Should a local public firm be privatized under unidirectional transboundary pollution?†

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Abstract

We examine whether a local regional government should privatize its local public firm in mixed duopoly when it faces the unidirectional transboundary pollution problem. We consider two regions in an economy, one located upstream and the other, downstream, and analyze the economy for all location patterns of the firms and the two types of transboundary pollution (transboundary pollution caused by consumption and that which is caused by production).

We consider the case where the fraction of transboundary pollution is such that the equilibrium outcome before and after privatization is the same. We also show that when there is a change in the fraction of transboundary pollution, (1) in some cases, privatization is desirable for both the local regional government that owns the local public firm and the central government of the economy; (2) however, there also exist cases where privatization is only desirable for the local government.

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1 Introduction

Phenomena attributed to transboundary pollution, such as acid rain and water or air pollution, have been attracting attention since the middle of the 19th century. Acid rain has been recognized as a serious environmental problem in Europe. Further, since the past few decades, acid rain has become a serious problem in East Asia, in particular, in China.[1] Such phenomena that are attributed to transboundary pollution are often considered to have been caused by production. However, such phenomena can also be caused by consumption. Recently, there has been a shift to a consumeristic way of life and consequently, trash generation has increased. Often we come across inshores where trash and medical waste emitted by an upstream country or region get transported to other downstream countries or regions. For example, for the past several years, trash thought to have been generated by Russia, China, and Korea has been regularly found to have washed up on the shores of northern Japan. In order to solve this problem, working-level talks between Japan and Korea look place in February, 2009.

Meanwhile, global warming continues to worsen all across the world. There is a possibility that global warming will affect the fraction of transboundary pollution. Global warming may result in the westerlies becoming meandering. This may result in the extreme weather; further natural caramities such as floods, heavy rains, and hurricanes may become more frequent and as a result, may become more of an issue in the future. The meandering of the westerlies will also affect the fraction of present transboundary air pollution and fraction of transboundary acid rain. Heavy rains transport the trash in a city that lies on a liverbed and the trash stored that is in waste-collection points dotting the riverfront into the river. Floods then transfer this trash from upstream regions to downstream ones. An increase in the atmospheric temperature and seawater surface level, and a decrease in the salinity of the seas because of melting glaciers may alter the flow of the oceans, and thus, affect the fraction of transboundary of sea trash pollution. From these facts, we conclude that there is a possibility that the influence of transboundary pollution varies even if the total pollution remains unchanged.

[1] See Nagase and Silva (2007) for more details as they have surveyed this extensively.

In some of the countries and regions mentioned above, we can observe that there still exist mixed markets where public firms and private firms compete. In mixed markets, privatization of a public firm is a major issue. If privatization occurs, public firm's objective changes. This alters the market equilibrium and leads to changes in pollution. Therefore, privatization in one country affects not only its welfare but also the welfare of other countries which are affected by transboundary pollution.

Keeping in mind these points, in our model, we have two regions, one upstream and the other, downstream, and one public firm and one private firm. The location of each firm and the type of pollution (pollution caused by consumption and that caused by production) are also considered. We examine whether privatization of the public firm in one region enhances welfare in the region and the welfare in the whole economy when the fraction of transboundary pollution (θ) varies for every location pattern of the firms and for all types of pollution.

The main results obtained in the paper are as follows. Suppose that there exists $\bar{\theta}$ such that the equilibrium outcome before and after privatization is the same. When the fraction of transboundary pollution changes marginally at $\bar{\theta}$, (1) in some cases, privatization enhances both local welfare (welfare of the region where the public firm is located) and whole welfare (welfare of the whole economy); (2) however, there also exists cases where it only enhances local welfare.

The intuition behind the results is as follows. When the fraction of transboundary pollution changes marginally at $\bar{\theta}$, the public firm changes its output to decrease environmental damage. Note that whether the public firm increases or decreases its output depends on the location of each firm and the type of pollution. This alters the output of the private firm (strategic substitution effect) and leads to a change in producer surplus. Though, in many cases, the environmental damage decreases in the region where the public firm exists, there is a possibility that the environmental damage in the entire economy increases. Thus, the effect of privatization on local welfare and whole welfare may differ in each case.

Many earlier works on the mixed oligopoly analyze within the framework of Defraja and Delbono (1989).[2] In recent years, some researches have addressed the environmental problem. Bárcena-Ruiz and Garzón (2007), Beladi and Chao (2006), Ohori (2006), and Kato (2006) examine environmental regulation in a mixed oligopoly and analyze the effect of privatization. Cato (2008) investigates the relationship between the degree of environmental damage and privatization. These works deal with the environmental problem in one region and therefore, do not consider transboundary pollution.

Many earlier works have discussed the transboundary pollution problem; in particular, Nagase and Silva (2007) are closely related to our paper. They consider the situation where there is an upstream region (China) and a downstream region (Japan) and examine an environmental policy-making game between the two under the bilateral transboundary pollution problem.[3]

The reminder of the paper is organized as follows. Section 2 describes our model. Section 3 derives the equilibrium in pure duopoly. Section 4 derives the equilibrium in mixed duopoly and compares the local welfare of each region and whole welfare before and after privatization when the pollution is caused by consumption. Section 5 derives and compares the same when the pollution is caused by production. Section 6 concludes the main text. Appendices provide detailed calculations for the equilibrium outcome in each case and the proof of Proposition 4.

2 Model

Suppose an economy of two regions; region A and B. Region A is located upstream and region B, downstream. In this economy, there is one local public firm (firm 0) owned by one local regional government and one private firm (firm 1). Both produce a homogeneous product that harms the environment. We call this product a "dirty good."

Firms 0 and 1 compete in quantity. The output of firm i is denoted by q_i $(i = 0, 1)$.

[2] Bös (1991) reviews a mixed market.

[3] Nagase and Silva (2007) consider a competitive market and allow abatement effort and an emission tax policy. They do not consider a mixed duopolistic market.

Total output is $Q = q_0 + q_1$. The cost function of firm i is $c_i(q_i)$. The profit of firm i is

$$\pi_i(q_0, q_1) = p(Q)q_i - c_i(q_i),$$

where $p(Q)$ is the inverse demand function of the dirty good. We assume that the dirty good can be transported from region k to region l ($k \neq l, k = 0, 1$) without any transportation costs.

A representative consumer exists in each region. The representative consumer in the region where firm 0 exists is called the "hot consumer" while the representative consumer in the other region is called the "cold consumer."[4] The hot consumer consumes the dirty good and a clean numeraire good. The cold consumer grows his/her own food and only exists. He/she does not consume the dirty good and does not discharge any pollution.

The hot consumer maximizes $U(Q) + y$ subject to $pQ + y = m$, where p denotes the price of the dirty good, y denotes the amount of the numeraire good whose price is normalized to 1, and m denotes the income of the representative consumer. We assume that $U(Q)$ is a thrice-continuously differentiable and strictly concave in $Q > 0$. Solving the maximization problem of the representative consumer, we get that $p = U'(Q)$. Further we define $p(Q)$ as $U'(Q)$, the inverse demand function. We make the following assumption with regard to $p(Q)$ and $c_i(q_i)$.

Assumption 1. *The inverse demand function and cost function satisfy the following properties.*

(*i*) $p(Q)$ *is twice-continuously differentiable for all* $Q > 0$ *and* $p'(Q) < 0$;

(*ii*) $c_i'(q_i) > 0,\ c_i''(q_i) \geq 0,$ *and* $c_i(0) = 0$; *and*

(*iii*) $c_0'(q) \geq c_1'(q),$ *for all* $q > 0$.

In our model, pollution is generated and is harmful to the environment. We consider the following two types of pollution: pollution because of consumption of the dirty good

[4] We consider that the raison d'etre of the local public firm is to protect the interests of the consumer in the region where it is located. Therefore, we consider the case where the local public firm and hot consumer exist in the same region.

and pollution because of the dirty good's production.[5] We refer to the former type of pollution as "consumption externality" and the latter as "production externality." Consuming/producing one unit of a dirty good generates one unit of pollution. The pollution is converted into environmental damage that reduces the consumer surplus via lump-sum transfer. The total pollution in region l is denoted by E_l; total environmental damage in region l is denoted by $D_l(E_l)$. We make the following assumption on the environmental damage function.

Assumption 2. *The environmental damage function satisfies the following property.*

$$D_l'(E_l) > 0, \;\; D_l''(E_l) > 0, \;\; \textit{and } D_l(0) = 0.$$

We assume that the pollution is transboundary and may also affect the environment of region B. Figure 1 shows the level of transboundary pollution and the location patterns of the two firms in consumption externality; Figure 2 shows the same in production externality. As an example, we pick case (AA) of consumption externality (explained later) and explain transboundary pollution. Pollution is generated only in region A; the amount of pollution generated is Q. We assume that region A is located upstream (by a river or in the path of a wind), and therefore, some of the pollution is transported to region B located downstream. The ratio of the pollution that remains in region A is θ; therefore, the ratio of the pollution that gets transported to region B is $(1-\theta)$. As a result, the pollution levels in region A and B are θQ and $(1-\theta)Q$, respectively.

