Strategic policy commitment in the open economy*

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Abstract

We investigate the welfare effect of the strategic regulation such that induces a collusive leadership of the organized domestic firms in an open market where foreign firms can enter. We formulate such a strategic regulation in the quantity-setting competition where the domestic firms can collusively make their decision before the entrance of foreign firms and demonstrate how strong this structuring works from the view point of the domestic welfare by comparing to the optimal import tariff/subsidy policy. As a result, we show when the products of firms are homogeneous, that strategic regulation always yields strictly higher welfare than the optimal import tariff does. This holds even when the policy maker perfectly engages in the domestic-industry protection and ignores the consumer surplus. We also investigate the robustness of this result by considering the decentralized decisions of the domestic firms or the differentiated products.

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1 Introduction

When a country opens its market and allows the free entry of foreign firms, it is often tempted to protect the incumbent domestic firms. On the one hand, the policies on the price side (import tariff, etc.) are one of the important tools for such a protectionism but on the other hand, many developing countries can/actually have/had some command-and-control policies that affect the quantity side (promotion of production, capacity, or investment, etc.) to develop their home immature industry. In transition economy, rapid industrial privatization (as "shock therapy" in Russia) is not usualy the case, command-and-control policies are often maintained in some fashion in medium term (as "gradualism" in China).[1] Not only that, in the field of political economy, it is called the "developmental state" the economic system for development, where the government who is motivated by desire for economic advancement strongly intervenes in industrial affairs (Woo-Cumings, 1999). The "developmental state" is/was typically observed in east Asia but it is also found in Islamic and Buddhist Asia as well as in Africa and, historically, in Europe (Leftwich, 1995). Particularly in the world-wide stream for open markets, the domestic industry does not sufficiently developed even after the domestic market is opened and thus, such an system that affect the quantity side may not be immediately abolished.[2] This paper investigates how strong such policies affects the domestic welfare in the open economy where the foreign firms endogenously enter.

In the context of international trade, Etro [8] and [9] investigate the strategic effects of tariff and subsidy in the endogenous market structure.[3] In particular, Etro [9] considers the market structure where the number of the foreign firms is determined endogenously (free entry of the foreign firms) in the market with the fixed number of domestic firms and shows the import tariff/subsidy can enhance the domestic welfare and explicitly induce the optimal level of the import tariff/subsidy. We adopt the similar market structure as Etro [9] and consider the policy/regulation[4] that control the domestic firm's output/capacity before the entries of

[1] See, among others, Buck et.al. [1].

[2] For instance, it is said, in Japan, the government (MITI) tended to regulate and conduct a industry before 1980's and instead, a trade association self-regulates a industry after 1980's and made/makes the decisions made in that industry to be collusive and committable, often, in order to use their domestic market as profit sanctuaries (Schaede 2000).

[3] Etro [10] also considers the endogenous market structure and investigates the effects of market integration.

[4] Because we consider the convex combination of the domestic social welfare and the domestic firms' profits as the objective

foreign firms. By comparing this policy to the optimal import tariff, we identify how strong it works on the domestic welfare.

Our result is as follows: When the products of firms are homogeneous, the regulation always yields strictly higher welfare than the optimal import tariff does. This holds even when the decision maker who controls the domestic firm's output/capacity perfectly engages in the domestic-industry protection and ignores the consumer surplus.

As for robustness of this result, we consider the decentralized decisions of the domestic firms or the differentiated products. Although we have assumed that the domestic firms collusively maximize their joint-profit in the main part, it is debatable how successfully the firms can collude in the trade association. However, even when each domestic firms commit to their output/capacity to maximize their own profit, our conclusion is robust. When the products of firms are differentiated, the regulation for the domestic-industry protection can also yield strictly higher welfare than the optimal import tariff does, in particular, when the degree of differentiation is relatively small. However, we can also find the cases where the optimal import tariff yields higher welfare.

The rest of this paper is organized as follows. Section 2 formulates the model. Section 3 investigates the case of homogeneous products and derives our main results. Section 4 investigates the case of differentiated products and discusses how our results are modified under product differentiation. Finally, Section 5 contains concluding remarks. Proofs of some results are provided in the Appendix.

2 Model

2.1 Basic setting

We consider an industry where m domestic firms (firm $1, \dots, m$) and n (≥ 0) foreign firms (firm $m+1, \dots, m+n$) produce homogeneous or differentiated products and compete in quantity. Let $q_i \in \mathbb{R}_+$ be the output of

function of policy maker, our model can also be interpreted as a mixed-oligopoly model where the domestic firms are controlled as a enterprise partially owned by the public sector. See Matsumura (1998) for the seminal work which investigates the partial privatized public enterprise as a firm whose objective function is the above-mentioned convex combination. For the analysis free entries of foreign firms in the context of mixed oligopolies, see Cato and Matsumura (2012, 2015).

firm i's products and $p_i \in \mathbb{R}_+$ be the price of firm i's products ($i = 1, \ldots, m+n$). The each firm produces its products according to a cost function $c_i : \mathbb{R}_+ \mapsto \mathbb{R}_+$. The foreign firms' cost functions are identical $c_{m+1} = \ldots, = c_{m+n} = c$ (world-standard technology) and the marginal cost of domestic firms are supposed to be higher than that of foreign firms $c_i'(q_i) \geq c'(q_i)$ for all $q_i \geq 0$ for all $i = 1, \ldots, m$ (including the symmetric case with equality). Each foreign firm must pay a fixed entry cost $f > 0$ in order to be active in the market, whereas the domestic firms are the incumbents in this industry and thus their entry costs have already been sunk. We suppose the linear-demand structure, that is, inverse demand function for firm i's product is

$$p_i(q_1, \ldots, q_{m+n}) = a - q_i - b \sum_{j \neq i} q_j,$$

where $a > c_i'(0)$ for all i and $b \in (0, 1]$. The firms' products are homogeneous if $b = 1$ and they are differentiated if $b \in (0, 1)$. The profit of firm i (excluding fixed cost) is defined as $\pi_i(q_1, \ldots, q_{m+n}) = p_i(q_1, \ldots, q_{m+n})q_i - c_i(q_i)$ for $i = 1, \ldots, m$ and $\pi_i(q_1, \ldots, q_{m+n}) = p_i(q_1, \ldots, q_{m+n})q_i - c_i(q_i) - tq_i$ for $i = m+1, \ldots, m+n$, where $t \in \mathbb{R}$ is an import tariff.

Utility function of a representative household is

$$U(q_1, \ldots, q_{m+n}) + q_0 = a \sum_{i=1}^{m+n} q_i - \frac{1}{2} \left(\sum_{i=1}^{m+n} q_i^2 + b \sum_{i=1}^{m+n} \sum_{j \neq i} q_i q_j \right) + q_0,$$

where q_0 is the numeraire. This utility function induces the above-mentioned linear-demand structure. The domestic social welfare (including tariff revenue) is given by

$$\begin{aligned} W(q_1, \ldots, q_{m+n}) &= U(q_1, \ldots, q_{m+n}) - \sum_{i=1}^{m+n} p_i(q_1, \ldots, q_{m+n})q_i + \sum_{i=1}^{m} \pi_i(q_1, \ldots, q_{m+n}) + t \sum_{i=m+1}^{m+n} q_i \\ &= U(q_1, \ldots, q_{m+n}) - \sum_{i=1}^{m+n} c_i(q_i) - \sum_{i=m+1}^{m+n} \pi_i(q_1, \ldots, q_{m+n}). \end{aligned} \tag{1}$$

Structure of the game under quantity control As an objective function of the policy maker, we consider the convex combination of the domestic social welfare and the domestic firms' profits: $(1-\alpha)W + \alpha \sum_{i=1}^{m} \pi_i = (1-\alpha)(U - \sum_{i=1}^{m+n} p_i q_i) + \sum_{i=1}^{m} \pi_i$, where $\alpha \in [0, 1]$. Note that $t = 0$ in this regime. When $\alpha = 0$, the objective of the policy maker is the domestic-welfare maximization, whereas when $\alpha = 1$, its objective is the joint-profit maximization of the domestic firms, that is, the policy maker ignores the consumer surplus and perfectly engages in domestic-industry protection. Since a controllability of quantities of firms depends on a context, we consider two extreme cases in this regard. First, as the most controllable case, we analyze a centralized economy where the policy maker can perfectly observe the quantity of each domestic firm and commit to plan/regulate it as follows.

1. Policy making: The policy maker sets the planned/regulated output (capacity) of the domestic firm, $(\overline{q}_1, \dots, \overline{q}_m) \in \mathbb{R}^m_+$, to maximize the defined convex combination for some $\alpha \in [0, 1]$.

2. Entries decisions: The foreign firms choose to enter the market or not. The number of the foreign firms $n \geq 0$ is determined by zero profit condition.[5]

3. Market competition: The domestic firms select $q_1 = \overline{q}_1, \dots, q_m = \overline{q}_m$ and the foreign firms select $q_{m+1}, \dots, q_{m+n}$.

Later, as the most uncontrollable case, we analyze the first stage by supposing that each domestic firm, instead of the policy maker, chooses $\overline{q}_i$ in order to maximize its own profit. This is the case where the domestic firms have already established dominant positions as the incumbents and assume leadership in the domestic market but their decision makings are perfectly decentralized ones. We denote with an asterisk $(*)$ the equilibrium values under quantity control or decentralized decision making.

Structure of the game under import tariff We compare the presented quantity-control regime with the case of a non-regulated open economy where the government can levy the import tariff on the products of foreign firms. We focus on the comparison to the optimal tariff that maximize the sum of domestic welfare and tariff revenue. This is because we would like to show that even if the policy maker can perfectly care about the welfare in tariff regime, the welfare can be larger in the quantity-control regime in the open economy, in particular, even when that quantity control intend domestic-industry protection. The game runs as follows.

[5]In this paper, we neglect the integer problem of the firms to enter the market.

1. Policy making: The policy maker sets the unit level of the tariff $t \in \mathbb{R}$ (including the subsidy) to maximize the sum of the domestic welfare and the tariff revenue from the foreign firms.

2. Entries decisions: The foreign firms choose to enter the market or not. The number of the foreign firms $n \geq 0$ is determined by zero profit condition.

3. Market competition: The domestic firms select $q_1, \ldots, q_m$ and with paying the tariff, the foreign firms select $q_{m+1}, \ldots, q_{m+n}$.

We denote with the superscript T the equilibrium values under import tariff.

2.2 When the number of foreign firms is exogenously given

Before proceeding with the main analysis, we briefly discuss the results when the number of foreign firms is not endogenously determined but exogenously fixed, in order to clarify the importance of endogenous market structure in this paper. Assuming that there is a single domestic firm ($m = 1$) and that the number of foreign firms is fixed at ten ($n = 10$) in the case of homogeneous product ($b = 1$), Figure 1 illustrates how the weight put on the domestic firm's profit by the policy maker (α) and the cost inefficiency of domestic firm (γ) affect the relative performance of quantity-control and import-tariff regimes ($W^* - W^T$).

The important point here is that even in such a simple numerical example, depending on α and γ, the quantity-control regime can yield *both higher and lower* domestic social welfare than the import-tariff regime does. For $\gamma = 0.2$ (0.6), the quantity-control (import-tariff) regime results in higher domestic welfare for any $\alpha \in [0, 1]$. On the other hand, for $\gamma = 0.3$, the quantity-control (import-tariff) regime results in higher domestic welfare when α is sufficiently small (large). However, as we will see below, this result drastically changes when the number of foreign firms is endogenous; in that case, the quantity-control regime *always* yields higher domestic welfare than the import-tariff regime, regardless of the policy maker's objective and the cost inefficiency of domestic firms.

3 Homogeneous Products

In this section, we consider the case of homogeneous product:

Assumption 1 *Firms' products are homogeneous, i.e.,* $b = 1$.

In this case, the inverse demand is reduced to $p(Q) = a - Q$ by denoting $p = p_1 = p_2 = \cdots = p_{m+n}$ and $Q = \sum_{i=1}^{m+n} q_i$. There are multiple domestic firms and we allow general convex cost functions and cost difference between the domestic firm and the foreign firms:

Assumption 2 $m \geq 1$ *and cost functions satisfy* $c_i'(q_i) \geq 0, c_i''(q_i) \geq 0$ *for all* $q_i \geq 0$ *and for all* i.

