Mechanism Design for Capacity Allocation under Price Competition

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Abstract

This paper considers mechanism design for capacity allocation in a selective distribution model under a price competition. We investigate how allocation mechanisms in an upstream market affect market behaviors in a downstream market. Unlike Kreps and Scheinkman [18], we show that an equilibrium order quantity is not always Cournot quantity, which is dependent on properties of allocation mechanisms in the upstream market.

1 Introduction

Since the emergence of electronic commerce, mechanism design has been spread out from the economical field to other fields such as computer science, artificial intelligence or information systems. Typical examples are mechanism designs for electronic market [6, 14, 12, 27], combinatorial auctions [19, 25] and task allocation [15, 17, 23, 24, 28].

Research on mechanism design mainly focuses on a single market. However, it has been shown (see for example [13]) that small changes of the market structure may lead to significant change of the market behavior. Furthermore, considering only a particular single market often fails to achieve expected outcomes. This is because outputs of the market behaviors by self-interested participants in a certain market are treated as inputs of the successive markets and they may cause a poor performance of the other supply chain partners. One of the typical example of such a phenomena is the so-called *bullwhip effect*, which shows that a smaller forecast error at the downstream in a supply chain is amplified toward the upper stream of supply chain. This could lead to inefficient inventory level at upper stream of suppliers, inaccurate size of capacity construction or inefficient allocation. This kind of problems can be prevented by acquiring accurate market information through the supply chain. However, incentive conflicts often obstruct information sharing in supply chain.

In this paper, we present allocation mechanism design in two connected markets. Unlike a single market model, we investigate how the properties of the connected markets affect the properties of allocation mechanisms and how allocation mechanisms affect the behavior of the connected markets. Typical allocation mechanisms, such as proportional allocation, uniform allocation and channel member selection, are applied in industries [5, 9, 16, 22]. Even if these allocation mechanisms have different properties, one of the key goals of mechanism design is to acquire truthful information from customers.

The first work on allocation mechanism design in the context of supply chain network have been made by Cachon and Lariviere [5]. In [5], the authors focus on a single supplier which supplies multiple retailers enjoying local monopoly (exclusive distribution). They formally represent the design of allocation mechanisms, which induces retailers (the supplier's customers) truthful order information. In [11], we have relaxed this Cachon and Lariviere's assumption, and we have investigated when competition exists in retail market. We assume that all retailers are in the Cournot competition in an oligopolistic market. Having direct competition in the retail market is more general and complicated. It has been shown that the allocation mechanisms used in the upstream market (wholesale market) are highly related to the characteristics of the downstream market [11]. For instance, the typical capacity allocation mechanism, *proportional allocation*, is no longer Pareto optimal in [11], whilst it is so in [5]. Other mechanism properties, such as *individual rationality* and *incentive compatibility*, are investigated in more closed to the real market situations.

In this paper, we further investigate the capacity allocation problem with the situation where price competition is encountered in the oligopolistic downstream market. Our model can be treated as an extension model of

Kreps and Scheinkman model (hereafter KS model) in [18], where sellers are in a price competition in a duopoly with pre-commitment of supply limit (capacity) for each seller. The main difference is our model has an allocation process. Once allocation is executed in our model, by treating allocated quantity as supply limit, the market behavior can be explained by using the KS model. While each seller is able to determine its supply limit in the KS model, each seller (in our context retailer) is not able to set desirable supply-limit (allocation quantity) independently in our model. Notice that our model deals with the case where the allocation does not satisfy the desirable supply limit.

Since Kreps and Scheinkman [18], several works on price competition with capacity constraints have been an active research. A first stream of research extends the results of the KS model. Vives [30] shows price equilibria in a symmetric oligopoly case with common capacity constraints among sellers. De Francesco [7] extends the KS model from a duopoly to an oligopoly. Madden [21] shows the conditions to obtain the same results with the KS model in elastic demand. A second stream of research shows the limits of the KS model by assuming asymmetric cases, including asymmetric cost in duopoly [8], imperfect capacity pre-commitment [3, 4], uncertain demand [26] and dynamic capacity accumulation [2]. The difference of the two streams is caused by the symmetric behavior in the models. Our model is relevant to both streams of these literatures, since feasible allocation can be symmetric and asymmetric.

This paper is organized as follows. Section 2 presents the model consisting of the capacity allocation in the upstream market and the price competition in the downstream market. In Section 3, we classify price equilibria in the downstream market according to the capacity size of the upstream market. In Section 4 we focus on how allocation mechanisms affect the total retailers profits. Then we show how retailers place orders according to allocation mechanisms and how retailers determine retail prices in Section 5. Section 6 briefly concludes this paper.