In subsequent analysis, we analyze eight cases: four cases in consumption externality (showed in Figure 1) and four cases in production externality (showed in Figure 2).

Welfare in region l and k is defined as the sum of consumer surplus (CS) and producer surplus, and the environmental damage.

If both firms 0 and 1 are located in region l, welfare in region l is given by

$$w_l = CS + \pi_0 + \pi_1 - D_l(E_l) + m. \tag{1}$$

We assume that the utility of the cold consumer is constant and normalized to 0. There-

[5] We do not consider a case where the pollution is caused by both consumption and production.

fore, welfare in region k is given by

$$w_k = -D_k(E_k). \tag{2}$$

If firm 0 is located in region l and firm 1, region k, welfare in the two regions is given by

$$w_l = CS + \pi_0 - D_l(E_l) + m \quad and \tag{3}$$

$$w_k = \pi_1 - D_k(E_k). \tag{4}$$

Note that E_l and E_k depend on the type of externality, and therefore, we derive E_l and E_k on a case-by-case basis.

Welfare in this economy is defined as the sum of welfare in region A and that in region B. Thus,

$$W(q_0, q_1) = w_l + w_k, \tag{5}$$

$$= \int_0^Q p(z)dz - \sum c_i(q_i) - \sum_l D_l(E_l) + m. \tag{6}$$

We denote welfare in region l as "local welfare l" and welfare in the entire economy as "whole welfare."

The objective of firm 0 is to maximize the local welfare of the region where it exists.[6] The objective of firm 1 is to maximize its own profits. We make the following assumptions to guarantee that the equilibrium exists and is unique.

[6] Earlier studies model the following two objectives of the public firm: maximization of social welfare in its region (that is, the region where it exists) and maximization of the sum of consumer surplus and producer surplus in its region. Beladi and Chao (2006) and Ohori (2006) model the latter. In particular, Ohori (2006) considers consumption externality. In this paper, however, we model the former, even though the objective of the public firm in Ohori (2006) is convincing when we consider consumption externality. This is the reason why there exist two distortions in terms of social welfare maximization: both the public firm and the private firm do not maximize social welfare. In this case, distinguishing the effects on social welfare is more complex than when the public firm is the social welfare maximizer. Furthermore, we consider that the characteristic differences between consumption externality and production externality exist only in the locations where the hot consumer and the firms are located. If we reconsider Beladi and Chao (2006) and Ohori (2006) using the setting of the former instead of the setting of the latter, the results might change.

Assumption 3.

$$1 > \left| \frac{p''(Q)Q}{p'(Q)} \right| \text{ for all } Q \geq 0.$$

The above assumption implies that the elasticity of the slope of the inverse demand function is less than 1. By this assumption, we can find that the second order condition of the maximization problem of each firm is negative.

First best allocation

We consider the first best allocation of this model. On maximizing whole welfare W with respect to q_i $(i = 0, 1)$, we obtain the following equation:

$$p(Q^*) - c_i'(q_i^*) - \sum D_l'(E_l^*) = 0,$$

where q_i^* is the solution of the above equations. $(q_0, q_1) = (q_0^*, q_1^*)$ is the first best allocation. Note that the output of each firm is chosen in order to equalize its own marginal cost and the marginal environmental damage to the market price.

In the rest section, we analyze the following four cases in consumption externality and in production externality: (AA), both firms 0 and 1 exist in region A; (BB), both firms 0 and 1 exist in region B; (AB), firm 0 exists in region A and firm 1 exists in region B; and (BA), firm 0 exists in region B and firm 1 exists in region A. In the above two-alphabet relations, the letter on the left gives the location of firm 0 and the letter on the right, the location of firm 1.

In order to observe the effect of privatization, we derive the equilibrium outcome of each case in pure duopoly (that is, after privatization) in the next section.

3 Pure duopoly (after privatization)

We derive the equilibrium output of each firm after privatization, when there are two private firms whose objectives are to maximize their own profits. In equilibrium, firm i chooses its output so as to satisfy

$$\frac{\partial \pi_0}{\partial q_0} = p(Q) + p'(Q)q_0 - c_0'(q_0) = 0 \ \ and \tag{7}$$

$$\frac{\partial \pi_1}{\partial q_1} = p(Q) + p'(Q)q_1 - c_1'(q_1) = 0. \tag{8}$$

Solving the above equations, we obtain the equilibrium output of each firm. We denote the equilibrium outputs of the firms 0 and 1 as q_0^N and q_1^N, respectively. N denotes the equilibrium outcome after privatization. From point (iii) of Assumption 1, we get $q_0^N \leq q_1^N$. Further, from (7) and (8), we find that these equilibrium outputs do not depend on θ. In equilibrium, a change in θ may affect at most only the pollution level of each region.

Note that the equilibrium output of each firm after privatization is the same in all cases of consumption externality and production externality.

4 Consumption externality

In this section, we derive the equilibrium outcome and compare the welfare before and after privatization in each case in consumption externality. Before the analysis, we assume that in terms of whole welfare, the equilibrium output of each firm after privatization is larger than that in the first best production allocation. In order to examine this situation, we assume the following.

Assumption 4.

$$p'(Q^N)Q^N + D_l'(Q^N) > 0. \tag{9}$$

Note that the fraction of transboundary pollution does not affect the equilibrium outcome before and after privatization in cases (BB) and (BA).[7] In the subsequent analyses in consumption externality, we examine cases (AA) and (AB).

4.1 Case (AA) in mixed duopoly (before privatization)

We consider the case where both firms 0 and 1 exist in region A.

[7]See Appendix A.

Local welfare A, local welfare B, and whole welfare are defined as

$$w_A = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_A(E_A) + m, \tag{10}$$

$$w_B = -\, D_B(E_B), \quad and \tag{11}$$

$$W = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_A(E_A) - D_B(E_B) + m, \quad respectively, \tag{12}$$

where $E_A = \theta Q$ and $E_B = (1-\theta)Q$.

In mixed duopoly, the first order condition of each firm is given by

$$\frac{\partial w_A}{\partial q_0} = p(Q) - c_0'(q_0) - \theta D_A'(E_A) = 0 \quad and \tag{13}$$

$$\frac{\partial \pi_1}{\partial q_1} = p(Q) + p'(Q)q_1 - c_1'(q_1) = 0. \tag{14}$$

By solving the above first order conditions, we obtain $q_0^{cAA} = q_0^{cAA}(\theta)$ and $q_1^{cAA} = q_1^{cAA}(\theta)$, the equilibrium outputs of firm 0 and 1. We use the superscript cAA to denote the equilibrium outcome in case (AA) before privatization in consumption externality. This superscript is also used to represent the equilibrium outcome in subsequent sections. Note that cr ($r = AA, BB, AB, BA$) denotes "case (r) before privatization in consumption externality;" $p(r)$, "case (r) before privatization in production externality;" Ncr, "case (r) after privatization in consumption externality;" and Npr, "case (r) after privatization in production externality."

Here, we analyze the comparative statics for the equilibrium output of each firm with respect to θ. We find that[8]

$$\frac{dq_0^{cAA}}{d\theta} < 0, \ \frac{dq_1^{cAA}}{d\theta} > 0, \ and \ \ \frac{dQ^{cAA}}{d\theta} < 0. \tag{15}$$

Should the local public firm be privatized when the fraction of transboundary pollution changes? (welfare comparison)

We compare the local welfare of each region and whole welfare before and after privatization. As we do not use the specific functional form of demand, cost, and environmental functions, we focus on the following points to examine the effect of privatization. First,

[8]For calculations, see Appendix B-1.

we find θ under which the equilibrium output of each firm is the same before and after privatization and denote it as $\bar{\theta}$. Needless to say, under $\bar{\theta}$, the local welfare of each region and whole welfare do not change regardless of whether or not the local public firm is privatized. Second, we examine whether privatization enhances welfare when θ shifts marginally from $\bar{\theta}$.

First, we identify $\bar{\theta}$. Comparing the first order conditions before and after privatization, we find that only the first order condition of firm 0 differs. On comparing the first order conditions of firm 0 before and after privatization, we obtain the following condition:

$$p'(Q^N)q_0^N + \bar{\theta}^{cAA}D_A'(E_A^N) = 0, \tag{16}$$

where $E_A^N = \bar{\theta}^{cAA}Q^N$ and $\bar{\theta}^{cAA}$ is the value of $\bar{\theta}$ in case (AA) before privatization in consumption externality. $\theta D_A'(E_A)$ is increasing and strictly convex in θ because $\partial\theta D_A'(E_A)/\partial\theta = D_A'(E_A) + \theta Q D_A''(E_A) > 0$ and $p'(Q^N)q_0^N$ is a finite negative constant. Therefore, we can prove that $\bar{\theta}^{cAA}$ exists in $[0,1]$ and is unique. When $\theta = \bar{\theta}^{cAA}$, $q_i^{cAA}(\bar{\theta}^{cAA}) = q_i^N$.

