

# Partial privatization under asymmetric multi-market competition\*

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## Abstract

This study examines the degree of privatization under asymmetric multi-market competition, where a public firm provides goods or services to two regions and a private firm provides goods or services to one market only. In addition, to examine how a difference in production technology affects the privatization policy, we assume the public firm's production technology is inferior to that of the private firm. The results show that when the market share of a duopoly market is large (small), partial privatization (nationalization) is socially preferable. Furthermore, the degree of privatization does not always increase as the public firm's production technology becomes increasingly inferior.

**Keywords:** partial privatization, multi-market competition

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# 1 Introduction

The privatization of public firms is becoming increasingly common. This trend is supported by many studies that argue for the necessity of privatizing public firms owing to their inefficient production.

The same trend is evident in Japan. For example, Japan National Railway (JNR), which supplies the railway network, was managed by the Japanese government until 1987. Thereafter, JNR was privatized and its transportation services began to be supplied by private firms. In addition, the postal service in Japan was run as a public firm until 2007, at which time, it too was privatized.

In Tokyo city, there are two subway companies: “Tokyo Metro Co., Ltd.,” which is a private firm, and “Toei Transportation,” which is a public firm. Some subway routes are served by both companies, while other routes are served by one company only. This is an example of asymmetric multi-market competition, where one good (or service) is provided by both a private and a public firm, and the other good is provided by the public firm only.

Previous studies on the (partial) privatization problem usually assume a single product market. One of the most important studies on partial privatization is that of Matsumura (1998), who finds that neither full privatization nor full nationalization is socially preferable, but partial privatization is socially preferable. The major reason for this finding is that the privatized firm considers only its profit and, thus, the quantity of the product produced becomes inadequate. On the other hand, the public firm has inferior production technology, which means its total production costs are significantly higher than those of the private firm when producing goods. As a result of this trade-off, partial privatization is socially preferable.

Another important study is that of Fujiwara (2007). He uses a model of a differentiated mixed oligopoly, extending the work of Ottavino et al. (2002) to demonstrate that partial privatization is socially preferable in both the short run and the long run.

Many other studies have addressed the partial privatization problems (e.g., Lee and Lee, 2014; Benassi et al., 2014; George et al., 1996; Tomaru, 2006; Bennett and Maw, 2003; Chen, 2017; Lee and Hwang, 2003), while others have applied partial privatization to specific problems. For example, the relationship between international trade and partial privatization has often been discussed (e.g., Han, 2012; Wang

and Lee, 2010; Wang et al., 2014; Chao and Yu, 2006). Other studies examine the relationship between partial privatization and environmental problems (Naito and Ogawa, 2009; Pal and Saha, 2015; Kato, 2013; Xu et al., 2016; Ohori, 2006). Then, Heywood and Ye (2009) consider the R&D problem, Ishibashi and Kaneko (2008) analyze a price–quantity competition model, Jain and Pal (2012) investigate the effect of cross-ownership on the degree of partial privatization, and Han and Ogawa (2009) and Ghosh et al. (2015) examine the influence of a foreign-owned private firm on the degree of privatization.

As noted above, while many studies consider the problem of partial privatization, they tend to assume a single market.<sup>1</sup> However, as mentioned earlier, in an actual economy, we often find that a public firm provides its goods or services in a domestic multi-market. Several studies have examined multi-market competition. For example, Bulow et al. (1985) investigate how a firm’s actions in one market affect a competitor’s strategy in a second market. Then, Kawasaki et al. (2014) investigate how a new firm that serves only one market affects the incentives of incumbent firms that serve multiple markets to invest in R&D. However, these studies do not analyze the (partial) privatization problem.

In considering multi-market competition and partial privatization, we examine whether the results obtained in previous studies on partial privatization still hold. In particular, we analyze the situation where one market is served by a public firm only, and the other market is served by both the public and private firms. In this case, if the public firm is privatized, although the inefficiency from the excess production of the public firm decreases, the consumer surplus in the monopolistic market also decreases. Therefore, in this case, privatization (including partial privatization) may not be socially preferable. Therefore, we construct a mixed duopoly model in an asymmetric multi-market and determine the optimal privatization level of the public firm.

In order to consider the above asymmetric multi-market competition, we consider that two regions exist in the economy, and two firms (a public firm and a private firm) supply goods or services. We further assume that the number of consumers in each region is different. The public firm supplies its goods or services to both regions, and the private firm supplies its goods or services to one region only. Therefore, one region is a monopoly market and the other region is a duopoly market. Following De Fraja and

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<sup>1</sup>Exceptions to this are Barcena-Ruiz and Garzon (2003) and Ye (2016), who address a multi-product situation in a mixed duopoly model.

Delbono (1989), both firms have an increasing marginal cost of production. At the same time, we assume that the public firm's marginal cost is larger than that of the private firm. Given these assumptions, we analyze whether the public firm should be privatized.

