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Duopoly Bundling: Multi-product vs Single-product Firms

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Abstract

In this paper, I show numerically that a multi-product firm competing against single-product firms will use linear pricing or pure bundling. Contrasted to the bundling strategy of monopolist, mixed bundling is dominated by either linear pricing or pure bundling. The complementarity or integrability between the products supplied by the multi-product firm cannot change this result. Even though the multi-product firm is better off from the enhanced product complementarity, mixed bundling will not be utilized as an optimal pricing strategy.

1 Introduction

Consumer's preference and firm's technology have been two workhorses in economic theory of choice. The demand for a good is the aggregate quantity demanded by rational consumers within their budgets at various prices. As noted, a price for a good would impact on the demand for another product, which depends on whether both goods are complements or substitutes to each other. Throughout this kind of demand analysis, the implicit assumption we might ignore is that only one price should be charged for a good.

However, it is easy for us to perceive that there are many other forms of pricing employed. In grocery stores, you can observe price tags that advertizes lower unit price when you buy the same item in bulk[1]. As you buy more units of the same item, what you pay for a unit purchase decreases (intra-product discount). Sometimes, the discount is placed on the purchase of different items together[2]. Buying predetermined bundle of goods together, you are eligible for a discount(inter-product discount). In a sense that this sort of discounts differs from the traditional uniform unit price scheme, they belong to a family of non-linear pricing. No matter what the reason is, modern firms utilize non-linear pricing actively.

With regard to inter-product discount, multi-product firm would offer the discount more flexibly than the single product firms since the bundle discount offer might cause a transaction cost of writing a contract that specifies the scope of bundle, the division of profit, and so forth. When it comes to the technological compatibility, only multi-product firm might supply the bundle of products successfully in some occasions.

Early literature of bundling mainly centers around the monopoly. Starting from Stigler(1968), monopoly bundling theories are grounded in heterogeneous customer valuation on products[3]. In this approach of papers, bundling is an effective tool for a firm to extract profit by sorting customer types indirectly. Relatively recent literature begins to focus on bundling effect in the framework of competitive markets[4].

[1] Buy One, Get One Free!

[2] Traditional example of perfect complements would fit this case: Left and right shoes.

[3] Adams and Yellen(1976) draw upon Stigler(1968)'s idea and suggest many examples in which pure or mixed bundling bring about maximum profit. McAfee, McMillan, and Whinston (1989) extends Adams and Yellen in continuous distribution of product valuation. Armstrong (1996) and Rochet and Choné (1998) studies bundling in broad perspective of multidimensional nonlinear pricing.

[4] See Spulber (1979), Stole (1995), Armstrong and Vickers (2001), Rochet and Stole (2002), Yin (2004), and Armstrong and Vickers (2007).

This paper is motivated by Gandal, Markovich, and Riordan (2005)'s empirical study of 'PC Office Software Market.' PC office software market from 1991 to 1998 has experienced the changes in marketing strategy. Release of Microsoft Office Suite in 1990 gave rise to the new generation of office software product bundles. Microsoft's use of mixed bundling strategy was going along its achievement of the dominant PC office suite producer even though other competing firms offered their own office suite cooperatively. According to product reviews[5], customers highly evaluated the integration of Microsoft Office products. In PC office software market, Microsoft is the multi-product firm using mixed bundling strategy competing against single-product firms. In order to model this competitive asymmetric bundling competition, I modified Armstrong and Vickers (2007)'s two-stop shopping duopoly model. Armstrong and Vickers (2007) consider the case in which consumers who 'mix-and-match' incur additional cost. This paper goes the other way around. A consumer who buys both products from the multi-product firm enjoys negative 'shopping cost' interpreted by 'integration' or 'product-complementarity.' Other than this type of consumers, 'shopping cost' or 'product-complementarity' will be zero.

The main results are as follows: 1) Mixed bundling is always dominated by linear pricing or pure bundling, 2) Where the product-complementarity is low, linear pricing will be used by the multi-product firm, and 3) Where the product-complementarity is high, pure bundling is better for the multi-product firm.

Section 2 explains the model in more detail. Section 3 studies equilibria in different modes of competition and compares them.

[5]Refer to the Product Reviews in Gandal, Markovich, and Riordan (2005)'s Appendix.

2 A Model

Consider two product markets with three firms. Firm A can produce both products (*product 1* and *product 2*). But firm *B1* produces only *product 1* while firm *B2* supplies only *product 2*. Unless firm *B1* and firm *B2* cooperate, what they can offer to consumers are stand-alone prices, say, P_{B1} and P_{B2}, in respective markets. Admittedly, firm A can set prices in both markets, P_{A1} and P_{A2}, competing against firm *B1* and *B2*. Moreover, firm A might offer 'bundle discount,' δ, for the consumers who buy both products from firm A. Or it can offer only bundles without selling single product so those who want to buy from firm A should buy both products. There are no fixed or variable cost for the production.

At the beginning of the time, firm A chooses how to compete. The first option is to offer stand-alone prices, P_{A1} and P_{A2}, without offering any discount for the bundle purchase [Linear Pricing]. In *product 1* market, firm A and firm *B1* compete in prices. Likewise firm A and firm *B2* do in *product 2* market. The second consideration is to offer 'bundle discount' without stand-alone prices [Pure Bundling], in which firm A supplies only bundles, *product 1* plus *product 2*. Therefore consumers are not able to 'mix-and-match' buying from different suppliers. The last possibility is to offer 'bundle discount' as well as stand-alone prices [Mixed Bundling]. Once firm A decides how it competes, firms are competing with prices which includes 'bundle discount' in mixed bundling. Firms' objective is to maximize its own profit.

A consumer is assumed to buy one unit of each product. The use of products gives gross utility v_1 from *product 1* and v_2 from *product 2*. Most literatures of monopoly bundling study heterogeneous product valuation across consumers[6], however, the paper here assumes the same value of (v_1, v_2) across individuals.

[6] Enumerate the literatures.

Moreover (v_1, v_2) is assumed sufficiently high in order for the purchase of both products.

Given prices, some people buy from firm A, some other people buy from firm B's or they might 'mix-and-match.' What makes consumers buy from different suppliers could be modeled by 'product differentiation.' A consumer is assumed to have preference for brands, A or B. For product $i \in \{1, 2\}$, a consumer represented by $x_i \in [0, 1]$ dislikes A's *product 1* by $t_i x_i$ and Bi's *product i* by $t_i(1-x_i)$ where t_i is a positive real number. For instance, people at 0.1 prefer A and those who at 0.8 prefers B. People at 0.5 do not have any brand preferences. The multiplier, t_i, describes the intensity of "lock-in" or "royalty" to a brand. As t_i increases, it is getting hard for a person at x_i to switch the supplier since the difference in brand preference increases. So a consumer's preference for brands is characterized by a pair $(x_1, x_2) \in X \subset [0, 1]^2$.

The main question of this paper is whether there is an incentive for the multi-product firm to utilize mixed bundling strategy when products are positively related or complementary. MS-Word and MS-Excel is a good example. Using both MS-Word and MS-Excel, consumers get more than the sum of utilities from separate uses since both products share the same command and a document written in MS-Word or MS-Excel is easily transferred to the other product. In this sense, the bundle purchase from a multi-product firm brings about an additional utility, say, α, upon the gross utility, $v_1 + v_2$. I define this additional gain or complementarity by $\alpha > 0$. However, the purchase from single-product firms does not give this sort of gain.

In sum, a type-(x_1, x_2) consumer's utility is defined by

$$u(x_1,x_2) = \begin{cases} v_1 + v_2 + \alpha - \{t_1x_1 + t_2x_2\} - \text{transfer to } A & [AA] \\ v_1 + v_2 - \{t_1x_1 + t_2(1-x_2)\} - \text{transfer to } A \text{ and } \mathit{B2} & [AB] \\ v_1 + v_2 - \{t_1(1-x_1) + t_2x_2\} - \text{transfer to } \mathit{B1} \text{ and } A & [BA] \\ v_1 + v_2 - \{t_1(1-x_1) + t_2(1-x_2)\} - \text{transfer to } \mathit{B1} \text{ and } \mathit{B2} & [BB] \end{cases}$$

For the analytical simplicity, the paper will suppose additional assumptions: 1) The support of consumer's type is $X = [0,1]^2$, the unit rectangle itself, and (x_1, x_2)'s are uniformly distributed on X, *i.e.*, $(x_1,x_2) \sim \text{Unif}\left([0,1]^2\right)$. 2) I will assume $t_1 = t_2 = 1$, that is, the intensity of "royalty" to a brand is the same across all the products and it is normalized to 1. Immediately, we can deduce that firm *B1* and *B2* are symmetric so they will charge the same price and earn the same profit no matter what competition firm A chooses. 3) For linear pricing and mixed bundling, I will confine the equilibrium to the case in which all of the four demand types exist[7].

3 Asymmetric Competition

Throughout this section, single-product firms *B1* and *B2* charges prices, P_{B1} and P_{B2}, respectively. Each single-product firm does not care about the other firm's profit since they are not allowed to cooperate. Given the mode of competition by the multi-product firm A – linear pricing, pure bundling, or mixed bundling – all of the firms charge prices simultaneously. Especially in the case of mixed bundling, firm A charges a bundle discount, δ, the minute it charges prices, P_{A1} and P_{A2}.

[7] Demands for [AA], [AB], [BA], and [BB] should exist.

3.1 Linear Pricing

Given prices, $(P_{A1}, P_{A2}; P_{B1}; P_{B2})$, type-$(x_1, x_2)$ consumer's utility function, $u(x_1, x_2)$, is described as follows:

$$u(x_1,x_2)=\begin{cases} v+\alpha-\{x_1+x_2\}-\{P_{A1}+P_{A2}\} & [AA] \\ v-\{x_1+(1-x_2)\}-\{P_{A1}+P_{B2}\} & [AB] \\ v-\{(1-x_1)+x_2\}-\{P_{B1}+P_{A2}\} & [BA] \\ v-\{(1-x_1)+(1-x_2)\}-\{P_{B1}+P_{B2}\} & [BB] \end{cases}$$

where $v = v_1 + v_2$. Figure 1 depicts the types of demands given these prices.