Next, we examine whether privatization enhances local welfare A, local welfare B, and whole welfare if θ shifts marginally from $\bar{\theta}$. We denote $w_l^{cAA}(\theta)$ as $w_l(q_0^{cAA}(\theta), q_1^{cAA}(\theta))$ and $W^{cAA}(\theta)$ as $W(q_0^{cAA}(\theta), q_1^{cAA}(\theta))$. In the subsequent analyses, we use similar notations in other cases. Using the first order condition of each firm, we can find that

$$\left.\frac{dw_A^{cAA}}{d\theta} - \frac{dw_A^{NcAA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{cAA}} = -\,p'(q_1^N - q_0^N)\frac{dq_1^{cAA}}{d\theta} \geq 0, \tag{17}$$

$$\left.\frac{dw_B^{cAA}}{d\theta} - \frac{dw_B^{NcAA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{cAA}} = -\,(1-\bar{\theta}^{cAA})D_B'\frac{dQ^{cAA}}{d\theta} > 0, \quad and \tag{18}$$

$$\left.\frac{dW^{cAA}}{d\theta} - \frac{dW^{NcAA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{cAA}} = -\,(1-\bar{\theta}^{cAA})D_B'\frac{dQ^{cAA}}{d\theta} - p'(q_1^N - q_0^N)\frac{dq_1^{cAA}}{d\theta} > 0, \tag{19}$$

where strictly inequality holds if $c_0' \neq c_1'$.[9] From the above results, we get the following proposition.

Proposition 1. *When θ marginally increases (decreases) at $\bar{\theta}^{cAA}$, privatization weakly decreases (increases) local welfare A. It also decreases (increases) local welfare B and whole welfare.*

[9]For calculations, see Appendix B-1.

The intuition behind proposition 1 is as follows. Let us consider the situation before privatization. When θ increases, the output of the local public firm decreases and that of the private firm increases; the total output decreases. Thus, consumer surplus decreases. Profit of the local public firm decreases and that of the private firm increases; producer surplus increases.[10] The effect of θ on the environmental damage is divided into two: direct effect and indirect effect. The direct effect is the effect that θ has on pollution. This effect is common not only under privatization but also when there is no privatization. The indirect effect is the effect that the change in the output of each firm has on pollution. Comparing welfare before and after privatization, we observe that the direct effect is offset, and therefore, we focus only on indirect effect. As total output decreases as θ increases, direct effect is positive for local welfare A and local welfare B. Given that the two positive effects are larger than or equal to the single negative effect, local welfare A increases or remain unchanged. If the production cost of the local public firm is strictly larger than that of the private firm, local welfare A increases since production inefficiency decreases. Since local welfare B also increases, whole welfare increases.[11]

If we explicitly suppose the existence of the following three governments: the local governments A and B (each is a local welfare maximizer of its own region), and the central government (it is a whole welfare maximizer), we can observe that their interests with regard to privatization of the local public firm coincide.

[10]Producer surplus before privatization is defined as $\sum \pi_l^{cAA}(\theta)$. When θ changes, producer surplus is obtained by using the first order conditions and evaluating at $\bar{\theta}^{cAA}$ as $p'\{q_0^N(dq_1^{cAA}/d\theta)+q_1^N(dq_0^{cAA}/d\theta)\}$. Since $q_1^N \geq q_0^N$, $dq_0^{cAA}/d\theta < 0$ and $|(dq_0^{cAA}/d\theta)| > |(dq_1^{cAA}/d\theta)|$, $p'\{q_0^N(dq_1^{cAA}/d\theta)+q_1^N(dq_0^{cAA}/d\theta)\} > 0$.

[11]Cato (2008) obtains a result similar to that in our Proposition 1 with regard to the examination of whether or not privatization increases welfare when the degree of environmental damage changes in autarky. However, he does not consider a transboundary pollution problem. Nevertheless, we can interpret the change in the degree of environmental damage in his model as corresponding to the change in transboundary pollution. The results of Cato (2008) are very similar to ours, though Cato (2008) considers a mixed oligopoly of n private firms and allows firms to abate their emissions. See Cato (2008) for details.

4.2 Case (AB) in mixed duopoly (before privatization)

We consider the case where firm 0 exists in region A and firm 1 exists in region B.

Local welfare A, local welfare B, and whole welfare are defined as

$$w_A = \int^Q p(s)ds - c_0(q_0) - p(Q)q_1 - D_A(E_A) + m, \tag{20}$$

$$w_B = \pi_1 - D_B(E_B), \quad and \tag{21}$$

$$W = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_A(E_A) - D_B(E_B) + m, \quad respectively, \tag{22}$$

where $E_A = \theta Q$ and $E_B = (1-\theta)Q$. The first order condition of each firm is given by

$$\frac{\partial w_A}{\partial q_0} = p(Q) - c_0'(q_0) - p'(Q)q_1 - \theta D_A'(E_A) = 0 \quad and \tag{23}$$

$$\frac{\partial \pi_1}{\partial q_1} = p(Q) + p'(Q)q_1 - c_1'(q_1) = 0. \tag{24}$$

Solving the above equations, we obtain the equilibrium outputs $q_0^{cAB} = q_0^{cAB}(\theta)$ and $q_1^{cAB} = q_1^{cAB}(\theta)$. Next, we analyze the comparative statics for the equilibrium output of each firm with respect to θ. We find that[12]

$$\frac{dq_0^{cAB}}{d\theta} < 0, \ \frac{dq_1^{cAB}}{d\theta} > 0, \ and \ \frac{dQ^{cAB}}{d\theta} < 0. \tag{25}$$

Here, we consider the case where $\bar{\theta}^{cAB}$ such that the equilibrium output of each firm before and after privatization is the same. Therefore, $\bar{\theta}^{cAB}$ satisfies the following equation:

$$p'(Q^N)Q^N + \bar{\theta}^{cAB} D_A'(E^N) = 0, \quad \text{where } E^N = \bar{\theta}^{cAB} Q^N. \tag{26}$$

The second term of the above equation is increasing and strictly convex in θ because $\partial \theta D_A'(E_A)/\partial \theta = D_A'(E_A) + \theta Q D_A''(E_A) > 0$ and the first term is a finite negative constant. Therefore, we can show that $\bar{\theta}^{cAB}$ exists in $[0, 1]$ and is unique.

Welfare comparison

[12] For calculations, see Appendix B-2.

We examine whether the privatization enhances local welfare A, local welfare B, and whole welfare if θ shifts marginally from $\bar{\theta}$.

$$\left.\frac{dw_A^{cAB}}{d\theta} - \frac{dw_A^{NcAB}}{d\theta}\right|_{\theta\ =\ \bar{\theta}^{cAB}} = p' q_0^N \frac{dq_1^{cAB}}{d\theta} < 0, \tag{27}$$

$$\left.\frac{dw_B^{cAB}}{d\theta} - \frac{dw_B^{NcAB}}{d\theta}\right|_{\theta\ =\ \bar{\theta}^{cAB}} = p' q_1^N \frac{dq_0^{cAB}}{d\theta} - (1-\bar{\theta}^{cAB}) D_B' \frac{dQ^{cAB}}{d\theta} > 0, \quad and \tag{28}$$

$$\left.\frac{dW^{cAB}}{d\theta} - \frac{dW^{NcAB}}{d\theta}\right|_{\theta\ =\ \bar{\theta}^{cAB}} = p' \left(q_1^N \frac{dq_0^{cAB}}{d\theta} + q_0^N \frac{dq_1^{cAB}}{d\theta} \right) - (1-\bar{\theta}^{cAB}) D_B' \frac{dQ^{cAB}}{d\theta} > 0. \tag{29}$$

From the above results, we obtain the following proposition.[13]

Proposition 2. *When θ marginally increases (decreases) at $\theta = \bar{\theta}^{cAB}$, privatization decreases (increases) both local welfare B and whole welfare, whereas it increases (decreases) local welfare A.*

The intuition behind proposition 2 is as follows. Consider the situation before privatization. First, we inspect the change in local welfare A. When θ increases, the equilibrium output of the local public firm decreases and that of the private firm increases. The equilibrium total output decreases. Therefore, consumer surplus decreases and indirect environmental damage decreases. From (26), we conclude that the decreases in consumer surplus and indirect environmental damage are offset, and hence, whether or not welfare increases depends on whether or not producer surplus increases. Producer surplus in region A is equal to the profit of the local public firm. When θ increases, the revenue of the local public firm decreases as there is an increase in the output of the private firm. As a result, producer surplus decreases. Therefore, local welfare A decreases. Next, we inspect the change in local welfare B. We can prove that local welfare B always increases when θ increases because the profit of the private firm increases and indirect environmental damage decreases as θ increases. Finally, we inspect the change in whole welfare. As the increase in the profit of the private firm is greater than the decrease in the profit of the local public firm, whole welfare increases.

In this case, we note that the interests of the local government which owns the local public firm and the other two governments are in conflict.

[13]For calculations, see Appendix B-2.

5 Production externality

In this section, we consider pollution that is caused by production. As in consumption externality, we consider four cases; these four cases are shown in Figure 2. However, Cases (AA) and (BB) in production externality is the same as cases (AA) and (BB), respectively, in consumption externality. Therefore, we examine cases (AB) and (BA) in the subsequent subsections.

5.1 Case (AB) in mixed duopoly (before privatization)

We consider the case where firm 0 exists in region A and firm 1 exists in region B.