The main results obtained in this study is as follows. When the market share of the duopoly market is large, partial privatization is socially preferable. As the market share of the duopoly market decreases (or, the market share of the monopolistic market increases), the degree of privatization becomes small. Ultimately, nationalization is socially preferable. In the following, this study examines how the difference of the size of marginal cost between the public and the private firms affects the degree of privatization. As a result, if the market share of the duopoly market is sufficiently large, as its difference increases, the optimal degree of privatization increases. However, when the market share of the duopoly market becomes somewhat small, the optimal degree of privatization does not always increase with the difference of the size of marginal cost. When its market share becomes furthermore small, contrary, the optimal degree of privatization decreases with its difference.

Intuitively, when the difference in the marginal costs of the two firms increases, because the public firm produces more, the degree of privatization increases in order to decrease the total production costs. This is true when the market share of the public firm in the duopoly market is sufficiently large. However, when its market share decreases, this expectation does not always hold, which is an interesting characteristic. The latter result stems from the assumption of increasing marginal costs. When the size of the marginal cost increases, the public firm decreases production in the monopolistic market, which decreases the marginal cost in the duopoly market. Therefore, even when the public firm is privatized, if its market share in the monopolistic market is large, because its marginal costs in the duopolistic market decrease, it produces more, which does not sufficiently decrease the total production costs. In other words, even when the public firm is privatized, because the benefit of decreasing total production costs is small, privatization is not always socially preferable. These findings are the main contribution of this study.

The remainder of this paper is organized as follows. Section 2 presents the proposed model and section 3 derives and analyzes the quantities decided by the private and public firms. Section 4 analyzes the characteristics of each firm's profit. In section 5, we derive the optimal degree of privatization, and then

examine its characteristics. Lastly, section 6 concludes the paper.

## 2 Model

We consider an economy with two regions (or markets): region  $A$  and region  $B$ . There are multiple residents in each region. We assume that the number of residents in region  $i$  ( $j$ ) is  $\phi_i$  ( $\phi_j$ ) ( $i, j = A, B, i \neq j$ ). Here, without loss of generality, we assume that  $\phi_i + \phi_j = 1$ . In addition, we assume there is no migration of residents between regions. There are two firms in the economy: a public firm and a private firm. Hereafter, we refer to the public firm as firm 0, and to the private firm as firm 1. Firm 0 provides goods or services in both regions  $A$  and  $B$ , while firm 1 provides goods or services in region  $B$  only. Therefore, region  $A$  is a monopoly market, and region  $B$  is a competitive market. Here, we assume homogenous Cournot competition in region  $B$ .

The residents in region  $i$  consume the goods provided by the firms. We denote the quantity of firm  $k$ 's goods consumed by the residents in region  $i$  as  $q_i^k$  ( $k = 0, 1$ ). The individual demand function of residents in region  $A$  is

$$p_A = a - q_A^0, \quad (1)$$

and that of residents in region  $B$  is

$$p_B = a - (q_B^0 + q_B^1). \quad (2)$$

Then, because the number of residents in region  $i$  is  $\phi_i$ , the total quantity supplied by firm  $k$  in region  $i$  is equal to  $\phi_i q_i^k \equiv Q_i^k$ . Consequently, the market demand functions in regions  $A$  and  $B$  are

$$p_A = a - \frac{Q_A^0}{\phi_A} \quad (3)$$

$$p_B = a - \frac{Q_B^0 + Q_B^1}{\phi_B}, \quad (4)$$

respectively. We consider that firm 0 has inferior production technologies to those of firm 1.<sup>2</sup> Therefore,

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<sup>2</sup>Following Pal (1998), we assume that the production technology of firm 0 does not change, even if firm 0 is privatized.

the quadratic cost functions of firms 0 and 1 are given by

$$C_0 = \frac{\gamma}{2}(Q_0^A + Q_0^B)^2 \quad (5)$$

$$C_1 = \frac{1}{2}(Q_1^B)^2, \quad (6)$$

respectively, where we assume  $\gamma \geq 1$ , which expresses that firm 0 has inferior production technology to that of firm 1. Therefore, the firms' profit functions are

$$\pi_0 = p_A Q_A^0 + p_B Q_B^0 - \frac{\gamma}{2}(Q_A^0 + Q_B^0)^2 \quad (7)$$

$$\pi_1 = p_B Q_B^1 - \frac{1}{2}(Q_B^1)^2, \quad (8)$$

respectively.

The consumer surpluses of regions  $A$  and  $B$  are

$$CS_A = \frac{(Q_A^0)^2}{2\phi_A}, \quad (9)$$

$$CS_B = \frac{(Q_B^0 + Q_B^1)^2}{2\phi_B}, \quad (10)$$

respectively. Therefore, the social welfare of this economy is defined as follows:

$$W = CS_A + CS_B + \pi_0 + \pi_1. \quad (11)$$

Following Matsumura (1998), we assume that the objective function of firm 0 is

$$V_0 = \theta\pi_0 + (1 - \theta)W. \quad (12)$$

Here,  $\theta(\in [0, 1])$  indicates the degree of privatization.