[Figure 1: Demands under Linear Pricing]

Then firms' profits are

$$\begin{aligned} \pi_A &= P_{A1}(AB+AA)+P_{A2}(BA+AA), \\ \pi_{B1} &= P_{B1}(BA+BB), \\ \pi_{B2} &= P_{B2}(AB+BB) \end{aligned}$$

where

$$\begin{aligned} AA &= \frac{1}{8}\big(2+4\alpha+\alpha^2+2P_{B1}+2\alpha P_{B1}-2P_{A2}(1+\alpha+P_{B1}) \\ &\quad +2P_{B2}(1+\alpha+P_{B1})-2P_{A1}(1+\alpha-P_{A2}+P_{B2})\big), \\ AB &= \frac{1}{4}\big((-1+P_{A1}-P_{B1})(-1+\alpha-P_{A2}+P_{B2})\big), \\ BA &= \frac{1}{4}\big((-1+P_{A2}-P_{B2})(-1+\alpha-P_{A1}+P_{B1})\big), \\ BB &= \frac{1}{8}\big(2-\alpha^2-2P_{A2}(-1+P_{B1})-2P_{B1}+2P_{A1}(1+P_{A2}-P_{B2}) \\ &\quad +2P_{B2}(-1+P_{B1})\big). \end{aligned}$$

Then the first-order conditions of profit maximization are

$$\begin{aligned}
(FOC: P_{A1}) &\quad : \quad \frac{1}{8}\big(4+\alpha(2+\alpha)-8P_{A1}-4\alpha P_{A2}+4P_{B1}+2\alpha P_{B2}\big)=0,\\
(FOC: P_{A2}) &\quad : \quad \frac{1}{8}\big(4+\alpha(2+\alpha)-8P_{A2}-4\alpha P_{A1}+4P_{B2}+2\alpha P_{B1}\big)=0,\\
(FOC: P_{B1}) &\quad : \quad \frac{1}{8}\big(4-\alpha(2+\alpha)+4P_{A1}+2\alpha P_{A2}-8P_{B1}-2\alpha P_{B2}\big)=0,\\
(FOC: P_{B2}) &\quad : \quad \frac{1}{8}\big(4-\alpha(2+\alpha)+4P_{A2}+2\alpha P_{A1}-8P_{B2}-2\alpha P_{B1}\big)=0.
\end{aligned}$$

So the equilibrium prices are determined by

$$P_A^L \equiv P_{A1}^L = P_{A2}^L = \frac{12+6\alpha+\alpha^2}{2(6+\alpha)}, \tag{1}$$

$$P_B^L \equiv P_{B1}^L = P_{B2}^L = \frac{12+6\alpha+\alpha^2}{(2+\alpha)(6+\alpha)}, \tag{2}$$

and the profits in equilibrium are pinned down

$$\pi_A^L = \frac{\big(12+\alpha(6+\alpha)\big)^2}{2(2+\alpha)(6+\alpha)^2}, \tag{3}$$

$$\pi_B^L \equiv \pi_{B1}^L = \pi_{B2}^L = \frac{\big(-12+\alpha(2+\alpha)\big)^2}{8(6+\alpha)^2}. \tag{4}$$

Especially, when products are independent, *i.e.*, $\alpha = 0$, the equilibrium is

$$\begin{aligned}
P_A^L = p_B^L &= 1,\\
\pi_A^L &= 1,\\
\pi_B^L &= \frac{1}{2},
\end{aligned}$$

so firm A and firm B's are dividing markets half and half as shown in figure 2

[Figure 2: Demands under Linear Pricing when $\alpha = 0$]

thus there is no disadvantage for firm *B1* or firm *B2*.

As the complementarity between two products increases, prices charged by firms fall, however, multi-product firm's profit is rising while single-product firms' profit is declining. Figure 3 and 4 show these numerical pattern[8].

Proposition 1 *Under a linear pricing scheme, the multi-product firm is better off as the product-complementarity improves while single-product competitors are worse off. When there is no complementarity between firm's products, single-product firms make a profit as much as the multi-product firm earns in each market.*

[Figure 3: Prices under Linear Pricing Competition]

[Figure 4: Profits under Linear Pricing Competition]

Over the region in which all types of consumers exist, single-product firm's price goes down faster than multi-product firm's price as the complementarity between products gets stronger. It makes sense that multi-product firm charges higher prices than single-product firms because more people want to buy a bundle of firm A's products due to enhanced complementarity. In order to attract consumers, single-product firms should act aggressively charging prices relatively low. In the end, all firms come to set prices low as the complementarity, α is large. Figure 4 shows the contrast between multi-product and single-product firms' profit. Despite the competition gets fierce as α increases, the multi-product firm earns more profit than it does without complementarity.

3.2 Pure Bundling

Similarly to linear pricing, consumers decide to choose whether to buy the firm A's bundle or the mix of single products, *B1*&*B2*. Given prices, $(P_A; P_{B1}; P_{B2})$,

[8] Algebraic expression can be easily derived from equations (1)–(4).

type-(x_1, x_2) consumer's utility function, $u(x_1, x_2)$, is described as follows:

$$u(x_1, x_2) = \begin{cases} v + \alpha - \{x_1 + x_2\} - P_A & [AA] \\ v - \{(1 - x_1) + (1 - x_2)\} - \{P_{B1} + P_{B2}\} & [BB] \end{cases}$$

where $v = v_1 + v_2$. As noticed, consumers cannot 'mix-and-match' so there are no demands for [AB] or [BA]. Figure 5 depicts the types of demands given these prices[9].

[Figure 5: Demands under Pure Bundling]

Then firms' profits are

$$\begin{aligned} \pi_A &= P_A \times AA, \\ \pi_{B1} &= P_{B1} \times BB, \\ \pi_{B2} &= P_{B2} \times BB \end{aligned}$$

where

$$\begin{aligned} AA &= 1 - \frac{1}{8}\big(-2 - P_A + P_{B1} + P_{B2} + \alpha\big)^2, \\ BB &= \frac{1}{8}\big(-2 - P_A + P_{B1} + P_{B2} + \alpha\big)^2. \end{aligned}$$

Then the first-order conditions of profit maximization are

$$\begin{aligned} (FOC : P_A) \quad &: \quad \frac{1}{8}\Bigg(-3P_A^2 + 4P_A(-2 + P_{B1} + P_{B2} + \alpha) \\ &\qquad -(P_{B1} + P_{B2} + \alpha)^2 + 4(1 + P_{B1} + P_{B2} + \alpha)\Bigg) = 0, \\ (FOC : P_{B1}) \quad &: \quad \frac{1}{8}(2 + P_A - 3P_{B1} - P_{B2} - \alpha)(2 + P_A - P_{B1} - P_{B2} - \alpha) = 0, \\ (FOC : P_{B2}) \quad &: \quad \frac{1}{8}(2 + P_A - 3P_{B2} - P_{B1} - \alpha)(2 + P_A - P_{B1} - P_{B2} - \alpha) = 0. \end{aligned}$$

[9] We can consider a case in which the demand for bundle from '*B1+B2*' is greater than the one for A. However, it does not survive in equilibrium.

So the equilibrium prices [10] are determined by

$$P_A^{PB} = \frac{1}{5}\Big(-6 + 3\alpha + 2\sqrt{44 + \alpha(\alpha - 4)}\Big), \tag{5}$$

$$P_B^{PB} \equiv P_{B1}^{PB} = P_{B2}^{PB} = \frac{1}{10}\Big(2 - \alpha + \sqrt{44 + \alpha(\alpha - 4)}\Big), \tag{6}$$

and the profits in equilibrium are

$$\begin{aligned} \pi_A^{PB} &= \frac{1}{500}\Big(-6 + 3\alpha + 2\sqrt{44 + \alpha(\alpha - 4)}\Big)\Big(76 \\ &\quad -2\sqrt{44 + \alpha(\alpha - 4)} + \alpha\big(4 - \alpha + \sqrt{44 + \alpha(\alpha - 4)}\big)\Big), \end{aligned} \tag{7}$$

$$\pi_B^{PB} = \frac{\big(2 - \alpha + \sqrt{44 + \alpha(\alpha - 4)}\big)^2}{2000}. \tag{8}$$

Figure 6 depicts pure bundling equilibrium prices, equation (5) and (6) and figure 7 draws equilibrium profits, equation (7) and (8).

[Figure 6: Prices under Pure Bundling Competition]

[Figure 7: Prices under Pure Bundling Competition]

When two products are independent, *i.e.*, $\alpha = 0$, figure 6 shows that multi-product firm A becomes tougher than single-product firms by charging prices lower than P_{B1}^{PB}, P_{B2}^{PB}, or $P_{B1}^{PB} + P_{B2}^{PB}$. By lowering P_A little bit, firm A induce consumers to buy both product *A1* and *A2*. Balancing increased revenue from new customers and decreased profit from incumbent consumers, firm A maximizes its profit. Consider a single-product firm's interest. Lowering price P_{B1} infinitesimally, firm *B1* can attract more customers to buy *product 1* from itself, which also increases the demand for *product 2* from firm *B2*. Since firm *B1* and firm *B2* shall not cooperate to benefit from this external price effect, a

[10] They satisfy second-order conditions.

single-firm underestimates the marginal benefit of lowering its own price. That is why single-product firms stay relatively higher in prices than the multi-product firm does.

Figure 6 shows that improved product-complementarity makes the multi-product firm raise price while it renders single-product firms aggressive setting prices low. Due to the enhanced complementarity of products, the multi-product firm is easily able to poach the consumers who buy from *B1* and *B2*. Facing this firm A's advantage, single-product firms offer lower prices to compete with the superior multi-product firm. A main difference between linear pricing and pure bundling is that the multi-product firm A raises bundle prices with improved product complementarity while it decreases stand-alone prices in linear pricing[11]. But in both pricing scheme, the multi-product firm A's profit is increasing while single-product firm's profit is declining as the complementarity gets improved[12]. More detailed comparison will be followed in subsection 3.4.

Proposition 2 *Under pure bundling strategy, the multi-product firm makes more profit than single-product firm do. As the multi-product firm's product-complementarity gets strengthened, multi-product firm is better off while single-product firms are worse off.*

3.3 Mixed Bundling

Compared to linear pricing, multi-product firm A has one more choice variable that is bundle discount, δ. Firm A sets prices P_{A1}, P_{A2} and δ at the same time firm *B1* and *B2* charge P_{B1} and P_{B2}, respectively.

Given prices, type-(x_1, x_2) consumer's utility function, $u(x_1, x_2)$, is summa-

[11] See figure 3 and 6.