Local welfare A, local welfare B, and whole welfare are defined as

$$w_A = \int^Q p(s)ds - c_0(q_0) - p(Q)q_1 - D_A(E_A) + m, \tag{30}$$

$$w_B = \pi_1 - D_B(E_B), \quad and \tag{31}$$

$$W = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_A(E_A) - D_B(E_B) + m, \quad respectively, \tag{32}$$

where $E_A = \theta q_0$ and $E_B = (1-\theta)q_0 + q_1$.

In mixed duopoly, the first order condition of each firm is given by

$$\frac{\partial w_A}{\partial q_0} = p(Q) - c_0'(q_0) - p'(Q)q_1 - \theta D_A'(E_A) = 0 \;\; and \tag{33}$$

$$\frac{\partial \pi_1}{\partial q_1} = p(Q) + p'(Q)q_1 - c_1'(q_1) = 0. \tag{34}$$

Solving the above equations, we obtain the equilibrium outputs $q_0^{pAB} = q_0^{pAB}(\theta)$ and $q_1^{pAB} = q_1^{pAB}(\theta)$. Next, we analyze the comparative statics for the equilibrium output of each firm with respect to θ. We find that[14]

$$\frac{dq_0^{pAB}}{d\theta} < 0, \; \frac{dq_1^{pAB}}{d\theta} > 0, \; and \; \frac{dQ^{pAB}}{d\theta} < 0. \tag{35}$$

Here, we consider the case where $\bar{\theta}^{pAB}$ is such that the equilibrium output of each firm before and after privatization is the same. Therefore, $\bar{\theta}^{pAB}$ satisfies the following

[14] For calculations, see Appendix B-3.

equation:

$$p'(Q^N)Q^N + \bar{\theta}^{pAB} D'_A(\bar{\theta}^{pAB} q_0^N) = 0. \tag{36}$$

We cannot ascertain whether or not $\bar{\theta}^{pAB}$ lies in $[0, 1]$ solely on the basis of Assumption 4. In order to ascertain that $\bar{\theta}^{pAB}$ exists in $[0, 1]$ and is unique, we make another assumption for case (AB) in production externality.

Assumption 5.

$$p'(Q^N)Q^N + D'_A(q_0^N) \geq 0. \tag{37}$$

Welfare comparison

We examine whether privatization enhances local welfare A, local B, and whole welfare if θ shifts marginally from $\bar{\theta}$.

$$\left.\frac{dw_A^{pAB}}{d\theta} - \frac{dw_A^{NpAB}}{d\theta}\right|_{\theta\,=\,\bar{\theta}} = -\,p'q_1^N \frac{dq_1^{pAB}}{d\theta} > 0. \tag{38}$$

$$\left.\frac{dw_B^{pAB}}{d\theta} - \frac{dw_B^{NpAB}}{d\theta}\right|_{\theta\,=\,\bar{\theta}} = p'q_1^N \frac{dq_0^{pAB}}{d\theta} - D'_B \left\{ (1 - \bar{\theta}^{pAB}) \frac{dq_0^{pAB}}{d\theta} + \frac{dq_1^{pAB}}{d\theta} \right\}, \quad \textit{and} \tag{39}$$

$$\left.\frac{dW^{pAB}}{d\theta} - \frac{dW^{NpAB}}{d\theta}\right|_{\theta\,=\,\bar{\theta}} = (p'q_1^N - (1 - \bar{\theta}^{pAB})D'_B) \frac{dq_0^{pAB}}{d\theta} - (p'q_1^N + D'_B) \frac{dq_1^{pAB}}{d\theta}. \tag{40}$$

From the above results, we can obtain the following proposition.[15]

Proposition 3. *When θ marginally increases (decreases) at $\bar{\theta}$, privatization decreases (increases) local welfare A, whereas the effect of privatization is ambiguous for local welfare B and whole welfare.*

The intuition behind proposition 3 is as follows. Consider the situation before privatization. First, we inspect local welfare A. As in case (AB) in consumption externality, we can show that when θ marginally increases at $\bar{\theta}$, indirect environmental damage decreases though both consumer surplus and producer surplus decrease. The difference between consumption externality and production externality can be attributed to the

[15] For calculations, see Appendix B-3.

total pollution in regions A and B. In region A, pollution levels in production externality and consumption externality are θq_0 and θQ, respectively. Since $0 > dQ/d\theta > dq_0/d\theta$, the decrease in indirect environmental damage is greater than the decrease in consumer surplus. Further the decrease in environmental damage is greater than the decrease in producer surplus. Consequently, local welfare A increases. Second, we inspect local welfare B. We can easily find that producer surplus increases. However, there is ambiguity as to whether indirect environmental damage increases or decreases. As total pollution is $(1-\theta)q_0 + q_1$, the change in indirect environmental damage is $(1-\theta)dq_0/d\theta + dq_1/d\theta$. Further the former term is negative and the latter, positive. If $\bar{\theta}^{pAB}$ is nearly equal to 1, that is, almost all of the pollution emitted by the local public firm remains in region A and little is transported to region B, total pollution in region B can be regarded as q_1. As $dq_1/\theta > 0$, indirect environmental damage increases. If $\bar{\theta}^{pAB}$ is nearly equal to 0, that is, almost all of the pollution emitted by the local public firm is transported to region B, total pollution in region B can be regarded as Q. As $dQ/\theta < 0$, indirect environmental damage decreases. The value of $\bar{\theta}^{pAB}$ depends on the functional form of the inverse demand function, the cost function of each firm, and the environmental damage function. Therefore, whether or not local welfare B increases is ambiguous. Consequently, the effect on whole welfare is also ambiguous.

5.2 Case (BA) in mixed duopoly (before privatization)

We consider the case where firm 0 exists in region B and firm 1 exists in region A. Local welfare A, local welfare B, and whole welfare are defined as

$$w_A = \pi_1 - D_A(E_A), \tag{41}$$

$$w_B = \int^Q p(s)ds - c_0(q_0) - p(Q)q_1 - D_B(E_B) + m, \quad and \tag{42}$$

$$W = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_A(E_A) - D_B(E_B) + m, \quad respectively, \tag{43}$$

where $E_A = \theta q_1$ and $E_B = (1-\theta)q_1 + q_0$.

In mixed duopoly, the first order condition of each firm is given by

$$\frac{\partial w_B}{\partial q_0} = p(Q) - c_0'(q_0) - p'(Q)q_1 - D_B'(E_B) = 0 \quad and \tag{44}$$

$$\frac{\partial \pi_1}{\partial q_1} = p(Q) + p'(Q)q_1 - c_1'(q_1) = 0. \tag{45}$$

Solving the above equations, we obtain the equilibrium outputs $q_0^{pBA} = q_0^{pBA}(\theta)$ and $q_1^{pBA} = q_1^{pBA}(\theta)$. Next, we examine the comparative statics for the equilibrium output of each firm with respect to θ. We find that[16]

$$\frac{dq_0^{pBA}}{d\theta} > 0, \ \frac{dq_1^{pBA}}{d\theta} < 0, \ and \ \frac{dQ^{pBA}}{d\theta} > 0. \tag{46}$$

Here, we consider the case where $\bar{\theta}^{pBA}$ is such that the equilibrium output of each firm before and after privatization is the same. Therefore, $\bar{\theta}^{pBA}$ satisfies the following equation:

$$p'(Q^N)Q^N + D_B'((1-\bar{\theta}^{pBA})q_1^N + q_0^N) = 0. \tag{47}$$

As in the previous case, we cannot ascertain whether or not $\bar{\theta}^{pBA}$ lies in $[0, 1]$ sole on the basis of Assumption 4. In order to ascertain that $\bar{\theta}^{pBA}$ exists in $[0, 1]$ and is unique, we make another assumption for case (BA) in production externality.

Assumption 6.

$$p'(Q^N)Q^N + D_B'(q_0^N) \leq 0. \tag{48}$$

Welfare comparison

We examine whether the privatization enhances local welfare A, local B, and whole welfare if θ shifts marginally from $\bar{\theta}$.

$$\left.\frac{dw_A^{pBA}}{d\theta} - \frac{dw_A^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} = p'q_1^N\frac{dq_0^{pBA}}{d\theta} - \bar{\theta}^{pBA}D_A'\frac{dq_1^{pBA}}{d\theta}, \tag{49}$$

$$\left.\frac{dw_B^{pBA}}{d\theta} - \frac{dw_B^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} = -\,p'\{-(1-\bar{\theta}^{pBA})q_0^N + \bar{\theta}^{pBA}q_1^N\}\frac{dq_1^{pBA}}{d\theta}, \quad and \tag{50}$$

$$\left.\frac{dW^{pBA}}{d\theta} - \frac{dW^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} = p'(q_1^N\frac{dq_0^{pBA}}{d\theta} + q_0^N\frac{dq_1^{pBA}}{d\theta}) - \bar{\theta}^{pBA}(D_A' - D_B')\frac{dq_1^{pBA}}{d\theta}. \tag{51}$$

[16]For calculations, see Appendix B-4.