### 3 The strategy of each firm

This section derives the quantity supplied by each firm, after which we examine the characteristics of the equilibrium quantities.

Now, because  $\phi_1 + \phi_2 = 1$ ,  $\phi_2 = 1 - \phi_1$  holds. Therefore, in the following, we express  $\phi_1 \equiv \phi$  and  $\phi_2 \equiv 1 - \phi$ . Because firm 0 decides  $Q_A^0$  and  $Q_B^0$ , maximizing  $V_0$ , and firm 1 decides  $Q_B^1$  to maximize  $\pi_1$ , the first-order conditions (FOCs) are as follows:

$$\alpha - \gamma(Q_A^0 + Q_B^0) - \frac{(1 + \theta)Q_A^0}{\phi} = 0 \quad (13)$$

$$\alpha - \gamma(Q_A^0 + Q_B^0) - \frac{(1 + \theta)Q_B^0 + Q_B^1}{1 - \phi} = 0 \quad (14)$$

$$\alpha - Q_B^1 - \frac{Q_B^0 + 2Q_B^1}{1 - \phi} = 0. \quad (15)$$

Solving these FOCs, we obtain the following equilibrium quantities:

$$Q_A^0 = \frac{\alpha\phi(-2 + \theta(-3 + \phi) + \gamma(-1 + \phi) + \phi)}{(1 + \theta)(-2 + \theta(3 + \phi) + \phi) + \gamma(-3 + \theta(-3 + \phi) + 2\phi)} \quad (16)$$

$$Q_B^0 = \frac{\alpha(1 - \phi)(-2 + \theta(-2 + \phi) + (1 + \gamma)\phi)}{(1 + \theta)(-2 + \theta(3 + \phi) + \phi) + \gamma(-3 + \theta(-3 + \phi) + 2\phi)} \quad (17)$$

$$Q_B^1 = \frac{\alpha(1 + \theta)(\gamma + \theta)(-1 + \phi)}{(1 + \theta)(-2 + \theta(3 + \phi) + \phi) + \gamma(-3 + \theta(-3 + \phi) + 2\phi)} \quad (18)$$

We perform a comparative static analysis for each quantity by  $\theta$ . The following lemma shows the results.

**Lemma 1** *When  $\theta$  increases,  $Q_A^0$  always decreases,  $Q_B^0$  increases (decreases) for  $\phi \geq (<) \phi_q$ , and  $Q_B^1$  increases (decreases) for  $\phi \geq (<) \phi_q$ .*

**Proof.** We omit the detailed calculation here.

Here,  $\phi_q \equiv \frac{2(1+\theta)^2}{\gamma(1+\gamma)+2\gamma\theta+(1+\theta)^2}$ , as shown in Figure 1.

**Figure 1 here.**

The three lines in Figure 1 are defined as follows: (i) the bold line is  $c = 1$ ; (ii) the dashed line is

$c = 1.5$ ; and (iii) the thin line is  $c = 2$ .<sup>3</sup> The first result is straightforward. Region  $A$  is a monopoly market for firm 0. Therefore, when firm 0 is privatized, it decreases its production in order to increase its price. The second and third results stem from the assumption of a multi-market. The case of  $\phi = 0$  corresponds to the findings of various previous studies. Here, because firm 0 wants to increase its profit, it decreases the production of its goods. At the same time, owing to the strategic substitution between  $Q_B^0$  and  $Q_B^1$ , firm 1 increases its production. This result still holds when  $\phi$  is small.

However, when  $\phi$  is large, the result reverses. This mechanism is as follows. First, note that the marginal cost of production is increasing (i.e.,  $MC = \gamma(Q_A^0 + Q_B^0)$ ) and, second, note that  $Q_A^0$  always decreases with  $\theta$ . Then, firm 0 has two opposite incentives with regard to its production in region  $B$ . First, it has an incentive to decrease its production in order to increase its price. Second, it has an incentive to increase its production owing to the lower marginal production costs resulting from the decrease in  $Q_A^0$ . When  $\phi$  is small, the influence of the decrease in production in region  $A$  is small, and the former incentive dominates. Therefore,  $Q_B^0$  decreases with  $\theta$ . However, when  $\phi$  is large, the influence of the decrease in production in region  $A$  is large, and the latter incentive dominates. In this case, firm 0 increases  $Q_B^0$  with  $\theta$ . Finally, the result for  $Q_B^1$  is obvious because it is a strategic substitute.

We refer to the second incentive as the “production substitute effect,” because a decrease in  $Q_A$  can increase  $Q_B$  (i.e.,  $Q_B$  is a substitute for  $Q_A$ ).<sup>4</sup> Here, we discuss how  $\gamma$  affects the results of the comparative statics analysis (see Figure 1), which is described in Lemma 2.