3.1 Quantity-control regime

In the quantity-control regime, the domestic firms' outputs (capacities) is given in the first stage. We begin with the subgame that follows given the total amount of these outputs $\bar{Q}_D = \bar{q}_1 + \cdots + \bar{q}_m$ (the second and third stages). We can show that this subgame has a unique symmetric equilibrium.[6] Thus, let us denote the equilibrium number of the foreign firms by $n^*(\bar{Q}_D)$ and the equilibrium output of each foreign firm by $q^*(\bar{Q}_D)$. To facilitate the analysis according to our interest, we additionally assume the following.

Assumption 3 $n^*(0) \geq m$, *that is, the market is enough fruitful in the sense that the firms with world-standard technology* c *that can be active outnumber the incumbents.*

Note that since $m \geq 1$, this assumption implies that $n^*(0) \geq 1$, that is, at least one firm can be active if there is no incumbents. Then, the necessary and sufficient[7] conditions to obtain the positive equilibrium outcomes such that $n^*(\bar{Q}_D) > 0$ and thus $q^*(\bar{Q}_D) > 0$ are the following zero-profit condition of a foreign firm in the second stage and first-order condition of a foreign firm in the third stage:

$$p(Q^*)q^* - c(q^*) - f = 0, \tag{2}$$

$$p(Q^*) + p'(Q^*)q^* - c'(q^*) = 0, \tag{3}$$

[6]See Ino and Matsumura [11] in detail.

[7]Sufficiency for the first-order condition immediately comes from under our assumptions (the second order condition is globally met). Sufficiency for the zero-profit condition comes from the fact that equilibrium profit of a foreign firm strictly decreases in n in the third stage.

where $Q^*(\bar{Q}_D) = \bar{Q}_D + n^*(\bar{Q}_D)q^*(\bar{Q}_D)$. From these conditions, we can show the following lemma.

Lemma 1 *Suppose that Assumptions 1-3 hold. Then, (i) if $\bar{Q}_D < Q^*(0)$, then $n^*(\bar{Q}_D) > 0$, $q^*(\bar{Q}_D) = q^*(0)$ and $Q^*(\bar{Q}_D) = Q^*(0)$; and (ii) if $\bar{Q}_D \geq Q^*(0)$, then $n^*(\bar{Q}_D) = 0$ and $Q^*(\bar{Q}_D) = \bar{Q}_D$.*

This is essentially the same result as is obtained in Ino and Matsumura (2012).[8] Since the domestic firms politically commit to $q_i = \bar{q}_i$ $(i = 1, \ldots, m)$ before the foreign firms' entries, they assume Stackelberg leadership towards the foreign firms in a free-entry market. The lemma indicates that as far as the given outputs of the domestic firms (leaders) are in the level that allows a foreign firm (follower) to enter as in case (i), each follower's output does not depend on the leader's output ($q^*(\bar{Q}_D) = q^*(0)$).[9] This further implies that the equilibrium total output and price also do not depend on the leader's output ($Q^*(\bar{Q}_D) = Q^*(0)$) since the equilibrium price is equal to the follower's average cost (zero-profit condition).

In the first stage, the optimization problem of the policy maker is

$$\begin{aligned}
&\max_{(\bar{q}_1,\ldots,\bar{q}_m)\in\mathbb{R}^m_+} (1-\alpha)W(\bar{q}_1,\ldots,\bar{q}_m,q^*,\ldots,q^*) + \alpha\sum_{i=1}^m \pi_i(\bar{q}_1,\ldots,\bar{q}_m,q^*,\ldots,q^*) \\
&\qquad = (1-\alpha)\left[U(\bar{q}_1,\ldots,\bar{q}_m,q^*,\ldots,q^*) - p(Q^*)Q^*\right] + \sum_{i=1}^m \pi_i(\bar{q}_1,\ldots,\bar{q}_m,q^*,\ldots,q^*). \qquad (4)
\end{aligned}$$

Taking Lemma 1 into consideration, the two components in the objective function are given by

$$\begin{aligned}
U(\bar{q}_1,\ldots,\bar{q}_m,q^*,\ldots,q^*) - p(Q^*)Q^* &= \begin{cases} \frac{1}{2}Q^*(0)^2 & \text{if } \bar{Q}_D < Q^*(0) \\ \frac{1}{2}\bar{Q}_D^2 & \text{if } \bar{Q}_D \geq Q^*(0), \end{cases} \\
\sum_{i=1}^m \pi_i(\bar{q}_1,\ldots,\bar{q}_m,q^*,\ldots,q^*) &= \begin{cases} (a - Q^*(0))\bar{Q}_D - \sum_{i=1}^m c_i(\bar{q}_i) & \text{if } \bar{Q}_D < Q^*(0) \\ (a - \bar{Q}_D)\bar{Q}_D - \sum_{i=1}^m c_i(\bar{q}_i) & \text{if } \bar{Q}_D \geq Q^*(0). \end{cases}
\end{aligned}$$

We can solve this problem by two steps. As the first step, consider the problem where for given $\bar{Q}_D$, we distribute $\bar{q}_1, \ldots, \bar{q}_m$ to minimize $\sum_{i=1}^m c_i(\bar{q}_i)$ subject to $\bar{q}_1 + \cdots + \bar{q}_m = \bar{Q}_D$ and then, denote the minimized

[8] See Lemma 1 of their paper, which does not depend on linear demand structure. As they show, Assumption 3 is redundant to obtain this result. However, we additionally make this assumption since later, we can avoid some troublesome procedure (but not so fruitful for this paper) that arises by neglecting the integer problem of the number of firms.

[9] We can see a plausible graphical explanation of this result in Figure 2 of Ino and Matsumura [11]. Recently, the similar neutral property in free entry market has been widely used in the literatures: e.g., Etro [6], Etro [7], Cato and Oki [2], and Matsumura and Matsushima [14].

cost by $C(\bar{Q}_D)$, that is,

$$C(\bar{Q}_D) = \min_{\bar{q}_1,\dots,\bar{q}_m} \sum_{i=1}^{m} c_i(\bar{q}_i) \text{ s.t. } \bar{Q}_D = \bar{q}_1 + \cdots + \bar{q}_m.$$

Graphically speaking, it is clear that $C'(\bar{Q}_D)$ must be the horizontal sum of $c'_1, \dots, c'_m$.[10] Thus, it is guaranteed that C' is positive and increasing. In the second step, by substituting this minimized cost $C(\bar{Q}_D)$, we can reduce the problem (4) to

$$\max_{\bar{Q}_D \in \mathbb{R}_+} (1-\alpha)CS^*(\bar{Q}_D) + \Pi^*(\bar{Q}_D), \tag{5}$$

where $CS^*(\bar{Q}_D) = U(\bar{q}_1, \dots, \bar{q}_m, q^*, \dots, q^*) - p(Q^*)Q^*$ and

$$\Pi^*(\bar{Q}_D) = \begin{cases} (a - Q^*(0))\bar{Q}_D - C(\bar{Q}_D) & \text{if } \bar{Q}_D < Q^*(0) \\ (a - \bar{Q}_D)\bar{Q}_D - C(\bar{Q}_D) & \text{if } \bar{Q}_D \geq Q^*(0). \end{cases}$$

This problem gives us an aggregate output of the domestic firms in a subgame perfect equilibrium. Note that the problem (5) has (at least one) solutions by the Weierstrass theorem since $CS^*(\bar{Q}_D)$ and $\Pi^*(\bar{Q}_D)$ are continuous in $\bar{Q}_D$ and we can truncate the domain as $\bar{Q}_D \in [0, a]$. Given α, we arbitrarily take one of these equilibrium outputs and denote it by $\bar{Q}^*_D(\alpha)$.

Let $Q^P > 0$ be the perfectly competitive output of the domestic firms that is given by $p(Q^P) = C'(Q^P)$. Then, the following lemma describes the equilibrium properties under the quantity-control regime.

Lemma 2 *Suppose that Assumptions 1-3 hold. Take* $\alpha \in [0, 1]$ *arbitrarily. Then, (i) if* $p(Q^*(0)) < C'(0)$, $\bar{Q}^*_D(\alpha) = 0$, *(ii) if* $C'(0) \leq p(Q^*(0)) \leq p(Q^P)$, $\bar{Q}^*_D(\alpha) \in [0, Q^*(0)]$ *such that satisfies* $p(Q^*(0)) = C'(\bar{Q}^*_D(\alpha))$, *and (iii) if* $p(Q^*(0)) > p(Q^P)$, $\bar{Q}^*_D(\alpha) \in [Q^*(0), Q^P]$.

Figure 2 depicts the cases stated in this lemma. (i) is the case where the marginal costs of domestic firms are so high that none of them can survive in the free-entry market, i.e., $\bar{Q}^*_D(\alpha) = 0$, as depicted in the left panel. Since the foreign firms enter and produce $Q^*(0)$, the equilibrium market outcome (total output and

[10]More formally, sum up the inverse functions of $c'_1, \dots, c'_m$ unless a marginal cost is constant, that is, define $I(y) = \{i \in \{1, \dots, m\} | c''_i(q_i) \neq 0 \text{ when } c'_i(q_i) = y\}$ and $F(y) = \sum_{i \in I(y)} c'^{-1}_i(y)$. Then, we obtain

$$C'(\bar{Q}_D) = \min[F^{-1}(\bar{Q}_D), c'_1(\bar{Q}_D), \dots, c'_m(\bar{Q}_D)].$$

price) is Point A. (iii) is the case where the marginal costs of domestic firms are sufficiently low to deter all the foreign firms as depicted in the right panel. If $\alpha = 0$, the welfare is maximized by choosing the perfectly competitive output $\bar{Q}_D^*(\alpha) = Q^P$ and if $\alpha = 1$, the joint profit of the domestic firms is maximized by choosing $\bar{Q}_D^*(\alpha) = Q^*(0)$ and exactly undercutting the free-entry price of the foreign firms. The equilibrium market outcome is between these two points depending on $\alpha \in [0, 1]$, i.e., $\bar{Q}_D^*(\alpha) \in [Q^*(0), Q^P]$, as is Point C. (ii) is the intermediate case where both the domestic firms and the foreign firms are active and the equilibrium market outcome is Point B as depicted in the middle panel. The domestic firms produce positive but do not undercut the price of the foreign firms, i.e, $\bar{Q}_D^*(\alpha) \in [0, Q^*(0)]$. In this case, since some foreign firms enter into the market, the price is independent of the outputs of the domestic firms and constant at $p(Q^*(0))$ as sated in Lemma 1 (i). Thus, the policy maker equates that price and a marginal cost $p(Q^*(0)) = C'(\bar{Q}_D^*(\alpha))$ no matter what α is.

The equilibrium domestic welfare of the whole game in the quantity-control regime is $W^*(\bar{Q}_D^*(\alpha)) = CS^*(\bar{Q}_D^*(\alpha)) + \Pi^*(\bar{Q}_D^*(\alpha))$ for given α. From the equilibrium properties induced in Lemmas 1 and 2, we obtain

$$W^*(\bar{Q}_D^*(\alpha)) \geq \int_0^{Q^*(0)} [p(Q) - \min[C'(Q), p(Q^*(0))]]\, dQ. \tag{6}$$

Observe in each panel of Figure 2, the shaded area represents the right hand side of this inequality and the welfare brought in the equilibrium (Point A, B, or C) is greater than or equal to this area.

3.2 Comparison to the import-tariff regime

In the import-tariff regime, the firms with heterogeneous costs, the domestic firms with $c_i(q_i)$ and the foreign firms with $c(q_i) + tq_i$, compete in the third stage and the foreign firms enter the market in the second stage. However, we can show that the subgames in these second and third stages have a equilibrium, where all the foreign firms produce an identical output.[11] $q^T(t)$ represents the equilibrium output of a foreign firm

[11]Under our setting, it is known as the third-stage game has a unique equilibrium. Therefore, there are no other equilibria except for the symmetric equilibrium with regards to the foreign firms that we focus on. See, among others, Chapter 4 of Vives [18], which provides an sufficiently general explanation about the conditions where Cournot model has a unique equilibrium with heterogeneous costs. Further, we can show in the third-stage equilibrium, the profit of a foreign firm is strictly negative if n is sufficiently large. Thus, there exists an equilibrium number of foreign firms in the second stage.

given the tariff level t, $q_i^T(t)$ the equilibrium output of the domestic firm i $(= 1, \dots, m)$, $n^T(t)$ the equilibrium number of the foreign firms, and $Q^T(t) = \sum_{i=1}^m q_i^T(t) + n^T(t)q^T(t)$. Then, the first-order conditions of a foreign firm and the domestic firms are

$$p(Q^T) + p'(Q^T)q^T - c'(q^T) - t \leq 0 \text{ with equality if } q^T > 0, \tag{7}$$

$$p(Q^T) + p'(Q^T)q_i^T - c_i'(q_i^T) \leq 0 \text{ with equality if } q_i^T > 0, \quad i = 1, \dots, m; \tag{8}$$

and when $n^T(t) > 0$, the zero-profit condition of a foreign firm must also be satisfied:

$$p(Q^T)q^T - c(q^T) - tq^T - f = 0. \tag{9}$$

The following lemma indicates that when $t = 0$, the equilibrium total output and output of a foreign firm are the same as $Q^*(0)$ and $q^*(0)$, respectively; and how the equilibrium outputs are affected by the change in t.