From the above results, we obtain the following relationships.[17]

Proposition 4. *If $D_A(E) = D_B(E)$ for all $E > 0$,*

$$\text{1-1.} \begin{cases} \left.\dfrac{dw_B^{pBA}}{d\theta} - \dfrac{dw_B^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} > 0 \\ \left.\dfrac{dW^{pBA}}{d\theta} - \dfrac{dW^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} < 0 \quad \text{for } \bar{\theta}^{pBA} \in \left[0, \dfrac{q_0^N}{Q^N}\right), \end{cases}$$

$$\text{1-2.} \begin{cases} \left.\dfrac{dw_B^{pBA}}{d\theta} - \dfrac{dw_B^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} \leq 0 \\ \left.\dfrac{dW^{pBA}}{d\theta} - \dfrac{dW^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} < 0 \quad \text{for } \bar{\theta}^{pBA} \in \left[\dfrac{q_0^N}{Q^N}, \dfrac{Q^N}{2q_1^N}\right), \quad \text{and} \end{cases}$$

$$\text{1-3.} \begin{cases} \left.\dfrac{dw_B^{pBA}}{d\theta} - \dfrac{dw_B^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} < 0 \\ \left.\dfrac{dW^{pBA}}{d\theta} - \dfrac{dW^{NpBA}}{d\theta}\right|_{\theta\,=\,\bar{\theta}^{pBA}} \quad \text{ambiguous} \quad \text{for } \bar{\theta}^{pBA} \in \left[\dfrac{Q^N}{2q_1^N}, 1\right], \end{cases}$$

2. Whether or not privatization increases local welfare A is ambiguous.

Proof. See Appendix C. □

The intuition behind proposition 4 is as follows. Consider the situation before privatization. First, we inspecct local welfare B. When θ increases, the output of the local public firm increases since the transboundary pollution from region A decreases; further, the output of the private firm decreases and total output increases. As in the previously-analyzed cases, the effect of the increase in θ on the output of each firm is opposite in this case; therefore, we can easily prove that consumer surplus and producer surplus increases with θ. With respect to indirect environmental damage, the pollution in region B always increases with θ, and thus indirect environmental damage also increases.[18] If $\bar{\theta}^{pBA}$ is nearly equal to 1, the total pollution in region B can be regarded as q_0^N. As $dq_0/d\theta > dQ/d\theta > 0$, the increase in pollution is large. In this case, the effect of environmental damage is greater than the other two positive effects, and therefore, local welfare

[17] For calculations, see Appendix B-4.

[18] $E_B = (1-\theta)q_1^{pBA} + q_0^{pBA}$. When θ increases, $dE_B/d\theta = -q_1^{pBA} + (1-\theta)(dq_1^{pBA}/d\theta) + dq_0^{pBA}/d\theta$. θ affects the first term directly and the other terms, indirectly. By comparing $dw_B^{pBA}/d\theta$ and $dw_B^{NpBA}/d\theta$, we conclude that the first term is canceled out, and therefore, we focus on $(1-\theta)(dq_1^{pBA}/d\theta) + dq_0^{pBA}/d\theta$ and find that it is positive since $dQ^{pBA}/d\theta > 0$.

B decreases. If $\bar{\theta}^{pBA}$ is nearly equal to 0, the total pollution in region B can be regarded as Q^N. In this case, the increase in pollution is small since $dq_0/d\theta > dQ/d\theta > 0$. As a result, the effect of environmental damage is less than the other two positive effects, and hence, local welfare B increases.[19] Second, we inspect local welfare A. We can easily find that producer surplus and indirect environmental damage decrease as θ increases. Whether or not local welfare A increases depends on the value of $\bar{\theta}^{pBA}$. If $\bar{\theta}^{pBA}$ is nearly equal to 0, the effect of the decrease in producer surplus is the only effect that remains. Therefore, local welfare A decreases. However, whether or not local welfare A increases is ambiguous when $\bar{\theta}^{pBA}$ is high. Finally, we inspect whole welfare. If $\bar{\theta}^{pBA}$ is nearly equal to 0, from (47), we find that the effect of the change in the producer surplus is the only effect that remains. As the increase in the producer surplus of region B is smaller than the decrease in the producer surplus of region A, whole welfare decreases. However, when $\bar{\theta}^{pBA}$ is high, whether or not whole welfare increases is ambiguous and depends on the functional form of the inverse demand function, cost function, and the environmental damage function.

In this case, the interests of the local government that owns the local public firm and the central government are in conflict if $\bar{\theta}^{pBA}$ is sufficiently low, but coincidence when $\bar{\theta}^{pBA}$ is moderately large. In this case, the threshold fraction of transboundary pollution is important.

6 Concluding remarks

This paper examines the effect that the privatization of a local public firm has on the local welfare of each region and the welfare of the entire economy when the fraction of unidirectional transboundary pollution varies. We analyze this problem by considering eight separate cases on the basis of the location of firms and the types of pollution.

We consider the case where there exists a certain fraction of transboundary pollution

[19]Note that, when $\bar{\theta}^{pBA}$ is nearly equal to 1, the effect of the increase in producer surplus is the only effect that remains since the increase in consumer surplus is offset by the increase in indirect environmental damage.

for which the equilibrium outcome before and after privatization is the same. When the fraction of transboundary pollution marginally decreases at this level, that is, the amount of pollution that is transported downstream increases, we obtain the following results.

1. When both firms exist upstream, privatization of the local public firm increases local welfare of each region and the whole welfare regardless of whether there is consumption externality or production externality.

2. When the local public firm exists upstream and the private firm, downstream, in consumption externality, privatization decreases local welfare upstream, and increases both local welfare in downstream and whole welfare. In production externality, privatization increases local welfare upstream; its effect is ambiguous for local welfare downstream and whole welfare.

3. When the local public firm exists downstream and the private firm, upstream, in production externality, privatization increases local welfare downstream, and decreases whole welfare if the threshold fraction of transboundary pollution is sufficiently low. However, it decreases not only local welfare downstream but also whole welfare if the threshold fraction of transboundary pollution is moderately large.

We find that the effects of the privatization might differ for (i) each location pattern even if the pollution type is the same, (ii) each pollution type even if the location pattern is the same, and (iii) the different values of the threshold fraction of transboundary pollution even if both the location pattern and pollution type are the same.

We consider the possible implication of out results. We consider the example of the relationship between China and Japan with regard to the trash that washes up on Japanese shores or air pollution. In this situation, case (AA) (in both consumption externality and production externality) seems applicable. From the results, we can see that the privatization of the Chinese public firm enhances not only welfare in China but also welfare in Japan. In this case, privatization is desirable for both China and Japan.

However, if Japanese firms enter the Chinese market and international mixed oligopoly occurs, privatization may lead to a conflict of interest. We have to pay attention to how privatization in a certain region would affect not only its region but also the other regions.

This paper considers the privatization problem under the unidirectional transboundary pollution problem in a simple framework, and therefore, we can consider several extensions of this analysis. We leave these for future research.

Appendix A

Case (BB) in consumption externality and production externality in mixed duopoly (before privatization)

We consider the case where both firms 0 and 1 exist in region B.

Local welfare A, local welfare B, and whole welfare are defined as

$$w_A = 0, \tag{52}$$

$$w_B = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_B(E_B) + m, \quad and \tag{53}$$

$$W = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_B(E_B) + m, \quad respectively, \tag{54}$$

where $E_B = Q$. The first order condition of each firm is given by

$$\frac{\partial w_B}{\partial q_0} = p(Q) - c_0'(q_0) - D_B'(E_B) = 0 \quad and \tag{55}$$

$$\frac{\partial \pi_1}{\partial q_1} = p(Q) + p'(Q)q_1 - c_1'(q_1) = 0. \tag{56}$$

By solving the above first order conditions, we obtain q_0^{cBB} and q_1^{cBB}, the equilibrium outputs of firms 0 and 1, respectively. We can easily find that the equilibrium outcomes do not depend on θ. Further, the fraction of transboundary pollution does not appear in case (BB). Therefore, a shift in the fraction of transboundary pollution does not influence the privatization decision.

Note that this case corresponds with the basic model of the environmental problem in a mixed duopoly: one region with no abatement effort and no transboundary pollution problem. Bárcena-Ruiz and Garzón (2007) analyze this case and show that privatization worsens local welfare B, and thus, also worsens whole welfare.

Case (BA) in consumption externality in mixed duopoly (before privatization)

We consider the case where firm 0 exists in region B and firm 1 exists in region A. Local welfare A, local welfare B, and whole welfare are defined as

$$w_A = \pi_1, \tag{57}$$

$$w_B = \int^Q p(s)ds - c_0(q_0) - pq_1 - D_B(E_B) + m, \quad and \tag{58}$$

$$W = \int^Q p(s)ds - c_0(q_0) - c_1(q_1) - D_B(E_B) + m, \quad respectively, \tag{59}$$

where $E_B = Q$. The first order condition of each firm is given by

$$\frac{\partial w_B}{\partial q_0} = p(Q) - c_0'(q_0) - p'q_1 - D_B'(E_B) = 0 \quad and \tag{60}$$

$$\frac{\partial \pi_1}{\partial q_1} = p(Q) + p'(Q)q_1 - c_1'(q_1) = 0. \tag{61}$$

By solving the above first order conditions, we obtain q_0^{cBA} and q_1^{cBA}, the equilibrium outputs of firms 0 and 1, respectively. Note that the equilibrium outcome does not depend on θ. Further, as in case (BB), a shift in the fraction of transboundary pollution does not influence the privatization decision.