**Lemma 2** *The range of  $\frac{\partial Q_B^0}{\partial \theta} > 0$  increases with  $\gamma$ .*

Lemma 2 is closely related to the main results of this study, as described later. When  $\gamma$  increases, because the marginal production cost of firm 0 increases, we expect this to strengthen the incentive for firm 0 to decrease production. In region  $A$ , this expectation is true. That is, because firm 0 is a monopoly, the incentive to decrease production increases significantly. On the other hand, in region  $B$ , this expectation is not true. This result stems from the production substitute effect. That is, because firm 0 faces competition with firm 1 and its marginal cost of production decreases owing to the large

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<sup>3</sup>We use these same definitions in all figures in the paper.

<sup>4</sup>A similar effect exists in Yu (2016).



decrease in production in region  $A$ , firm 0 increases its production in region  $B$ .

Figure 2 shows a comparison of  $Q_B^0$  and  $Q_B^1$ . These results lead to the following lemma.

**Figure 2 here.**

In Figure 2,  $\phi_B \equiv 2 - \theta - \frac{3\gamma}{1+\gamma+\theta}$ .

**Lemma 3** *When both  $\theta$  and  $\phi$  are small,  $Q_B^0 \geq Q_B^1$  holds. Otherwise,  $Q_B^0 < Q_B^1$  holds. In addition, when  $\gamma$  increases, the range holding  $Q_B^0 \geq Q_B^1$  decreases, and ultimately disappears.*

**Proof** We omit the detailed calculation here.

The result of this comparison is derived from the following factors. First, because firm 0 (partially) considers the consumer surplus, it has an incentive to increase production. On the other hand, because firm 0's marginal cost of production is large, it has an incentive to decrease production. We first consider the extreme case ( $\phi = 0$ ), which corresponds to the single market case. When  $\theta$  is small, because firm 0 considers the consumer surplus, it produces more than firm 1 does for a small size of  $\gamma$ . However, as  $\theta$  increases, because firm 0 tends not to consider the consumer surplus, it decreases production. In addition, because of the high marginal costs, firm 0 decreases production even further. Consequently, for small (large)  $\theta$ ,  $Q_B^0 \geq (<)Q_B^1$  holds. Then, when  $\gamma$  increases, because the marginal cost of production increases, the range of  $Q_B^0 < Q_B^1$  increases.

Next, we discuss the influence of  $\phi$ , given small  $\theta$  and small  $\gamma$ . As  $\phi$  increases, the size of the monopolistic market (region  $A$ ) increases. Therefore, firm 0 must produce goods for the monopolistic market. Then, the marginal cost of production in region  $B$  increases owing to the increasing marginal cost. Therefore, when  $\phi$  is large, because the marginal cost of firm 0's production becomes large,  $Q_B^0$  decreases and, thus,  $Q_B^0 < Q_B^1$  holds.

## 4 Profit

This section analyzes how each firm's profit changes as the degree of privatization changes.<sup>5</sup> First, we analyze firm 0's profit. From the comparative static analysis by  $\theta$ , we obtain Figure 3.

**Figure 3 here.**

From Figure 3, we can obtain following proposition.

**Proposition 1** *The profit of firm 0 increases (decreases) with  $\theta$  for small (large)  $\theta$ . The range in which privatization increases firm 0's profit widens with  $\phi$ .*

**Proof** We omit the detailed calculation here owing to the complexity of the calculations.

First, we discuss the extreme case of  $\phi = 0$ , which corresponds to the findings of previous studies. When the degree of privatization increases, firm 0 decreases its production of goods so as to set a higher price. On the other hand, as a result of strategic substitution, firm 1 increases production. Because overall production decreases with  $\theta$ , the price of the product increases. Therefore, intuitively, we expect that firm 0's profit always increases with  $\theta$ .

However, our exception is not always correct. When  $\theta$  increases, if  $\phi = 0$ , firm 0 decreases its production and the price of its product increases. Therefore, if the influence of the increase in price is larger (smaller) than that of the decrease in production, firm 0's profit increases (decreases). In other words, for small (large)  $\theta$ , the influence of the increase in price is larger (smaller) than that of the decrease in production. Given this situation, as  $\phi$  increases, because firm 0 gains the profit from the monopolistic market, privatization brings additional profit to firm 0.

Next, we discuss the influence of  $\gamma$  on the result of the comparative static analysis about  $\pi_0$ . Figure 3 also shows the results. In addition, in order to show the relationship of each line clearly, Figure 3-b shows the restrictive range.

Thus, we obtain the following lemma.

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<sup>5</sup>Surprisingly, prior studies have not considered the influence of partial privatization on a firm's profit sufficiently.

