

Relational Political Contribution under Common Agency

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Introduction

- ▶ Monetary transfer in political process
 - ▶ Lobbyists often affect the politician through the (in)pecuniary compensation
 - ▶ e.g., Each industry is willing to control the trade tariff through campaign contribution (Grossman and Helpman (1994), Goldberg and Maggi (1999))
- ▶ Framework: Common agency model (Bernheim and Whinston (1986, hereafter BW), Grossman and Helpman (1994))
 - ▶ Multi principals (Lobbyists) and one common agent (politician)
 - ▶ Compensation contract contingent on the agent's decision
 - ▶ It can be fully committed or enforced by a third-party
- ▶ Rather we should consider **self-enforced agreements**

Relational Contract

- ▶ Relational contracts:
 - ▶ Implicit agreements being self-enforced due to dynamic concern
 - ▶ Formally, infinitely repeated interaction of decision making by the agent and voluntary compensation by the principal
- ▶ They have already studied in employment relations or procurements (MacLeod and Malcomson (1989), Baker et al. (2002), Levin (2003), etc)
- ▶ This paper applies them into
 - ▶ [political contribution](#)
 - ▶ [allowing multiple principals](#)
 - ▶ (no asymmetric information)

Question (1): How to Punish

- ▶ **Credible punishment** is necessary to lead the voluntary payment
- ▶ E.g., in employment relation,
 - ▶ each player can terminate their relation and go to her outside option if she wants
 - ▶ Under some mild assumptions, the optimal punishment on a deviator would be to go to the outside option forever
- ▶ Between the lobbyist and the politician, **no such outside options**
 - ▶ E.g., in case of protection for sale,
 - ▶ the industry group and politicians always cares about imports and/or exports from foreign countries
 - ▶ It is influenced by the tariff policy but not the event to access the politician
 - ▶ → reasonable to presume that the players cannot escape from the agent's decision

Issue (1): How to Punish

- ▶ The punishment is a little bit complicated, especially for principals
- ▶ Results; the punishment on a deviating principal is either
 - ▶ “One-shot sanction” (or so-called “stick and carrot”):
 - ▶ the terrible decision for the deviator is made first
 - ▶ if the deviator pays some “fine”, go to a desirable decision
 - ▶ “Exclusion”:
 - ▶ the terrible decision for the deviator is chosen forever
- ▶ both could be optimal, depending on the agent’s decision space and the payoff function.

Issue (2): Equilibrium Payoff Set

- ▶ Is the static outcome (in BW) valid in the relational situation?
- ▶ To answer it,
 - ▶ characterize the set of the stationary equilibrium payoff
 - ▶ compare it with the static common agency (*a la* BW)
- ▶ Results
 - ▶ The static equilibrium is more likely to be supported if the players are patient
 - ▶ If they are not patient, the amount paid by principals is less than the static model
 - ▶ It could be the case that **the static equilibrium payoff can never supported no matter how patient the players are**

Outline

1. Setting
2. Characterize the punishment
3. Characterize Stationary Eq. and compare it with the corresponding static equilibrium
4. Conclusion
5. (Related Literature)

Players, Action, and Timing

- ▶ $N + 1$ players, indexed by $0, 1, \dots, N$
 - ▶ $j \in \mathcal{N} := \{1, \dots, N\}$: Principals
 - ▶ 0: one agent
- ▶ All of them live for infinite periods $t = 0, 1, \dots$ with common discount factor $\delta \in [0, 1)$
- ▶ Each period has 2 stages;
 - ▶ Stage 1: The agent chooses $a_t \in \mathcal{A}$ (the decision set)
 - ▶ Stage 2: Each of the principals pays $b_t^j \geq 0$ to the agent
- ▶ Both a_t and b_t^j are observable but not contractible

Payoff

- ▶ Player $i \in \mathcal{N} \cup \{0\}$'s one-shot benefit $v^i(a_t) \in \mathbb{R}$ if the agent chooses $a_t \in \mathcal{A}$
- ▶ \rightarrow the one-shot payoffs are
 - ▶ Principal j : $v^j(a_t) - b_t^j$
 - ▶ Agent: $v^0(a_t) + B_t$ where $B_t := \sum_{k=1}^N b_t^k$
- ▶ Each maximizes his/her average payoff of the discounted sum;
 - ▶ Principal j : $(1 - \delta) \sum_{\tau=0}^{\infty} \delta^\tau [v^j(a_\tau) - b_\tau^j]$
 - ▶ Agent: $(1 - \delta) \sum_{\tau=0}^{\infty} \delta^\tau [v^0(a_\tau) + B_\tau]$