Appendix B

B-1. Case (AA) in consumption externality

Comparative statics for $q_0^{cAA}(\theta)$, $q_1^{cAA}(\theta)$, and $Q^{cAA}(\theta)$

$$\begin{pmatrix} w_{00}^{cAA} & w_{01}^{cAA} \\ \pi_{10}^{cAA} & \pi_{11}^{cAA} \end{pmatrix} \begin{pmatrix} \frac{dq_0^{cAA}}{d\theta} \\ \frac{dq_1^{cAA}}{d\theta} \end{pmatrix} = \begin{pmatrix} D_A' + \theta Q^{cAA} D_A'' \\ 0 \end{pmatrix},$$

where we denote $w_{ij}^{cAA} = \partial^2 w_A^{cAA}/\partial q_i \partial q_j$ and $\pi_{ij}^{cAA} = \partial^2 \pi_1^{cAA}/\partial q_i \partial q_j$, $(i,j = 0,1)$. In the subsequent analyses in Appendix B, we use the terms w_{ij}^{cr}, w_{ij}^{pr}, π_{ij}^{cr}, and π_{ij}^{pr} $(r = AA, BB, AB, BA)$ similarly. Note that $w_{00}^{cAA}, w_{01}^{cAA}, \pi_{10}^{cAA}$, and $\pi_{11}^{cAA} < 0$.[20] We denote that $\Delta^{cAA} = w_{00}^{cAA}\pi_{11}^{cAA} - w_{01}^{cAA}\pi_{10}^{cAA} > 0$. We find that

$$\frac{dq_0^{cAA}}{d\theta} = \frac{\pi_{11}^{cAA}}{\Delta^{cAA}}(D' + \theta Q^{cAA} D_A'') < 0, \tag{62}$$

$$\frac{dq_1^{cAA}}{d\theta} = -\frac{\pi_{10}^{cAA}}{\Delta^{cAA}}(D' + \theta Q^{cAA} D_A'') > 0, \quad and \tag{63}$$

$$\frac{dQ^{cAA}}{d\theta} = \frac{\pi_{11}^{cAA} - \pi_{10}^{cAA}}{\Delta^{cAA}}(D' + \theta Q^{cAA} D_A'') < 0. \tag{64}$$

The last inequality holds since $|\pi_{11}^{cAA}| > |\pi_{10}^{cAA}|$.

Comparative statics for $w_A^{cAA}(\theta)$, $w_B^{cAA}(\theta)$, and $W^{cAA}(\theta)$

Using the first order condition of each firm, we can find that before privatization,

$$\frac{dw_A^{cAA}}{d\theta} = -(p'q_1^{cAA} + \theta D_A')\frac{dq_1^{cAA}}{d\theta} - Q^{cAA} D_A', \tag{65}$$

$$\frac{dw_B^{cAA}}{d\theta} = -D_B'\left\{-Q^{cAA} + (1-\theta)\frac{dQ^{cAA}}{d\theta}\right\} > 0, \quad and \tag{66}$$

$$\frac{dW^{cAA}}{d\theta} = -(1-\theta)D_B'\frac{dq_0^{cAA}}{d\theta} - \{p'q_1^{cAA} + \theta D_A' + (1-\theta)D_B'\}\frac{dq_1^{cAA}}{d\theta} - Q^{cAA}(D_A' - D_B'). \tag{67}$$

Comparative statics for $w_A^{NcAA}(\theta)$, $w_B^{NcAA}(\theta)$, and $W^{NcAA}(\theta)$

After privatization, the equilibrium output of each firm is $q_i = q_i^N$ and does not depend on θ. Thus, we obtain

$$\frac{dw_A^{NcAA}}{d\theta} = -Q^N D_A' < 0, \tag{68}$$

$$\frac{dw_B^{NcAA}}{d\theta} = Q^N D_B' > 0, \quad and \tag{69}$$

$$\frac{dW^{NcAA}}{d\theta} = -Q^N(D_A' - D_B'). \tag{70}$$

[20] $w_{00}^{cAA} = p' - c_0'' - \theta^2 D_A''$, $w_{01}^{cAA} = p' - \theta^2 D_A''$, $\pi_{10}^{cAA} = p' + p'' q_1$, and $\pi_{11}^{cAA} = 2p' + p'' q_1 - c_1''$.

B-2. Case (AB) in the consumption externality

Comparative statics for $q_0^{cAB}(\theta)$, $q_1^{cAB}(\theta)$, and $Q^{cAB}(\theta)$

$$\begin{pmatrix} w_{00}^{cAB} & w_{01}^{cAB} \\ \pi_{10}^{cAB} & \pi_{11}^{cAB} \end{pmatrix} \begin{pmatrix} \dfrac{dq_0^{cAB}}{d\theta} \\ \dfrac{dq_1^{cAB}}{d\theta} \end{pmatrix} = \begin{pmatrix} D'_A + \theta Q^{cAB} D''_A \\ 0 \end{pmatrix}.$$

We denote that $\Delta^{cAB} = w_{00}^{cAB}\pi_{11}^{cAB} - w_{01}^{cAB}\pi_{10}^{cAB} > 0$.[21] We find that

$$\frac{dq_0^{cAB}}{d\theta} = \frac{\pi_{11}^{cAB}}{\Delta^{cAB}}(D' + \theta Q^{cAB} D''_A) < 0, \tag{71}$$

$$\frac{dq_1^{cAB}}{d\theta} = -\frac{\pi_{01}^{cAB}}{\Delta^{cAB}}(D' + \theta Q^{cAB} D''_A) > 0, \quad and \tag{72}$$

$$\frac{dQ^{cAB}}{d\theta} = \frac{\pi_{11}^{cAB} - \pi_{10}^{cAB}}{\Delta^{cAB}}(D' + \theta Q^{cAB} D''_A) < 0. \tag{73}$$

Comparative statics for $w_A^{cAB}(\theta)$, $w_B^{cAB}(\theta)$, and $W^{cAB}(\theta)$

Using the first order condition of each firm, we can find that before privatization,

$$\frac{dw_A^{cAB}}{d\theta} = -\left(p' q_1^{cAB} + \theta D'_A\right)\frac{dq_1^{cAB}}{d\theta} - Q^{cAB} D'_A, \tag{74}$$

$$\frac{dw_B^{cAB}}{d\theta} = p' q_1^{cAB}\frac{dq_0^{cAB}}{d\theta} - D'_B\left\{-Q^{cAB} + (1-\theta)\frac{dQ^{cAB}}{d\theta}\right\} > 0, \quad and \tag{75}$$

$$\frac{dW^{cAB}}{d\theta} = \{p' q_1^{cAB} - (1-\theta)D'_B\}\frac{dq_0^{cAB}}{d\theta} - \{p' q_1^{cAB} + \theta D'_A + (1-\theta)D'_B\}\frac{dq_1^{cAB}}{d\theta} - Q^{cAB}(D'_A - D'_B). \tag{76}$$

Comparative statics for $w_A^{NcAB}(\theta)$, $w_B^{NcAB}(\theta)$, and $W^{NcAB}(\theta)$

After privatization,

$$\frac{dw_A^{NcAB}}{d\theta} = -Q^{NcAB} D'_A < 0, \tag{77}$$

$$\frac{dw_B^{cAB}}{d\theta} = Q^{NcAB} D'_B > 0, \quad and \tag{78}$$

$$\frac{dW^{NcAB}}{d\theta} = -Q^{NcAB}(D'_A - D'_B). \tag{79}$$

[21] $w_{00}^{cAB} = p' - p'' q_1 - c''_0 - \theta^2 D''_A < 0$ and $w_{01}^{cAB} = -p'' q_1 - \theta^2 D''_A$.