**Lemma 4** *When  $\phi$  is sufficiently small, the range in which firm 0's profit increases with  $\theta$  narrows with  $\gamma$ . When  $\phi$  becomes slightly larger, its range does not always narrow with  $\gamma$ . However, as  $\phi$  increases further, its range widens with  $\gamma$ .*

First, we consider the case of  $\phi = 0$ . As  $\gamma$  increases, firm 0's marginal cost increases. Therefore, as the degree of privatization increases, although firm 0 decreases production, because firm 1 increases production, the price of the goods does not increase sufficiently. Consequently, the range of  $\frac{\partial \pi_0}{\partial \theta} > 0$  decreases with  $\gamma$ . As  $\phi$  begins to increase, firm 0 has a monopoly market. Then, as  $\gamma$  increases, the range of  $\frac{\partial \pi_0}{\partial \theta} > 0$  does not always decrease, because the decrease in production becomes small as a result of the production substitute effect. Then, as  $\phi$  increases further, the influence of the monopolistic market (region A) increases. That is, as  $\gamma$  increases, because the production substitute effect becomes significant, the range of  $\frac{\partial \pi_0}{\partial \theta} > 0$  increases with  $\gamma$ .

In the following, we analyze how the degree of privatization affects firm 1's profit. From the comparative static analysis about  $\pi_1$  by  $\theta$ , we obtain following proposition.

**Proposition 2** *The profit of firm 1 increases (decreases) with  $\theta$  for small (large)  $\phi$  (or, smaller (large) than  $\phi_q$ )*

**Proof** We omit the detailed calculation here.

Here,  $\phi_q$  is defined as in Lemma 1. The result of Proposition 2 is shown in Figure 4.

**Figure 4 here.**

When considering the result of Lemma 1, Proposition 2 seems to be straightforward. That is, because firm 1 decreases its production with  $\theta$  for large  $\phi$ , firm 1's profit decreases with  $\theta$ . According to the single market situation (i.e.,  $\phi = 0$ ), the profit of a private firm increases with the degree of privatization of the public firm. However, in the multi-market case, this characteristic can reverse, which is an interesting result.

## 5 Optimal degree of privatization

This section analyzes the optimal degree of privatization of firm 0. The degree of privatization is determined by maximizing social welfare. Because the calculation results are too complex to show here, we instead perform a simulation analysis. First, we assume  $\gamma = 1$ , which means that firm 0 and firm 1 have a same production technology.

Figure 5 shows the FOC that maximizes social welfare.

**Figure 5 here.**

The line of  $\theta(\phi)$  shown in Figure 5 expresses the optimal degree of partial privatization, which depends on  $\phi$ . From Figure 5, we obtain Proposition 3.

**Proposition 3** *The socially preferable degree of privatization decreases with  $\phi$ . When  $\phi$  becomes sufficiently large, privatization is not socially preferable.*

The case of  $\phi = 0$  corresponds to the single market situation. Therefore, when  $\phi = 0$ , this study's result is consistent with the conclusion of Matsumura (1998), which argues that partial privatization is socially preferable. Thus, as  $\phi$  increases, the influence of the monopolistic market increases. That is, if firm 0 is privatized, because it has market power in region  $A$ , social welfare decreases. In order to avoid this situation, the optimal degree of privatization decreases with  $\phi$  and, ultimately, for sufficiently large  $\phi$ , it becomes socially preferable for firm 0 to be a public firm.

In the following, we discuss how the difference between the production technologies of the two firms affects the optimal degree of privatization. In order to discuss this interesting problem, we show simulation results in which three cases are analyzed. First, both firm 0 and firm 1 have an identical production technology ( $\gamma = 1$ ), which has been shown before. Second, we show the optimal degree of privatization when firm 0's production technology becomes somewhat inferior (i.e.,  $\gamma = 1.5$ ). Third, we show the optimal degree of privatization when firm 0's production technology drops further behind that of firm 1 ( $\gamma = 2$ ). Figure 6 shows the results.

**Figure 6 here.**

As a result, we have Proposition 4.

**Proposition 4** *As the production technology of public firm becomes inferior, the optimal degree of privatization does not always increase.*

Proposition 4 is a very surprising result. When the production technology of firm 0 becomes inferior (i.e., the marginal cost of the public firm increases), we expect the degree of privatization to increase in order to decrease the firm's production. When  $\phi$  is zero, this expectation is correct. However, according to Proposition 4, as  $\phi$  increases, this expectation is not always correct. In the following, we discuss the reason for this.

First, we repeat the advantage and disadvantage of privatization. The advantage of privatization is that because the production by the inefficient firm decreases, the total production costs decrease. The disadvantage is that the consumer surplus is not considered when deciding the amount of production.

When  $\phi = 0$ , because there is no monopolistic market, we consider only the mixed duopoly market (region  $B$ ). As  $\gamma$  increases, from the viewpoint of social welfare, the production by firm 0 should decrease. Therefore, the optimal degree of privatization decreases with  $\gamma$ . This result is straightforward.