Lemma 3 *Suppose that Assumptions 1-3 hold. Then, $Q^T(0) = Q^*(0) > 0$ and $q^T(0) = q^*(0) > 0$. As far as $q^T(t) > 0$, $dQ^T(t)/dt < 0$ and $dq^T(t)/dt = 0$. Further if $q_i^T(t) > 0$, $dq_i^T(t)/dt > 0$.*

Let $W^T(t) = W(q_1^T(t), \dots, q_m^T(t), q^T(t), \dots, q^T(t))$ be the equilibrium domestic welfare in the import-tariff regime given t. Note that $W^T(t)$ includes the equilibrium tariff revenue $tn^T(t)q^T(t)$ by the definition (1). Then, we have the following proposition.

Proposition 1 *Suppose that Assumptions 1-3 hold. Take $\alpha \in [0, 1]$ arbitrarily. Then, for all $t \in \mathbb{R}$, $W^*(\bar{Q}_D^*(\alpha)) \geq W^T(t)$ with equality if and only if $p(Q^*(0)) \leq C'(0)$ and $t = 0$.*

Proof Denote $Q_D^T(t) = \sum_{i=1}^m q_i^T(t)$. Then, we can decompose $W^T(t)$ into three parts (see also Figure 3 for supplementary explanation[12]):

$$W^T(t) = \left[\int_0^{Q_D^T(t)} p(Q)dQ - \sum_{i=1}^m c_i(q_i^T(t)) \right] + \int_{Q_D^T(t)}^{Q^T(t)} \left[p(Q) - p(Q^T(t)) \right] dQ + t[Q^T(t) - Q_D^T(t)]. \tag{10}$$

The first term that is in the bracket is the consumer surplus associated with the area less than $Q_D^T(t)$ (Area ABDC in Figure 3) and the domestic firms' profit. The second term is the consumer surplus associated with

[12]Although the figure depicts and exemplifies the case where $t > 0$ and all the equilibrium outcomes are strictly positive, the following our proof is valid for all the cases including the cases where $t < 0$ or some outcomes are zero.

the area greater than $Q_D^T(t)$ (Area CDE in Figure 3) and the foreign firms' profit (which is zero by (9)). The third term is the tariff revenue from the foreign firms' outputs, which is denoted as $TR(t) = tn^T(t)q^T(t) = t(Q^T(t) - Q_D^T(t))$ in this proof.

As for the first part, we obtain

$$\int_0^{Q_D^T(t)} p(Q)dQ - \sum_{i=1}^{m} c_i(q_i^T(t)) \leq \int_0^{Q_D^T(t)} p(Q)dQ - C(Q_D^T(t))$$
$$= \int_0^{Q_D^T(t)} [p(Q) - C'(Q)]dQ \leq \int_0^{Q_D^T(t)} [p(Q) - \min[C'(Q), p(Q^*(0))]]\, dQ, \quad (11)$$

where the first line is because $\sum_{i=1}^m c_i(q_i^T(t)) \geq C(Q_D^T(t))$ by the definition of C, and the this right hand side of the first line (Area AFGHC in Figure 3) is further elaborated as in the second line since $p(Q) > C'(Q)$ for all $Q \in [0, Q_D^T(t)]$.

As for the second part, the following rearrangement helps our comparison:

$$\int_{Q_D^T(t)}^{Q^T(t)} \left[p(Q) - p(Q^T(t))\right] dQ = \int_{Q_D^T(t)}^{Q^*(0)} [p(Q) - p(Q^*(0))]\, dQ$$
$$- \int_{Q_D^T(t)}^{Q^T(t)} \left[p(Q^T(t)) - p(Q^*(0))\right] dQ - \int_{Q^T(t)}^{Q^*(0)} [p(Q) - p(Q^*(0))]\, dQ$$

The first term of the right hand side (Area CIK in Figure 3, which the consumer surplus associated with the area greater than $Q_D^T(t)$ under the command-and -control regime) further satisfies

$$\int_{Q_D^T(t)}^{Q^*(0)} [p(Q) - p(Q^*(0))]\, dQ \leq \int_{Q_D^T(t)}^{Q^*(0)} [p(Q) - \min[C'(Q), p(Q^*(0))]]\, dQ \quad (12)$$

since $p(Q) \geq p(Q^*(0))$ for all $Q \in [Q_D^T(t), Q^*(0)]$ (Note that $Q_D^T(t) < Q^*(0)$ by Lemma 3). The second term's integral (Area DIJE in Figure 3) equals the tariff revenue $TR(t)$. This is because when $q^T(t) > 0$ ($Q_D^T(t) < Q^T(t)$), $q^T(t) = q^T(0) = q^*(0)$ by Lemma 3. Thus, (9) can be arranged to be $p(Q^T(t))q^*(0) - c(q^*(0)) - tq^*(0) - f = 0$. Subtracting (2) with $\bar{Q}_D = 0$, we obtain $p(Q^T(t)) - p(Q^*(0)) = t$. Therefore, the integral is $t(Q^T(t) - Q_D^T(t)) = TR(t)$. Note that when $q^T(t) = 0$ ($Q_D^T(t) = Q^T(t)$), the second term also equals $TR(t) = 0$. The third term's integral (Area EJK in Figure 3) is zero when $t = 0$ since $Q^T(0) = Q^*(0)$ by Lemma 3 and strictly positive when $t \neq 0$ since $Q^T(t) \neq Q^*(0)$ by Lemma 3; Note that since $p' < 0$,

$p(Q) - p(Q^*(0)) > 0$ for all $Q < Q^*(0)$ if $Q^T(t) < Q^*(0)$ and $p(Q) - p(Q^*(0)) < 0$ for all $Q > Q^*(0)$ if $Q^T(t) > Q^*(0)$. From these,

$$\int_{Q_D^T(t)}^{Q^T(t)} \left[p(Q) - p(Q^T(t))\right] dQ + TR(t) \leq \int_{Q_D^T(t)}^{Q^*(0)} [p(Q) - \min[C'(Q), p(Q^*(0))]] \, dQ \tag{13}$$

with inequality if $t \neq 0$.

By (6), (10), (11) and (13), we obtain $W^*(\bar{Q}_D^*(\alpha)) \geq W^T(t)$ with inequality if $t \neq 0$. Finally, consider the case where $t = 0$. Note $TR(0) = 0$ in this case. When $p(Q^*(0)) \leq C'(0)$, since $\bar{Q}_D^*(\alpha) \leq Q^*(0)$ by Lemma 2(i) and (ii), $Q^*(\bar{Q}_D^*(\alpha)) = Q^*(0)$. Also $Q^T(0) = Q^*(0)$ by Lemma 3 and thus, the consumer surplus is the same in both the regimes. Further, in this case, the profits of domestic firms are zero in both the regimes. This is because, when $p(Q^*(0)) \leq C'(0)$ $(p(Q^*(0)) \leq c_i'(0))$, $p(Q^*(0)) + p'(Q^*(0))q_i - c_i'(q_i) < 0$ for all $q_i > 0$ since $p' - c_i'' < 0$. Thus, $Q^*(0) = Q^T(0)$ and (8) implies $q_i^T(0) = 0$ $(Q_D^T(0) = 0)$. As for the regulation regime, when $p(Q^*(0)) < C'(0)$, $\bar{Q}_D^*(\alpha) = 0$ by Lemma 2(i); and when $p(Q^*(0)) = C'(0)$, $C'(\bar{Q}_D) = p(Q^*(0))$ for all $\bar{Q}_D \in [0, \bar{Q}_D^*(\alpha)]$ by Lemma 2(ii) (the marginal cost has to be constant and equal to the price in the relevant range). Therefore, we have $W^T(0) = W^*(\bar{Q}_D^*(\alpha))$ when $p(Q^*(0)) \leq C'(0)$. If $p(Q^*(0)) > C'(0)$ $(p(Q^*(0)) > c_i'(0))$, there uniquely exists $q_i' > 0$ such that $p(Q^*(0)) + p'(Q^*(0))q_i' - c_i'(q_i') = 0$ since $p' - c_i'' < 0$. Therefore, since $Q^*(0) = Q^T(0)$ by Lemma 3, (8) implies $q_i^T(0) = q_i' > 0$. Thus, by noting that $C'(Q_D^T(0)) < p(Q^*(0))$ by (8), (12) must satisfies with inequality– resulting in $W^*(\bar{Q}_D^*(\alpha)) > W^T(t)$. **Q.E.D.**

This proposition indicates that as far as $C'(0) \leq p(Q^*(0))$,[13] the quantity-control policy always yields strictly higher welfare than the import tariff does regardless of t, and thus, the optimal import tariff does. An interesting thing is that this result holds for all $\alpha \in [0, 1]$. Thus, even when the policy maker perfectly engages in the domestic-industry protection and ignores the consumer surplus ($\alpha = 1$), the quantity-control policy induces a greater domestic welfare than the import tariff can.

[13] Otherwise, we have the case where all the domestic firms shut down by the entries of foreign firms both under the command-and-control regime (Lemma 2 (i)) and the optimal tariff $t = 0$.

This result is obtained because the mechanisms to expand the profit of domestic firms are different between the two regimes. Etro [9] formalizes the mechanism in the import-tariff regime and induces the optimal tariff level.[14] Import tariff affects the average cost of each foreign firm and thus, limits the entries of foreign firms and increases the price. When there are the fixed number of incumbent domestic firms, this price increase expands the market share of the domestic firms and shifts profits toward them. However, it also causes the loss in consumer surplus (some part of the loss is canceled by tariff revenue). The optimal import tariff must balance these effects. On the other hand, the mechanism behind quantity-control policy causes commitment effect: the domestic firms' outputs/capacities are restricted at the policy making stage. Since this policy does not affect the average cost of each foreign firm, the policy maker can commit to expand outputs/capacities of the domestic firms without affecting the price. Since the loss of consumer surplus does not occur, in order to shift profit toward the domestic firms, this commitment can induce more aggressive expansion of the domestic firms' market share and limitation of foreign firms' entries than tariff. This results in a higher domestic welfare. Furthermore, since the consumer surplus is not affected, profits of the domestic firms are only the matter to maximize the welfare. This is why even the domestic-industry protection works better than the optimal tariff.

3.3 Decentralized decision making of domestic firms

In this subsection, we suppose that domestic firm $i = 1, \ldots, m$ chooses $\bar{q}_i$ and maximize its *own* profit in the first stage. Since the second and third stages are the same as in Section 3.1, Lemma 1 is valid also in this analysis. Taking the result of this lemma into account, the domestic firm i's problem is

$$\max_{\bar{q}_i \in \mathbb{R}_+} = \pi_i(\bar{q}_1, \ldots, \bar{q}_m, q^*, \ldots, q^*) = \begin{cases} (a - Q^*(0))\bar{q}_i - c_i(\bar{q}_i) & \text{if } \bar{Q}_D < Q^*(0) \\ (a - \bar{Q}_D)\bar{q}_i - c_i(\bar{q}_i) & \text{if } \bar{Q}_D \geq Q^*(0). \end{cases}$$

Let $\hat{q}_i \in \mathbb{R}_+ \cup \{\infty\}$ be firm i's price-taking output at the price $p(Q^*(0))$ that is defined as the maximum element of $\{q_i | p(Q^*(0)) = c_i'(\hat{q}_i)\}$ or $\hat{q}_i = 0$ ($\hat{q}_i = \infty$) if $c_i'(q_i) > p(Q^*(0))$ ($c_i'(q_i) < p(Q^*(0))$) for all q_i. Let $\check{q}_i \in \mathbb{R}_+$ be firm i's partial-monopoly output at the price $p(Q^*(0))$ that is given by $p(Q^*(0)) + p'(Q^*(0))\check{q}_i =$

[14] The optimal tariff level is positive when the demand is linear as in our setting. However, it is worth noting that when the demand is highly convex, the optimal tariff can be negative. See Etro's paper in detail.