[12] Nalebuff (2000) reports a similar result under a monopoly pure bundling.

rized as follows:

$$u(x_1, x_2) = \begin{cases} v + \alpha - (x_1 + x_2) - (P_{A1} + P_{A2} - \delta) & [AA] \\ v - (x_1 + (1 - x_2)) - (P_{A1} + P_{B2}) & [AB] \\ v - ((1 - x_1) + x_2) - (P_{B1} + P_{A2}) & [BA] \\ v - ((1 - x_1) + (1 - x_2)) - (P_{B1} + P_{B2}) & [BB] \end{cases}$$

Now we can derive four types of demands – buying both from firm A (AA), 1 from A and 2 from B (AB), 1 from B1 and 2 from A (BA), and 1 from B1 and 2 from B2 (BB) – given prices $(P_{A1}, P_{A2}, \delta; P_{B1}; P_{B2})$.

[Figure 8: Demands under Mixed Bundling]

Now firms are to maximize profits

$$\begin{aligned} \pi_A &= P_{A1}(AB + AA) + P_{A2}(BA + AA) - \delta AA, \\ \pi_{B1} &= P_{B1}(BA + BB), \\ \pi_{B2} &= P_{B2}(AB + BB) \end{aligned}$$

where

$$\begin{aligned} AA &= -\frac{1}{8}\Big(\alpha + \delta\Big)^2 + \frac{1}{4}\Big(1 - P_{A1} + P_{B1} + \alpha + \delta\Big)\Big(1 - P_{A2} + P_{B2} + \alpha + \delta\Big), \\ AB &= \frac{1}{4}\Big(1 - P_{A1} + P_{B1}\Big)\Big(1 + P_{A2} - P_{B2} - \alpha - \delta\Big), \\ BA &= \frac{1}{4}\Big(1 - P_{A2} + P_{B2}\Big)\Big(1 + P_{A1} - P_{B1} - \alpha - \delta\Big), \\ BB &= \frac{1}{8}\Big(2 - 2P_{A2}(-1 + P_{B1}) + 2P_{A1}(1 + P_{A2} - P_{B2}) \\ &\quad +2P_{B1}(-1 + P_{B2}) - 2P_{B2} - (\alpha + \delta)^2\Big). \end{aligned}$$

The first-order conditions of profit maximization are

$$
\begin{aligned}
(FOC: P_{A1}) \quad &: \quad \frac{1}{8}\Big(4 - 8P_{A1} + 4P_{B1} + \alpha^2 \\
&\quad +\delta(4 - 6P_{A2} + 4P_{B2} + 3\delta) + \alpha(2 - 4P_{A2} + 2P_{B2} + 4\delta)\Big) = 0, \\
(FOC: P_{A2}) \quad &: \quad \frac{1}{8}\Big(4 - 8P_{A2} + 4P_{B2} + \alpha^2 \\
&\quad +\delta(4 - 6P_{A1} + 4P_{B1} + 3\delta) + \alpha(2 - 4P_{A1} + 2P_{B1} + 4\delta)\Big) = 0, \\
(FOC: \delta) \quad &: \quad \frac{1}{8}\Big(-2 - 2P_{B1} - 2P_{B2} - 2P_{B1}P_{B2} - 4\alpha - 2P_{B1}\alpha - 2P_{B2}\alpha - \alpha^2 \\
&\quad -4(2 + P_{B1} + P_{B2} + \alpha)\delta - 3\delta^2 + P_{A2}\big(4(1 + P_{B1} + \alpha) + 6\delta\big) \\
&\quad +P_{A1}\big(-6P_{A2} + 4(1 + P_{B2} + \alpha) + 6\delta\big)\Big) = 0, \\
(FOC: P_{B1}) \quad &: \quad \frac{1}{8}\Big(4 + 4P_{A1} - 8P_{B1} - \alpha(2 - 2P_{A2} + 2P_{B2} + \alpha) \\
&\quad -2\delta - 2(-P_{A2} + P_{B2} + \alpha)\delta - \delta^2\Big) = 0, \\
(FOC: P_{B2}) \quad &: \quad \frac{1}{8}\Big(4 + 4P_{A2} - 8P_{B2} - \alpha(2 - 2P_{A1} + 2P_{B1} + \alpha) \\
&\quad -2\delta - 2(-P_{A1} + P_{B1} + \alpha)\delta - \delta^2\Big) = 0.
\end{aligned}
$$

All of the first-order conditions other than $(FOC: \delta)$ are decreasing in prices. However, the first-order condition $(FOC: \delta)$ is a quadratic function of δ. Thus we need to verify whether the interior solutions satisfy the second-order condition. More than the interior solution, we need to investigate the corner solution when $\delta = 0$. Comparing the first-order conditions of linear pricing with the ones of mixed bundling, the corner solution is equivalent to the linear pricing equilibrium. Therefore in this subsection, I will focus on the interior solutions satisfying first- and second-order conditions where there are four demand types exist.

The derivative of $(FOC : \delta)$ is

$$(SOC : \delta) \quad : \quad \frac{1}{4}\Big(3P_{A1} + 3P_{A2} - 2(2 + P_{B1} + P_{B2} + \alpha) - 3\delta\Big) < 0.$$

Solving first-order conditions except for $(FOC : \delta)$, we obtain

$$P_{A1} = P_{A2} \quad = \quad \frac{\delta}{2} + \frac{6(2+\delta) + \alpha(6+\alpha+2\delta)}{12+\alpha^2+2\alpha(4+\delta)+\delta(10+\delta)}, \tag{9}$$

$$P_{B1} = P_{B2} \quad = \quad -\frac{\alpha}{2} + \frac{2(2+\alpha)(3+\alpha)+(8+3\alpha)\delta}{12+\alpha^2+2\alpha(4+\delta)+\delta(10+\delta)}. \tag{10}$$

Plugging equation (9) and (10) into $(FOC : \delta)$, we can simplify the condition $(FOC : \delta)$ such as

$$\begin{aligned} 0 \quad = \quad & 576 + 576\alpha - 48\alpha^2 - 48\alpha^2 + 32\alpha^4 + 12\alpha^5 + \alpha^6 \\ & -768\alpha\delta - 192\alpha^2\delta + 256\alpha^2\delta + 92\alpha^4\delta + 8\alpha^5\delta \\ & -912\delta^2 - 336\alpha\delta^2 + 616\alpha^2\delta^2 + 256\alpha^3\delta^2 + 25\alpha^4\delta^2 \\ & -192\delta^3 + 608\alpha\delta^3 + 336\alpha^2\delta^3 + 40\alpha^3\delta^3 \\ & +216\delta^4 + 212\alpha\delta^4 + 35\alpha\delta^4 \\ & +52\delta^5 + 16\alpha\delta^5 \quad + 3\delta^6 \end{aligned} \tag{11}$$

with respect to δ. Roots of polynomial (11) will be found numerically. Figure 9 to 12 portray the solution satisfying the second-order conditions as well as assumptions at the beginning of a model. There are two equilibria: Figure 9 and 10 show an equilibrium price strategies and profits under the equilibrium. Figure 11 and 12 indicate the other equilibrium.

[Figure 9: Equilibrium (1) Prices under Mixed Bundling]

[Figure 10: Mixed Bundling Profits under Equilibrium (1)]

[Figure 11: Equilibrium (2) Prices under Mixed Bundling]

[Figure 12: Mixed Bundling Profits under Equilibrium (2)]

Figure 9 and 11 show that the multi-product firm A's stand-alone prices are higher than single-product firms' prices as long as the complementarity α is less than 0.39. In both equilibrium (1) and (2), single-product firms' prices, P_{B1}^{MB} and P_{B2}^{MB} is decreasing as the product-complementarity is going up. But multi-product firm's stand-alone prices differ. As shown in figure 9, in equilibrium (1), stand-alone prices, P_{A1}^{MB} and P_{A2}^{MB}, decline as the product-complementarity is strengthened and bundle discount, δ, is also decreasing. But in the other equilibrium (2), multi-product firm raise stand-alone prices and bundle discount alongside improved product-complementarity.

As shown in figure 10, in equilibrium (1), augmented product-complementarity leads to higher profit to the multi-product firm A but less profit to single-product firms. In this sense, multi-product firm takes advantage of product-complementarity. However, the joint profit of single-product firm *B1* and *B2* stays always higher than the multi-product firm A's profit paradoxically. Figure 12 verifies this paradox in equilibrium (2), too. Moreover, product-complementarity depreciates the multi-product firm A's profit, π_A^{MB}, thus, equilibrium (2) is bit more puzzling than equilibrium (1). Comparison of profits between linear pricing will be followed in subsection 3.4.

Proposition 3 *There exist three mixed bundling equilibria. One is equivalent to linear pricing equilibria and the other two ones are mixed equilibria with positive bundle discounts.*

3.4 Comparison and Conclusion

Consider the multi-product firm A's choice of competition at the beginning of the period. Firm A can choose 'linear pricing', 'pure bundling', or 'mixed bundling'. The profit realized in each mode of competition is depicted in figure 13.

[Figure 13: Comparison of Profits under Different Mode of Competition – Multi-product Firm A]

As noted, mixed bundling equilibrium profits are dominated by linear pricing or pure bundling equilibria. Thus mixed bundling competition will not be chosen by multi-product firm at the beginning of the period so mixed bundling is out of equilibrium in the model. Now the only options for firm A are either 'linear pricing' or 'pure bundling.' Figure 13 indicates that firm A chooses 'linear pricing' where the product-complementarity is low, more precisely $\alpha < 0.3050$ in the model. But firm A decides to offer 'pure bundle' where the product-complementarity is relatively high, $0.3050 < \alpha < 0.3900$.

Proposition 4 *Multi-product firm decides to compete in linear pricing where the product-complementarity is less than* 0.3050*, however it determines to compete in pure bundling strategy where the complementarity is higher than* 0.3050*. Mixed bundling is dominated by other competing schemes.*

The corresponding single-product firm's profit is depicted in figure 14.

[Figure 14: Comparison of Profits under Different Mode of Competition – Single-product Firm B]

Single-product firm's profit is deteriorating as firm A's product-complementarity gets improved. At a threshold value, $\alpha = 0.3050$, the profit drops sharply since the competition mode switches from 'linear pricing' to 'pure bundling.'

4 Concluding Remarks

Adams and Yellen (1976) notice that "whenever the exclusion requirement[13] is violated in a pure bundling equilibrium, mixed bundling is necessarily preferred to pure bundling." However, contrary to monopoly bundling, mixed bundling is inferior to pure bundling when we consider duopoly price competition between multi-product firm and single-product firms.