Assumptions

- ▶ Assume that
 - ▶ $\mathcal{A} \subset \mathbb{R}^M$ is compact
 - ▶ $v^i(\cdot)$ is continuous for all i
- ▶ The maximum and minimum are well-defined;

$$\bar{A}^i := \arg \max_{a \in \mathcal{A}} v^i(a), \quad \underline{A}^i := \arg \min_{a \in \mathcal{A}} v^i(a),$$

$$\bar{v}^i := \max_{a \in \mathcal{A}} v^i(a), \quad \underline{v}^i := \min_{a \in \mathcal{A}} v^i(a)$$

- ▶ A representative element is written as $\underline{a}^i \in \underline{A}^i$ and $\bar{a}^i \in \bar{A}^i$
- ▶ Let $s(a) := \sum_{i=0}^N v^i(a)$ and $A^* := \arg \max_{a \in \mathcal{A}} s(a)$

Example 1

- ▶ $N = 2$
- ▶ $\mathcal{A} = \{l, c, r\}$
- ▶ $v^j(\cdot)$ is given as follows ($G, C, D > 0$)

Decision	l	c	r
$v^1(a)$	$-D$	0	G
$v^2(a)$	G	0	$-D$
$v^0(a)$	$-C$	0	$-C$

- ▶ E.g. $\underline{A}^1 = \{l\}$ and $\underline{v}^1 = -D$. $\bar{A}^0 = \{c\}$ and $\bar{v}^0 = 0$

Strategy and Equilibrium

- ▶ Define the history as following;
 - ▶ History: $h_t := (a_0, \mathbf{b}_0, a_1, \mathbf{b}_1, \dots, a_{t-1}, \mathbf{b}_{t-1})$ where
 $\mathbf{b}_\tau = (b_\tau^1, b_\tau^2, \dots, b_\tau^N)$
- ▶ (Pure) strategy
 - ▶ Agent: $\sigma^0(h_t) \in \mathcal{A}$
 - ▶ Principal j : $\sigma^j(h_t, a_t) \in \mathbb{R}_+$
- ▶ Let $u^i(\sigma)$ be the average payoff when the strategy profile is σ
- ▶ Subgame perfect equilibrium (SPE); each strategy on SPE is best response to the opponents' strategy given any history

Strategy and Equilibrium

- ▶ Suppose that $\hat{\sigma}$ generates $(\hat{a}_0, \hat{\mathbf{b}}_0, \hat{\sigma}_1)$ on the path where $\hat{\sigma}_1$ is the continuation strategy
- ▶ Let $\sigma(i)$ be a SPE which yields the lowest equilibrium payoff for player i : “**optimal penal code**” (OPC)
- ▶ Abreu (1988) shows that $\hat{\sigma}$ is SPE iff
 - ▶ \hat{a}_0 is incentive compatible (see next slide)
 - ▶ \hat{b}_0^j is incentive compatible for all $j \in \mathcal{N}$ (see next slide)
 - ▶ $\hat{\sigma}_1$ is SPE

Incentive Compatibility

- ▶ \hat{a}_0 is IC iff

$$\begin{aligned} u^0(\hat{\sigma}) &\equiv (1 - \delta) \left[v^0(\hat{a}_0) + \hat{B}_0 \right] + \delta u^0(\hat{\sigma}_1) \\ &\geq (1 - \delta) \bar{v}^0 + \delta u^0(\sigma(0)) \end{aligned}$$

- ▶ RHS: the payoff when

- ▶ the agent deviates \hat{a}_0 to $\bar{a}^0 \in \bar{A}^0$ (maximum gain by deviation)
- ▶ the players punish him by the lowest SPE payoff (pay nothing and play $\sigma(0)$)