$c_i'(\check{q}_i)$ or $\check{q}_i = 0$ if $c_i'(0) > p(Q^*(0))$. By using these values, we can describe the equilibrium properties under the decentralized decisions as in the following lemma. The output of domestic firm $i = 1, \ldots, m$ in an equilibrium is denoted by $\overline{q}_i^*$.

Lemma 4 *Suppose that Assumptions 1-3 hold. Then, (i) if $p(Q^*(0)) < C'(0)$, the equilibrium is given by $\overline{q}_i^* = 0$ for all $i = 1, \ldots, m$, (ii) if $C'(0) \leq p(Q^*(0)) \leq p(Q^P)$, the equilibrium is all the combinations that satisfy $\sum_{i=1}^m \overline{q}_i^* \in [0, Q^*(0)]$ and $p(Q^*(0)) = c_i'(\overline{q}_i^*)$ for all $i = 1, \ldots, m$, and (iii) if $p(Q^*(0)) > p(Q^P)$, the equilibrium is all the combinations that satisfy $\sum_{i=1}^m \overline{q}_i^* = Q^*(0)$ and $\overline{q}_i^* \in [\check{q}_i, \hat{q}_i]$ for all $i = 1, \ldots, m$*

This is essentially the extension of the result obtained in Ino and Matsumura (2012)[15] to the case where cost difference exists between domestic firms (leaders) and foreign firms (followers). The proof is quite similar to their's, thus we omit it.

Case (i) of Lemma 4 indicates that none of domestic firm can survive and the equilibrium structure in this case is exactly the same as in the case (i) of Lemma 2. Case (ii) of Lemma 4 tells us that all the domestic firms behave like price takers at the price $p(Q^*(0))$.[16] This is because when some foreign firms enter into the market, the price is constant regardless of the actions of domestic firms by Lemma 1(i). This implies that the total cost is minimized, i.e., $p(Q^*(0)) = C'(\sum_{i=1}^m \overline{q}_i^*)$. Thus, the equilibrium structure in this case is also exactly the same as in the case (ii) of Lemma 2. Hence, in these cases, that is, when some foreign firms enter in the equilibrium, the equilibrium domestic welfare under the decentralized decisions, which is denoted as $W^*(\overline{q}_1^*, \ldots, \overline{q}_m^*) = W(\overline{q}_1^*, \ldots, \overline{q}_m^*, q(\overline{Q}_D^*), \ldots, q(\overline{Q}_D^*))$, is exactly the same as under the centralized decision $W^*(\overline{Q}_D^*)$, where $\overline{Q}_D^* = \sum_{i=1}^m \overline{q}_i^*$. As a result, the same result holds as in Proposition 1 as follows.

Proposition 2 *Suppose that Assumptions 1-3 hold. If $p(Q^*(0)) \leq p(Q^P)$, for all $t \in \mathbb{R}$, $W^*(\overline{q}_1^*, \ldots, \overline{q}_m^*) \geq W^T(t)$ with equality if and only if $p(Q^*(0)) \leq C'(0)$ and $t = 0$.*

It must be noted that we can have $W^*(\overline{q}_1^*, \ldots, \overline{q}_m^*) < W^T(t)$ if $p(Q^*(0)) > p(Q^P)$, which is the case (iii)

[15] See Lemma 2 of their paper.

[16] Some readers may think that the expression $\sum_{i=1}^m \overline{q}_i^* \in [0, Q^*(0)]$ in the lemma is redundant because $p(Q^*(0)) = c_i'(\overline{q}_i^*)$ seems to identify $\overline{q}_i^*$. This is true if c_i' is strictly increasing. However, when some firm's marginal cost is possibly constant at $c_i'(q_i) = p(Q^*(0))$ for some $q_i \leq Q^*(0)$ as in our setting, this firm can select any levels of output in that range as an equilibrium output.

of Lemma 4 where the domestic firms are so efficient that they can deter all the foreign firms. The following example suffices to show this.

Example Suppose $m = 2$, $p = 4 - Q$, and $f = 1$. Firm 1 is still uses a laggard technology with marginal cost 1, $c_1(q_1) = q_1$ for all q_1, but Firm 2 catches up with the world standard technology with marginal cost 0, $c_2(q_2) = c(q) = 0$ for all q_2 and q. Then, if there is no quantity-control regulation and no import tariff, the equilibrium outcomes are $p(Q^T(0)) = 1$ ($Q^T(0) = 3$) and $q_2^T(0) = q^T(0) = 1$.[17] Firm 1 cannot be active $q_1^T(0) = 0$ under this price by (8) and thus, $n^T(0) = 2$. As a result, the equilibrium welfare is $W^T(0) = 9/2 + 1 = 5.5$, which is the sum of the consumer surplus and Firm 2's profit. Under the regulation regime, $\overline{q}_1^* = 2$ and $\overline{q}_2^* = 1$ are supported in one of the equilibria and these committed quantities of the domestic firms, $\overline{q}_1^* + \overline{q}_2^* = 3$, cause the entry deterrence, $n^*(3) = 0$, under the price $p(Q^*(0)) = p(Q^T(0)) = 1$ ($Q^*(0) = 3$). Since Firm 1's profit is zero because of the marginal cost pricing, the welfare in this case is obtained as $W^*(\overline{q}_1^*, \overline{q}_2^*) = 9/2+1 = 5.5$, which is the same as $W^T(0)$ calculated above. However, if we impose the import tariff $t = 1$, the equilibrium price is $p(Q^T(1)) = 2$ ($Q^T(1) = 2$) and Firm 2 occupies all the market under this price, i.e., $q_2^T(1) = 2$, $q_1^T(1) = 0$ and $n^T(1) = 0$. Thus, the equilibrium welfare under this level of import tariff $W^T(1) = 4/2 + 4 = 6$, the sum of the consumer surplus and Firm 2's profit, exceeds that in the regulation regime.[18] Intuitively, this situation occurs because in the regulation regime, there is a equilibrium where the firm who uses a laggard technology (Firm 1) commit to a large portion of the market. When some firm catches up with the world technology, without the decision maker who can coordinately allocate market shares among the domestic firms, the welfare improvement in the regulation regime may fail.

[17]The readers can easily check that these outcomes satisfies (7)-(9) with $t = 0$.

[18]Indeed, $t = 1$ is an optimal level of import tariff in this example, since $W^T(t) = 11/2 + t - t^2/2$ and $dW^T(t)/dt = 1 - t$ (which is consistent to the expression (27) calculated later) when $t < 1$ and $W^T(t) = 6$ (which is the welfare obtained by Firm 2's monopoly) when $t \geq 1$.

4 Product Differentiation

In the previous section, we showed that the quantity-control regulation (quantity policy) for the domestic firms always brings higher welfare than the optimal import tariff (price policy) in a homogeneous product setting. We discussed that the rationale behind this result is the fact that in the regulation regime, more aggressive behavior of the domestic firms restricts the entries of foreign firms more severely than in the tariff regime. This aspect indicates that in the presence of product differentiation, the variety of products becomes smaller in the regulation regime than in the tariff regime. Since this is a negative welfare effect in the regulation regime, it is important to investigate how the previous result in a homogeneous product setting is modified under product differentiation.[19]

For this purpose, we introduce some alternative assumptions in the following analysis of this section. First, as mentioned above, we consider the case of differentiated product:

Assumption 4 *Firms' products are differentiated, i.e.,* $b \in (0, 1)$.

Next, to effectively focus on the effect of product differentiation, we simplify the relatively general setting of the last section as follows:

Assumption 5 *There is a single domestic firm, i.e.,* $m = 1$, *and cost functions of domestic and foreign firms satisfy* $c_i'(q_i) = c'(q_i) = 0$ *for all* $q_i \geq 0$ *and for all i.*

The latter assumption implies that the domestic and foreign firms have identical and constant marginal cost, which is normalized to zero. Finally, we assume that the size of domestic market is sufficiently large or the entry costs of foreign firms are sufficiently small that at least one foreign firm can profitably enter the domestic market:

Assumption 6 $a \geq 2\sqrt{f}$.

[19] For the model that considers endogenous number of followers under the product differentiation, see a recent paper Žigić [20], which elaborates the performance of the model by using similar linear demand and cost setting as our paper.

In other words, this assumption corresponds to Assumption 3 with $m = 1$. Under these assumptions, the domestic social welfare given by (1) becomes

$$W(q_1, \dots, q_{n+1}) = U(q_1, \dots, q_{n+1}) - \sum_{i=2}^{n+1} \pi_i(q_1, \dots, q_{n+1}). \tag{14}$$

In this section, we compare the equilibrium domestic social welfare under the regulation regime with that under the tariff regime. As for the regulation regime, we consider the two extreme cases: "welfare-maximizing regulation" and "profit-maximizing regulation." In the former case, the policy maker maximizes the domestic social welfare when deciding the level of quantity regulation. On the other hand, in the latter case, the policy maker maximizes the domestic firm's profit when setting the regulation. These two cases respectively correspond to those with $\alpha = 0$ and $\alpha = 1$ in the last section. Under the tariff regime, we assume that the objective of the policy maker is to maximize the domestic welfare (including tariff revenue).

4.1 Welfare-maximizing regulation regime

First, we consider a regulation regime where the policy maker sets the output of domestic firm in order to maximize the domestic social welfare. In the third stage of market competition with $n > 0$, for given committed output of the domestic firm $\bar{q}_1 \in [0, a]$, the equilibrium output of each foreign firm is

$$q^*(\bar{q}_1) = \frac{a - b\bar{q}_1}{2 - b + bn}. \tag{15}$$

Therefore, the equilibrium profit of the domestic firm is

$$\pi_1^*(\bar{q}_1) = \pi_1(\bar{q}_1, q^*, \dots, q^*) = \frac{a(2-b)\bar{q}_1 - (2 - b + bn(1-b))(\bar{q}_1)^2}{2 - b + bn}, \tag{16}$$

and that of each foreign firm is $\pi^*(\bar{q}_1) = \pi_i(\bar{q}_1, q^*, ..., q^*) = (q^*(\bar{q}_1))^2$ for $i = 2, \dots, n+1$.

Next, we consider the second stage of entry decision. Let the equilibrium number of the foreign firms be $n^*(\bar{q}_1)$ for given $\bar{q}_1 \in [0, a]$. Then, since $\pi^*(\bar{q}_1)$ is decreasing in n, $n^*(\bar{q}_1)$ is uniquely determined by the zero-profit condition of the foreign firms, $\pi^*(\bar{q}_1) = f$, when $\lim_{n \to 0} \pi^*(\bar{q}_1) > f$ and zero when $\lim_{n \to 0} \pi^*(\bar{q}_1) \leq f$.

Therefore,

$$n^*(\bar{q}_1) = \begin{cases} \dfrac{a-(2-b)\sqrt{f}-b\bar{q}_1}{b\sqrt{f}} & \text{if } \bar{q}_1 < (a-(2-b)\sqrt{f})/b, \\ 0 & \text{if } \bar{q}_1 \geq (a-(2-b)\sqrt{f})/b. \end{cases} \tag{17}$$

Note that when $n^*(\bar{q}_1) > 0$, we always have $q^*(\bar{q}_1) = \sqrt{f}$ by the zero-profit condition $\pi^*(\bar{q}_1) = f$. Substituting $n = n^*(\bar{q}_1)$ into (16) yields the domestic firm's profit:

$$\pi_1^*(\bar{q}_1) = \begin{cases} (2-b)\sqrt{f}\bar{q}_1 - (1-b)(\bar{q}_1)^2 & \text{if } \bar{q}_1 < (a-(2-b)\sqrt{f})/b, \\ (a-\bar{q}_1)\bar{q}_1 & \text{if } \bar{q}_1 \geq (a-(2-b)\sqrt{f})/b. \end{cases} \tag{18}$$

In addition, by substituting (15) and (17) into (14), we obtain the equilibrium domestic welfare for given $\bar{q}_1$:

$$W^*(\bar{q}_1) = W(\bar{q}_1, q^*, \ldots, q^*) = \frac{a^2 + (2-b)f - (a-b\bar{q}_1)(3-b)\sqrt{f} - (1-b)b(\bar{q}_1)^2}{2b} \tag{19}$$

if $\bar{q}_1 < (a-(2-b)\sqrt{f})/b$, and $W^*(\bar{q}_1) = a\bar{q}_1 - (\bar{q}_1)^2/2$ otherwise.

