Losses from competition policies in a dynamic oligopoly

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Abstract

We prove an intriguing effect of increasing efficient firms in a differential game model of asymmetric oligopoly. It is demonstrated that an increase in *efficient* firms reduces welfare in the feedback Nash equilibrium, which is impossible in a static oligopoly. We discuss that the closed-loop property of feedback strategies plays a major role in our finding.

Keywords: dynamic game, asymmetric oligopoly, welfare-reducing competition policy.

JEL classification: C73, L13.

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1 Introduction

Cournot-Nash oligopoly theory is one of the basic models in economics and is applied in a variety of fields. Under symmetric oligopoly where all the firms share an identically constant marginal cost and no fixed cost, increasing firms benefits welfare while the profit of each individual firm decreases. However, it becomes stringent whether welfare improves as a result of increasing competition once we allow asymmetric costs among firms. In a seminal paper, Lahiri and Ono (1988, Proposition 2, p. 1201) finds that 'national welfare increases if a firm with a sufficiently low share is removed from the market.' This result has long had a great influence on the policymaking of competition.

On the other hand, relaxing the assumption of statics, Benchekroun (2008) proves that increasing firms reduces the long-run industry output and welfare by constructing a differential game model of productive asset oligopoly.[1] There are two forces behind his finding. The first effect is positive as static oligopoly theory tells. Besides, an increase in the number of firms induces a decrease in the steady state resource stock and industry output. This second effect dominates in his model and hence increasing competition becomes detrimental even in a symmetric oligopoly. Benchekroun's (2008) argument motivates us to examine whether a parallel logic also applies to asymmetric oligopoly.

Incorporating asymmetric costs into Benchekroun's (2008) model, this paper reconsiders welfare effects of increasing competition. It is proved that an increase in *efficient* firms harms steady state welfare in the feedback Nash equilibrium, which is impossible in a static setting. Two notes on this result are in order. First, this result can straightforwardly apply to increasing inefficient firms which Lahiri and Ono (1988) address. Second, the share of efficient firms does not matter for our result whereas the assumption of 'a sufficiently low share' of inefficient firms is needed in Lahiri and Ono's (1988) proposition. We will relate Benchekroun's (2008) argument to ours.

The paper is organized as follows. Section 2 presents a model. Section 3 states the main result and compares it with a static counterpart. Section 4 concludes and Appendix proves the result stated in Section 3.

[1] It should be commented that the short-run effect is also covered in Benchekroun (2008). Benchekroun (2003) and Benchekroun and Long (2006) also prove that some of the results in static models are no longer valid in a context of dynamic oligopoly.

2 A model

While we basically follow Benchekroun (2008) in modeling, we deviate from him in two respects. First, we decompose firms into m efficient firms with zero marginal cost and n inefficient firms with a positive marginal cost c. Second, these firms exploit a renewable resource whose dynamics is

$$\dot{S} = kS - \sum x_i - \sum x_j, \tag{1}$$

which is more specified than in Benchekroun (2008). In (1), S is a stock of productive asset, x_i a representative efficient firm's output and x_j a representative inefficient firm's output.[2]

Assuming that inverse demand is linear so that $p = a - \sum x_i - \sum x_j$, where p is the price, each firm's profit maximization problem is formulated as

$$\begin{aligned} \max_{x_i} \quad & \int_0^\infty e^{-rt} \left(a - \sum x_i - \sum x_j\right) x_i dt \\ \max_{x_j} \quad & \int_0^\infty e^{-rt} \left(a - c - \sum x_i - \sum x_j\right) x_j dt, \end{aligned}$$

under the constraint of (1) with $r > 0$ denoting a constant rate of discount.