- ▶ b_0^j is IC iff

$$\begin{aligned} u^j(\hat{\sigma}) &\equiv (1 - \delta) \left[v^j(\hat{a}_0) - b_0^j \right] + \delta u^j(\hat{\sigma}_1) \\ &\geq (1 - \delta) v^j(\hat{a}_0) + \delta u^j(\sigma(j)) \end{aligned}$$

What Will Be Done

1. Derive $u^i(\sigma(i))$
2. Characterize stationary equilibrium payoff
3. Compare it with the static common agency equilibrium

OPC on Agent

- ▶ What is $u^0(\sigma(0))$?
- ▶ SPE when the game is one-shot
 - ▶ the principals pay nothing
 - ▶ the agent chooses $\bar{a}^0 \in \bar{A}^0$
 - ▶ \rightarrow the agent payoff: $v^0(\bar{a}^0) = \bar{v}^0$
- ▶ Repeating the one-shot SPE is also SPE in the repeated game
- ▶ This is the minimax value for the agent
- ▶ In general, no SPE leads less than the minimax value of the stage game.
- ▶ Then $u^0(\sigma(0)) = \bar{v}^0$

OPC on Principal j

- ▶ Principal j
 - ▶ The minimax value of the one-shot game:
 - ▶ the agent chooses \underline{a}^j
 - ▶ (all the principals pay nothing.)
 - ▶ → the minimax value: \underline{v}^j
 - ▶ In general, this is not SPE payoff in the one-shot game
 - ▶ → not clear as $u^0(\sigma(0))$
- ▶ Go to the **minimization problem**

Minimization Problem

- ▶ $\sigma(j)$ minimizes j 's payoff subject to that $\sigma(j)$ is SPE
- ▶ Let $(a_0(j), b_0(j), \sigma_1(j))$ be the realization on the eq. path
- ▶ It has to satisfy;

$$\min_{(a_0(j), \{b_0^i(j)\}_{i=1}^N, \sigma_1(j))} u^j(\sigma(j)) \equiv (1 - \delta)[v^j(a_0(j)) - b_0^j(j)] + \delta u^j(\sigma_1(j)) \quad (1)$$

$$\text{subject to} \quad (1 - \delta)[v^0(a_0(j)) + B_0(j)] + \delta u^0(\sigma_1(j)) \geq \bar{v}^0 \quad (2)$$

$$0 \leq b_0^i(j) \leq \frac{\delta}{1 - \delta} [u^i(\sigma_1(j)) - u^i(\sigma(i))] \\ \text{for } i \in \mathcal{N}$$

$$\sigma_1(j) \in \Sigma^*.$$

where Σ^* is the set of SPE

Minimization Problem

- ▶ WLOG $b_0^j(j) = (\delta/(1 - \delta))[u^j(\sigma_1(j)) - u^j(\sigma(j))]$ for all $i \in \mathcal{N}$
- ▶ Substituting it into (1) yields;

Lemma (3)

$$u^j(\sigma(j)) = v^j(a_0(j)) \quad \text{for all } j \in \mathcal{N}.$$

- ▶ Lemma 3 tells us that **the punishment payoff is completely characterized by $a_0(j) \in \mathcal{A}$**

Minimization Problem

- ▶ The problem can be reduced to

$$\min_{(a_0(j), \sigma_1(j))} v^j(a_0(j)) \quad \text{subject to } \sigma_1(j) \in \Sigma^*$$

and

$$(1 - \delta)v^0(a_0(j)) + \delta \left[\sum_{i=1}^N (u^i(\sigma_1(j)) - u^i(\sigma(i))) \right] + \delta u^0(\sigma_1(j)) \geq \bar{v}^0$$

$$\iff \frac{\delta}{1 - \delta} \left[\sum_{i=0}^N u^i(\sigma_1(j)) - \left(\bar{v}^0 + \sum_{i=1}^N u^i(\sigma(i)) \right) \right] \geq \bar{v}^0 - v^0(a_0(j))$$

- ▶ $\sigma_1(j)$ should maximize the total surplus subject to that it is SPE

Minimization Problem

- ▶ Such $\sigma_1(j)$ can be found in the class of the “stationary” strategy SPE;
 - ▶ the agent chooses the same decision repeatedly and
 - ▶ each principal pays the same amount every period
- ▶ It means that
 - ▶ $a_t(j) = a_1(j)$ for all $t \geq 1$
 - ▶ $b_t^j(j) = b_1^j(j)$ for all $t \geq 1$