Finally, we consider the first stage of policy maker's commitment. The policy maker maximizes $W^*(\bar{q}_1)$ with respect to $\bar{q}_1$ and this yields the equilibrium output of the domestic firm $\bar{q}_{1W}^*$ as in the following lemma.

Lemma 5 *Suppose that Assumptions 4-6 hold and define*

$$f_W(b) = \frac{4a^2[(1-b)^2(10-9b+4b^2-b^3) - 2(3-b)(1-b)^{7/2}]}{(8-3b-2b^2+b^3)^2}. \tag{20}$$

Then, (i) if $f < f_W(b)$, we have

$$\bar{q}_{1W}^* = \frac{(3-b)\sqrt{f}}{2(1-b)}, \quad n^*(\bar{q}_{1W}^*) = \frac{2a(1-b)-(4-3b+b^2)\sqrt{f}}{2(1-b)b\sqrt{f}}, \quad \pi_1^*(\bar{q}_{1W}^*) = \frac{(3-b)f}{4};$$

and (ii) if $f \geq f_W(b)$, we have

$$\bar{q}_{1W}^* = a, \quad n^*(\bar{q}_{1W}^*) = 0, \quad \pi_1^*(\bar{q}_{1W}^*) = 0.$$

This is a plausible result. When the entry cost of foreign firms f is so large as in (ii), the domestic firm deters the entry of all the foreign firms, that is, $n^*(\bar{q}_{1W}^*) = 0$. Otherwise as in (i), the foreign firms are active in the market, that is, $n^*(\bar{q}_{1W}^*) > 0$ and thus $q^*(\bar{q}_{1W}^*) = \sqrt{f}$.

From the results in Lemma 5, the equilibrium welfare under the regulation regime is

$$W^*(\bar{q}_{1W}^*) = \frac{4a^2(1-b) - 4a(3-4b+b^2)\sqrt{f} + (8-3b-2b^2+b^3)f}{8(1-b)b} \tag{21}$$

if $f < f_W(b)$, and $W^*(\bar{q}_{1W}^*) = a^2/2$ otherwise.

4.2 Profit-maximizing regulation regime

Next, we consider another regulation regime where the objective of the policy maker is to maximize the domestic firm's profit. Note that the equilibrium outcomes of the third and second stages are the same as in the previous subsection and given by (15) and (17) respectively. Therefore, we begin with the first stage of policy maker's commitment. The policy maker chooses $\bar{q}_1$ in order to maximize the domestic firm's profit given by (18). Then, we have the equilibrium output of the domestic firm $\bar{q}_{1P}^*$ as in the following lemma.

Lemma 6 *Suppose that Assumptions 4-6 hold and define* $f_P(b) = (2a(1-b))^2/(2-b)^4$. *Then, (i) if* $f < f_P(b)$, *we have*

$$\bar{q}_{1P}^* = \frac{(2-b)\sqrt{f}}{2(1-b)}, \quad n^*(\bar{q}_{1P}^*) = \frac{2a(1-b)-(2-b)^2\sqrt{f}}{2(1-b)b\sqrt{f}}, \quad \pi_1^*(\bar{q}_{1P}^*) = \frac{(2-b)^2 f}{4(1-b)};$$

and (ii) if $f \geq f_P(b)$, *we have*

$$\bar{q}_{1P}^* = \frac{a-(2-b)\sqrt{f}}{b}, \quad n^*(\bar{q}_{1P}^*) = 0, \quad \pi_1^*(\bar{q}_{1P}^*) = \frac{(a(b-1)+(2-b)\sqrt{f})(a-(2-b)\sqrt{f})}{b^2}.$$

This is a similar result to Lemma 5. When the entry cost of foreign firms f is sufficiently large, the policy maker deters the entry of all the foreign firms, that is, $n^*(\bar{q}_{1P}^*) = 0$. Otherwise, it is optimal for the policy maker to allow some foreign firms to enter into the domestic market, that is, $n^*(\bar{q}_{1P}^*) > 0$ and $q^*(\bar{q}_{1P}^*) = \sqrt{f}$.

From the results in Lemma 6, the equilibrium welfare under the profit-maximizing regulation is

$$W^*(\bar{q}_{1P}^*) = \frac{4a^2(1-b)-4a(3-4b+b^2)\sqrt{f}+(2-b)^2(2+b)f}{8(1-b)b} \tag{22}$$

if $f < f_P(b)$ and

$$W^*(\bar{q}_{1P}^*) = \frac{(a(2b-1)+(2-b)\sqrt{f})(a-(2-b)\sqrt{f})}{2b^2} \tag{23}$$

otherwise.

4.3 Import-tariff/subsidy regime

Finally, we derive the equilibrium outcomes under the optimal import-tariff. In the third stage of market competition with $n > 0$, for given t, the equilibrium outputs of the domestic firm (firm 1) and each foreign firm (firm $i = 2, \ldots, n+1$) are

$$q_1^T(t) = \frac{a(2-b)+bnt}{(2-b)(2+bn)}, \quad q^T(t) = \frac{a(2-b)-2t}{(2-b)(2+bn)}, \tag{24}$$

respectively.[20] Then, the equilibrium profits of the domestic and foreign firms are respectively given by $\pi_1^T(t) = \pi_1(q_1^T, q^T, \ldots, q^T) = (q_1^T(t))^2$ and $\pi^T(t) = \pi_i(q_1^T, q^T, \ldots, q^T) = (q^T(t))^2$ for $i = 2, \ldots, n+1$.

In the second stage, let $n^T(t)$ denote the equilibrium number of foreign firms for given t. Then, since $\pi^T(t)$ is decreasing in n, $n^T(t)$ is uniquely determined by the zero-profit condition of the foreign firms, $\pi^T(t) = f$, when $\lim_{n\to 0} \pi^T(t) > f$ and zero when $\lim_{n\to 0} \pi^T(t) \leq f$. Therefore, we obtain

$$n^T(t) = \begin{cases} \dfrac{(a-2\sqrt{f})(2-b)-2t}{(2-b)b\sqrt{f}} & \text{if } t < (a-2\sqrt{f})(2-b)/2, \\ 0 & \text{if } t \geq (a-2\sqrt{f})(2-b)/2. \end{cases} \tag{25}$$

Note that when $n^T(t) > 0$, we always have $q^T(t) = \sqrt{f}$ by the zero-profit condition. By substituting $n = n^T(t)$ into (24), we obtain the domestic firm's equilibrium output for given t as follows:

$$q_1^T(t) = \begin{cases} \sqrt{f} + \dfrac{t}{2-b} & \text{if } t < (a-2\sqrt{f})(2-b)/2, \\ \dfrac{a}{2} & \text{if } t \geq (a-2\sqrt{f})(2-b)/2. \end{cases} \tag{26}$$

From these results, we can calculate the equilibrium domestic welfare $W^T(t) = W(q_1^T, q^T, \ldots, q^T)$:

$$W^T(t) = \frac{(2-b)^2(a^2 - a(3-b)\sqrt{f} + (2+b)f) + 2(2-b)(2b-1)\sqrt{f}t - (4-3b)t^2}{2(2-b)^2 b} \tag{27}$$

if $t < (a-2\sqrt{f})(2-b)/2$, and $W^T(t) = 3a^2/8$ otherwise.

In the first stage, the policy maker sets the import tariff t in order to maximize the domestic welfare $W^T(t)$. This yields the optimal level of the import tariff t^T as in the following lemma.

Lemma 7 *Suppose that Assumptions 4-6 hold and define* $f_T(b) = a^2(4-3b)^2/(4(3-b)^2)$. *Then, (i) if* $f < f_T(b)$, *we have*

$$t^T = \frac{(2-b)(2b-1)\sqrt{f}}{4-3b}, \quad n^T(t^T) = \frac{a(4-3b)-2(3-b)\sqrt{f}}{b(4-3b)\sqrt{f}}, \quad \pi_1^T(t^T) = \frac{(3-b)^2 f}{(4-3b)^2}; \tag{28}$$

[20]Later, we can confirm that both $q_1^T(t)$ and $q^T(t)$ in (24) become positive in the equilibrium with $n > 0$. See Footnote 21.

and (ii) if $f \geq f_T(b)$*, the optimal import tariff is any t such that*

$$t \geq \frac{(a - 2\sqrt{f})(2 - b)}{2},$$

and we have $n^T(t^T) = 0$ *and* $\pi_1^T(t^T) = a^2/4$.

Note that the optimal import tariff given by (28) becomes negative (i.e., import subsidy) if $b < 1/2$.[21] When products are sufficiently differentiated such that $b < 1/2$, it is optimal for the domestic government to attract more foreign firms by providing import subsidies in order to increase product variety in the domestic market.

From the result in this lemma, the maximum domestic welfare under the import-tariff regime is

$$W^T(t^T) = \frac{a^2(4 - 3b) - a(12 - 13b + 3b^2)\sqrt{f} + (3 - b)^2 f}{2b(4 - 3b)} \tag{29}$$

if $f < f_T(b)$, and $W^T(t^T) = 3a^2/8$ otherwise.

4.4 Comparison

Now, we compare the domestic social welfare under each regulation regime with that under the optimal import tariff/subsidy. From Lemmas 5-7, entry by foreign firms is completely deterred if $f \geq f_W(b)$ under the welfare-maximizing regulation, if $f \geq f_P(b)$ under the profit-maximizing regulation, and if $f \geq f_T(b)$ under the tariff policy. The following lemma summarizes the properties of these thresholds.

Lemma 8 $f_T(b)$*,* $f_W(b)$*, and* $f_P(b)$ *have the following properties:*

(i) $f_T(b) < a^2/4$ *if and only if* $b \in (1/2, 1)$.

(ii) For $b \in (0, 1)$, $0 < f_R(b) < a^2/4$ *and* $f_R(b) < f_T(b)$*, where* $R = W, P$.

These three thresholds and the equilibrium entry behavior under each regime are illustrated in Figure 4. According to Figure 4(a), in the comparison between welfare-maximizing regulation and tariff/subsidy regimes, there are three cases when $b > 1/2$: entry accommodation realizes under both regimes if $0 < f < f_W(b)$,

[21] By substituting t^T in (28) into (26), we obtain the equilibrium output of domestic firm as $q_1^T(t^T) = (3 - b)\sqrt{f}/(4 - 3b) > 0$.

entry accommodation under tariff and entry deterrence under regulation if $f_W(b) \leq f < f_T(b)$, and entry deterrence under both regimes if $f_T(b) \leq f \leq a^2/4$. When $b \leq 1/2$, there are the following two cases: entry accommodation realizes under both regimes if $0 < f < f_W(b)$, and entry accommodation under subsidy and entry deterrence under regulation if $f_W(b) \leq f < a^2/4$. From Figure 4(b), we can see that the situation is similar in the comparison between profit-maximizing regulation and optimal tariff/subsidy regimes.

First, we explore the relative performance of the welfare-maximizing regulation and the optimal import tariff/subsidy. In the previous section, we showed that in a homogeneous product setting, the welfare-maximizing regulation (i.e., $\alpha = 0$) always achieves higher domestic welfare than the import tariff/subsidy policy. The next proposition shows that even in the presence of product differentiation, such regulation can (but not always) lead to greater domestic welfare than the optimal import tariff/subsidy policy.

Proposition 3 *Suppose that Assumptions 4-6 hold and define*

$$\hat{b} = \frac{(54\sqrt{79} - 433)^{2/3} + 8(54\sqrt{79} - 433)^{1/3} - 35}{9(54\sqrt{79} - 433)^{1/3}} \approx 0.212.$$

Then, the relative performance of the welfare-maximizing regulation and the optimal import tariff/subsidy is as follows:

(i) for $\hat{b} \leq b < 1$, we have $W^*(\overline{q}^*_{1W}) \geq W^T(t^T)$ for any $f \in (0, a^2/4]$, with equality when $b = \hat{b}$ and $f \leq f_W(b)$,

(ii) for $1/7 \leq b < \hat{b}$, there exists a threshold $\underline{f}_W(b) \in (0, a^2/4]$ such that we have $W^*(\overline{q}^*_{1W}) \geq W^T(t^T)$ if and only if $f \geq \underline{f}_W(b)$,

(iii) for $0 < b < 1/7$, we have $W^*(\overline{q}^*_{1W}) < W^T(t^T)$ for any $f \in (0, a^2/4]$.