We seek stationary feedback strategies of this game. To this end, let us employ a derivation technique by Tsutsui and Mino (1990) and Shimomura (1991). It begins by defining firm i's Hamilton-Jacobi-Bellman equation:

$$rV_i(S) = \max_{x_i} \left\{ \left[a - x_i - \sum_{l \neq i} x_l(S) - \sum x_j(S)\right] x_i + V_i'(S) \left[kS - x_i - \sum_{l \neq i} x_l(S) - \sum x_j(S)\right] \right\}, \tag{2}$$

where $V_i(\cdot)$ is a value function of firm i:

$$\begin{aligned} V_i(S) \quad \equiv \quad & \max_{x_i} \left\{ \int_t^\infty e^{-r(\tau - t)} \left[a - x_i - \sum_{l \neq i} x_l(S) - \sum x_j(S)\right] x_i d\tau \; \middle| \right. \\ & \left. \dot{S} = kS - x_i - \sum_{l \neq i} x_l(S) - \sum x_j(S) \right\}. \end{aligned}$$

[2]Benchekroun (2008, 2003) allow the dynamics of resource accumulation to take an inverted-V shaped, which implies that the resource decreases if its amount is sufficiently large. In contrast, some previous studies, e.g., Tornell and Velasco (1992), Benchekroun and Long (2002) and Sorger (2005) assume the same type of resource dynamics as ours.

Maximizing the right-hand side of (2) and using an assumption that all the efficient firms choose $x_i(S)$ and all the inefficient firms choose $x_j(S)$, we have the first-order condition:

$$V_i'(S) = a - (m+1)x_i(S) - nx_j(S).$$

Substituting this into (2) yields an identity in S:

$$rV_i(S) = [a - mx_i(S) - nx_j(S)]\, x_i(S) + [a - (m+1)x_i(S) - nx_j(S)]\, [kS - mx_i(S) - nx_j(S)]\,.$$

Differentiating both sides with respect to S and rearranging terms, we have an auxiliary equation:

$$\begin{aligned} & \left[2m^2 x_i(S) + 2mnx_j(S) - (m-1)a - (m+1)kS\right] x_i'(S) \\ & + [2mx_i(S) + 2nx_j(S) - a - kS]\, nx_j'(S) \\ = \;& (k-r)\, [(m+1)x_i(S) + nx_j(S) - a]\,. \end{aligned} \tag{3}$$

Applying the same procedure to efficient firm j's maximization problem, we have an auxiliary equation of firm j which parallels (3):

$$\begin{aligned} & [2mx_i(S) + 2nx_j(S) - (a-c) - kS]\, mx_i'(S) \\ & + \left[2mnx_i(S) + 2n^2 x_j(S) - (n-1)(a-c) - (n+1)kS\right] x_j'(S) \\ = \;& (k-r)\, [mx_i(S) + (n+1)x_j(S) - (a-c)]\,. \end{aligned} \tag{4}$$

Feedback Nash equilibrium strategies are determined by solving the system of differential equations (3) and (4) together with the boundary conditions: $\lim_{t\to\infty} e^{-rt}V_i(S) = \lim_{t\to\infty} e^{-rt}V_j(S) = 0$.

In what follows, we derive the linear feedback strategies: $x_i(S) = \alpha_i S + \beta_i$ and $x_j(S) = \alpha_j S + \beta_j$, where $\alpha_i, \alpha_j, \beta_j$ and β_j are undetermined coefficients. Under these specifications, (3) and (4) become

$$\begin{aligned} & \left[2m^2(\alpha_i S + \beta_i) + 2mn(\alpha_j S + \beta_j) - (m-1)a - (m+1)kS\right] \alpha_i \\ & + [2m(\alpha_i S + \beta_i) + 2n(\alpha_j S + \beta_j) - a - kS]\, n\alpha_j \\ = \;& (k-r)\, [(m+1)(\alpha_1 S + \beta_i) + n(\alpha_j S + \beta_j) - a] \end{aligned} \tag{5}$$