How to Punish

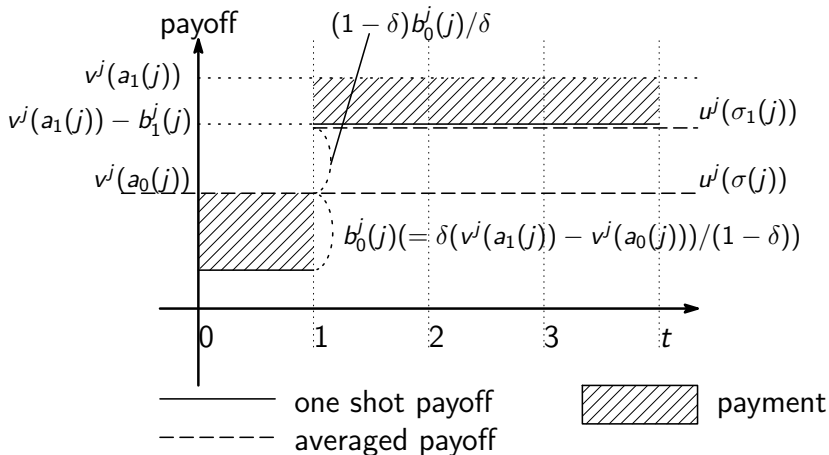
- ▶ Notice that (provided $\delta > 0$)

$$v^j(a_0(j)) (= v^j(\sigma(j))) = (1 - \delta)(v^j(a_0(j)) - b_0^j(j)) + \delta(v^j(\hat{a}) - b_1^j(j))$$

$$\iff v^j(a_0(j)) = v^j(a_1(j)) - \frac{(1 - \delta)}{\delta} w_0^j(j) - b_1^j(j)$$

- ▶ $\rightarrow v^j(a_0(j)) \leq v^j(a_1(j))$ (since $b_0^j(j), b_1^j(j) \geq 0$)
- ▶ If $v^j(a_0(j)) < v^j(a_1(j))$, the following “one-shot sanction” happens

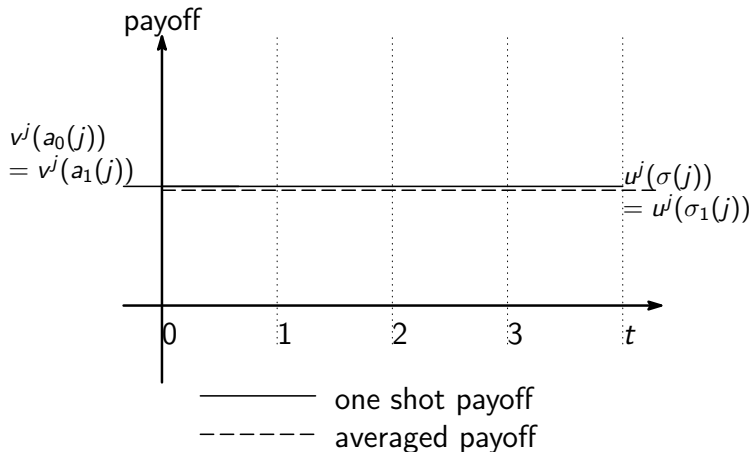
Punishment by One-shot Sanction



- Similar to the so-called “stick and carrot” strategy by Abreu

Punishment by Exclusion

- ▶ If $v^j(a_0(j)) = v^j(a_1(j))$, \rightarrow repeat the same things



When Sanction / Exclusion

- ▶ Interestingly,
 - ▶ in some cases a Sanction-type is strictly optimal and
 - ▶ in other cases an Exclusion-type is strictly optimal
- ▶ How to determine it?
- ▶ → Roughly speaking,
 - ▶ if there is a decision which is bad for the deviator but good for the other players, to choose such decision repeatedly is still stable → Exclusion
 - ▶ if not, choose the terrible decision for the deviator and after that reward every one by another decision → Sanction