2 Model

We consider a supply chain model with two connected markets: a monopolistic *upstream market* (wholesale market) and an oligopolistic *downstream market* (retail market) as shown on Figure 1. The upstream market uses an allocation mechanism with respect to a predefined capacity. The downstream market applies a competitive market mechanism. In the upstream market, a supplier sells products to intermediaries, called retailers. Note that we assume that the customers of the intermediaries are end-users. When the sum of order quantities from the retailers exceeds the capacity size of the supplier, the supplier allocates products according to a selected allocation mechanism. In the downstream market, the retailers resale products to a range of end-users under some competitive mechanisms. We investigate how the competitive model in the downstream market affects the properties of the allocation mechanism in the upstream market and how the allocation mechanism affects behaviors of the successive downstream market. In [11], we have considered the situation where the retailers are under quantity competitions. In this paper, we consider the more complicated situation where retailers encounter price competition.

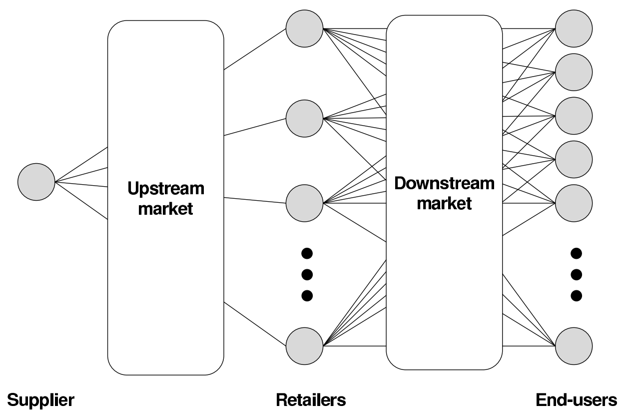

Figure 1: Supply Chain Model

With the supply chain model, the whole capacity allocation game consists of two stages: order placement and allocation in the upstream market and price setting and resale to end-users in the downstream market.

2.1 Upstream Market

In the first stage, the supplier sets capacity K and selects allocation mechanism g by which the supplier allocates product when the capacity is bound.

The supplier notifies the selected allocation mechanism to all retailers. Let $N = \{1, \ldots, n\}$ be the set of retailers usually noted i, j or k. Retailer i determines order quantity m_i with respect to market demand, allocation mechanism g, and other retailers' orders in order to maximize its profit π_i. Let us denote revenue of retailer i as Π_i. All retailers submit their orders, $m = (m_1, \ldots, m_n)$, simultaneously and independently with respect to fixed cost w. If the total order quantity exceeds K, the supplier allocates products according to the adopted allocation mechanism. Let $\mathcal{A} = \{a \in \Re^n : a \geq 0 \ \ \& \ \ \sum_{i=1}^{n} a_i \leq K\}$, where a vector $a \geq 0$ means for any component a_i of the vector, $a_i \geq 0$. We call each $a \in \mathcal{A}$ a *feasible allocation*.

Definition 1. *An allocation mechanism is a function $g : \Re^n \rightarrow \mathcal{A}$ which assigns a feasible allocation to each vector of orders such that for any retailers' order vector m, $g_i(m) \leq m_i$ for each $i = 1, \cdots, n$.*

Let $g_i(m^*)$ be the allocation quantity of retailer i under allocation mechanism g with respect to the vector of the equilibrium order quantity m^*. An allocation g is said to be *efficient* if $\sum_{i=1}^{n} g_i(m) = K$ whenever $\sum_{i=1}^{n} m_i \geq K$. For the supplier, the efficient allocation is the preferable one since the capacity has been fully used.

2.2 Downstream Market

In the second stage of the capacity allocation game, the retailers are in the price competition in the downstream market. Let $D(p)$ be the demand of the end-users at price p and $P(q)$ be its inverse function where q stands for the total supply quantity. We assume that the function $P(q)$ is strictly positive on some bounded interval $(0, \hat{q})$, on which it is twice-continuously differentiable, strictly decreasing and concave. For $q \geq \hat{q}$, we simply assume $P(q) = 0$. Let a_i be the allocated quantity for retailer i which has been determined in the first stage. Like Once retailers are allocated by the supplier, the retailers determine prices p_i simultaneously and independently. Like [20] and [18], we assume *surplus maximizing rule* where the end-users choose from the lowest price-offering retailers.