The exact expression of $\underline{f}_W(b)$ is provided in the Appendix. In Figure 4(a), the colored area indicates the parameter ranges where the welfare-maximizing regulation regime yields higher domestic welfare than the optimal import tariff/subsidy regime. Note that under optimal import tariff/subsidy regime, the domestic government imposes an import tariff for $b \geq 1/2$, whereas it provides an import subsidy for $b < 1/2$ (see

Lemma 7). Therefore, in this proposition, the welfare level under regulation regime is compared with that under import tariff regime for $b \geq 1/2$ and with that under import subsidy regime for $b < 1/2$.

When products are less differentiated ($b \geq 1/2$), since product variety is less important for domestic consumers, the main concern of the government which maximizes domestic social welfare is to expand the domestic firm's market share. In this case, the regulation regime can achieve higher domestic welfare than the optimal import-tariff regime for the same reason as in the homogeneous product setting: while the import tariff leads to a price increase which harms domestic consumers, the quantity regulation does not have such a negative impact on consumer surplus.

However, when products are relatively differentiated ($b < 1/2$) and domestic consumers place higher value on product variety, this superiority of regulation regime is not necessarily guaranteed. In this situation, the advantage of regulation regime becomes smaller because aggressive expansion of domestic firm's output can have a large negative impact on consumer surplus by limiting the number of foreign entries. In contrast, the import-subsidy regime becomes more advantageous because the government can attract more foreign entries and allow consumers to enjoy product variety. When products are sufficiently differentiated, since the variety expansion effect induced by the import subsidy is predominant, the optimal import tariff regime results in higher domestic welfare than the welfare-maximizing regulation regime.

Next, we compare the domestic welfare under the profit-maximizing regulation regime with that under the optimal import-tariff/subsidy regime. In the homogeneous product case, we showed that profit-maximizing regulation (i.e., $\alpha = 1$) has the same welfare consequences as the welfare-maximizing regulation and always yields higher domestic welfare than the import tariff/subsidy. As the next proposition shows, also in the presence of product differentiation, the profit-maximizing regulation has similar welfare effects as the welfare-maximizing regulation.

Proposition 4 *Suppose that Assumptions 4-6 hold. Then, the relative performance of the profit-maximizing regulation and the optimal import tariff/subsidy is as follows:*

(i) for $1/2 \leq b < 1$, *we have* $W^*(\bar{q}^*_{1P}) \geq W^T(t^T)$ *for any* $f \in (0, a^2/4]$, *with equality when* $f = a^2/4$,

(ii) *for* $(4-\sqrt{10})/3 < b < 1/2$, *there exists a threshold* $\underline{f}_P(b) \in (0, a^2/4]$ *such that* $W^*(\overline{q}^*_{1P}) \geq W^T(t^T)$ *if and only if* $0 < f \leq \underline{f}_P(b)$.

(iii) *for* $0 < b \leq (4-\sqrt{10})/3$, *we have* $W^*(\overline{q}^*_{1P}) \leq W^T(t^T)$ *for any* $f \in (0, a^2/4]$, *with equality when* $b = (4-\sqrt{10})/3$ *and* $f \leq f_P(b)$.

The exact expression of $\underline{f}_P(b)$ is provided in the Appendix. The colored area in Figure 4(b) represents the parameter ranges where the profit-maximizing regulation regime leads to higher domestic welfare than the optimal import-tariff/subsidy regime. By comparing Figures 3(a) and (b), we can see that the performance of profit-maximizing regulation relative to the optimal tariff/subsidy is similar to that of welfare-maximizing regulation; that is, the profit-maximizing regulation yields higher domestic welfare than the optimal import tariff/subsidy policy if and only if b is sufficiently large.

However, we can find a difference between the properties of two types of regulations in Propositions 2(ii) and 3(ii), that is, when b falls within an intermediate range. In these two cases, while the welfare-maximizing regulation is superior to the optimal import subsidy when f is large enough, the profit-maximizing regulation dominates the import subsidy when f is not so large. The intuition behind this difference is as follows. Under welfare-maximizing regulation for $f > f_W(b)$, the government deters foreign firms by implementing marginal cost pricing (i.e., setting $\overline{q}^*_{1W} = a$) and achieves $W^*(\overline{q}^*_{1W}) = a^2/2$, which is independent of the size of entry costs. On the other hand, under optimal import subsidy, larger entry costs make it more costly to induce foreign entry and result in lower domestic welfare. Therefore, for sufficiently large entry costs, the relative advantage of welfare-maximizing regulation becomes stronger.

In contrast, under profit-maximizing regulation for $f > f_P(b)$, since the government sets the domestic firm's output at the minimum level necessary to deter foreign firms, larger entry costs induce the government to reduce domestic firm's output. In particular, when entry costs are sufficiently large (i.e., close to $a^2/4$), the domestic welfare level is almost the same as under unconstrained monopoly. Therefore, as entry costs are larger, the profit-maximizing regulation is less likely to achieve higher welfare than the import subsidy.[22]

[22] Also in the presence of product differentiation, we can discuss the case of decentralized decision making where there are multiple domestic firms and each of them chooses its output level to maximize its own profit. In this case, as in 3.3 of homogeneous

5 Concluding remarks

We have investigated the quantity-control policy/regulation that controls the domestic firms' output/capacity before the entries of foreign firms. By comparing this policy to the optimal import tariff, we have found that such a policy can have stronger impact on the domestic welfare than the import tariff.

As an alternative policy option to quantity control or import tariff/subsidy policy, we can consider imposing price regulation or price ceilings on domestic incumbents.[23] Binding price ceilings could work as a commitment device that induces the domestic incumbents to expand their outputs and thereby limit the entries of foreign firms. Although we expect that price ceilings would result in similar outcomes to those of quantity control in this study, we leave a detailed analysis of this issue for future research.

product setting, when some foreign firms enter the market in the equilibrium, the equilibrium structure and domestic social welfare are the same as under the (centralized) profit-maximizing regulation. A more detailed analysis of this case is available upon request.

[23]There are several studies analyzing the effects of price ceilings under imperfect competition. For example, Molho [16] and Chang [5] respectively explore the effects of price ceilings in a closed economy under Cournot and Stackelberg oligopoly. Matsushima [15] focuses on a monopolist's location choice between two countries and analyzes how imposing a binding price ceiling in one country affects the location choice and overall social welfare.

Appendix

Proof of Lemma 1

(i) $(Q^*(0), q^*(0))$ is one of the solution of the system of equations (2)(3). Suppose this system of equation have a solution (Q', q') other than $(Q^*(0), q^*(0))$. Since $0 = p(Q^*(0)) + p'(Q^*(0))q^*(0) - c'(q^*(0)) = p(Q')+p'(Q')q'-c'(q')$ and the left-hand side of (3) is strictly decreasing both in Q^* and q^*, $Q^*(0) \lesseqgtr Q'$ if and only if $q^*(0) \gtreqless q'$. Therefore, $(Q^*(0), q^*(0)) \neq (Q', q')$ implies that $Q^*(0) < Q'$ and $q^*(0) > q'$, or $Q^*(0) > Q'$ and $q^*(0) < q'$. Suppose that $Q^*(0) < Q'$ and $q^*(0) > q'$. Then, $0 = p(Q^*(0))q^*(0) - c(q^*(0)) - f > p(Q^*(0))q' - c(q') - f \geq p(Q')q' - c(q') - f$, which is a contradiction. Note here that the first inequality is valid since $p(Q^*(0)) - c'(q) > 0$ for all $q \in [q', q^*(0)]$ by (3) and the second one is since $p'(Q)q' \leq 0$. Similarly $Q^*(0) > Q'$ and $q^*(0) < q'$ leads a contradiction. Thus, $(Q^*(0), q^*(0))$ is the unique solution of the system of equations (2)(3) with regard in (Q^*, q^*). When $\overline{Q}_D < Q^*(0)$, since there uniquely exists $n' > 0$ that satisfies $Q^*(0) = \overline{Q}_D + n'q^*(0)$, that is $n' = (Q^*(0) - \overline{Q}_D)/q^*(0) > 0$, $(n', q^*(0))$ is the unique solution of (2)(3) with regard in (n^*, q^*). Therefore, $n^*(\overline{Q}_D) = n'$, $q^*(\overline{Q}_D) = q^*(0)$, and $Q^*(\overline{Q}_D) = Q^*(0)$ must holds.

(ii) Suppose that $n^*(\overline{Q}_D) > 0$. Then, we must have $Q^*(\overline{Q}_D) > Q^*(0)$ since $q^*(\overline{Q}_D) > 0$ and $\overline{Q}_D \geq Q^*(0)$. Then, from (3), $q^*(\overline{Q}_D) < q^*(0)$ since $0 = p(Q^*(0)) + p'(Q^*(0))q^*(0) - c'(q^*(0)) > p(Q^*(\overline{Q}_D)) + p'(Q^*(\overline{Q}_D))q^*(0) - c'(q^*(0))$ and $p' - c'' < 0$. From (2), $f = p(Q^*(0))q^*(0) - c(q^*(0)) > p(Q^*(\overline{Q}_D))q^*(0) - c(q^*(0)) > p(Q^*(\overline{Q}_D))q^*(\overline{Q}_D) - c(q^*(\overline{Q}_D))$, which is a contradiction, where the last inequality is because $p(Q^*(\overline{Q}_D)) - c'(q) > 0$ for $q \in (q^*(\overline{Q}_D), q^*(0))$ by (3). Therefore, $n^*(\overline{Q}_D) = 0$. It immediately follows that $Q^*(\overline{Q}_D) = \overline{Q}_D$. **Q.E.D.**

Proof of Lemma 2

(i) Suppose that $p(Q^*(0)) < C'(0)$. Then, $p(Q^*(0)) < C'(\overline{Q}_D)$ for all $\overline{Q}_D \geq 0$. By Lemma 1, $p(Q^*(\overline{Q}_D)) \leq p(Q^*(0))$ for all $\overline{Q}_D \geq 0$. From these, $p(Q^*(\overline{Q}_D)) \leq p(Q^*(0)) < C'(\overline{Q}_D)$ for all $\overline{Q}_D \geq 0$. Therefore, any additional increase in $\bar{Q}_D$ strictly reduce both the welfare and the profits of the domestic firms as well as the convex combination of them. Thus, $\bar{Q}_D^*(\alpha) = 0$ for all $\alpha \in [0, 1]$.

(ii) Suppose $C'(0) \leq p(Q^*(0)) \leq p(Q^P)$. When $\bar{Q}_D \geq Q^*(0)$, $Q^*(\bar{Q}_D) = \bar{Q}_D$ by Lemma 1. Since $Q^*(0) \geq Q^P$ by the assumption of the present case, the welfare and the profits of the domestic firms are strictly smaller when $\bar{Q}_D > Q^*(0)$ than $\bar{Q}_D = Q^*(0)$ by the definition of Q^P. Therefore, $(1-\alpha)CS^*(Q^*(0)) + \Pi^*(Q^*(0)) > (1-\alpha)CS^*(\bar{Q}_D) + \Pi^*(\bar{Q}_D)$ for all $\bar{Q}_D > Q^*(0)$. Thus, by the existence, $\bar{Q}_D^*(\alpha)$ must be in $[0, Q^*(0)]$. For all $\bar{Q}_D \leq Q^*(0)$, $Q^*(\bar{Q}_D) = Q^*(0)$ by Lemma 1, that is, the total output (price) is constant. Thus, the first-order condition $p(Q^*(0)) = C'(\bar{Q}_D^*(\alpha))$ must be satisfied.