Example

Decision	l	c	r
$v^1(a)$	$-D$	0	G
$v^2(a)$	G	0	$-D$
$v^0(a)$	$-C$	0	$-C$

- ▶ Assume that $C > 0$ and $G - C - D \leq 0$
- ▶ When P2 would be punished,
 - ▶ to choose r is required, but
 - ▶ r is also costly for the agent
- ▶ if $\delta \geq C/(2D + C)$, the punishment on P2 would be;
 - $r \rightarrow$ (P2 pays something) $\rightarrow c \rightarrow c \dots$: Sanction
- ▶ higher δ is required to implement $r \rightarrow r \rightarrow r \dots$
 - (specifically $\delta \geq C/(G - D)$ provided $G > D$)

Example Modified

Decision	l	c	r
$v^1(a)$	$-D$	0	G
$v^2(a)$	G	0	$-D$
$v^0(a)$	0	$-C$	0

- ▶ r is costly for P2, beneficial for the others
- ▶ Exclusion ($r \rightarrow r \rightarrow r \dots$) can be always the punishment on P2
- ▶ even assumed that c is socially efficient, (i.e. $G - D < -C$), positive $\delta \geq C/2D$ is required to implement the sanction type punishment ($r \rightarrow c \rightarrow c \dots$)

Remarks

- ▶ For the later analysis, keep in mind that
 1. The punishment could be more severe when $\delta \uparrow$
 - ▶ Constraint (2) is relaxed when $\delta \uparrow$
 2. $u^j(\sigma(j)) = v^j(a_0(j)) \geq \underline{v}^j$ for any $\delta \in [0, 1)$
- ▶ In what follows, denote it by $a_0(j; \delta)$

Stationary SPE

- ▶ Now consider SPE $\hat{\sigma}$ where the agent chooses $\hat{a} \in \mathcal{A}$ repeatedly on the equilibrium path
- ▶ WLOG, **the payment can also be stationary**
- ▶ The payment is stationary if

$$\hat{\sigma}^j(\hat{h}_t, \hat{a}_t) = \hat{\beta}^j(\hat{a}_t)$$

for $j \in \mathcal{N}$ and the on-path history (\hat{h}_t, \hat{a}_t) .

- ▶ If the payment is stationary, the payment schedule only depends on the current action

Stationary Payment

- ▶ (Proposition 1): Suppose that $\hat{\sigma}$ is a decision stationary SPE of $\hat{a} \rightarrow \exists$ strategy profile which satisfies the following;
 - ▶ payment stationarity and action stationarity of \hat{a}
 - ▶ same payoff vector as $\hat{\sigma}$
 - ▶ SPE
- ▶ \rightarrow We will focus on the strategy with stationary payment
- ▶ It can reduce the equilibrium conditions to be checked because
 - ▶ $\hat{\sigma}_1 = \hat{\sigma}$
 - ▶ \rightarrow enough to check IC at period 0
- ▶ Let $\hat{\beta}^j(a)$ be the stationary payment and $\hat{B}(a) := \sum_{j=1}^N \hat{\beta}^j(a)$

Incentive Compatibility

- ▶ Incentive compatibility of the agent at period 0:

$$(1 - \delta) [v^0(\hat{a}) + \hat{B}(\hat{a})] + \delta u^0(\hat{\sigma}) \geq (1 - \delta)\bar{v}^0 + \delta\bar{v}^0,$$

$$\iff \hat{B}(\hat{a}) \geq \frac{1}{1 - \delta}\bar{v}^0 - v^0(\hat{a}) - \frac{\delta}{1 - \delta}u^0(\hat{\sigma}) \quad (3)$$

- ▶ Incentive compatibility of principal j at period 0 (given $\hat{a} \in \mathcal{A}$):

$$(1 - \delta)[v^j(\hat{a}) - \hat{\beta}^j(\hat{a})] + \delta u^j(\hat{\sigma}) \geq (1 - \delta)v^j(\hat{a}) + \delta v^j(a_0(j; \delta))$$

$$\iff \hat{\beta}^j(\hat{a}) \leq \frac{\delta}{1 - \delta} [u^j(\hat{\sigma}) - v^j(a_0(j; \delta))] \quad (4)$$