When  $\phi > 0$ , the influence from the monopolistic market (region  $A$ ) appears. As  $\gamma$  increases, the production of firm 0 in region  $B$  changes through the production substitute effect, the degree of which matters.

When  $\phi$  is sufficiently small, although the production substitute effect appears, its degree is very small. That is, similarly to the case of  $\phi = 0$ , as a result of privatization, firm 0 decreases production in both regions. Therefore, as  $\gamma$  increases, because the benefit of the decrease in production increases, the optimal degree of privatization increases.

When  $\phi$  is somewhat large, its effect is slightly large. That is, even when firm 0 is (partially) privatized, production in the competitive market (region  $B$ ) does not decrease sufficiently. Therefore, the total production cost also does not decrease sufficiently, which means the benefit of privatizing firm 0 becomes small. This tendency strengthens with  $\gamma$ . As a result, the optimal degree of privatization when  $\gamma = 2$  is smaller than that when  $\gamma = 1.5$ .

As  $\phi$  increases further, the production substitute effect also becomes large. That is, when firm 0 is privatized, production in the competitive market barely changes. Therefore, as  $\gamma$  increases, as a result of privatization, total production costs may increase. Consequently, the optimal degree of privatization decreases with  $\gamma$ .

Finally, when  $\phi$  is sufficiently large, because the consumer surplus of the monopolistic market (region A) matters for social welfare, the publicly run firm is always socially preferable.

These results have an important implication for economies. In general, the major reason for privatizing the public firm stems from the firm's inefficient production technology. That is, in order to avoid excess production by the public firm, the privatization of the public firm becomes necessary. When a single market is assumed, this argument is correct. However, if the public firm provides its goods or services in a multi-market, the above argument may not be correct. In other words, in order to avoid excess production by the public firm, nationalization can be socially preferable.

## 6 Concluding remarks

Almost all previous studies on partial privatization assume a single market. However, we sometimes observe that a public firm provides goods or services in multiple markets. Therefore, this study constructed a mixed duopoly model for an asymmetric multi-market, and analyzed the optimal degree of privatization of the public firm. Here, we made two important assumptions. First, we assume increasing marginal costs. Second, we assume that the public firm has inferior technology to that of the private firm. There are two regions in the economy, and different numbers of consumers live in each region. The public firm provides its goods or services to both regions, and the private firm provides its goods or services to one region only.

Our results were as follows. When the market share of the duopoly market is large, partial privatization is socially preferable. As the market share of the duopoly market decreases (or the market share of the monopolistic market increases), the degree of privatization becomes small. Ultimately, nationalization becomes socially preferable. Next, we examined how the difference between the marginal costs of the public and the private firm affects the degree of privatization. If the market share of the duopoly market

is sufficiently large, as its difference increases, the optimal degree of privatization increases. However, when its market share becomes somewhat small, the optimal degree of privatization does not always increase with the difference in marginal costs. However, when its market share becomes still smaller, the optimal degree of privatization decreases with the difference. In other words, by introducing multi-market competition, we present a new mechanism, which is the main contribution of this study.

Finally, a few problems remain. First, we assume that one market is a monopoly. Thus, we need to determine what happens if this market is also a duopoly market. We expect that partial privatization is still socially preferable, and that the higher marginal costs result in a larger degree of privatization. Similarly, we need to determine how our main result changes if the number of private firms differs between regions. Second, we do not consider the entry of private firms. It sometimes occurs that private firms enter a market in which the public firm is a monopoly. In this case, we need to determine whether the entry of private firms improves social welfare. Third, we can also consider the entry of public firms to the markets. In future research, we will consider these problems.

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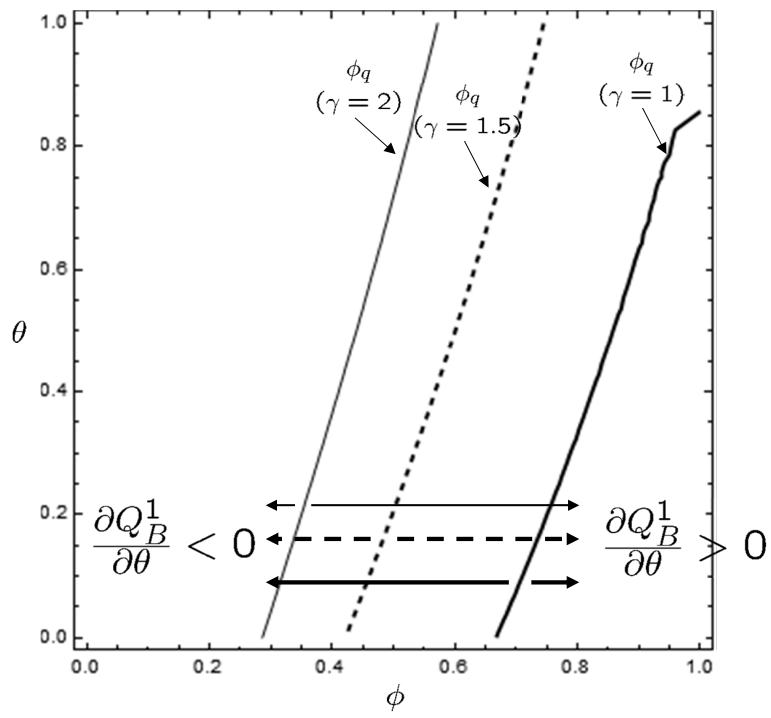