B-3. Case (AB) in production externality

Comparative statics for $q_0^{pAB}(\theta)$, $q_1^{pAB}(\theta)$, and $Q^{pAB}(\theta)$

$$\begin{pmatrix} w_{00}^{pAB} & w_{01}^{pAB} \\ \pi_{10}^{pAB} & \pi_{11}^{pAB} \end{pmatrix} \begin{pmatrix} \frac{dq_0^{pAB}}{d\theta} \\ \frac{dq_1^{pAB}}{d\theta} \end{pmatrix} = \begin{pmatrix} D_A' + \theta q_0^{pAB} D_A'' \\ 0 \end{pmatrix}$$

We denote that $\Delta^{pAB} = w_{00}^{pAB}\pi_{11}^{pAB} - w_{01}^{pAB}\pi_{10}^{pAB} > 0$.[22] We find that

$$\frac{dq_0^{pAB}}{d\theta} = \frac{\pi_{11}^{pAB}}{\Delta^{pAB}}(D_A' + \theta q_0^{pAB} D_A'') < 0, \tag{80}$$

$$\frac{dq_1^{pAB}}{d\theta} = -\frac{\pi_{10}^{pAB}}{\Delta^{pAB}}(D_A' + \theta q_0^{pAB} D_A'') > 0, \quad and \tag{81}$$

$$\frac{dQ^{pAB}}{d\theta} = \frac{\pi_{11}^{pAB} - \pi_{10}^{pAB}}{\Delta^{pAB}}(D_A' + \theta q_0^{pAB} D_A'') < 0. \tag{82}$$

Comparative statics for $w_A^{pAB}(\theta)$, $w_B^{pAB}(\theta)$, and $W^{pAB}(\theta)$

Using the first order condition of each firm, we can find that before privatization,

$$\frac{dw_A^{pAB}}{d\theta} = -\, p' q_1 \frac{dq_1^{pAB}}{d\theta} - q_0 D_A', \tag{83}$$

$$\frac{dw_B^{pAB}}{d\theta} = p' q_1 \frac{dq_0^{pAB}}{d\theta} - D_B' \left\{ -q_0^{pAB} + (1-\theta)\frac{dq_0^{pAB}}{d\theta} + \frac{dq_1^{pAB}}{d\theta} \right\}, \quad and \tag{84}$$

$$\frac{dW^{pAB}}{d\theta} = \{p' q_1 - (1-\theta) D_B'\} \frac{dq_0^{pAB}}{d\theta} - \{p' q_1 + D_B'\} \frac{dq_1^{pAB}}{d\theta} - q_0 (D_A' - D_B'). \tag{85}$$

Comparative statics for $w_A^{NpAB}(\theta)$, $w_B^{NpAB}(\theta)$, and $W^{NpAB}(\theta)$

After privatization,

$$\frac{dw_A^{NpAB}}{d\theta} = -\, q_0^N D_A' < 0, \tag{86}$$

$$\frac{dw_B^{NpAB}}{d\theta} = q_0^N D_B' > 0, \quad and \tag{87}$$

$$\frac{dW^{NpAB}}{d\theta} = -\, q_0^N (D_A' - D_B'). \tag{88}$$

[22] $w_{00}^{pAB} = p' - c_0'' - p'' q_1 - \theta^2 D_A'' < 0$ and $w_{01}^{pAB} = -p'' q_1$.

B-4. Case (BA) in production externality

Comparative statics for $q_0^{pBA}(\theta)$, $q_1^{pBA}(\theta)$, and $Q^{pBA}(\theta)$

$$\begin{pmatrix} w_{00}^{pBA} & w_{01}^{pBA} \\ \pi_{10}^{pBA} & \pi_{11}^{pBA} \end{pmatrix} \begin{pmatrix} \frac{dq_0^{pBA}}{d\theta} \\ \frac{dq_1^{pBA}}{d\theta} \end{pmatrix} = \begin{pmatrix} -q_1^{pBA} D_B'' \\ 0 \end{pmatrix}$$

We denote that $\Delta^{pBA} = w_{00}^{pBA}\pi_{11}^{pBA} - w_{01}^{pBA}\pi_{10}^{pBA} > 0$.[23] We find that

$$\frac{dq_0^{pBA}}{d\theta} = -\frac{\pi_{11}^{pBA}}{\Delta^{pBA}}(q_1^{pBA} D_B'') > 0, \tag{89}$$

$$\frac{dq_1^{pBA}}{d\theta} = \frac{\pi_{10}^{pBA}}{\Delta^{pBA}}(q_1^{pBA} D_B'') < 0, \quad and \tag{90}$$

$$\frac{dq_0^{pBA}}{d\theta} + \frac{dq_1^{pBA}}{d\theta} = -\frac{\pi_{11}^{pBA} - \pi_{10}^{pBA}}{\Delta^{pBA}}(q_1^{pBA} D_B'') > 0. \tag{91}$$

Comparative statics for $w_A^{pBA}(\theta)$, $w_B^{pBA}(\theta)$, and $W^{pBA}(\theta)$

Using the first order condition of each firm, we can find that before privatization,

$$\frac{dw_A^{pBA}}{d\theta} = p' q_1^{pBA} \frac{dq_0^{pBA}}{d\theta} - D_A' \left(q_1^{pBA} + \theta \frac{dq_1^{pBA}}{d\theta} \right), \tag{92}$$

$$\frac{dw_B^{pBA}}{d\theta} = -\left\{ p' q_1^{pBA} + (1-\theta) D_B' \right\} \frac{dq_1^{pBA}}{d\theta} + q_1^{pBA} D_B', \quad and \tag{93}$$

$$\frac{dW^{pBA}}{d\theta} = p' q_1^{pBA} \frac{dq_0^{pBA}}{d\theta} - \left\{ p' q_1^{pBA} + \theta D_A' + (1-\theta) D_B' \right\} \frac{dq_1^{pBA}}{d\theta} - q_1^{pBA}(D_A' - D_B'). \tag{94}$$

Comparative statics for $w_A^{NpBA}(\theta)$, $w_B^{NpBA}(\theta)$, and $W^{NpBA}(\theta)$

After privatization,

$$\frac{\partial w_A^{NpBA}}{\partial \theta} = -\, q_1^N D_A' < 0, \tag{95}$$

$$\frac{\partial w_B^{NpBA}}{\partial \theta} = q_1^N D_B' > 0, \quad and \tag{96}$$

$$\frac{\partial W^{NpBA}}{\partial \theta} = -\, q_1^N (D_A' - D_B'). \tag{97}$$

[23] $w_{00}^{pBA} = p' - c_0'' - p'' q_1 - D_B'' < 0$ and $w_{01}^{pBA} = -p'' q_1 - (1-\theta) D_B''$.

Appendix C

Proof of Proposition 4

Proof. First, we examine $dw_B^{pBA}/d\theta - dw_B^{NpBA}/d\theta\Big|_{\theta=\bar{\theta}^{pBA}}$. We can easily find that $-p' > 0$ and $dq_1^{pBA}/d\theta < 0$. We focus on the sign of the brace. If the brace is negative (positive), that is, if

$$
\begin{gathered}
-(1-\bar{\theta}^{pBA})q_0^N + \bar{\theta}^{pBA}q_1^N < 0 \Leftrightarrow \bar{\theta}^{pBA} < \frac{q_0^N}{Q^N} \\
\left(-(1-\bar{\theta}^{pBA})q_0^N + \bar{\theta}^{pBA}q_1^N > 0 \Leftrightarrow \bar{\theta}^{pBA} > \frac{q_0^N}{Q^N}\right),
\end{gathered} \tag{98}
$$

$dw_B^{pBA}/d\theta - dw_B^{NpBA}/d\theta\Big|_{\theta\,=\,\bar{\theta}^{pBA}}$ is positive (negative).

Second, we examine $dW^{pBA}/d\theta - dW^{NpBA}/d\theta\big|_{\theta\,=\,\bar{\theta}^{pBA}}$. We find that

$$
p'\left(q_1^N\frac{dq_0^{pBA}}{d\theta} + q_0^N\frac{dq_1^{pBA}}{d\theta}\right) < 0. \tag{99}
$$

Therefore, $dW^{pBA}/d\theta - dW^{NpBA}/d\theta\big|_{\theta\,=\,\bar{\theta}^{pBA}} < 0$ if $-\bar{\theta}^{pBA}(D'_A - D'_B)(dq_1^{pBA}/d\theta) < 0$. Given that $-\bar{\theta}^{pBA} \leq 0$ and $dq_1^{pBA}/d\theta < 0$, we need to ascertain the sign of $D'_A - D'_B$. To simplify the analysis, we assume that $D_A(E) = D_B(E)$ for all $E \geq 0$.

Given that $E_A^N = \bar{\theta}^{pBA}q_0^N$ and $E_B^N = (1-\bar{\theta}^{pBA})q_0^N + q_1^N$, we get $E_A < E_B$, that is,

$$
\bar{\theta}^{pBA}q_0^N < (1-\bar{\theta}^{pBA})q_0^N + q_1^N \Leftrightarrow \bar{\theta}^{pBA} < \frac{Q^N}{2q_1^N}. \tag{100}
$$

In this case, $dW^{pBA}/d\theta - dW^{NpBA}/d\theta\big|_{\theta\,=\,\bar{\theta}^{pBA}} < 0$. Note that this is the just enough condition.