2.2.1 Quantity Competition

Before describing market behaviors in the downstream market, we show some assumptions and facts in Cournot quantity competition that are used in our price competition. Let us assume that retailers are in Cournot competition. Let q_{-i} stands for $\sum_{j \neq i \in N} q_j$. We define the best response function for retailer i at cost w is defined as:

$$r_w(q_{-i}) = \arg\max_{q_i}\{q_i P(q_i + q_{-i}) - w q_i\}.$$

If all retailers take the best responses to other retailers, we have the Cournot equilibrium q^{cw} and the total Cournot quantity $q^{Cw} = \sum_{i=1}^{N} q^{cw}$. Note that $r_w(q_{-i}^{cw}) = q^{cw}$. At the special case where cost is zero, we denote q^c as the Cournot equilibrium at zero cost and q^C as the total Cournot quantity.

For the existence of the Cournot equilibrium, Frank Jr. and Quandt [10] assume that $q_i P(q_i + q_{-i}) - w q_i$ is concave in q_i. In the following, we also assume this concavity. Based on this assumption, $r_w(q_{-i})$ is a unique solution of

$$P(q_i + q_{-i}) + q_i \frac{\mathrm{d}P(q_i + q_{-i})}{\mathrm{d}q_i} - w = 0, \tag{1}$$

and satisfies

$$-1 < \frac{\partial r_w(q_{-i})}{\partial q_{-i}} < 0, \tag{2}$$

Hence, $r_w(q_{-i}) + q_{-i}$ is increasing in q_{-i}. In order to ensure that the profit of each retailer is positive, we assume that $w < P(q^{Cw})$.

2.2.2 Price Competition

Now we consider how retailers set the price. We start from the case when the supplier allocates the product exclusively. Let i be the exclusively allocated retailer. In this case, the exclusively allocated retailer enjoys the benefits of the monopoly price p^M. Since $qP(q)$ is concave in q, where $q \in (0, \hat{q})$, we have,

$$q^M = \operatorname*{argmax}_q qP(q). \tag{3}$$

Therefore, the selling quantity is $x_i = \min\{q^M, a_i\}$. Hence retailer i sets the monopoly price p^M to maximize its profit π_i,

$$p^M = \begin{cases} P(a_i) & \text{if } a_i \leq q^M \\ P(q^M), & \text{otherwise} \end{cases} \tag{4}$$

$$\pi^M = \begin{cases} a_i P(a_i) - wa_i & \text{if } a_i \leq q^M \\ q^M P(q^M) - wa_i, & \text{otherwise} \end{cases} \tag{5}$$

Now let us consider the case where the capacity is allocated to several different retailers. Suppose a feasible allocation has been executed. The remaining part of the problem is how the retailers determine retail prices according to the allocation which is a pricing subgame. Since the allocation has already been executed, we treat the purchase cost of the retailers as a sunk cost. This situation is similar to the model of De Francesco [7]: several manufacturers are in a price competition with capacity pre-commitments. The manufacturers in [7] correspond to the retailers in our model and the pre-committed capacity corresponds to the allocation quantity. When the allocation of the retailers exceeds the best response quantity, the retailer considers two options which are: (i) selling all allocated products at a lower price or (ii) selling a limited quantity at a higher price. Before we describe the relation between allocation quantity and the best response, we show the relation between allocation and best response (the lemma is based on [3, 4]).

Lemma 1. *Suppose an efficient allocation mechanism and let a be the allocated quantities. Suppose $a_i > a_j$. If $a_i \leq r(a_{-i})$ then $a_j < r(a_{-j})$.*

Proof. Since the allocation is efficient, if $a_i > a_j$ then $a_{-i} < a_{-j}$. According to Equation (2), we have $r(a_{-i}) < r(a_{-j})$. Hence we have $a_j < a_i \leq r(a_{-i}) < r(a_{-j})$. □

According to Lemma 1, if the allocation is less than the best response for the largest allocated retailer l, then all other allocated retailers have the same characteristic. By considering the allocation quantity of the largest allocated retailer, De Francesco [7] classifies the price equilibria, the pure strategy equilibrium and the mixed strategy equilibrium, according to the patterns of the pre-committed capacity sizes. Now let us denote, $\bar{p}$ and $\underline{\mathrm{p}}$ as an upper and a lower bound of the price equilibrium of the mixed strategy. The following lemma characterizes the price equilibrium in the pricing subgame, which is a straightforward conversion from the capacity sizes of the manufacturers in the model of De Francesco [7] in terms of the allocated quantities of retailers,