$$
\begin{aligned}
& [2m(\alpha_i S+\beta_i)+2n(\alpha_j S+\beta_j)-(a-c)-kS]\, m\alpha_i \\
& +\left[2mn(\alpha_i S+\beta_i)+2n^2(\alpha_j S+\beta_j)-(n-1)(a-c)-(n+1)kS\right]\alpha_j \\
= \; & (k-r)\,[m(\alpha_i S+\beta_i)+(n+1)(\alpha_j S+\beta_j)-(a-c)]\,, \qquad (6)
\end{aligned}
$$

which are alternatively written as

$$
\begin{aligned}
& \Delta_i S+\left[2m^2\alpha_i+2mn\alpha_j-(m+1)(k-r)\right]\beta_i \\
& +[2m\alpha_i+2n\alpha_j-k+r]\, n\beta_j+[k-r-(m-1)\alpha_i-n\alpha_j]\, a=0 \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
& \Delta_j S+[2m\alpha_i+2n\alpha_j-k+r]\, m\beta_i \\
& +\left[2mn\alpha_i+2n^2\alpha_j-(n+1)(k-r)\right]\beta_j+[k-r-m\alpha_i-(n-1)\alpha_j]\,(a-c)=0 \\
& \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_i &\equiv \left[2m^2\alpha_i+2mn\alpha_j-(m+1)(2k-r)\right]\alpha_i+(2m\alpha_i+2n\alpha_j-2k+r)\, n\alpha_j \\
\Delta_j &\equiv (2m\alpha_i+2n\alpha_j-2k+r)\, m\alpha_i+\left[2mn\alpha_i+2n^2\alpha_j-(n+1)(2k-r)\right]\alpha_j.
\end{aligned}
$$

The four coefficients are determined as follows. First, α_i and α_j are determined so that the terms multiplied by S is zero, i.e., $\Delta_i=\Delta_j=0$. While it is possible that this system of equations has contains multiple pairs of (α_i,α_j), two of them are given by

$$
\alpha_i=\alpha_j=\alpha=0, \frac{(2k-r)(m+n+1)}{2(m+n)^2}. \qquad (9)
$$

On the other hand, β_i and β_j are determined through the system of equations which is obtained by setting the other terms in (7) and (8) to zero. Then, we have

$$
\begin{aligned}
\beta_i &= \frac{[k-r-(m+n-1)\alpha]\,\{(k-r)a+[k-r-2(m+n)\alpha]nc\}}{(k-r)\,[(m+n+1)(k-r)-2(m+n)^2\alpha]} \qquad (10) \\
\beta_j &= \frac{[k-r-(m+n-1)\alpha]\,\{(k-r)a-[(m+1)(k-r)-2m(m+n)\alpha]\, c\}}{(k-r)\,[(m+n+1)(k-r)-2(m+n)^2\alpha]}. \qquad (11)
\end{aligned}
$$

By substituting (9) into (10) and (11), the closed form of β_i and β_j is computed.

Henceforth, we restrict attention to the limiting case with $r\to 0$ since it is extremely difficult to obtain an analytical result for an arbitrary r. In this ‘limit game’, the coefficients obtained above simplify to

$$\alpha = \frac{k(m+n+1)}{(m+n)^2} \tag{12}$$

$$\beta_i = \frac{-(m+n)a + n(m+n+2)c}{(m+n)^3(m+n+1)} \tag{13}$$

$$\beta_j = \frac{-(m+n)a - [2m + (m+n)(m-1)]c}{(m+n)^3(m+n+1)}. \tag{14}$$

In the rest of the paper, we assume that a is sufficiently larger than c so as to ensure $\beta_i, \beta_j < 0$. Substituting (12)-(14) into $x_i(S) = \alpha S + \beta_i$ and $x_j(S) = \alpha S + \beta_j$, each firm's output under the feedback strategy is characterized as follows.