Equilibrium Conditions

- ▶ Notice that

- ▶ $v^0(\hat{a}) + \hat{B}(\hat{a}) = u^0(\hat{\sigma})$

- ▶ $v^j(\hat{a}) - \hat{\beta}^j(\hat{a}) = u^j(\hat{\sigma})$

- ▶ → By eliminating \hat{B} and $\hat{\beta}^j$, equation (3) and (4) would be

$$(3) \iff u^0(\hat{\sigma}) \geq \bar{v}^0$$

$$(4) \iff u^j(\hat{\sigma}) \geq (1 - \delta)v^j(\hat{a}) + \delta v^j(a_0(j; \delta))$$

- ▶ Finally, notice that

- ▶ $u^j(\hat{\sigma}) \leq v^j(\hat{a})$ for $j \in \mathcal{N}$ and $u^0(\hat{\sigma}) \geq v^0(\hat{a})$ since $\hat{\beta}^j \geq 0$

- ▶ $\sum_{j=0}^N u^j(\hat{\sigma}) = s(\hat{a})$ if $\hat{\sigma} \in \hat{\Sigma}^*(\hat{a})$

- ▶ The stationary net SPE payoff set when the agent chooses \hat{a} is;

Stationary Equilibrium Payoff

Proposition (7)

$$\hat{U}(\hat{a}; \delta) := \left\{ \left(\begin{array}{c} u^0 \\ u^1 \\ \vdots \\ u^N \end{array} \right)' \mid \begin{array}{l} \sum_{j=0}^N u^j = s(\hat{a}), \\ u^0 \geq \bar{v}^0, \\ v^j(\hat{a}) \geq u^j \geq (1 - \delta)v^j(\hat{a}) + \delta v^j(a_0(j; \delta)), \forall j \in \mathcal{N} \end{array} \right\}$$

Stationary Equilibrium Payoff

- ▶ For $u \in \hat{U}(\hat{a}; \delta)$, there exists a vector $(\hat{b}^1, \dots, \hat{b}^N) \in \mathbb{R}_+^N$ such that on a stationary equilibrium of \hat{a} ,
 - ▶ the agent chooses \hat{a}
 - ▶ principal j pays \hat{b}^j to the agent
 - ▶ the net payoff vector is

$$u = (v^0(\hat{a}) + \hat{B}, v^1(\hat{a}) - \hat{b}^1, \dots, v^N(\hat{a}) - \hat{b}^N)$$
 where

$$\hat{B} = \sum_{k=1}^N \hat{b}^k$$
- ▶ We will compare it with the one-shot common agency game by BW to investigate the validity of menu auction for political contribution

Comparison to the Static Model

- ▶ Let us call our equilibrium “Relational Political Contribution (RPC)” equilibrium
- ▶ Basic question: When can equilibria in the static common agency be replicated by RPC-equilibria?
- ▶ Consider one-shot game where action a is verifiable (as BW)
 - ▶ Stage 1: The principals offers the compensation plan contingent on a $w^j : \mathcal{A} \rightarrow \mathbb{R}_+$ for $j \in \mathcal{N}$
 - ▶ Stage 2: the agent chooses $a \in \mathcal{A}$
 - ▶ (Stage 3: The compensation is enforced.)

SMA Equilibrium

- ▶ Outcome: $(a, \{w^1(a)\}_{a \in \mathcal{A}}, \dots, \{w^N(a)\}_{a \in \mathcal{A}})$
 - ▶ We call it the “Static Menu Auction (SMA)” equilibrium
- ▶ (Lemma 2 in BW) $(\hat{a}, \hat{w}^1(\cdot), \dots, \hat{w}^N(\cdot))$ is a SMA-Eq. \iff
 1. $\hat{w}^j(a) \geq 0$ for all $a \in \mathcal{A}$ and $j = 1, \dots, N$
 2. $\hat{a} \in \arg \max_{a \in \mathcal{A}} [v^0(a) + \hat{W}(a)]$
 3. $\hat{a} \in \arg \max_{a \in \mathcal{A}} ([v^j(\hat{a}) - \hat{w}^j(\hat{a})] + [v^0(\hat{a}) + \hat{W}(\hat{a})])$ for $j = 1, \dots, N$
 4. For $j = 1, \dots, N$, there exists $a_j \in \arg \max_a [v^0(a) + \hat{W}(a)]$ such that $\hat{w}^j(a_j) = 0$

where $\hat{W}(\cdot) := \sum_{j=1}^N \hat{w}^j(\cdot)$

When Is the Static Eq. Supported?