This paper proves that there are multiple mixed bundling equilibria with positive bundle discounts contrasting to the symmetric equilibrium of Armstrong and Vickers (2007). Asymmetric competition between multi-product firm and single-product firms does not guarantee nice and neat unique equilibrium even with the assumption of uniformly distributed product differentiation.

In the beginning of the paper, I referred to 'PC Office Software Market' of Gandal, Markovich, and Riordan (2005). Although their discrete choice model estimation suggest that word processors and spreadsheets are complementary products, this paper concludes that product-complementarity does not lead to the use of mixed bundling.

I analyze the equilibria only when there exist 'mix-and-match' consumers except pure bundling. When stand-alone prices are high enough, 'mix-and-match' will not be employed so the analysis I do here is needed to be extended even for this case. More interesting study would be the case in which stand-alone firms can cooperate to offer bundles by making merger and acquisition or simply forming a business coalition. In the light of regulation, we need to investigate how consumer welfare or social welfare change in different equilibrium, moreover, how to introduce regulation to foster efficiency or welfare.

[13] No individual consumes a good if the cost of that good exceeds its cost in fact consumes that good.

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Figure 1: Demands under Linear Pricing

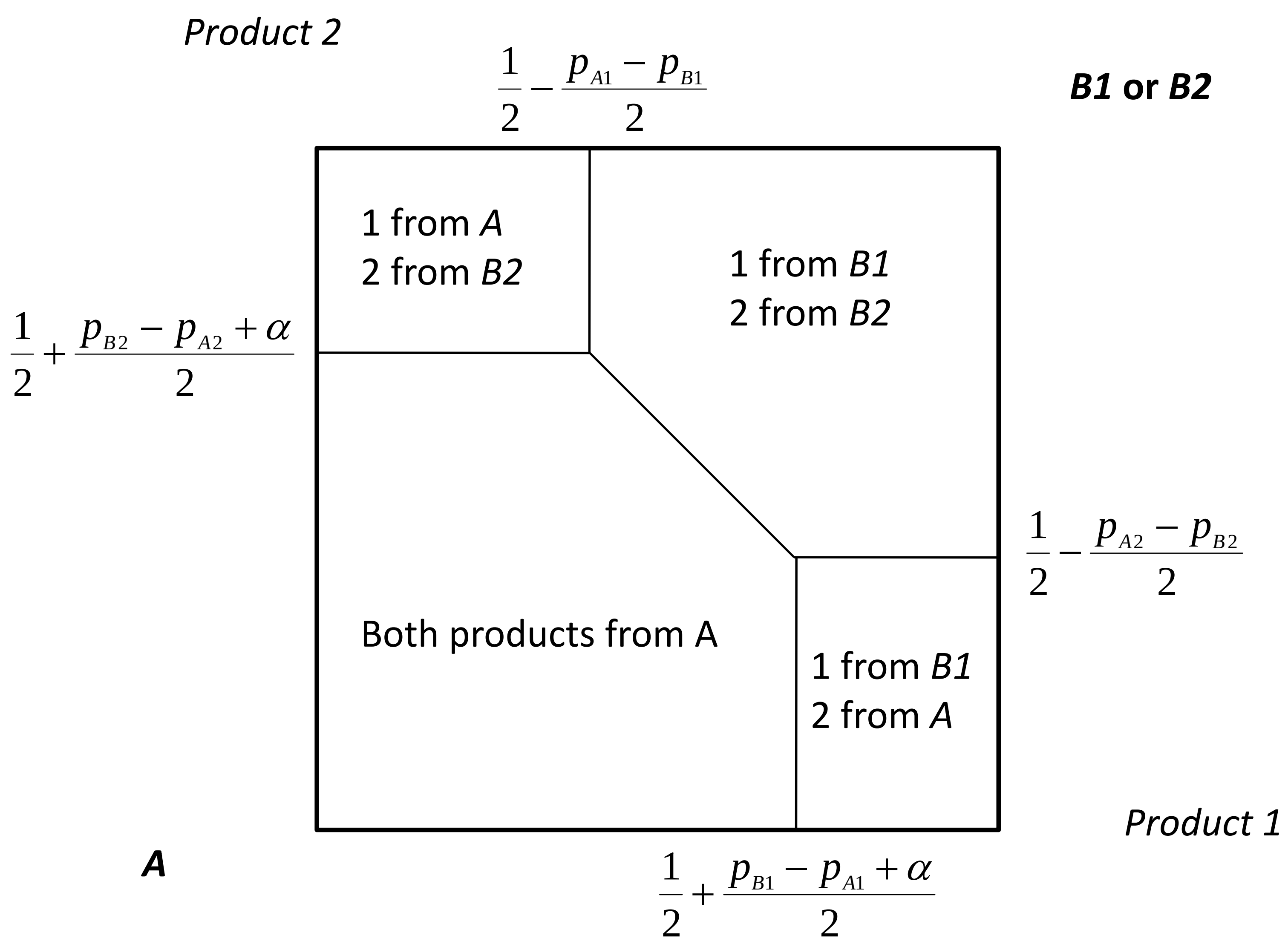


Figure 2: Demands under Linear Pricing ($\alpha = 0$)