(iii) Suppose $p(Q^P) < p(Q^*(0))$. When $\bar{Q}_D < Q^*(0)$, $Q^*(\bar{Q}_D) = Q^*(0)$ by Lemma 1 and $C'(\bar{Q}_D) < p(Q^*(0))$ by $p(Q^P) < p(Q^*(0))$. Therefore, since $CS^*(Q^*(0)) = CS^*(\bar{Q}_D)$ and $\Pi^*(Q^*(0)) > \Pi^*(\bar{Q}_D)$ hold, $(1-\alpha)CS^*(Q^*(0)) + \Pi^*(Q^*(0)) > (1-\alpha)CS^*(\bar{Q}_D) + \Pi^*(\bar{Q}_D)$ for all $\bar{Q}_D < Q^*(0)$. When $\bar{Q}_D > Q^P$, since $Q^P > Q^*(0)$ by the assumption of the present case, $Q^*(\bar{Q}_D) = \bar{Q}_D$ by Lemma 1. Thus, by the definition of Q^P, the welfare and the profits of the domestic firms are strictly smaller when $\bar{Q}_D > Q^P$ than $\bar{Q}_D = Q^P$. Therefore, $(1-\alpha)CS^*(Q^P) + \Pi^*(Q^P) > (1-\alpha)CS^*(\bar{Q}_D) + \Pi^*(\bar{Q}_D)$ for all $\bar{Q}_D > Q^P$. Thus, by the existence, $\bar{Q}_D^*(\alpha)$ must be in $[Q^*(0), Q^P]$. **Q.E.D.**

Proof of Lemma 3

Since $c_i'(q_i) \geq c'(q_i)$ for all $i = 1, \ldots, m$, $q^T(0) > 0$ by Assumption 3. Thus, when $t = 0$, $(Q^T(0), q^T(0))$ satisfies (7) and (9) with equality. $(Q^*(0), q^*(0))$ satisfies (2) and (3), and this system of equation is exactly the same as (7) and (9) with equality. Since the said system of equation never has multiple solutions, $(Q^T(0), q^T(0)) = (Q^*(0), q^*(0))$.

As far as $q^T(t) > 0$ ($n^T(t) > 0$), (7) and (9) are satisfied with equality. Totally differentiating these two equations yields

$$\begin{bmatrix} -1 & -1-c'' \\ -q^T & p-c'-t \end{bmatrix} \begin{bmatrix} dQ^T(t)/dt \\ dq^T(t)/dt \end{bmatrix} = \begin{bmatrix} 1 \\ q^T \end{bmatrix}.$$

Note that $p - c' - t > 0$ by (7). Hence, we have $dQ^T(t)/dt = [(p - c' - t) + q^T(1 + c'')]/\Delta < 0$, where $\Delta = -(p - c' - t) - q^T(1 + c'') < 0$, and $dq^T(t)/dt = 0$.

Further, suppose $q_i^T(t) > 0$. Then, (8) is satisfied with equality and differentiating this yields

$$\frac{dq_i^T}{dt} = -\frac{1}{1 + c_i''}\frac{dQ^T}{dt} > 0.$$

Q.E.D.

Proof of Lemma 5

Define q_1', q_{1W}^a, and q_{1W}^d respectively as

$$q_1' = \frac{a - (2-b)\sqrt{f}}{b}, \quad q_{1W}^a = \frac{(3-b)\sqrt{f}}{2(1-b)}, \quad q_{1W}^d = \underset{\bar{q}_1}{\operatorname{argmax}}\left[a\bar{q}_1 - \frac{(\bar{q}_1)^2}{2}\right] = a,$$

where q_{1W}^a is the maximizer of (21) with respect to $\bar{q}_1$. Note that $W^*(\bar{q}_1)$ is continuous at $\bar{q}_1 = q_1'$. Since we have

$$q_1' - q_{1W}^a = \frac{2a(1-b) - (4 - 3b + b^2)\sqrt{f}}{2(1-b)b},$$

q_{1W}^a is valid as long as $f \leq 4a^2(1-b)^2/(4 - 3b + b^2)^2$. Similarly, since we have

$$q_{1W}^d - q_1' = \frac{(2-b)\sqrt{f} - a(1-b)}{b},$$

q_{1W}^d is valid as long as $f \geq a^2(1-b)^2/(2-b)^2$. Note that since

$$\frac{4a^2(1-b)^2}{(4-3b+b^2)^2} - \frac{a^2(1-b)^2}{(2-b)^2} = \frac{a^2b(1-b)^3(8-5b+b^2)}{(2-b)^2(4-3b+b^2)^2} > 0$$

for all $b \in (0,1)$, both q_{1W}^a and q_{1W}^d are valid for $a^2(1-b)^2/(2-b)^2 < f < 4a^2(1-b)^2/(4-3b+b^2)^2$. By solving $W^*(q_{1W}^a) = W^*(q_{1W}^d)$ with respect to f, we obtain $f_W(b)$ given by (20) and we can confirm that for all $b \in (0,1)$,

$$\frac{a^2(1-b)^2}{(2-b)^2} < f_W(b) < \frac{4a^2(1-b)^2}{(4-3b+b^2)^2}.$$

Therefore, if $f < f_W(b)$ where $W^*(q^a_{1W}) > W^*(q^d_{1W})$ holds, we have $\bar{q}^*_{1W} = q^a_{1W}$. On the other hand, if $f \geq f_W(b)$ where $W^*(q^a_{1W}) \leq W^*(q^d_{1W})$ holds, we have $\bar{q}^*_{1W} = q^d_{1W}$. By substituting these domestic firm's output into (17) and (18), we obtain the equilibrium number of foreign firms and domestic firm's profit. **Q.E.D.**

Proof of Lemma 6

Define q^a_{1P} and q^d_{1P} respectively as

$$q^a_{1P} = \underset{\bar{q}_1}{\operatorname{argmax}}[(2-b)\sqrt{f}\bar{q}_1 - (1-b)(\bar{q}_1)^2] = \frac{(2-b)\sqrt{f}}{2(1-b)},$$
$$q^d_{1P} = \underset{\bar{q}_1}{\operatorname{argmax}}(a-\bar{q}_1)\bar{q}_1 = \frac{a}{2}.$$

Note that $\pi^*_1(\bar{q}_1)$ is continuous at $\bar{q}_1 = q'_1$. Since $a \geq 2\sqrt{f}$ by Assumption 6, we obtain $q'_1 - q^d_{1P} = (2-b)(a-2\sqrt{f})/2b > 0$. This implies that $\pi^*_1(\bar{q}_1)$ is strictly decreasing for $\bar{q}_1 > q'_1$. Thus, $\bar{q}^*_{1P} = q^a_{1P}$ if $q^a_{1P} < q'_1$ and $\bar{q}^*_{1P} = q'_1$ if $q^a_{1P} \geq q'_1$. By substituting these domestic firm's output into (17) and (18), we obtain the equilibrium number of foreign firms and domestic firm's profit. **Q.E.D.**

Proof of Lemma 7

Denote the maximizer of (27) with respect to t by

$$t^a = \frac{(2-b)(2b-1)\sqrt{f}}{4-3b}.$$

Note that $q^T_1(t)$, $n^T(t)$, and $\pi^T_1(t)$ are continuous at $t = (a-2\sqrt{f})(2-b)/2$ and so is $W^T(t)$. In addition, when $t \geq (a-2\sqrt{f})(2-b)/2$, we have $W^T(t) = 3a^2/8$, which is constant with respect to t. Therefore, if $t^a < (a-2\sqrt{f})(2-b)/2$, we have $t^T = t^a$. On the other hand, if $t^a \geq (a-2\sqrt{f})(2-b)/2$, the maximum value of $W^T(t)$ is $3a^2/8$ and any tariff level higher than or equal to $(a-2\sqrt{f})(2-b)/2$ is optimal for the policy maker. By substituting these optimal tariff level into (25), we obtain the equilibrium number of foreign firms.

In addition, since $\pi_1^T(t) = (q_1^T(t))^2$, we can derive the equilibrium domestic firm's profit by using (26). **Q.E.D.**

Proof of Lemma 8

(i) From the definition of $f_T(b)$, we have $f_T'(b) = -5a^2(4-3b)/(2(3-b)^3) < 0$ for all $b \in (0,1)$ and $f_T(1/2) = a^2/4$. These imply that $f_T(b) < a^2/4$ if and only if $b \in (1/2, 1)$.

(ii) First, we show the properties of $f_P(b)$. From its definition, we have $f_P'(b) = -8a^2b(1-b)/(2-b)^5 < 0$ for all $b \in (0,1)$, $f_P(0) = a^2/4$, and $f_P(1) = 0$. These imply that $0 < f_P(b) < a^2/4$ for all $b \in (0,1)$. In addition, we have

$$f_T(b) - f_P(b) = \frac{a^2(28-44b+20b^2-3b^3)(4-12b+12b^2-3b^3)}{4(3-b)^2(2-b)^4}. \tag{30}$$

Since this is positive for $b \in (0,1)$, we conclude that $f_T(b) > f_P(b)$ for $b \in (0,1)$.

Next, we focus on $f_W(p)$. From (20), we have $f_W(0) = a^2/4$, $f_W(1) = 0$, and

$$f_W'(b) = -\frac{4a^2(1-b)}{(8-3b-2b^2+b^3)^3}\Big[172-333b+333b^2-186b^3+54b^4-9b^5+b^6 - \sqrt{1-b}(148-289b+210b^2-92b^3+26b^4-3b^5)\Big]. \tag{31}$$

Since the expression in the bracket of (31) is positive for all $b \in (0,1)$, we have $f_W'(b) < 0$ for $b \in (0,1)$. Therefore, we can see that $0 < f_W(b) < a^2/4$ for all $b \in (0,1)$. In addition, we have

$$f_T(b) - f_W(b) = \frac{a^2}{4(3-b)^2(8-3b-2b^2+b^3)^2}\Big[(4-3b)^2(8-3b-2b^2+b^3)^2 -16(3-b)^2((1-b)^2(10-9b+4b^2-b^3) - 2(1-b)^{7/2}(3-b))\Big].$$

We can confirm that the expression in the bracket of this equation is positive for all $b \in (0,1)$. Therefore, we have $f_T(b) > f_W(b)$ for $b \in (0,1)$. **Q.E.D.**

Proof of Proposition 2

First, we focus on the parameter range of $b \in (1/2, 1)$ and $f \in [f_T(b), a^2/4]$. In this case, Lemma 8 implies that foreign entry is deterred under both regimes and that the equilibrium welfare is given by $W^*(\overline{q}^*_{1W}) = a^2/2$ and $W^T(t^T) = 3a^2/8$. Therefore, we always have $W^*(\overline{q}^*_{1W}) > W^T(t^T)$.

Second, we focus on the parameter range of $b \in (0, 1)$ and $f \in [0, f_W(b))$, where some foreign firms enter in the equilibrium under both regimes and the equilibrium welfare is given by (21) and (29). Then, we have

$$W^*(\overline{q}^*_{1W}) - W^T(t^T) = \frac{(-4 + 24b - 27b^2 + 14b^3 - 3b^4)f}{8(1-b)b(4-3b)}. \tag{32}$$

The sign of the right-hand side of (32) is determined by the sign of $-4 + 24b - 27b^2 + 14b^3 - 3b^4$ in the numerator. Since this is positive if and only if $b > \hat{b}$, we have $W^*(\overline{q}^*_{1W}) > W^T(t^T)$ if and only if $b > \hat{b}$.

Finally, we focus on the parameter range of $b \in (0, 1)$ and $f \in [f_W(b), \min\{a^2/4, f_T(b)\})$, where the equilibrium welfare is given by $W^*(\overline{q}^*_{1W}) = a^2/2$ and (29). Then, we have

$$W^*(\overline{q}^*_{1W}) - W^T(t^T) = \frac{a^2(-4 + 7b - 3b^2) + a(12 - 13b + 3b^2)\sqrt{f} - (3-b)^2 f}{2b(4-3b)}. \tag{33}$$

Note that the sign of the right-hand side of (33) is determined by the sign of the numerator. Then, this is positive if and only if $\underline{f}_W(b) < f < \overline{f}_W(b)$, where

$$\underline{f}_W(b) = \frac{a^2(4-3b)(2 - b - \sqrt{b(4-3b)})}{2(3-b)^2}, \quad \overline{f}_W(b) = \frac{a^2(4-3b)(2 - b + \sqrt{b(4-3b)})}{2(3-b)^2}.$$

By direct comparison, we can confirm that $\overline{f}_W(b) > a^2/4$ for all $b \in (0, 1)$. Then, we only have to focus on $\underline{f}_W(b)$ and we have the following three cases. First, when $b \in (0, 1/7]$, we can confirm that $\underline{f}_W(b) \geq a^2/4$. Therefore, in this case, we always have $W^*(\overline{q}^*_{1W}) < W^T(t^T)$. Second, for $b \in (1/7, \hat{b})$, we have $f_W(b) < \underline{f}_W(b) < a^2/4 < f_T(b)$. Therefore, we have $W^*(\overline{q}^*_{1W}) \gtreqless W^T(t^T)$ if and only if $f \gtreqless \underline{f}_W(b)$. Finally, when $b \in [\hat{b}, 1)$, we can see that $\underline{f}_W(b) \leq f_W(b)$. Therefore, in this case, we always have $W^*(\overline{q}^*_{1W}) \geq W^T(t^T)$. **Q.E.D.**

Proof of Proposition 3

First, we consider the parameter range of $b \in (1/2, 1)$ and $f \in [f_T(b), a^2/4]$. In this case, Lemma 8 implies that foreign entry is deterred under both regimes and that the equilibrium welfare is given by (23) and $W^T(t^T) = 3a^2/8$. Then, we have

$$W^*(\overline{q}^*_{1P}) - W^T(t^T) = \frac{(2-b)(a-2\sqrt{f})(a(3b-2)+2(2-b)\sqrt{f})}{8b^2}. \tag{34}$$

When $a = 2\sqrt{f}$, we can easily see that $W^*(\overline{q}^*_{1P}) = W^T(t^T)$. On the other hand, when $a > 2\sqrt{f}$, the sign of the right-hand side of (34) is equivalent to the sign of $g_{dd}(b, f) \equiv a(3b-2) + 2(2-b)\sqrt{f}$. Since $g_{dd}(b, f)$ is increasing in f and $g_{dd}(b, f_T(b)) = a(2+b)/(3-b) > 0$ for $b \in (1/2, 1)$, $g_{dd}(b, f)$ is positive for any $b \in (1/2, 1)$ and $f \in [f_T(b), a^2/4]$. Therefore, we have $W^*(\overline{q}^*_{1P}) > W^T(t^T)$.

