕ_m is the degree of compatibility between machines.

[5] This surplus is modeled similarly to Baake and Boom (2001).

For simplicity, we assume that the parameter of the degree of compatibility in both cases, $\phi_\delta \in \{0, 1\}(\delta = c, m)$ takes just 0 or 1. Thus, when the value of each parameter is 0 (1), it implies that consumers are incompatible (compatible) in each case.

We do not explicitly model the process through which consumers' expectations are formed. However, we impose the requirement that in equilibrium, consumers' expectations are fulfilled. That is, we assume the following *fulfilled expectations Cournot equilibrium*: when consumers form rational expectations, in equilibrium, the consumers' expected quantity is equal to actual quantity. Each firm chooses its output level under the following assumptions:

(a) Consumers' expectations about the size of networks are given.

(b) The actual output level of the other firm is fixed.

Assumption (b) is the standard Cournot assumption. Assumption (a) implies that in this model, the firms are unable to commit themselves, so that only the output levels of the *fulfilled expectations Cournot equilibrium* are credible announcements.

Furthermore, we assume that consumers must make their purchase decisions before the actual network sizes are known. Thus, the timing of the game is as follows.

1st Stage: Consumers form expectations about the size of the network with which each firm is associated.
2nd Stage: The firms decide transaction fee and output of two kinds of goods, taking consumers' expectations as given. This game generates a set of prices. Consumers then make their purchase decisions by comparing their reservation prices with the prices set by the two firms ($i = 1, 2$). and both smartphone producers make a decision to either entry or quit this market.

Each consumer determines to buy nothing, or one unit of the good α, from firm i to maximize his/her surplus.

Before deriving the inverse demand of each good, we assume that for an arbitrary type-θ_α consumer,

$$U_{1\alpha}(\theta_\alpha) = U_{2\alpha}(\theta_\alpha), \quad \alpha = H, L. \tag{6}$$

This assumption states that the net surplus from buying the good from firm 1 or firm 2 must be equal, as long as the two firms produce the good with the same quality and have positive sales. From (3) and (6), we obtain

$$\begin{aligned} V_\alpha\hat{\theta}_\alpha + \nu V_\alpha g^e_{1\alpha} - p_{1\alpha} &= V_\alpha\hat{\theta}_\alpha + \nu V_\alpha g^e_{2\alpha} - p_{2\alpha} \\ \Longleftrightarrow \quad p_{1\alpha} - \nu V_\alpha g^e_{1\alpha} &= p_{2\alpha} - \nu V_\alpha g^e_{2\alpha}. \end{aligned} \tag{7}$$

Here, $p_{1\alpha} - \nu V_\alpha g^e_{1\alpha} = p_{2\alpha} - \nu V_\alpha g^e_{2\alpha}$ is *the expected hedonic price* of brand α, that is, the price adjusted for the network size. This hedonic price is used by Katz and Shapiro (1985). Thus, I may let

$$p_\alpha \equiv p_{1\alpha} - \nu V_\alpha g^e_{1\alpha} = p_{2\alpha} - \nu V_\alpha g^e_{2\alpha}, \quad \alpha = H, L. \tag{8}$$

I assume that there exists a consumer who is indifferent between the two goods of the same firm. This consumer's type is denoted by $\hat{\theta}_i$. Then, we have

$$U_{iH}\left(\hat{\theta}_i\right) = U_{iL}\left(\hat{\theta}_i\right) > 0 \tag{9}$$

$$\begin{aligned} &\Longleftrightarrow \quad (1+\mu)\hat{\theta}_i + \nu(1+\mu)g^e_{iH} - p_{iH} = \hat{\theta}_i + \nu g^e_{iL} - p_{iL} \\ &\Longleftrightarrow \quad \hat{\theta}_i = \frac{1}{\mu}\{p_{iH} - p_{iL} - (\nu(1+\mu)g^e_{iH} - \nu g^e_{iL})\} \quad i = 1, 2. \end{aligned} \tag{10}$$