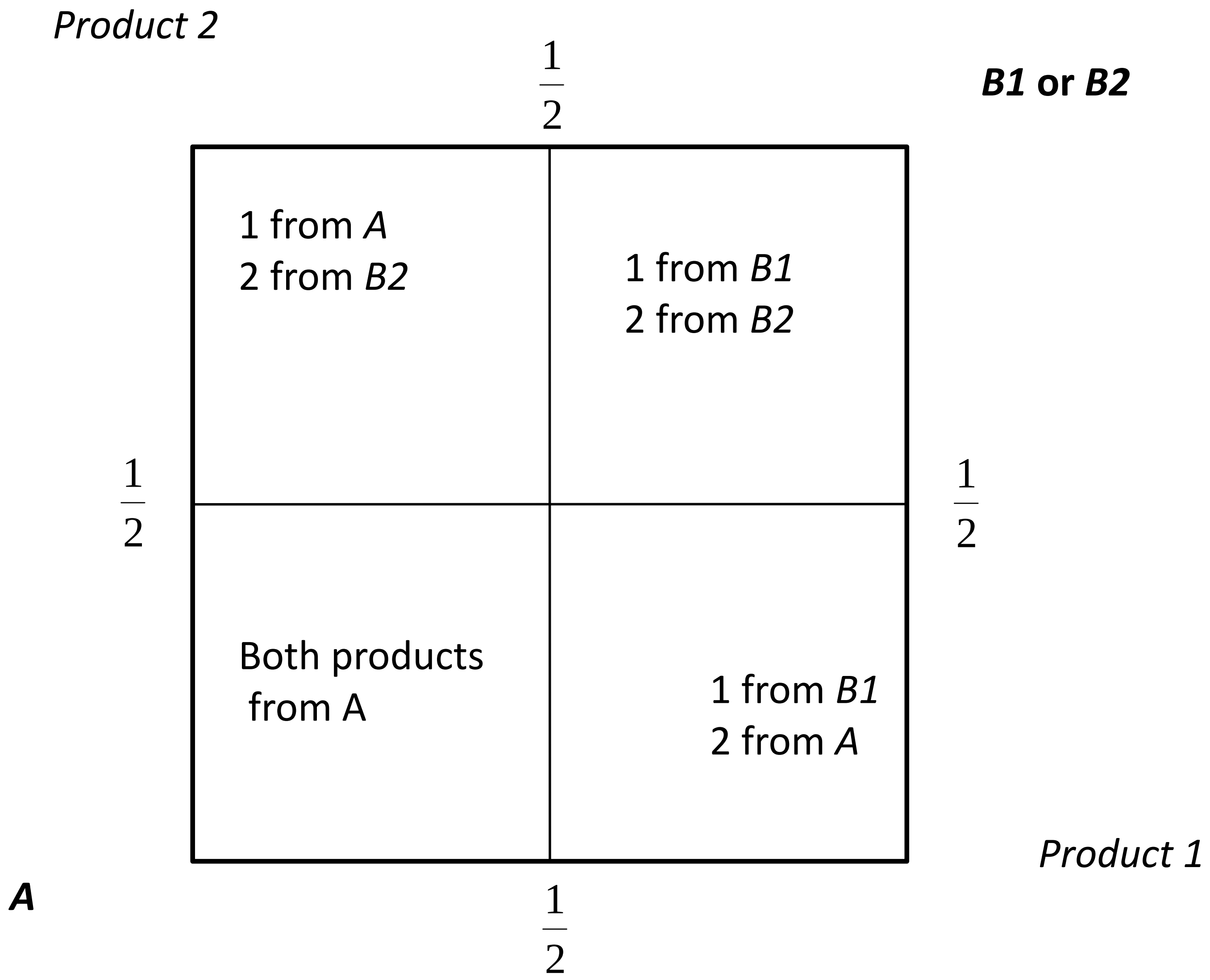


Figure 3: Prices under Linear Pricing Competition

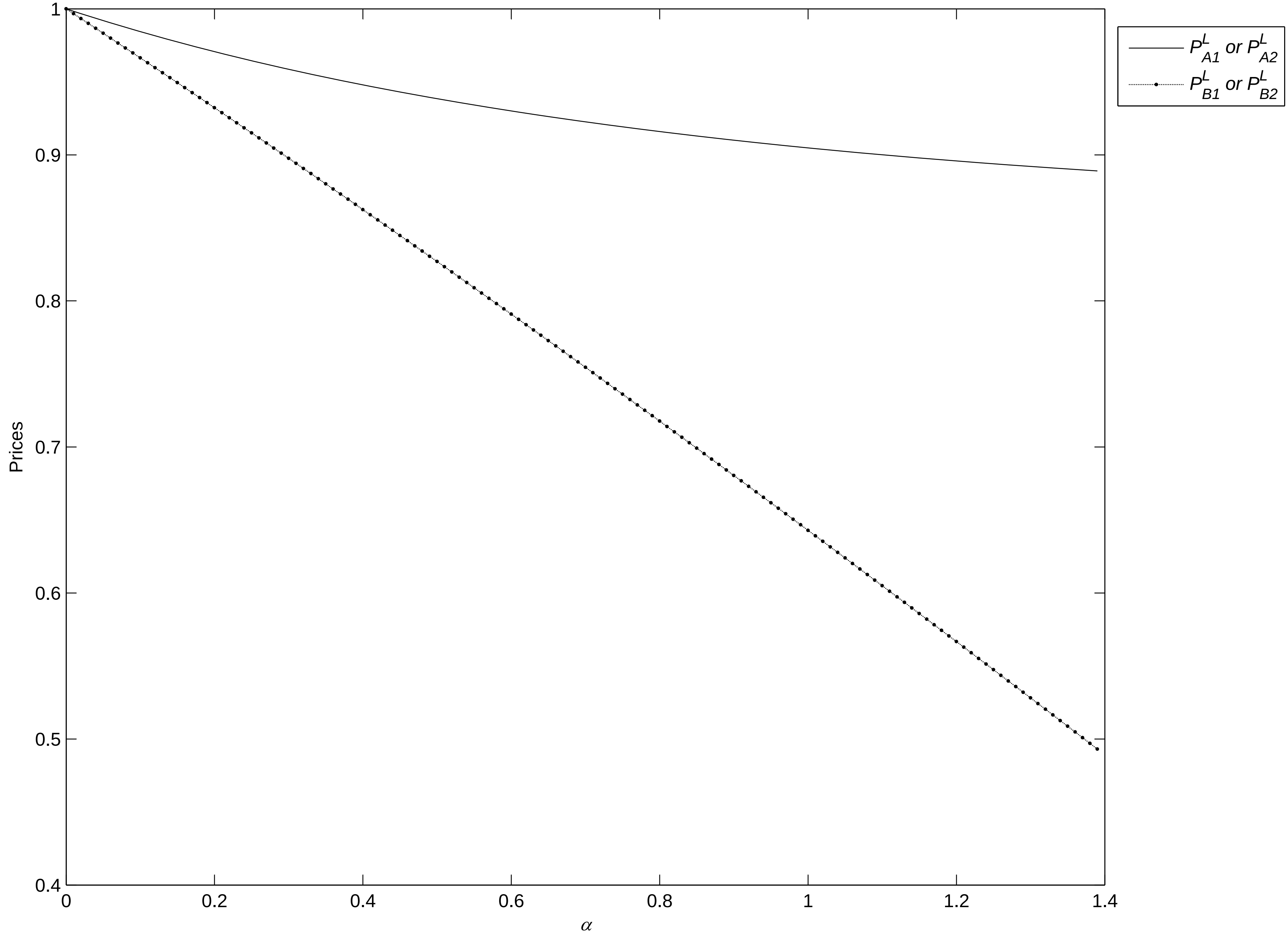


Figure 4: Profits under Linear Pricing Competition

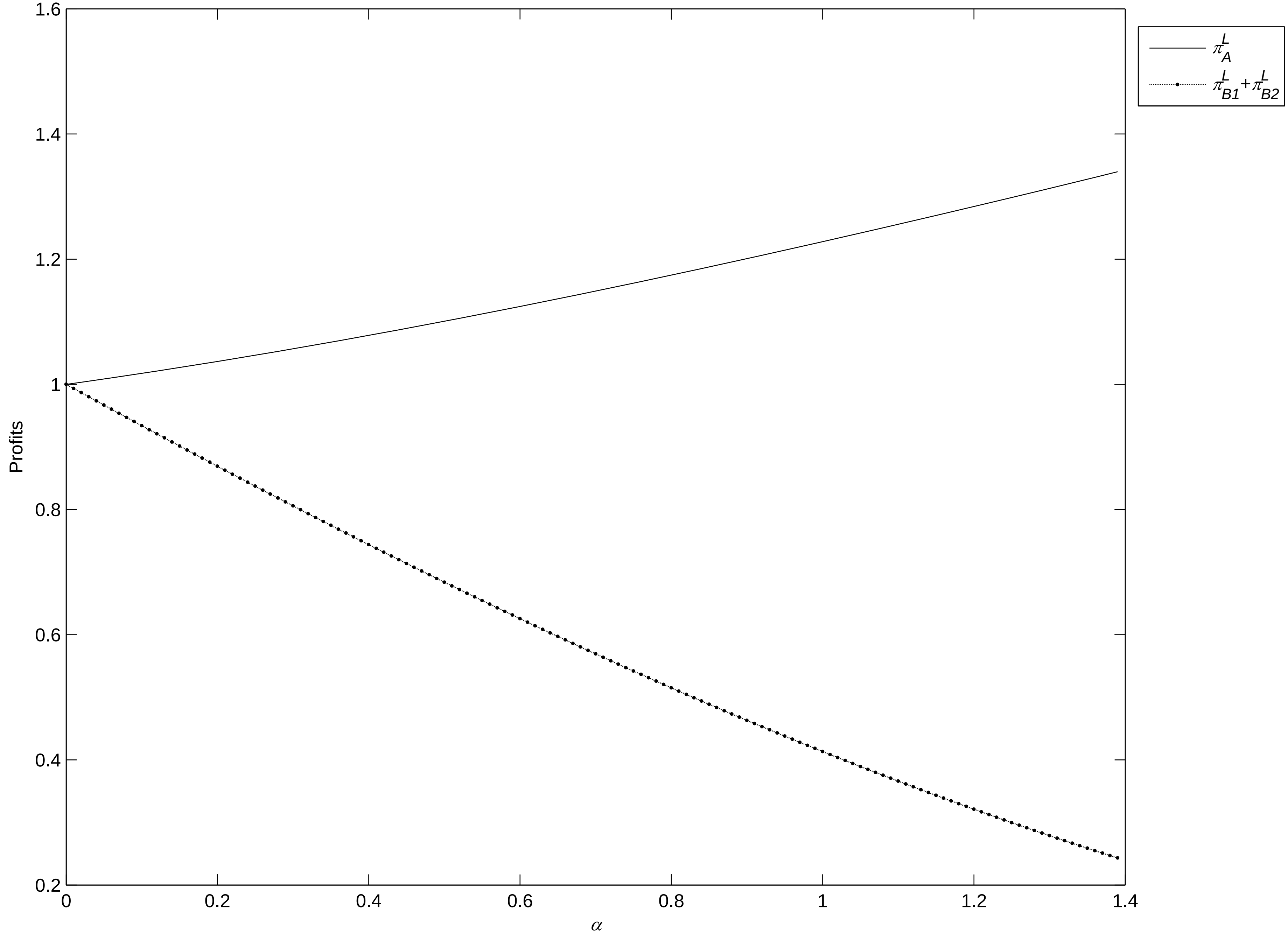
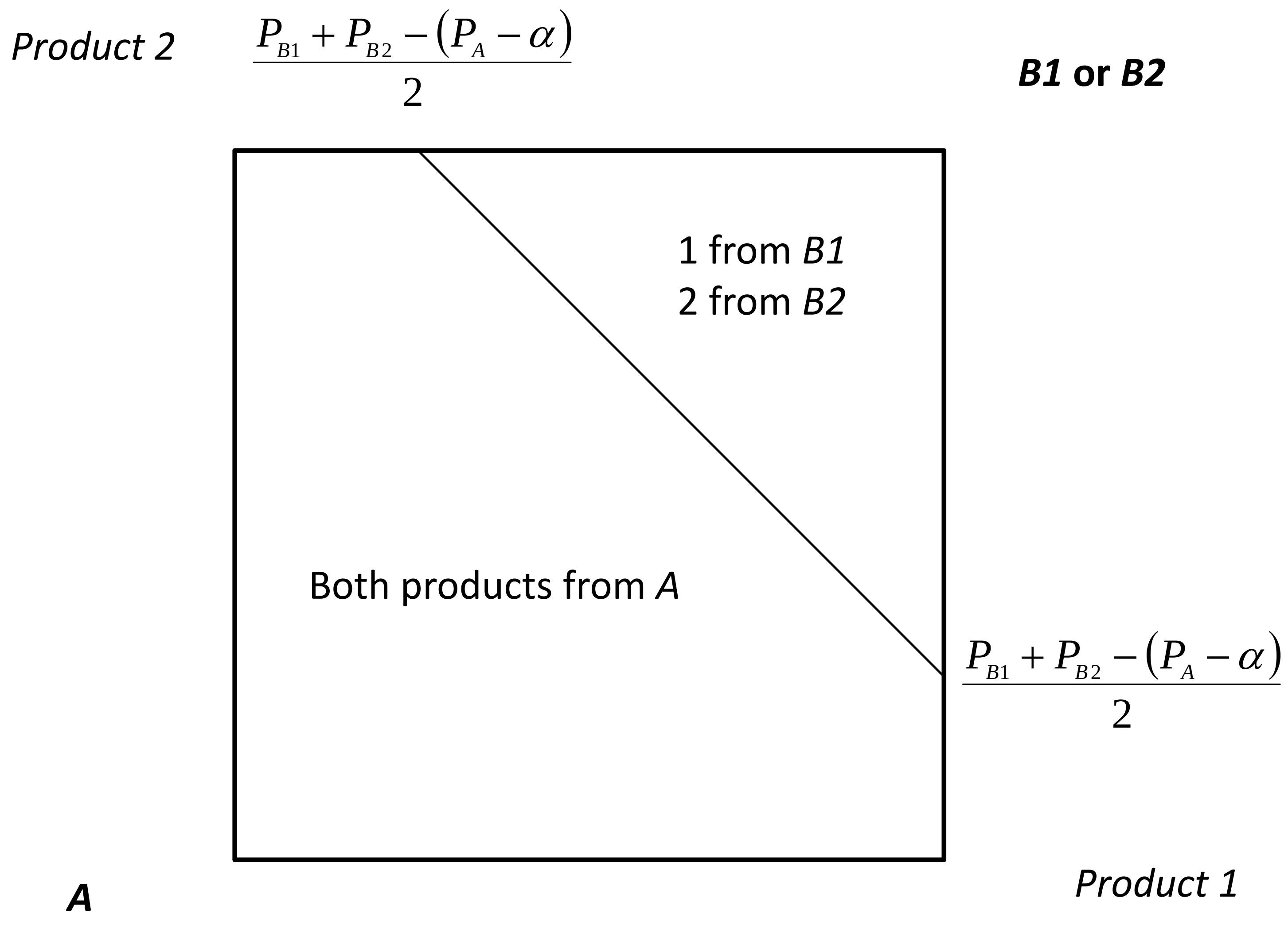