Second, we focus on the parameter range of $b \in (0, 1)$ and $f \in [0, f_P(b))$, where some foreign firms enter in the equilibrium under both regimes and the equilibrium welfare is given by (22) and (29). Then, we have

$$W^*(\overline{q}^*_{1P}) - W^T(t^T) = \frac{(-4+20b-24b^2+14b^3-3b^4)f}{8(1-b)b(4-3b)}. \tag{35}$$

Note that the sign of the right-hand side of (35) is determined by the sign of $-4+20b-24b^2+14b^3-3b^4$ in the numerator. Since this is positive if and only if $b > (4-\sqrt{10})/3$, we have $W^*(\overline{q}^*_{1P}) > W^T(t^T)$ if and only if $b > (4-\sqrt{10})/3$.

Finally, we focus on the parameter range of $b \in (0, 1)$ and $f \in [f_P(b), \min\{a^2/4, f_T(b)\})$, where the equilibrium welfare is given by (23) and (29). Then, we have

$$W^*(\overline{q}^*_{1P}) - W^T(t^T) = \frac{a^2X + aY\sqrt{f} - Zf}{2b^2(4-3b)}, \tag{36}$$

where $X = -4+7b-3b^2$, $Y = 16-24b+13b^2-3b^3$, and $Z = 16-19b+10b^2-2b^3$. Let the numerator of (36) be denoted by $g_{da}(b, f)$. Note that for $b \in (0, \hat{b})$, the discriminant of $g_{da}(b, f) = 0$, Y^2+4XZ, is negative. In this case, since we have $g_{da}(b, f) < 0$ for any $f > 0$, we always have $W^*(\overline{q}^*_{1P}) < W^T(t^T)$.

On the other hand, for $b \in [\hat{b}, 1)$, we have $g_{da}(b, f) > 0$ if and only if $\underline{f}_P(b) < f < \overline{f}_P(b)$, where

$$\underline{f}_P(b) = \frac{a^2(Y^2+2XZ-Y\sqrt{Y^2+4XZ})}{2Z^2}, \quad \overline{f}_P(b) = \frac{a^2(Y^2+2XZ+Y\sqrt{Y^2+4XZ})}{2Z^2}.$$

By direct comparison, we can confirm that $\underline{f}_P(b) < f_P(b)$ for all $b \in [\hat{b}, 1)$. Then, we only have to focus on $\overline{f}_P(b)$ and we have the following three cases. First, when $b \in [\hat{b}, (4 - \sqrt{10})/3]$, we can confirm that $\overline{f}_P(b) \leq f_P(b)$. This implies that we always have $W^*(\overline{q}^*_{1P}) \leq W^T(t^T)$ in this case. Second, for $b \in ((4 - \sqrt{10})/3, 1/2)$, we have $f_P(b) < \overline{f}_P(b) < a^2/4 < f_T(b)$. Therefore, we have $W^*(\overline{q}^*_{1P}) \gtreqless W^T(t^T)$ if and only if $f \lesseqgtr \overline{f}_P(b)$. Finally, for $b \in [1/2, 1)$, we can see that $f_T(b) \leq \overline{f}_P(b)$. Then, in this case, we always have $W^*(\overline{q}^*_{1P}) \geq W^T(t^T)$. **Q.E.D.**

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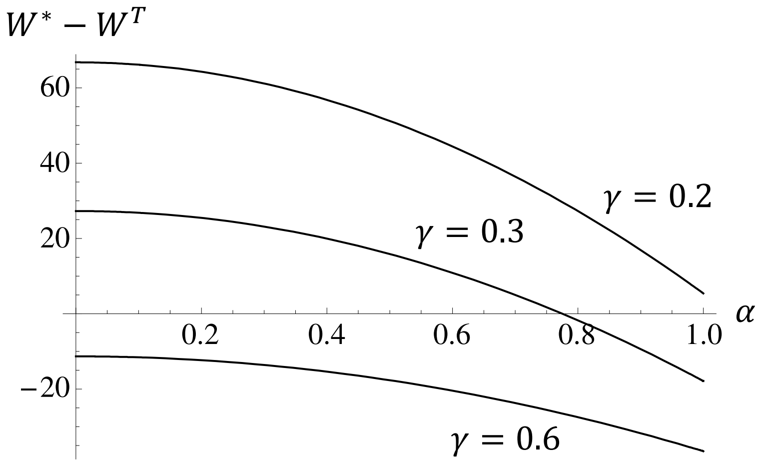


Figure 1: Comparison of domestic social welfare when the number of foreign firms is exogenously fixed. This is a numerical example assuming that $a = 100$, $b = 1$, $m = 1$, $n = 10$, $c_1(q_1) = \gamma q_1^2$, and $c_i(q_i) = 0$ for $i = 2, ..., 11$.

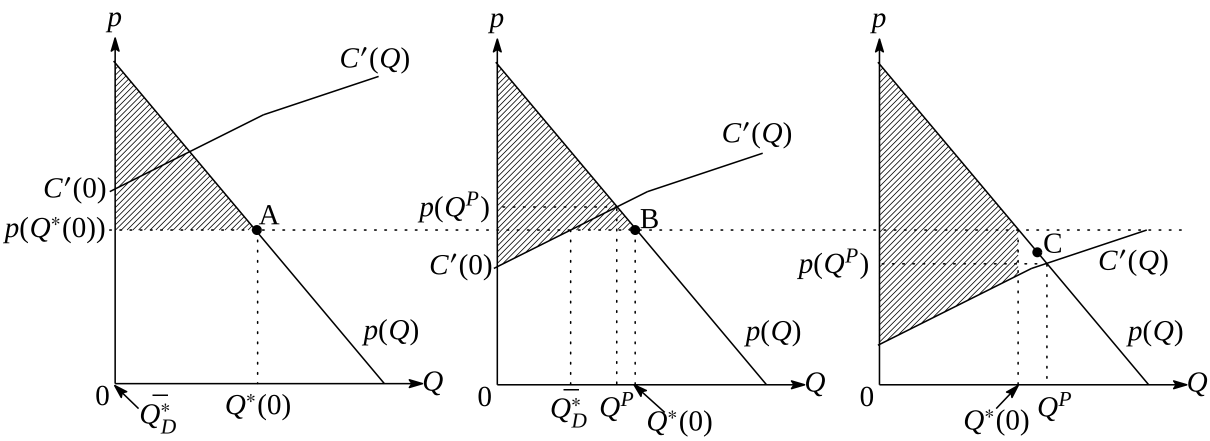


Figure 2: The equilibrium properties under quantity-control regime.

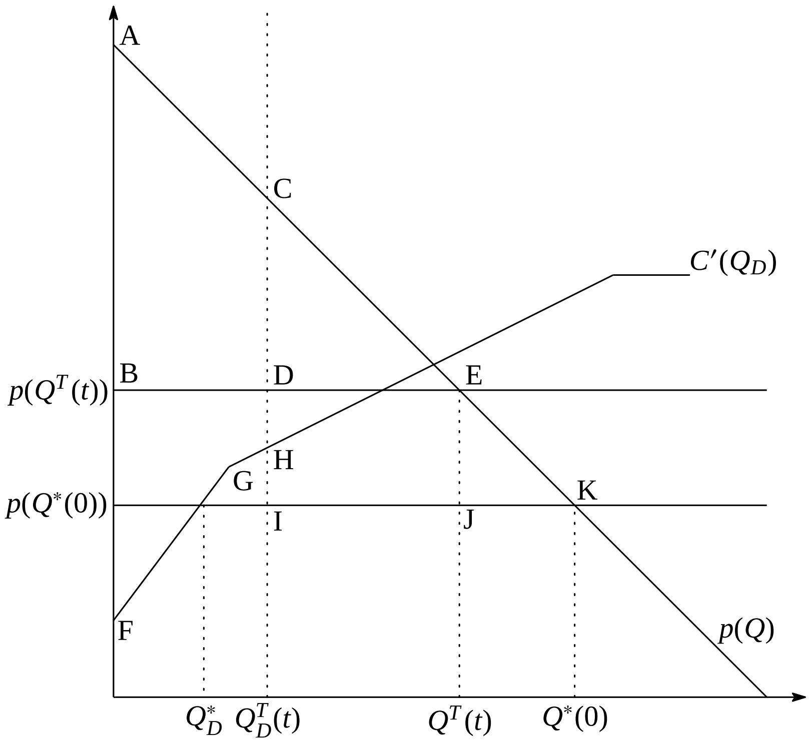
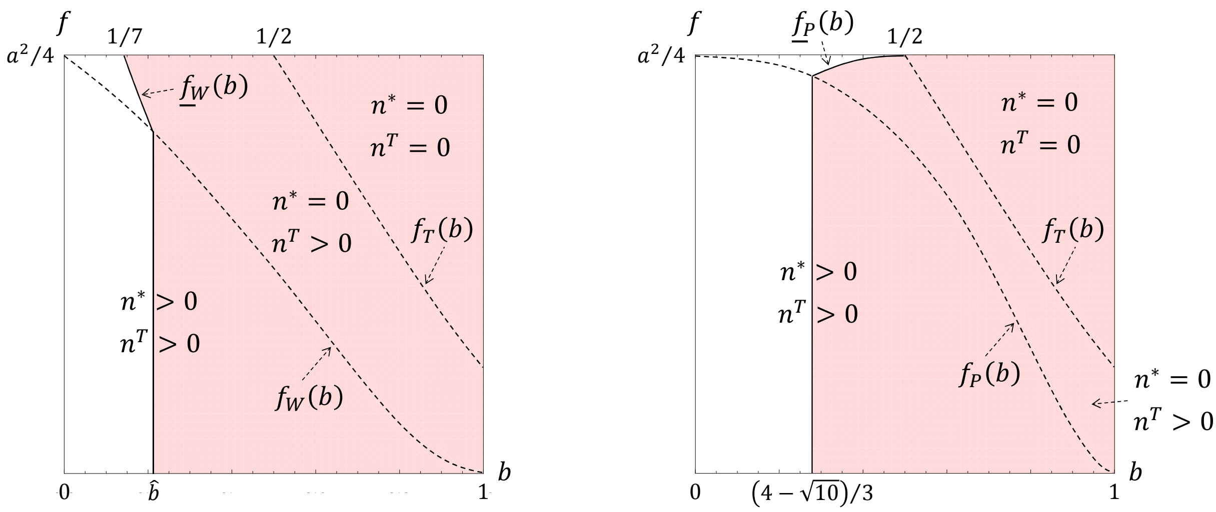


Figure 3: Comparison of domestic social welfare under quantity-control and tariff regimes.



(a) Welfare-maximizing regulation vs. Optimal import tariff/subsidy

(b) Profit-maximizing regulation vs. Optimal import tariff/subsidy

Figure 4: Comparison of domestic social welfare in the presence of product differentiation. The colored area indicates the parameter ranges where the regulation dominates the optimal import-tariff/subsidy.