Equations (8) and (10) yield

$$\hat{\theta}_1 = \frac{1}{\mu}\{p_{1H} - p_{1L} - (\nu(1+\mu)g^e_{1H} - \nu g^e_{1L})\} = \frac{1}{\mu}\{p_{2H} - p_{2L} - (\nu(1+\mu)g^e_{2H} - \nu g^e_{2L})\} = \hat{\theta}_2,$$

and therefore,

$$\hat{\theta}_1 = \hat{\theta}_2.$$

So I may let

$$\hat{\theta} \equiv \hat{\theta}_i \quad i = 1, 2. \tag{11}$$

Furthermore, as in the preceding chapter, we suppose that there exists a type of consumer $\underline{\theta}_L$, who is indifferent between purchasing good L and purchasing nothing. Then, the following equation holds:

$$\begin{aligned} U_{iL}\left(\underline{\theta}_L\right) = U_{2L}\left(\underline{\theta}_L\right) = 0 \\ \Leftrightarrow \ \underline{\theta}_L = p_{iL} - \nu g^e_{iL}. \end{aligned} \tag{12}$$

Then, from (3), (9), (12) and the increasing function of $U_{iL}(\cdot)$, we see that

$$U_{iH}(\hat{\theta}) = U_{iL}(\hat{\theta}) > U_{1L}(\underline{\theta}_L) = U_{2L}(\underline{\theta}_L) = 0.$$

So, equivalently we have

$$\hat{\theta} > \underline{\theta}_L. \tag{13}$$

Thus, I obtain the next lemma[6].

Lemma 1. *Any consumer $\theta \in (-\infty, \underline{\theta}_L)$ buys nothing, consumer $\theta \in (\ \underline{\theta}_L, \widehat{\theta})$ $(\theta \in [\widehat{\theta}, r])$* buys *good L (good H), respectively.*

[6]For proof of the lemma, see Appendix.

From Lemma 1, we obtain the following system of equations:

$$\begin{cases} r - \hat{\theta} = X_H \\ r - \underline{\theta}_L = X_H + X_L \equiv x_{1H} + x_{2H} + x_{1L} + x_{2L}, \end{cases} \tag{14}$$

where $X_\alpha = x_{1\alpha} + x_{2\alpha}, \alpha = H, L$.

Substituting (10) and (12) into these equations and solving them for p_{iH} and p_{iL}, the inverse demand functions are obtained as

$$\begin{cases} p_{iH} = (1+\mu)(r + \nu g^e_{iH} - X_H) - X_L \\ p_{iL} = r - X_H - X_L + \nu g^e_{iL}. \end{cases} \tag{15}$$

For smartphone producers, they decide to entry this market as long as $\pi_\alpha \geq 0$. Thus, carriers must set $P_{i\alpha}$ as

$$\pi_\alpha = \sum_{i=1}^{2} (p_{i\alpha} - P_{i\alpha} - c_\alpha) x_{i\alpha} = 0 \qquad i = 1, 2 \;\; \alpha = H, L. \tag{16}$$

Where, I assume that in equilibrium, $p_{i\alpha} - P_{i\alpha} - c_\alpha \geq 0$ and $x_{i\alpha} \geq 0$. Then from (16), I obtain following condition;

$$p_{i\alpha} - P_{i\alpha} - c_\alpha = 0 \tag{17}$$

Substituting (15) into this equation,

$$\begin{cases} (1+\mu)(r + \nu g^e_{iH} - X_H) - X_L - c_H = P_{iH} \\ r - X_H - X_L + \nu g^e_{iL} = P_{iL} \end{cases} \tag{18}$$

Thus, in 2nd stage, carriers' profit function is as follows;

$$\Pi_i = \{(1+\mu)(r+\nu g^e_{iH} - X_H) - X_L - c_H\}x_{iH} + \{r - X_H - X_L + \nu g^e_{iL}\}x_{iL} \qquad i = 1, 2. \tag{19}$$

Both carriers decide to quantity of two goods H and L to maximize this profit function.

2.1 Derivation of Equilibrium

To maximize the profit function, each firm determines each quantity x_{iH} and x_{iL}, given consumers' expectations,

$$\max_{x_{iH}, x_{iL}} \Pi_i.$$

The first-order conditions for profit maximization[7] are

$$\begin{cases} \frac{\partial \Pi_i}{\partial x_{iH}} = -(1+\mu)x_{iH} + (1+\mu)(r+\nu g^e_{iH} - X_H) - X_L - x_{iL} - c_H = 0 \\ \frac{\partial \Pi_i}{\partial x_{iL}} = -x_{iH} - x_{iL} + r + \nu g^e_{1L} - X_H - X_L = 0, \qquad i = 1, 2. \end{cases} \tag{20}$$