Figure 5: Demands under Pure Bundling



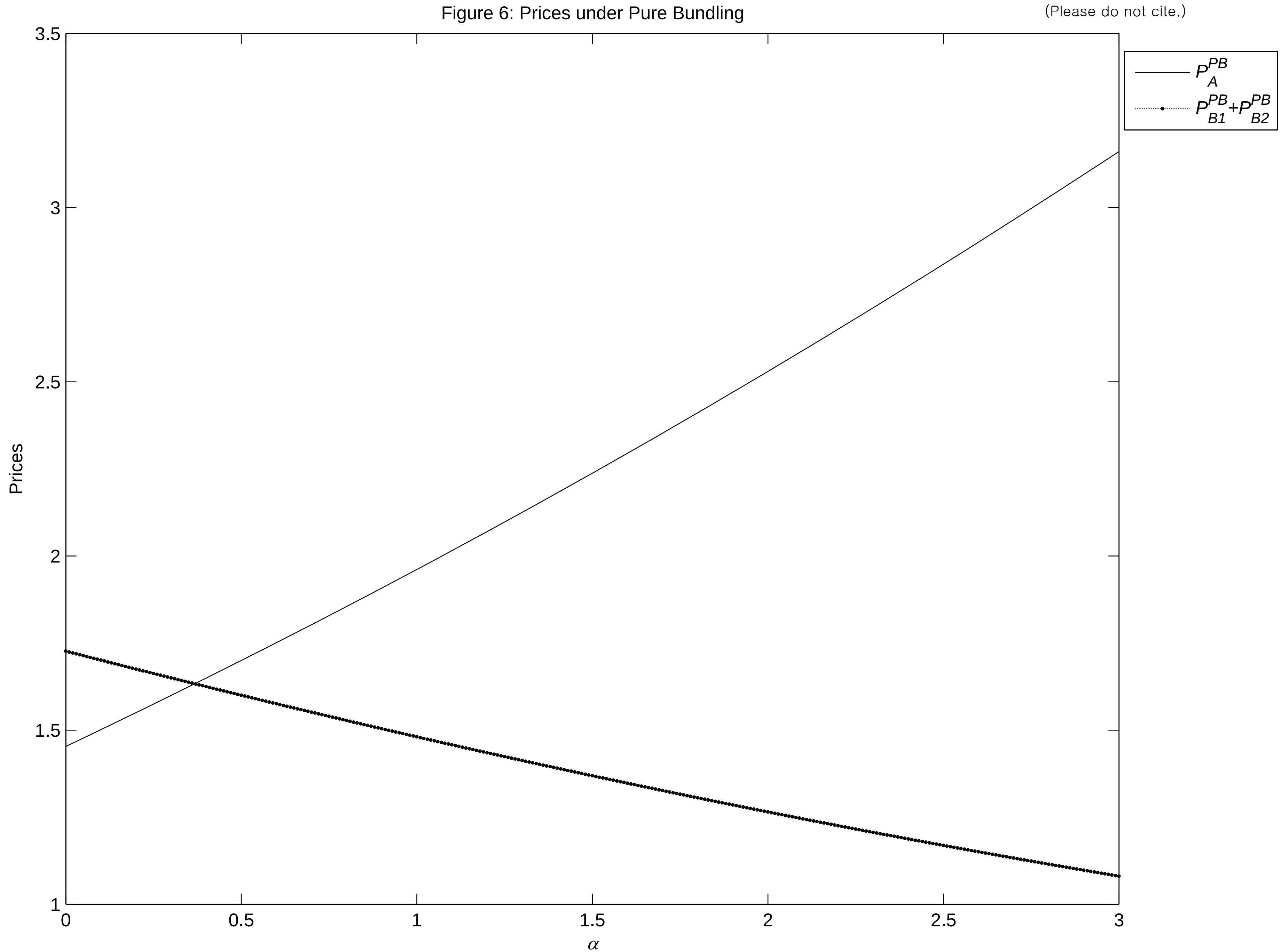


Figure 6: Prices under Pure Bundling
Prices
α
P_A^{PB}
$P_{B1}^{PB}+P_{B2}^{PB}$
3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3

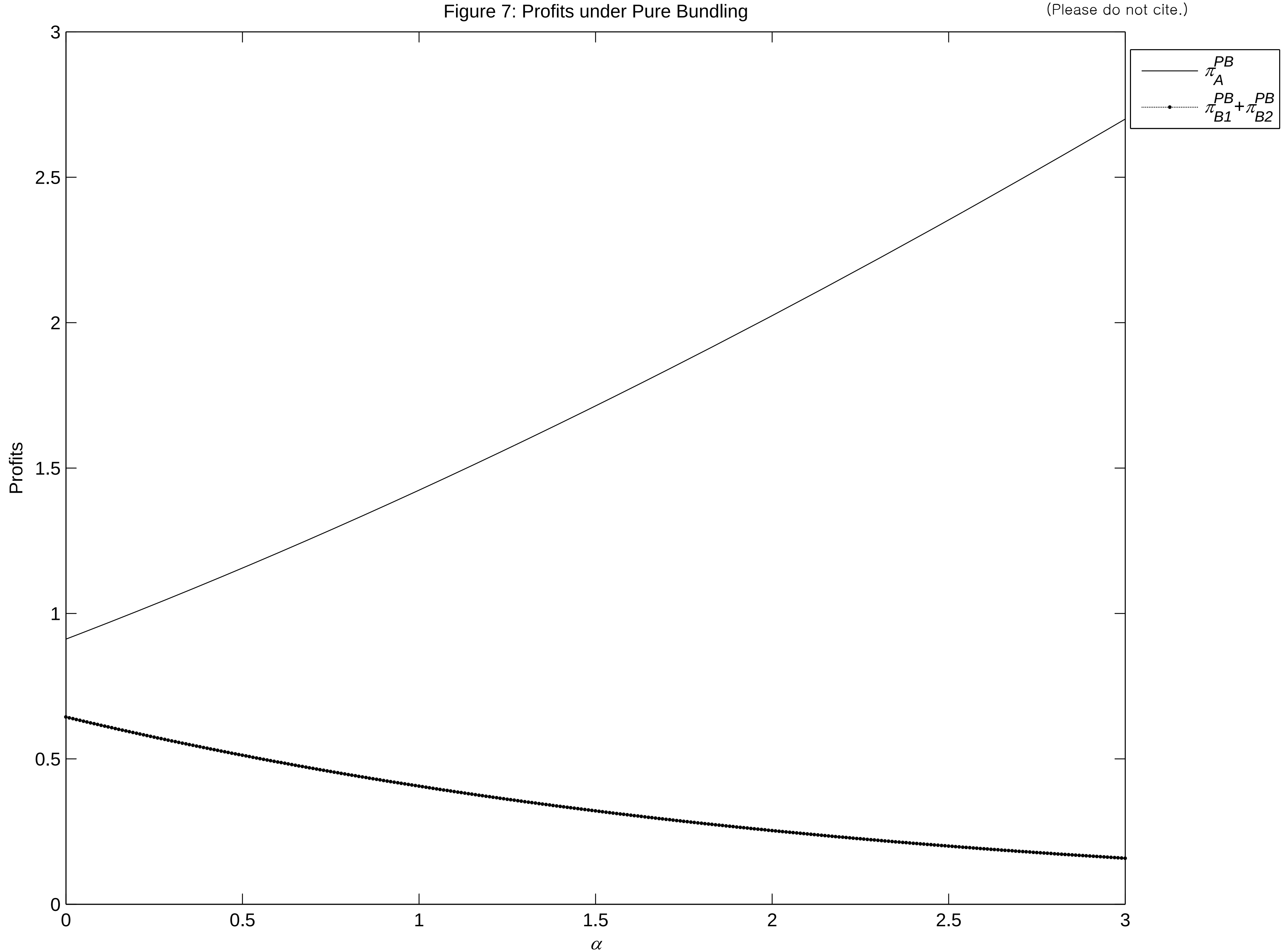


Figure 7: Profits under Pure Bundling
Profits
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
α
π_A^{PB}
$\pi_{B1}^{PB}+\pi_{B2}^{PB}$