$$x_i(S) = \begin{cases} 0 & \text{if} \quad S \leq \frac{-\beta_i}{\alpha} \\ \alpha S + \beta_i & \text{if} \quad \frac{-\beta_i}{\alpha} < S \leq \frac{a+nc-(m+n+1)\beta_i}{(m+n+1)\alpha} \\ \frac{a+nc}{m+n+1} & \text{if} \quad S > \frac{a+nc-(m+n+1)\beta_i}{\alpha} \end{cases} \tag{15}$$

$$x_j(S) = \begin{cases} 0 & \text{if} \quad S \leq \frac{-\beta_j}{\alpha} \\ \alpha S + \beta_j & \text{if} \quad \frac{-\beta_j}{\alpha} < S \leq \frac{a-(m+1)c-(m+n+1)\beta_j}{(m+n+1)\alpha} \\ \frac{a-(m+1)c}{m+n+1} & \text{if} \quad S > \frac{a-(m+1)c-(m+n+1)\beta_i}{\alpha} \end{cases}. \tag{16}$$

These strategies are depicted in Figure 1, which allows us to know that feedback strategies are zero (resp. the static output) if S is sufficiently small (resp. large) and linearly increasing in S if it is in certain closed interval.[3]

3 Main results

This section states and discusses the main result: an increase in the number of efficient firms m worsens welfare in the steady state. For this purpose, let us define welfare U:

$$\begin{aligned} U &= \frac{(mx_i + nx_j)^2}{2} + m(a - mx_i - nx_j)x_i + n(a - c - mx_i - nx_j)x_j && (17) \\ &= \frac{(kS)^2}{2} + (a - kS)mx_i + (a - c - kS)nx_j && (18) \\ &= \frac{(kS)^2}{2} + (a - kS)(mx_i + nx_j) - ncx_j \\ &= \frac{(kS)^2}{2} + (a - kS)kS - ncx_j \\ &= \frac{kS(2a - kS)}{2} - ncx_j, && (19) \end{aligned}$$

[3] Lohoues (2006) provides a much more detailed characterization of feedback strategies.

where the first term in (17) is consumer surplus and the other terms are the aggregate profits. Eq. (18) uses the steady state condition: $\dot{S} = kS - mx_i - nx_j = 0$.

Then, we can state:

Proposition 1. *An increase in m reduces the steady state welfare if a is sufficiently larger than c.*

Proof. See Appendix A.

Proposition 1 sharply contrasts to conventional wisdom that increasing efficient firms necessarily benefits welfare. Let us discuss what causes this result. An increase in m has two effects. First, it expands the industry output, which is expected to increase consumer surplus and welfare. Second, increasing firms accelerates overexploitation of the resource and S is likely to decrease. Responding to this decrease in S, all firms contract output since feedback strategies are monotonically increasing in S. If this closed-loop effect dominates the first static effect, the total supply in the steady state will be lower. In other words, an increase in m has an anti-competitive effect. This can be also verified by looking at (23) in Appendix A. As a result, consumer surplus inevitably declines and so does welfare.

Relating Proposition 1 to Lahiri and Ono's (1988) finding, we see:

Corollary 1. *An increase in n reduces the steady state welfare if a is sufficiently larger than c.*

We should draw attention concerning Corollary 1. In Lahiri and Ono's (1988) argument, increasing inefficient firms has two competing effects. One is a procompetitive effect and the other is a rent-shifting effect from efficient firms to inefficient firms. Their conclusion that 'helping minor firms reduces welfare' rests on an additional assumption that the efficient firms' share is initially large enough. When this is not satisfied, it is still possible that increasing inefficient firms improves welfare. In contrast, not only Proposition 1 but also Corollary 1 needs no such additional assumption. What is significant is that any competition policy becomes anticompetitive due to the closed-loop effects.

At this stage, it is helpful to see what would happen if static Cournot outputs were to be chosen.[4] This confirms the conventional wisdom:

Proposition 2. *If static Cournot outputs were to be chosen, an increase in m improves welfare.*

Proof. See Appendix B.