Furthermore, to guarantee positive quantities and downward-sloping demand in all situations, we assume that

$$0 < \nu < 1 \quad and \quad \frac{2\nu(1+\mu)r}{3} < c_H < \frac{(3\mu - 2\nu - 2\nu\mu)r}{3 - 2\nu}. \tag{21}$$

From the first-order condition (20), we have the following reaction functions for $x_{iH}$and

[7]Note that I look $g^e_{i\alpha}$ upon as constant when I derive the first order condition, but in equlibrium $g^e_{i\alpha}(x^e_{iH}, x^e_{iL}, x^e_{jH}, x^e_{jL}, \phi_\delta) = g_{i\alpha}(x^e_{iH}, x^e_{iL}, x^e_{jH}, x^e_{jL}, \phi_\delta)$ from fulfilled expectation assumption.

x_{iL}:[8]

$$x_{iH} = -\frac{3-\nu(1+\mu)\frac{\partial g_{iH}}{\partial x_{iL}}}{(1+\mu)(3-\nu\frac{\partial g_{iH}}{\partial x_{iH}})} x_{iL} + \frac{(1+\mu)r - c_H}{(1+\mu)(3-\nu\frac{\partial g_{iH}}{\partial x_{iH}})}, \tag{22}$$

$$x_{iL} = -\frac{3-\nu\frac{\partial g_{iL}}{\partial x_{iH}}}{3-\frac{\nu\partial g_{iL}}{\partial x_{iL}}} x_{iH} + \frac{r}{3-\nu\frac{\partial g_{iL}}{\partial x_{iL}}} \ . \tag{23}$$

- Case 1 (Network externality between carriers)

  In this case of a network externality between carriers, we consider two extreme settings: $\phi_c = 0$ and $\phi_c = 1$.

  - Case of full compatibility ($\phi_c = 1$)

    From (4) and the assumption of fulfilled expectations, in equilibrium we have

$$g_{i\alpha} = X_i + X_j, \ i, j = 1, 2, i \neq j. \tag{24}$$

    Thus, from the first-order conditions (20),

$$\begin{cases} x_{iH}^{*CF} = \frac{r}{3-2\nu} - \frac{c_H}{3\mu} \\ x_{iL}^{*CF} = \frac{c_H}{3\mu}. \end{cases} \tag{25}$$

    Then, the equilibrium price is determined as follows:

$$\begin{cases} P_H^{*CF} = \frac{r(1+\mu)}{3-2\nu} - \frac{c_H}{3}. \\ P_L^{*CF} = \frac{r}{3-2\nu}. \end{cases} \tag{26}$$

[8]Then, we solve these reaction functions given by $x_{1\alpha} = x_{2\alpha}$. Furthermore, $\frac{\partial g_{i\alpha}}{\partial x_{i\alpha}}, \frac{\partial g_{i\alpha}}{\partial x_{i\beta}}$ don't indicate usual partial derivatives of network externality function $g_{i\alpha}^e$ in (4), (5). From the fulfilled expectation assumption, in equilibrium, $g_{i\alpha}^e(x_{iH}^e, x_{iL}^e, x_{jH}^e, x_{jL}^e, \phi_\delta) = g_{i\alpha}(x_{iH}, x_{iL}, x_{jH}, x_{jL}, \phi_\delta)$. So $\frac{\partial g_{i\alpha}}{\partial x_{i\alpha}}, \frac{\partial g_{i\alpha}}{\partial x_{i\beta}}$ imply the partial derivatives of these function $g_{i\alpha}(x_{iH}, x_{iL}, x_{jH}, x_{jL}, \phi_\delta)$, thus $\frac{\partial g_{i\alpha}}{\partial x_{i\beta}} = \phi_c$ or ϕ_m.

From (25) and (26), carriers' profit is obtained as follows;

$$\Pi_i = \frac{9\mu(1+\mu)r^2 - 6(3-2\nu)\mu c_H r + (3-2\nu)^2 c_H^2}{9\mu(3-2\nu)^2}. \tag{27}$$

– Case of incompatibility ($\phi_c = 0$)

From (4) and the assumption of fulfilled expectations, in equilibrium we have

$$g_{i\alpha} = X_i, \ \ i, = 1, 2, . \tag{28}$$

Thus, from the first-order conditions (20),

$$\begin{cases} x_{iH}^{*CI} = \frac{r}{3-\nu} - \frac{c_H}{3\mu} \\ x_{iL}^{*CI} = \frac{c_H}{3\mu}. \end{cases} \tag{29}$$

This leads to the following equilibrium price:

$$\begin{cases} P_H^{*CI} = \frac{r(1+\mu)}{3-\nu} - \frac{c_H}{3} \\ P_L^{*CI} = \frac{r}{3-\nu}. \end{cases} \tag{30}$$

From (29) and (30), carriers' profit is obtained as follows;

$$\Pi_i = \frac{9\mu(1+\mu)r^2 - 6(3-\nu)\mu c_H r + (3-\nu)^2 c_H^2}{9\mu(3-\nu)^2}. \tag{31}$$

- Case 2 (Network externality between machines or devices)

  As with Case 1, the following two settings can be considered.