Figure 8: Demands under Mixed Bundling

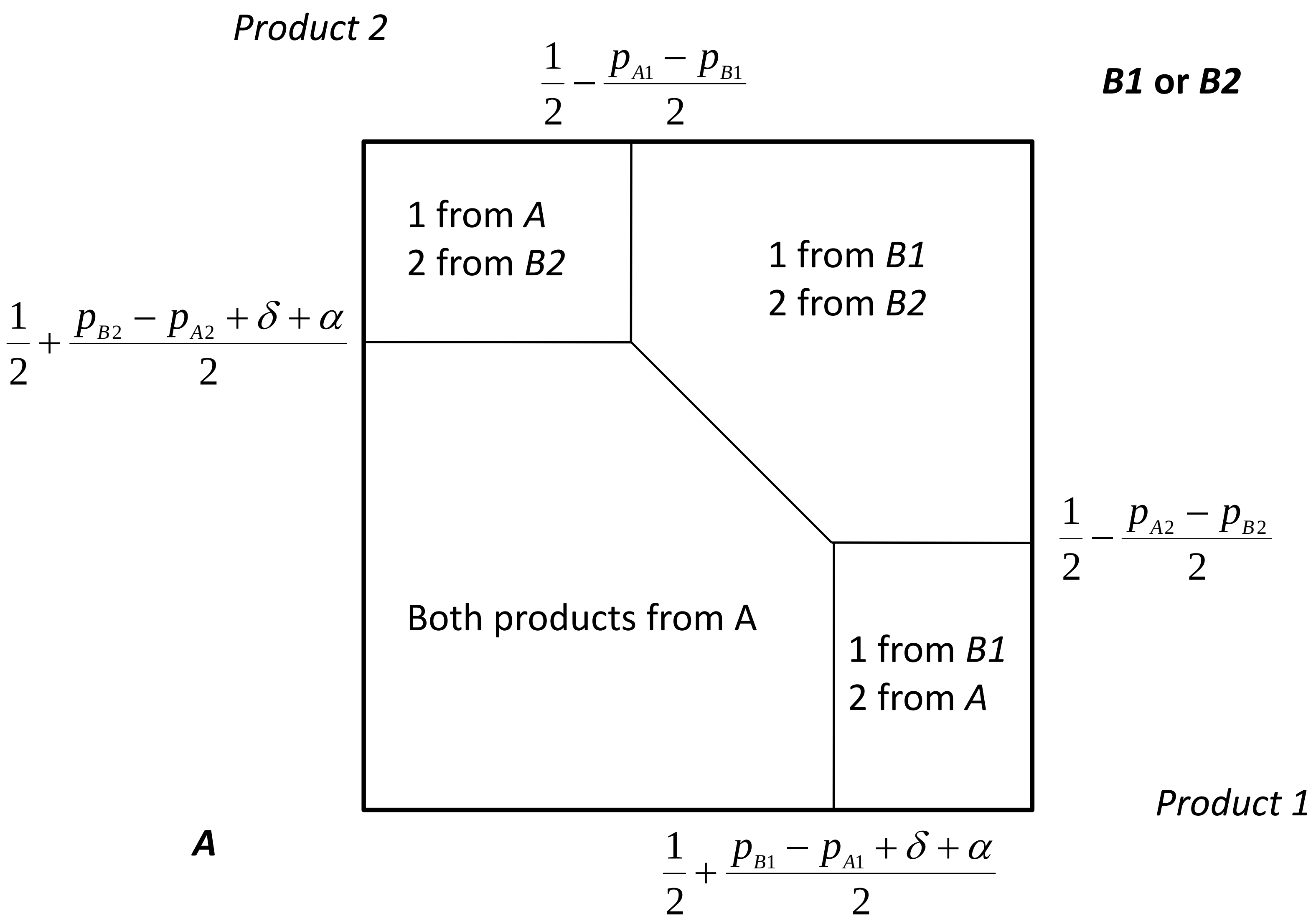


Figure 9: Equilibrium (1) Prices under Mixed Bundling

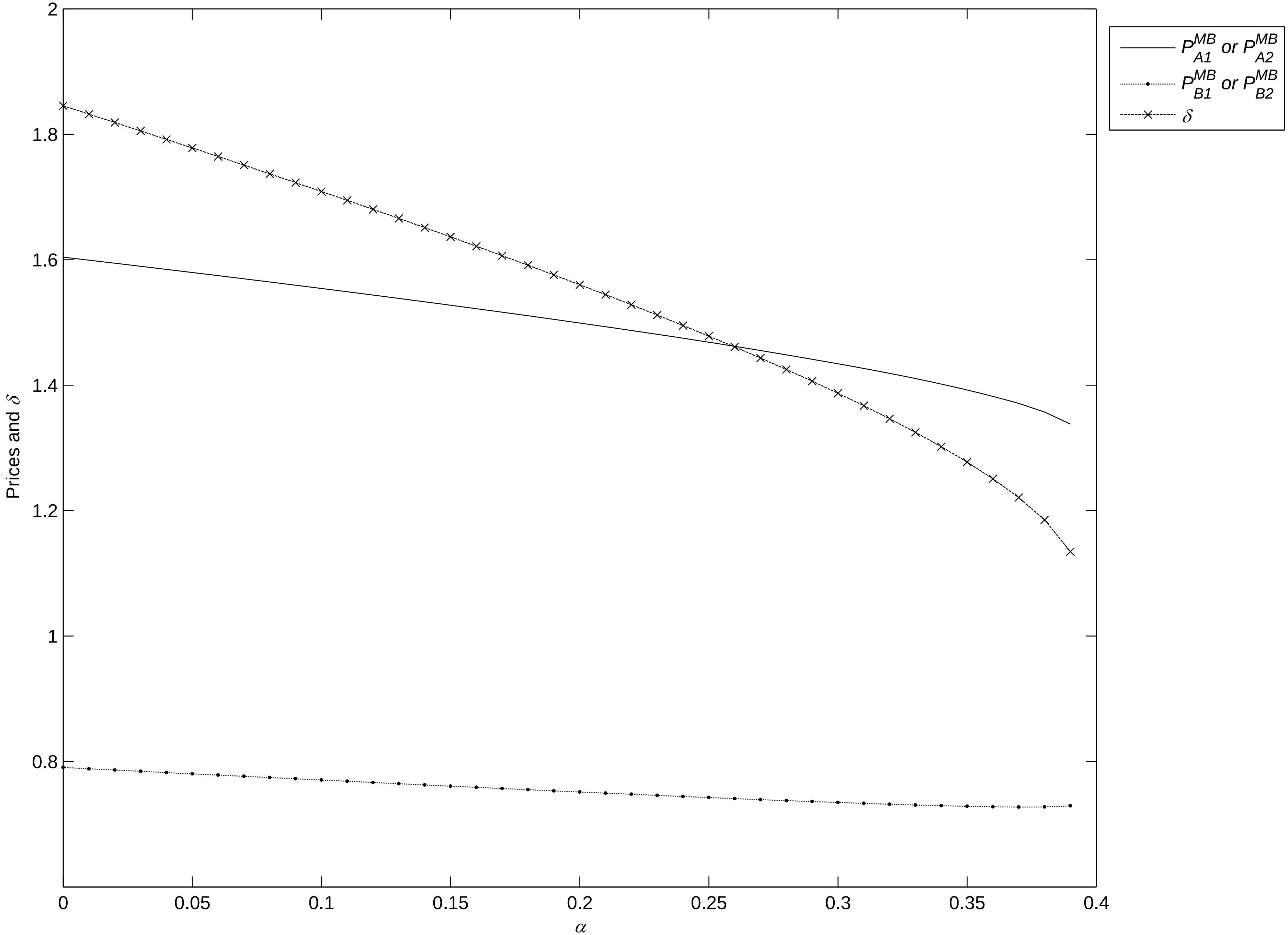


Figure 10: Mixed-Bundling Profits under Equilibrium (1)

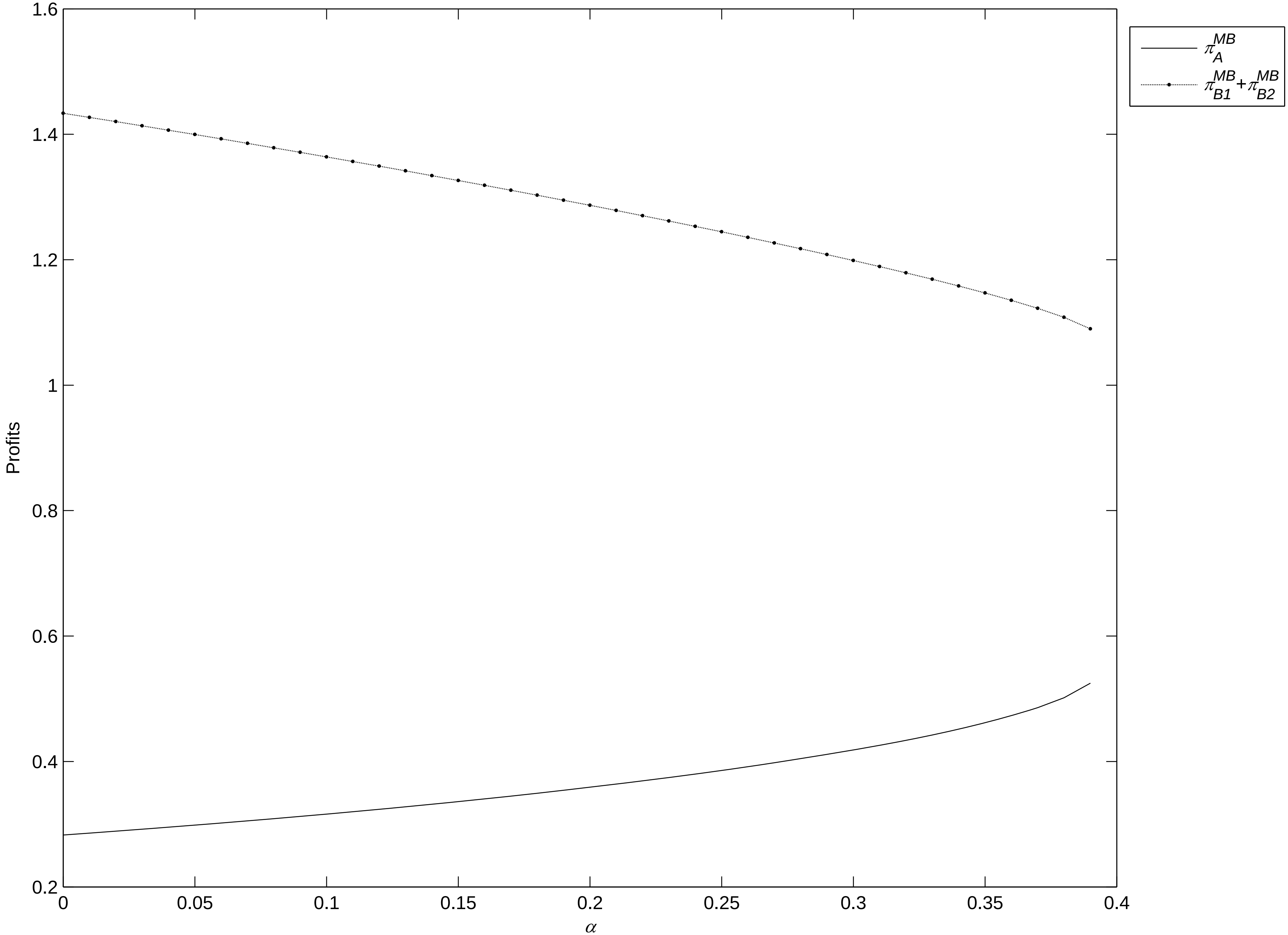
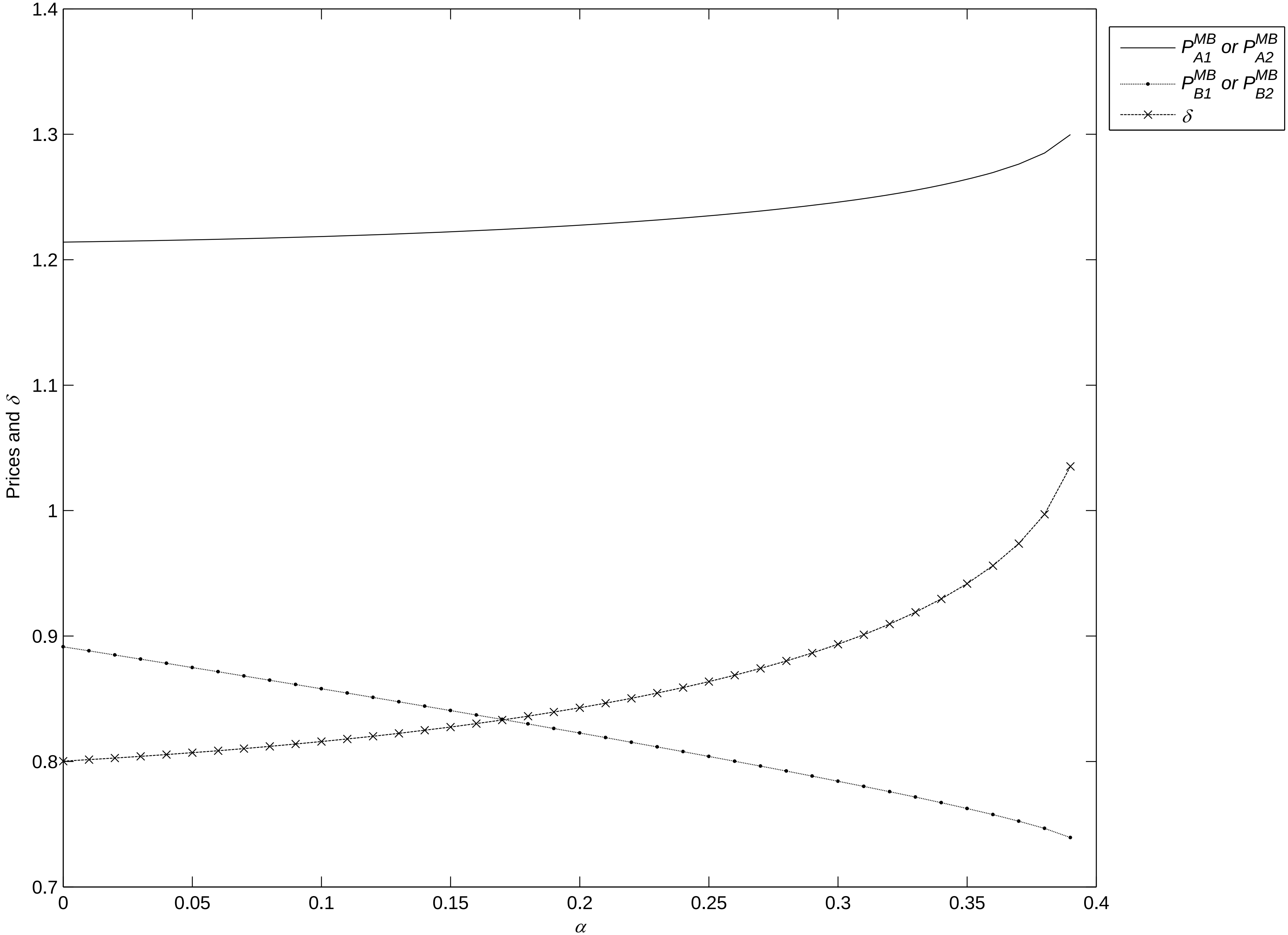