- ▶ Let a SMA-equilibrium payoff vector
 $\hat{y} := (v^0(\hat{a}) + \hat{W}(\hat{a}), v^1(\hat{a}) - \hat{w}^1(\hat{a}), \dots, v^N(\hat{a}) - \hat{w}^N(\hat{a}))$
- ▶ Question 1; When $\hat{y} \in \hat{U}(\hat{a}; \delta)$?

Proposition (8)

$\hat{y} \notin \hat{U}(\hat{a}; \delta) \iff$ for some $j \in \mathcal{N}$,

$$\hat{w}^j(\hat{a}) > \delta[v^j(\hat{a}) - v^j(a_0(j; \delta))].$$

When Is the Static Eq. Supported?

- ▶ Proof (Sketch);
 - ▶ In RPC, IC for principal j is

$$(1 - \delta)(v^j(\hat{a}) - \hat{b}^j) + \delta(v^j(\hat{a}) - \hat{b}^j) \geq (1 - \delta)v^j(\hat{a}) + \delta v^j(a_0(j; \delta))$$

$$\iff \hat{b}^j \leq \delta [v^j(\hat{a}) - v^j(a_0(j; \delta))]$$

- ▶ RHS; **upper bound of payment to be credible**
- ▶ If the payment in SMA-eq. exceeds this upper bound, it cannot credibly be paid in the relational case.
- ▶ Recall that $v^j(a_0(j; \delta)) \downarrow$ as $\delta \uparrow$
- ▶ \rightarrow If δ is higher, the static eq is more likely to be supported by relational eq

If Patient

- ▶ Question 2: Is it always on a RPC-equilibrium as long as δ is high enough? → **No**

Proposition (9)

For any $\delta \in [0, 1)$, $\hat{y} \notin \hat{U}(\hat{a}; \delta) \iff \exists j \in \mathcal{N}$ s.t.

1. $\hat{w}^j(\hat{a}) = v^j(\hat{a}) - \underline{v}^j$ and
2. $\underline{v}^j < v^j(\hat{a})$

- ▶ Proof (sketch): Again, the upper bound of the credible payment $\delta [v^j(\hat{a}) - v^j(a_0(j; \delta))]$ is (strictly) less than $v^j(\hat{a}) - \underline{v}^j$ for any $\delta \in [0, 1)$

Could It BE the Case?

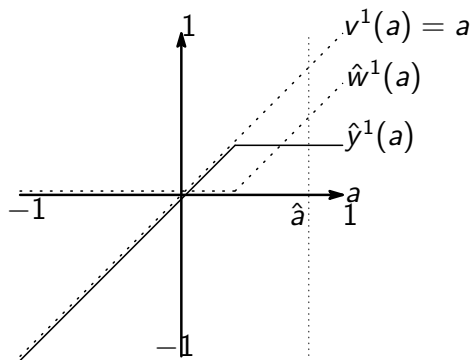
- ▶ Question 3: When are the conditions in Proposition 9 established ?
- ▶ Now focus on the SMA truthful (SMAT) equilibria where

$$\hat{w}^j(a) = \max\{v^j(a) - \hat{y}^j, 0\}$$

for all $j \in \mathcal{N}$

Example of Truthful Payment

- ▶ Suppose that $v^1(a) = a$ for $a \in [-1, 1]$
- ▶ If $\hat{w}^1(\cdot)$ consists a truthful equilibria, it is **parallel to $v^1(a)$ whenever $\hat{w}^1(a) > 0$**



SMAT and RPC Equilibrium

- ▶ Property of SMAT equilibrium (BW Theorem 2);
 - ▶ SMAT is **efficient**
 - ▶ Equilibrium payoff is **no more than marginal contribution**;

$$v^j(\hat{a}) - \hat{w}^j(\hat{a}) \leq \max_{a \in \mathcal{A}} s(a) - \max_{a \in \mathcal{A}} [s(a) - v^j(a)]$$

- ▶ If $\underline{A}^j \cap A^* \notin \emptyset$,
 - ▶ RHS is $\underline{v}^j \rightarrow v^j(\hat{a}) - \hat{w}^j(\hat{a}) \leq \underline{v}^j$
 - ▶ (Note that $v^j(\hat{a}) - \hat{w}^j(\hat{a}) \geq \underline{v}^j$ (\underline{v}^j is j 's minimax value))
- ▶ \rightarrow Condition 1 in Proposition 9 holds