  – Case of full compatibility ($\phi_m = 1$)

In equilibrium, we obtain

$$g_{i\alpha} = X_H + X_L, \ . \tag{32}$$

Thus, from the first-order conditions (20),

$$\begin{cases} x_{iH}^{*MF} = \frac{r}{3-2\nu} - \frac{c_H}{3\mu} \\ x_{iL}^{*MF} = \frac{c_H}{3\mu}. \end{cases} \tag{33}$$

Thus, the equilibrium price of good H is the same as in equation (26), that is,

$$\begin{cases} P_H^{*MF} = \frac{r(1+\mu)}{3-2\nu} - \frac{c_H}{3}. \\ P_L^{*MF} = \frac{r}{3-2\nu}. \end{cases} \tag{34}$$

From (33) and (34), carriers' profit is obtained as follows;

$$\Pi_i = \frac{9\mu(1+\mu)r^2 - 6(3-2\nu)\mu c_H r + (3-2\nu)^2 c_H^2}{9\mu(3-2\nu)^2}. \tag{35}$$

– Case of incompatibility ($\phi_m = 0$)

Similarly, from fulfilled expectation assumption, we have

$$g_{i\alpha} = X_\alpha, \ \alpha, = H, L. \tag{36}$$

Where, I assume that the equilibrium of this cournot game is stable;

$$\begin{aligned} &(1+\mu)(3-2\nu)^2 - 9 > 0 \qquad (37) \\ \Longleftrightarrow\ &0 < \nu < \frac{3(\sqrt{1+\mu}-1)}{2(\sqrt{1+\mu})}. \end{aligned}$$

From the first-order conditions (20),

$$\begin{cases} x_{iH}^{*MI} &= \frac{(3-2\nu)\{(1+\mu)r-c_H\}-3r}{(1+\mu)(3-2\nu)^2-9} \\ x_{iL}^{*MI} &= \frac{-2(1+\mu)\nu r+3c_H}{(1+\mu)(3-2\nu)^2-9}. \end{cases} \tag{38}$$

The equilibrium prices are

$$\begin{cases} p_H^{*IM} &= \frac{r(1+\mu)(3\mu-4\nu-2\mu\nu)-(3\mu-2\nu-2\mu\nu)c_H}{(1+\mu)(3-2\nu)^2-9} \\ p_L^{*IM} &= \frac{r(3\mu-4\nu-4\mu\nu)+2\nu c_H}{(1+\mu)(3-2\nu)^2-9}. \end{cases} \tag{39}$$

From (38) and (39), carriers' equilibrium profit is obtained as follows;

$$\Pi_i = \frac{(1+\mu)\{\mu Z+4\nu(4\nu+4\mu\nu-3\mu)\}r^2-2\{\mu Z+8\nu^2(1+\mu)\}c_H r+(Z+12\nu)c_H^2}{Z^2} \tag{40}$$

where, $Z=(1+\mu)(3-2\nu)^2-9$.

3 Comparative Statics

In this section, I analyze the comparative statics for quantity and carriers' profit.

The effects of an increase in the quality of the high-quality good on each quantity can be confirmed as follows:

$$\begin{cases} \frac{\partial x_{iH}^{*CF}}{\partial\mu} = \frac{\partial x_{iH}^{*CI}}{\partial\mu} = \frac{\partial x_{iH}^{*MF}}{\partial\mu} = \frac{c_H}{3\mu^2} > 0 \\ \frac{\partial x_{iL}^{*CF}}{\partial\mu} = \frac{\partial x_{iL}^{*CI}}{\partial\mu} = \frac{\partial x_{iL}^{*MF}}{\partial\mu} = -\frac{c_H}{3\mu^2} < 0 \\ \frac{\partial x_{iH}^{*MI}}{\partial\mu} = \frac{(3-2\nu)\{(3-2\nu)^2c_H-6\nu r\}}{\{(1+\mu)(3-2\nu)^2-9\}^2} > 0, \\ \frac{\partial x_{iL}^{*MI}}{\partial\mu} = \frac{-3\{(3-2\nu)^2c_H-6\nu r\}}{\{(1+\mu)(3-2\nu)^2-9\}^2} < 0. \end{cases}$$