Figure 1: Comparative static analysis for  $Q_B^1$

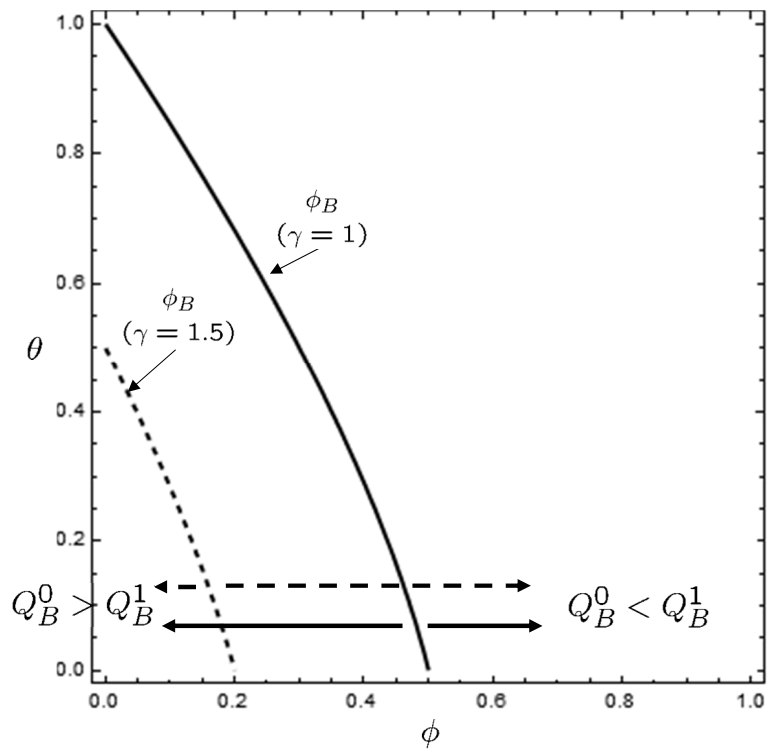
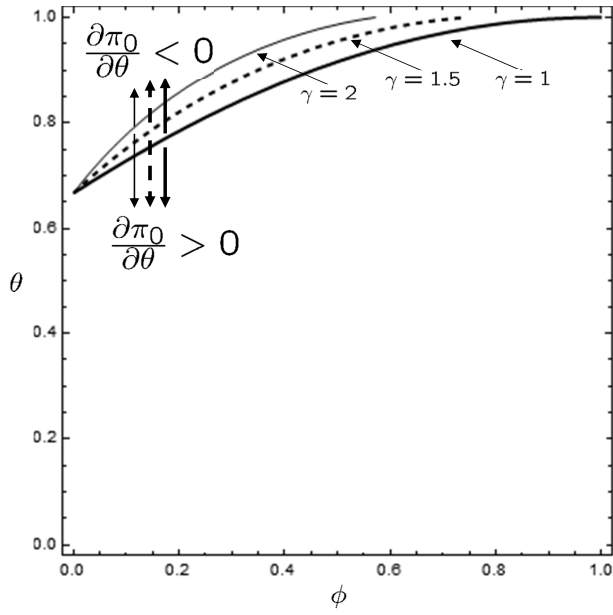
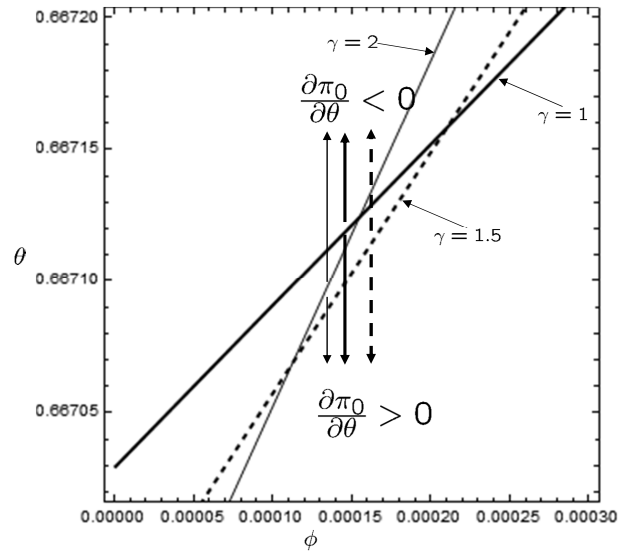