Finally, we compare q_0^N/Q^N and $Q^N/2q_1^N$, and easily find that $0 < q_0^N/Q^N < Q^N/2q_1^N \leq 1$. □

Appendix D

Examples of case (AB) and (BA) in production externality

Here, we consider the following specific functional form of the demand, cost, and environmental damage functions; $p(Q) = 1 - q_0 - q_1$, $c_0(q_0) = cq_0^2/2$, $c_1(q_1) = q_1^2/2$,

$D_A(E) = D_B(E) = dE^2/2$. After the privatization, the equilibrium output of each firm is $q_0^N = 2/(5+3c)$ and $q_1^N = 1 + c/(5+3c)$. In case (AB) and (BA) in production externality, we can find that $\bar{\theta}^{pAB} = \sqrt{3 + c/2d}$ and $\bar{\theta}^{pBA} = (3+c)(d-1)/\{(1+c)d\}$. The results are shown in Table 1 and 2.

c \ d	2	4.5	8	12.5	18
1	w_A: 1/48 w_B: 1/48 W: 1/24 $\bar{\theta}^{pAB}$=1	w_A: 1/32 w_B: 3/32 W: 1/8 $\bar{\theta}^{pAB}$=2/3	w_A: 1/24 w_B: 3/8 W: 5/12 $\bar{\theta}^{pAB}$=1/2	w_A: 5/96 w_B: 95/96 W: 25/24 $\bar{\theta}^{pAB}$=2/5	w_A: 1/16 w_B: 33/16 W: 17/8 $\bar{\theta}^{pAB}$=1/3
13	n.a.	n.a.	w_A: 14/3993 w_B: −70/3993 W: −56/3993 $\bar{\theta}^{pAB}$=1	w_A: 35/7986 w_B: −25/2662 W: −20/3993 $\bar{\theta}^{pAB}$=4/5	w_A: 7/1331 w_B: 21/1331 W: 28/1331 $\bar{\theta}^{pAB}$=2/3

Table 1: The relationship in Proposition 3 under several values of c and d

c \ d	1	2	4	8
1	w_A: −3/128 w_B: 1/128 W: −1/64 $\bar{\theta}^{pBA}$=0	w_A: −1/96 w_B: −1/96 W: −1/48 $\bar{\theta}^{pBA}$=1	n.a.	n.a.
13	w_A: −147/21296 w_B: 7/21296 W: −35/5324 $\bar{\theta}^{BA}$=0	w_A: −161/15972 w_B: −35/15972 W: −49/3993 $\bar{\theta}^{pBA}$=4/7	w_A: −21/45254 w_B: −287/45254 W: −14/2057 $\bar{\theta}^{pBA}$=6/7	w_A: 245/3993 w_B: −49/3993 W: 196/3993 $\bar{\theta}^{pBA}$=1

Table 2: The relationship in Proposition 4 under several values of c and d

Note that "n.a." implies that $\bar{\theta}$ does not exist in $[0, 1]$ under these parameters of c and d.

In case (BA), we can find that $q_0^N = q_1^N = 1/4$ and $Q^N = 1/2$ when $c = 1$, and that $q_0^N = 1/22$, $q_1^N = 7/22$, and $Q^N = 4/11$ when $c = 13$. We can also find that $q_0^N/Q^N = 1/2$ and $Q^N/2q_1^N = 1$ when $c = 1$, and that $q_0^N/Q^N = 1/8$ and $Q^N/2q_1^N = 4/7$ when $c = 13$. Therefore, when $c = 1$, 1-3 of Proposition 4 vanishes.

From Table 1 and 2, with regard to "ambiguous" in Prospotion 3 and 4, there are cases that the sign is positive and there are also cases that its sign is negative.

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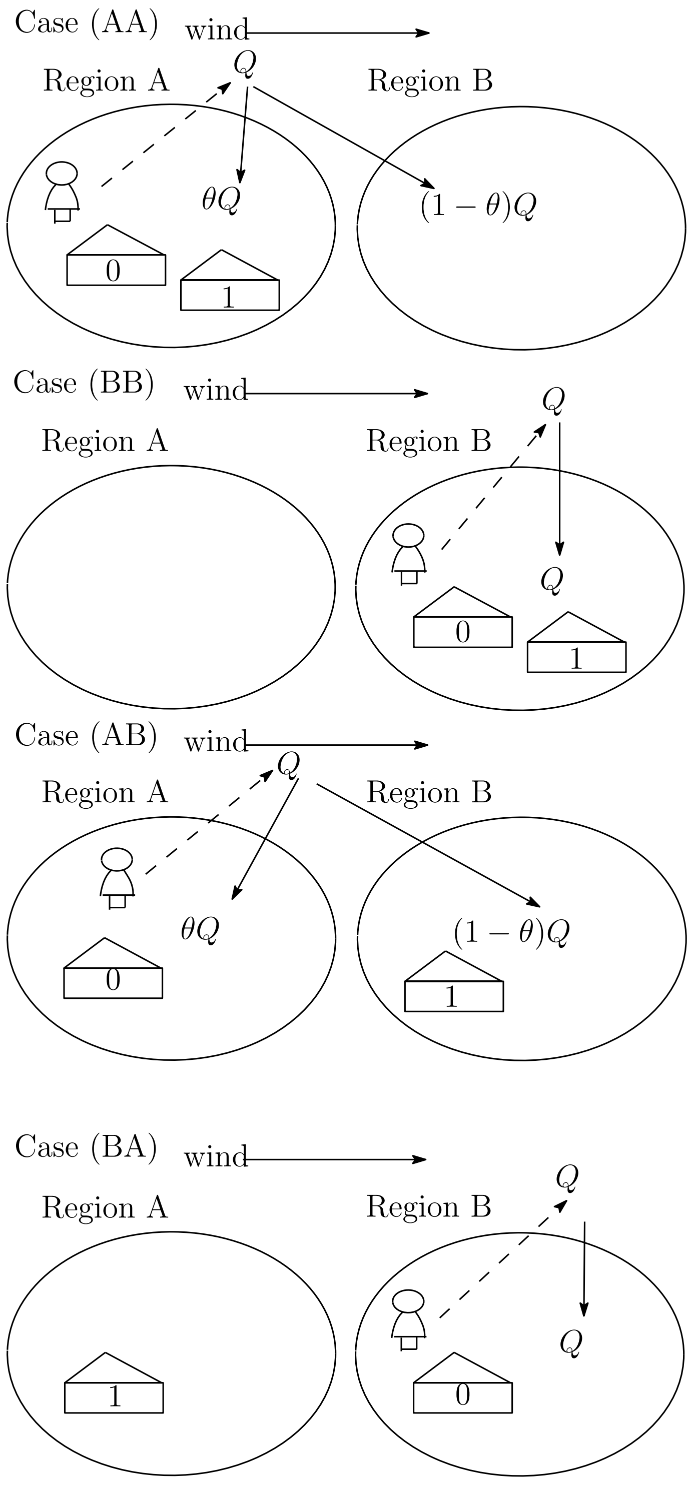

Figure 1: Four cases in consumption externality

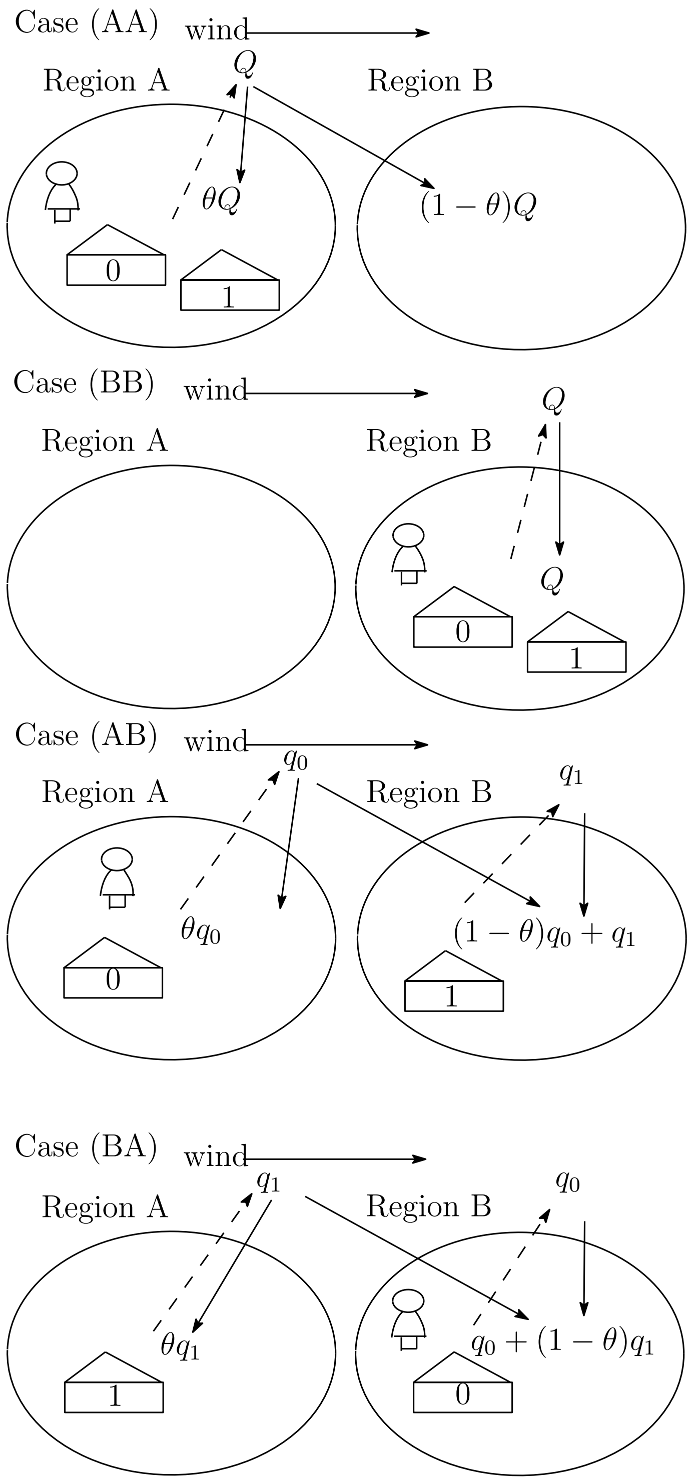

Figure 2: Four cases in production externality