Proposition 1 *Suppose there is one kinds of network externality, between carriers or machines or devices. Then, an increase in the quality difference between two goods leads to a increase in the quantity of high-quality goods and an decrease in that of low-quality goods.*

That is an example of cannibalization. That is, good $H(L)$ is supplied more with decrease in supply of good $L(H)$. Similarly, The effects of an increase in the value(strength) of the network externality on each quantity are as follows:

$$\begin{cases} \frac{\partial x_{iH}^{*CF}}{\partial \nu} = \frac{\partial x_{iH}^{*MF}}{\partial \nu} = \frac{2r}{(3-2\nu)^2} > 0 \\ \frac{\partial x_{iH}^{*CI}}{\partial \nu} = \frac{r}{(3-\nu)^2} > 0 \\ \frac{\partial x_{iL}^{*CF}}{\partial \nu} = \frac{\partial x_{iL}^{*CI}}{\partial \nu} = \frac{\partial x_{iL}^{*MF}}{\partial \nu} = 0 \\ \frac{\partial x_{iH}^{*MI}}{\partial \nu} > 0 \quad if \quad c_H < c_H^{xH\nu}, \\ \frac{\partial x_{iH}^{*MI}}{\partial \nu} < 0 \quad if \quad c_H^{xH\nu} < c_H \\ \frac{\partial x_{iL}^{*MI}}{\partial \nu} < 0 \quad if \quad c_H < c_H^{xL\nu} \\ \frac{\partial x_{iL}^{*MI}}{\partial \nu} > 0 \quad if \quad c_H^{xL\nu} < c_H. \end{cases} \tag{41}$$

Where, $c_H^{xH\nu}$ and $c_H^{xL\nu}$ satisfy $\frac{\partial x_{iH}^{*MI}}{\partial \nu} = 0$, $\frac{\partial x_{iL}^{*MI}}{\partial \nu} = 0$, respectively.

Proposition 2 *Suppose there is a network externality, between not carriers but machines. Then, an increase in the strength of network externality leads to a decrease in the quantity of high-quality goods and an increase in that of low-quality goods if production cost for high-quality good is high enough.*

This proposition implies that two kinds of network externaties should be distinct. This is because in case that carrier network externality exists or that two devices are compatible, an increase in ν always gives positive effect on the quantity of high-quality goods and no effect on low-quality goods, while in case that machines network externality exists and two devices are incompatible the effects of increase in ν on quantity of two goods depend on production cost for high-quality goods and are negative if the cost is high enough. Moreover, in case that machines network externality exists, cannibalization may occur from an increase in ν. From the reaction functions (22) and (23), in case that machines network externality exists and two mobile devices are incompatible $\frac{\partial g_{i\alpha}}{\partial x_{i\beta}} = 0$. This makes the slope of the reaction functions steeper. Thus, if there is only this case, the two differentiated goods is very substitutable. However, an increase in the strength of network externality (ν) makes the slope of the reaction function (22) steeper and the x_{iL}-intercept of one not change in the $x_{iH} - x_{iL}$ plane and it makes slope of the reaction function (23) gentler and increases the x_{iL}-intercept of one in the $x_{iH} - x_{iL}$ plane. Consequently, the increase in ν makes the intersection points of the reaction functions move toward the upper left in the $x_{iH} - x_{iL}$ plane if c_H is large enough. Thus, the equilibrium output of the high-quality good decreases and more of the low-quality good is produced. This is another example of cannibalization, where the low-quality good L drives the high-quality good H out of the market. That is, an increase in the quality difference between the two goods gives rise to relaxing competition in these goods. It also has a positive effect on the equilibrium output of the low-quality goods; however, this change in the output of the low-quality good leads to lower production of the high-quality good.