Figure 2: Comparison of  $Q_B^0$  and  $Q_B^1$



a. Overall



b. Expanded area

Figure 3: Comparative static analysis for  $\pi_0$

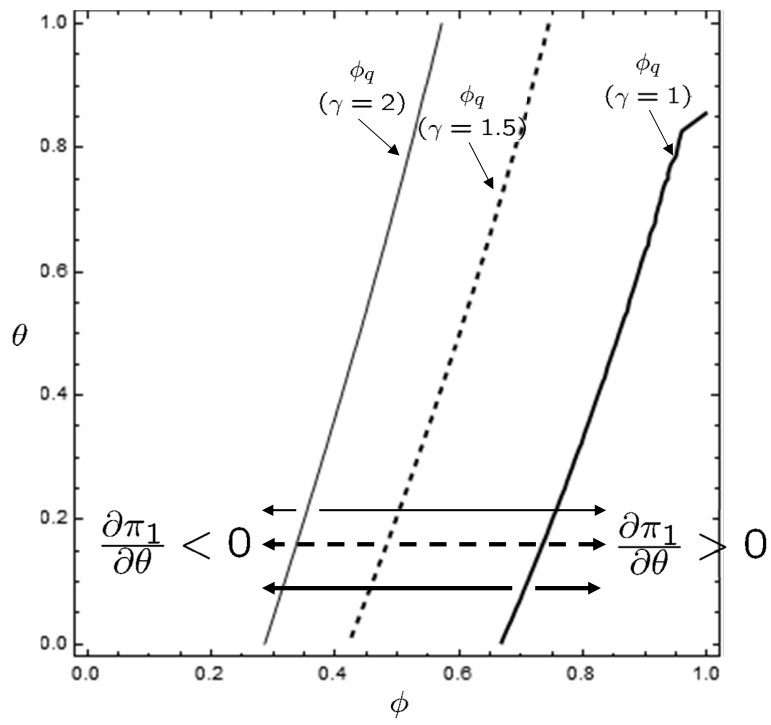


Figure 4: Comparative static analysis for  $\pi_1$

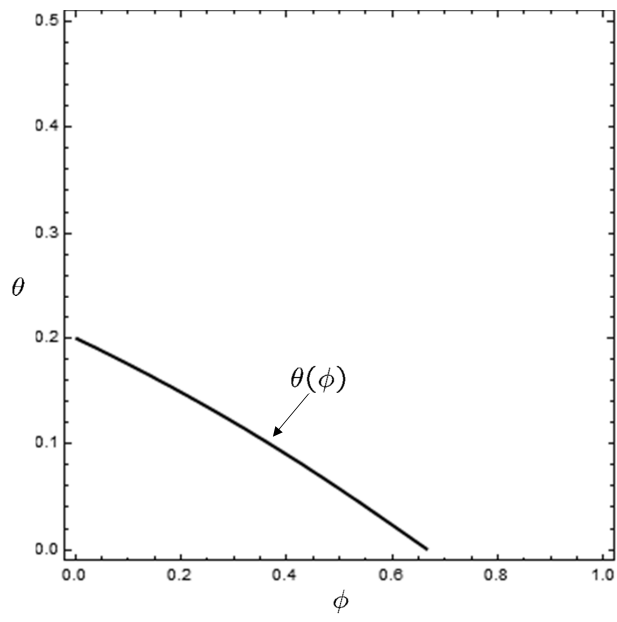


Figure 5: Optimal degree of privatization

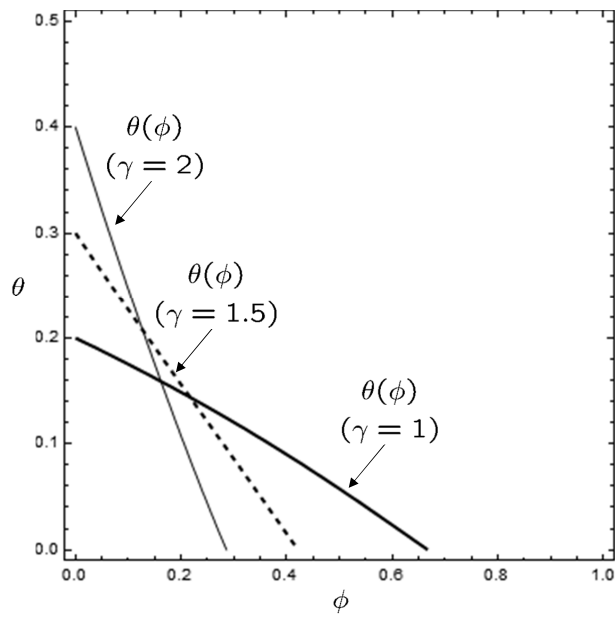


Figure 6: The influence of  $\gamma$  on the optimal degree of privatization