Even if static outputs were to be an equilibrium strategy, a result parallel to Proposition 1 would not be the case. This is because static outputs have no closed-loop property. That is, no firm optimally responds to a change in S. In this case, a static procompetitive effect dominates and unambiguously results in a welfare improvement as in the traditional theory.

4 A concluding remark

Using an asymmetric version of Benchekroun's (2008) model of productive asset oligopoly, we have proved that 'helping major firms reduces welfare.' This yields a natural corollary that 'helping any firm reduces welfare.' We have discussed that the Benchekroun's (2008) argument of welfare-reducing competition policy can be extended to our model of asymmetric oligopoly.

Of course, our result rests on numerous simplifying assumptions some of which are stricter than Benchekroun (2008). For instance, dynamics of the resource is monotonically increasing and the analysis is restricted to steady states. Moreover, we have confined attention to the most relevant case where both efficient and inefficient firms employ a linear feedback strategies $x_i = \alpha S + \beta_i$ and $x_j = \alpha S + \beta_j$. We do not know what happens in the other cases except for the case in which all firms choose a static Cournot-Nash output. It is our future research agenda to cover such cases as well as to relax the

[4] As shown in Benchekroun (2003, 2008) in a context of symmetric oligopoly, the steady state associated with static Cournot outputs is asymptotically unstable. The same is true of our model.

simplifying assumptions mentioned above.

Appendix A: Proof of Proposition 1

Differentiating (19) with respect to m, we have

$$\frac{dU}{dm} = k(a - kS)\frac{dS}{dm} - nc\frac{dx_j}{dm}. \tag{20}$$

The steady state in which $\dot{S} = kS - m(\alpha S + \beta_i) - n(\alpha S + \beta_j) = 0$ involves

$$S = \frac{m\beta_i + n\beta_j}{k - (m+n)\alpha}. \tag{21}$$

Substituting this into $x_j(S) = \alpha S + \beta_j$, an inefficient firm's steady state output is

$$x_j = \alpha S + \beta = \alpha \cdot \frac{m\beta_i + n\beta_j}{k - (m+n)\alpha} + \beta_i = \frac{m\alpha(\beta_i - \beta_j) + k\beta_j}{k - (m+n)\alpha}. \tag{22}$$

Substituting (12)-(14) into (21) and (22), the closed-form of S and x_j in the steady state is

$$\begin{aligned} S &= \frac{(m+n)a - nc}{k(m+n)(m+n+1)} \\ x_j &= \frac{(m+n)^2 a - \left[m + (m+n)^2\right] c}{(m+n)^3(m+n+1)}. \end{aligned} \tag{23}$$

Thus, differentiating these with respect to m yields

$$\begin{aligned} \frac{dS}{dm} &= \frac{-(m+n)^2 a + n(2m+2n+1)c}{k(m+n)^2(m+n+1)^2} < 0 \\ \frac{dx_j}{dm} &= \frac{-(m+n)^2(2m+2n+1)a + \left[2(m+n)^3 + (4m-1)(m+n) + 3m\right] c}{(m+n)^4(m+n+1)^2} < 0, \end{aligned}$$

when a is much larger than c. Substituting these into (20), rearranging terms and letting $N \equiv m+n$, dU/dm becomes

$$\begin{aligned} \frac{dU}{dm} &= \frac{\Gamma}{kN^4(N+1)^3} \\ \Gamma &= -N^5 a^2 + N^2 n \left[2N^2 + k(N+1)(2N+1)\right] ac \\ &\quad + n\left\{N(2N+1)n - k(N+1)\left[2N^3 + (4m-1)N + 3m\right]\right\} c^2 < 0, \end{aligned}$$

under the condition that a is sufficiently larger than c. Consequently, we have concluded that $dU/dm < 0$.