SMAT and RPC Equilibrium

- ▶ Suppose that \exists two principals j and k s.t. $\underline{A}^j \cap A^* \neq \emptyset$ and $\underline{A}^k \cap A^* \neq \emptyset$
- ▶ Furthermore if $\underline{A}^j \cap \underline{A}^k = \emptyset$,
 - ▶ $\rightarrow A^* \cap \underline{A}^j \cap \underline{A}^k = \emptyset$
 - ▶ \rightarrow for any $a^* \in A^*$, either $a^* \notin \underline{A}^j$ or $a^* \notin \underline{A}^k$ (or both).
 - ▶ Since $\hat{a} \in A^*$, either $v^j(\hat{a}) > \underline{v}^j$ or $v^k(\hat{a}) > \underline{v}^k$
 - ▶ Condition 2 also holds

Proposition (10)

$\hat{y} \notin \hat{U}(\hat{a}; \delta)$ for any $\delta \in [0, 1)$ if $\exists j, k \in \mathcal{N}$ such that

$$\underline{A}^j \cap A^* \neq \emptyset, \underline{A}^k \cap A^* \neq \emptyset, \text{ and } \underline{A}^j \cap \underline{A}^k = \emptyset.$$

Example

Decision	l	c	r
$v^1(a)$	$-D$	0	G
$v^2(a)$	G	0	$-D$
$v^0(a)$	$-C$	0	$-C$

- ▶ Assume that $G - C - D = 0$.
- ▶ In this case $A^* = \{l, c, r\}$, $\underline{A}^1 = \{l\}$ and $\underline{A}^2 = \{r\}$
- ▶ \rightarrow the conditions in Proposition 10 hold.

Equilibria in Ex.1

- ▶ (Unique) SMAT equilibrium:
 - ▶ $(\hat{w}^1(l), \hat{w}^1(c), \hat{w}^1(r)) = (0, D, G + D)$ (symmetrical to P2)
 - ▶ A chooses 0 and is paid D from Ps
 - ▶ Net payoff: $(\hat{y}^0, \hat{y}^1, \hat{y}^2) = (2D, -D, -D)$
- ▶ Notice that $\hat{w}^1(c) = v^1(c) - \underline{v}^1 (= D)$ and $v^1(c) > -D$
- ▶ What happens in RPC eq.?
 - ▶ From IC conditions for the principal the upper bound of the credible payment is

$$\delta [v^j(c) - v^j(a_0(j; \delta))]$$

which is less than $-\delta \underline{v}^j = \delta D$

- ▶ Ps cannot credibly pay D

Intuition

- ▶ A decision $\underline{a}^1 \in \underline{A}^1 \cap A^*$ is
 - ▶ the worst for Principal 1 but
 - ▶ the best for P2 and the agent
- ▶ In SMA,
 - ▶ since binding contract is possible, P2 and the agent would be willing to agree the contract to choose \underline{a}^1
 - ▶ → the agent can use \underline{a}^1 as a credible threat to exploit P1
- ▶ If, further, $\underline{A}^2 \cap A^* \neq \emptyset$, then the agent can exploit both of the principals

Intuition

- ▶ But, if $\underline{A}^1 \cap \underline{A}^2 = \emptyset$,
 - ▶ \nexists decision beneficial only for the agent
 - ▶ \rightarrow either of the principals must be exploited *via* much (positive) payment which leads her net payoff equal to her minimax value
- ▶ In RPC,
 - ▶ the punishment is always delayed
 - ▶ if such a large amount of the transfer is required, principal is willing to renege

Conclusion

- ▶ Analyze infinitely repeated common agency without verifiability in political process
- ▶ Main Result
 - ▶ Characterization of the punishment;
 - ▶ either “Exclusion” or “One-shot Sanction” would be the punishment strategy
 - ▶ “Micro foundation” of the binding-contract model
 - ▶ Compare the payoff of relational stationary equilibrium with that of the static equilibrium
 - ▶ dependence on the discount factor
 - ▶ not always replicated even patient enough
- ▶ Future research
 - ▶ Introducing election process

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