Figure 11: Equilibrium (2) Prices under Mixed Bundling



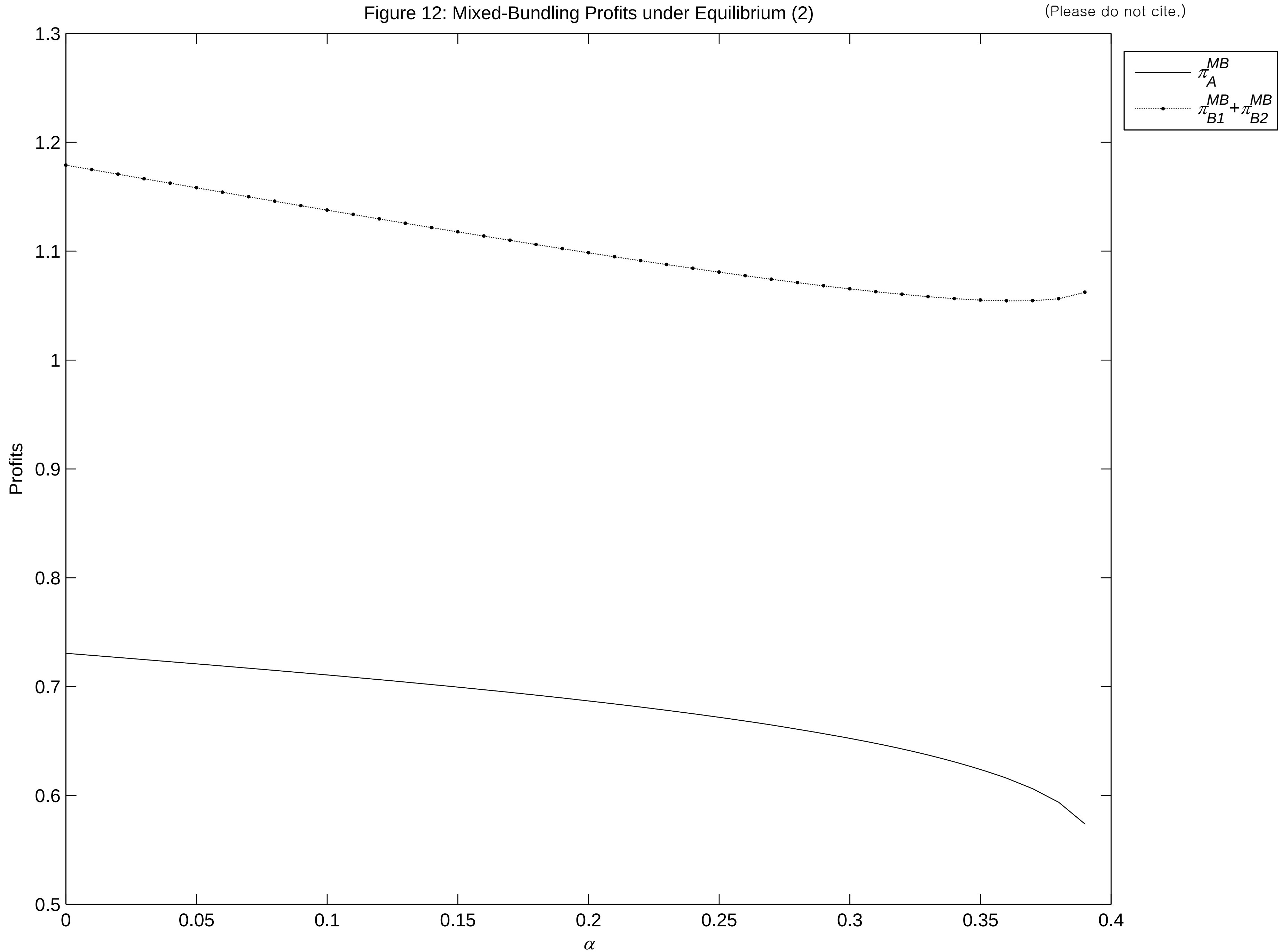


Figure 12: Mixed-Bundling Profits under Equilibrium (2)
Profits
α
π_A^{MB}
$\pi_{B1}^{MB}+\pi_{B2}^{MB}$
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

Figure 13: Comparison of Profits under Different Mode of Competition: Multi-product Firm A

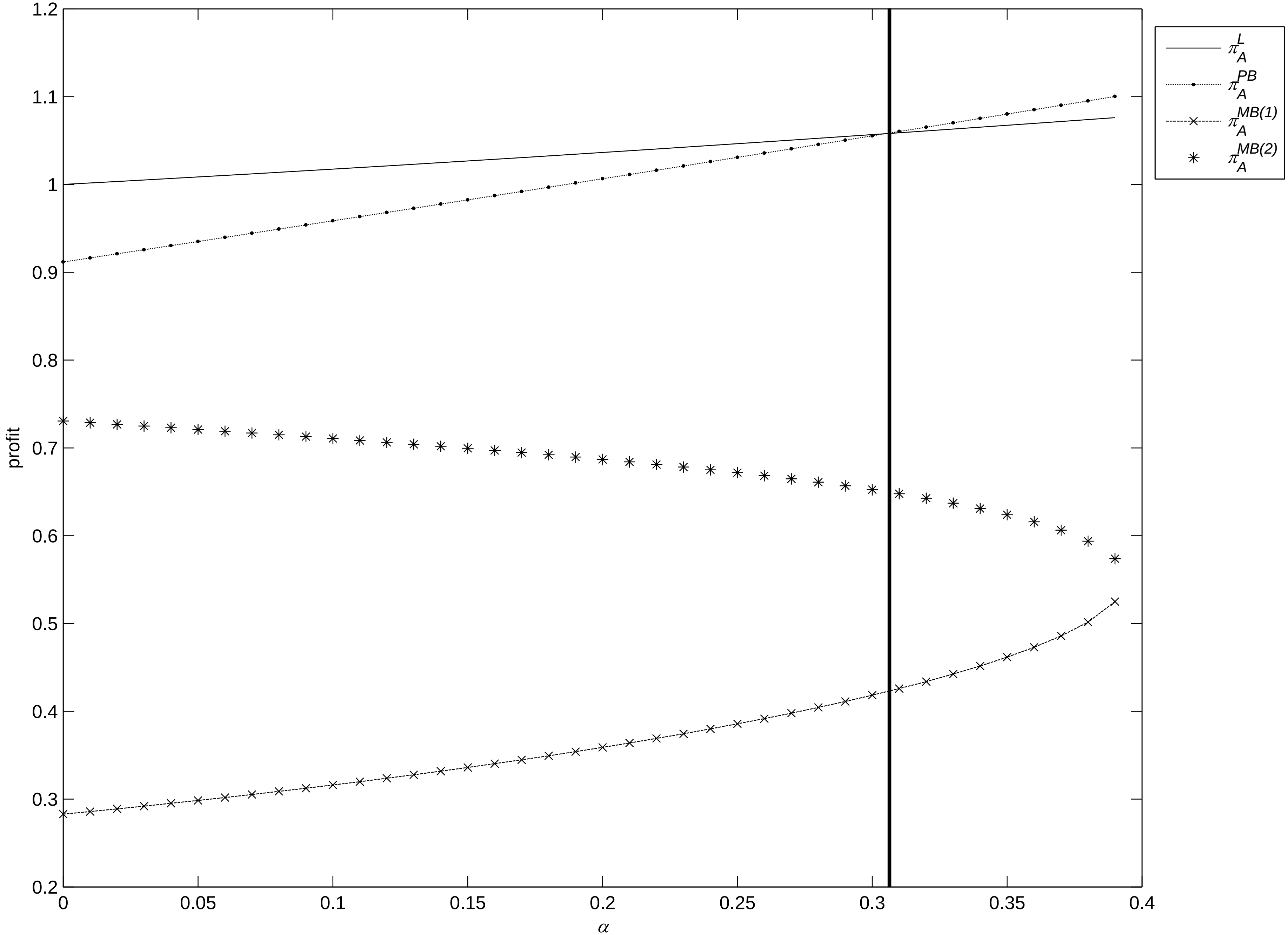


Figure 14: Comparison of Profits under Different Mode of Competition: Single-product Firm B