Finally, I analyze the effect of the increase in the production cost for high-quality goods on carriers' profit;

$$
\begin{cases}
\frac{\partial \Pi_i^{*CF}}{\partial c_H} = \frac{\partial \Pi_i^{*MF}}{\partial c_H} = \frac{2\{(3-2\nu)c_H - 3\mu r\}}{9\mu(3-2\nu)} < 0 \\
\frac{\partial \Pi_i^{*CI}}{\partial c_H} = \frac{2\{(3-\nu)c_H - 3\mu r\}}{9\mu(3-\nu)} < 0 \\
\frac{\partial \Pi_i^{*MI}}{\partial c_H} < 0 \quad if \quad c_H < c_H^{\Pi c} \\
\frac{\partial \Pi_i^{*MI}}{\partial c_H} > 0 \quad if \quad c_H^{\Pi c} < c_H.
\end{cases}
\tag{42}
$$

Where, $c_H^{\Pi c}$ satisfies $\frac{\partial \Pi_i^{*MI}}{\partial c_H} = 0$.

Proposition 3 *Suppose there is a network externality, not between carriers, but machines or devices. Then, an increase in the marginal cost of good H increases the profit of the carrier if the cost is high enough.*

This proposition is counterintuitive and implies that if production cost of high-quality good c_H is too high, then the increase in c_H makes both carriers earn more and they supply low-quality good more because of equilibrium output (38).

4 Concluding Remarks

Extending Katz and Shapiro's (1985) model, this paper theoretically analyzed firm behavior and the resulting market configuration in the smartphone industry.

In section 2, I constructed a duopoly model where two firms sell two differentiated products and there is a network externality between either carriers or machines. Then I derived two polar full compatible or incompatible equilibria in cases that there exists only network externaility between carriers or only network externaity between mobile devices.

In section 3, I derived proposition 1 and proposition 2 that highlights the effects of a change in the quality of goods and strength of network externality on the quantity of each good. Here, we also explained cannibalization behavior of firms in terms of the change in the difference in quality of goods and in strength of network externality. Then, I find that only in case that machines network externality exists and two devices are incompatible, the effects of increase in strength of network externality on quantity of two goods depend on production cost for high-quality goods and are negative if the cost for production of high-quality goods is high enough. Furthermore, only in this case, cannibalization occurs from an change in strength of network externality, while in all cases an change in difference in quality of two goods bring about cannibalization.

Finally, I analyzed the effect of the change in the production cost for high-quality goods on carriers' equlibrium profit. Then, I showed that only in case that machines network externality exists and two devices are incompatible, an increase in the production cost of high-quality goods makes the carrier earn more if the cost is high enough and supply low-quality goods more. These propositions implies that two kinds of network externalities work differently in equlibrium and should be distinct.

However, in this study, I considered a duopoly model without carriers' phone services costs so that two carriers have no cost and are symmetry. Thus, future studies must analyze the case where firms have some phone services costs, including the costs of making carriers or machines compatible.

Appendix: Lemma 1

Proof: By equation (3) and (9), for arbitrary type $\theta > \hat{\theta}_i$, From (3) and (13), we also have, for arbitrary type $\theta \in (\underline{\theta}_L, \widehat{\theta})$,

$$
\begin{aligned}
U_{iL}(\widehat{\theta}) - U_{iL}(\underline{\theta}_L) &= \hat{\theta} + \nu g^e_{iL} - p_L - (\underline{\theta}_L + \nu g^e_{iL} - p_L) \\
&= \hat{\theta} - \underline{\theta}_L > 0.
\end{aligned}
$$

$$
\begin{aligned}
U_{iH}(\theta) - U_{iL}(\theta) &= (1+\mu)\theta + \nu(1+\mu)g^e_{iH} - p_{iH} - \theta - \nu g^e_{iL} + p_{iL} \\
&= \mu\theta - \{p_{iH} - p_{iL} - (\nu(1+\mu)g^e_{iH} - \nu g^e_{iL})\} \\
&> \mu\hat{\theta}_i - \{p_{iH} - p_{iL} - (\nu(1+\mu)g^e_{iH} - \nu g^e_{iL})\} \\
&= 0.
\end{aligned}
$$

From (3) and (13), we also have, for arbitrary type $\theta \in (\underline{\theta}_L, \widehat{\theta})$,

$$
\begin{aligned}
U_{iL}(\widehat{\theta}) - U_{iL}(\underline{\theta}_L) &= \hat{\theta} + \nu g^e_{iL} - p_{iL} - (\underline{\theta}_L + \nu g^e_{iL} - p_{iL}) \\
&= \hat{\theta} - \underline{\theta}_L > 0.
\end{aligned}
$$

■

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Figure 1