Appendix B: Proof of Proposition 2

It is convenient to slightly rewrite (19) as follows.

$$U = \frac{X(2a - X)}{2} - ncx_j,$$

where $X \equiv mx_i + nx_j$. Therefore, a change in m induces

$$\frac{dU}{dm} = (a - X)\frac{dX}{dm} - nc\frac{dx_j}{dm}. \tag{24}$$

Static Cournot outcomes give

$$\begin{aligned} X &= \frac{(m+n)a - nc}{m+n+1}, \quad & x_j &= \frac{a - (m+1)c}{m+n+1} \\ \frac{dX}{dm} &= \frac{a + nc}{(m+n+1)^2}, \quad & \frac{dx_j}{dm} &= \frac{-a - nc}{(m+n+1)^2}. \end{aligned}$$

Substituting these into (24), a welfare change associated with a change in m is

$$\frac{dU}{dm} = \frac{[a + n(m+n+2)c](a + nc)}{(m+n+1)^2} > 0,$$

that is, increasing efficient firms favorably affects welfare.

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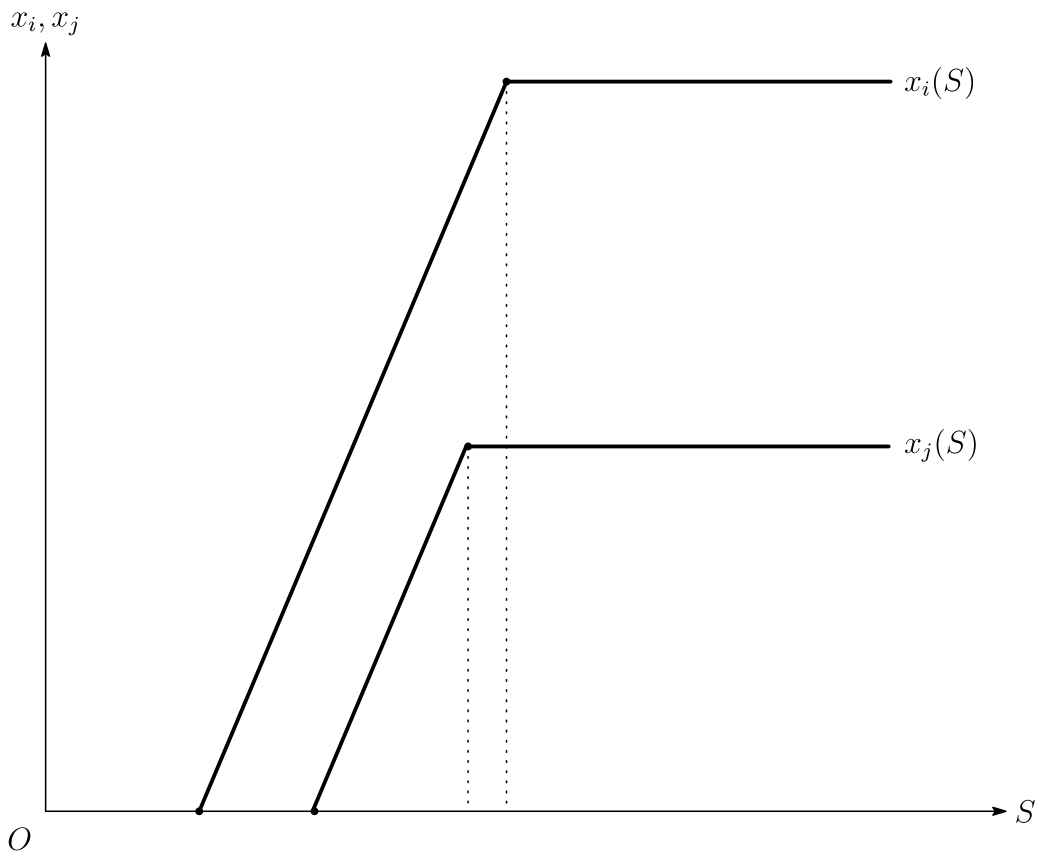

Figure 1: Feedback strategies