Lemma 2. *[7] Given an allocation mechanism g, and order m, let $a_i = g_i(m)$. Let the largest allocated retailer $l = \underset{i \in N}{\operatorname{argmax}}\, a_i$. Then*

1. *if for all i, $a_i \leq r(a_{-i})$, $P(a_i + a_{-i})$ is a unique equilibrium in the pricing subgame.*

2. *if $a_l > r(a_{-l})$ and $D(0) > a_{-l}$, then*
   $\bar{p}_l = P(r(a_{-l}) + a_{-l})$,
   $\underline{p}_i = \frac{P(r(a_{-l})+a_{-l})r(a_{-l})}{a_l}$ *for all i, and*
   $\Pi_l = r(a_{-l})P(r(a_{-l}) + a_{-l})$,

3. *if $D(0) \leq a_{-l}$, $p^* = 0$ is the unique price equilibrium.*

Proof. Let i and a_i be a certain manufacturer and its respective capacity choice (see [7]). The pricing subgame can be seen as the same problem as [7]. See proofs of Propositions 1 and 2 in [7]. □

The result of De Francesco is applicable in our model to describe the relationship between allocation quantities of the retailers and the price equilibrium. We extend the link between the capacity size of the supplier and the price equilibrium with the help of Lemma 2 in the next section.

According to the result of the pricing subgame, Kreps and Scheinkman show that the equilibrium for the capacity size becomes Cournot equilibrium $q_i^* = q^{cw}$. In the KS model, manufacturers do not have any constraints to determine their capacity sizes. In our model, each retailer is not able to determine allocation quantity, which is determined by the order quantity and the allocation mechanism. Therefore, the equilibrium analysis on purchase quantity becomes more complex in our model.

3 The Pricing Subgame

Kreps and Scheinkman [18] and De Francesco [7] show the link between supply-limits and the pricing equilibria in a single market. In this section, we construct the link between the capacity of the supplier in the upstream market and the pricing equilibrium in the downstream market, i.e. allocations act as bridges.

Suppose allocations are efficient. Once the supplier sets its capacity K, the patterns for the feasible allocations are limited. By using Lemma 2, we classify possible price equilibria in the downstream market with respect to the capacity size in the upstream market.

As a primary supply chain member, the supplier focuses on two aspects: market price in the downstream market and the total profits of all supply chain members. Even if the supplier has no direct control over the retail price in the downstream market, we show the possible circumstance that the supplier is able to make influence on the market prices via allocations.

First we consider the situation where the capacity of supplier is very limited (i.e. its capacity size is less than the monopoly quantity).

Theorem 1. *Suppose that $K \leq q^M$ and g is efficient. $p^* = P(K)$ is an equilibrium of the pricing subgame under any feasible allocation g.*

Proof. According to Lemma 2, it is sufficient to show that for any i, $g_i(m) \leq r(g_{-i}(m))$. Since g is efficient and $K \leq q^M$, we have $g_i(m) + g_{-i}(m) = K \leq q^M = \operatorname*{argmax}_q qP(q)$, with respect to Equation (3).

Case 1: If $r(g_{-i}(m)) + g_{-i}(m) \geq q^M$, we have

$$g_i(m) + g_{-i}(m) \leq r(g_{-i}(m)) + g_{-i}(m).$$

It follows that $g_i(m) \leq r(g_{-i}(m))$ as desired.

Case 2: Assume that $r(g_{-i}(m)) + g_{-i}(m) < q^M$. According to the definition of Cournot best response function, we have

$$\begin{aligned} r(g_{-i}(m)) &= \operatorname*{argmax}_{q_i} q_i P(q_i + g_{-i}(m)) \\ &= \operatorname*{argmax}_{q_i} ((q_i + g_{-i}(m))P(q_i + g_{-i}(m)) - g_{-i}(m)P(q_i + g_{-i}(m))). \end{aligned}$$

Let $y = q_i + g_{-i}(m)$ and $y^* = \operatorname*{argmax}_y (yP(y) - g_{-i}(m)P(y))$; we have:

$$\begin{aligned} r(g_{-i}(m)) &= \operatorname*{argmax}_y (yP(y) - g_{-i}(m)P(y)) - g_{-i}(m) \\ &= y^* - g_{-i}(m) \end{aligned} \tag{6}$$

According to the assumption of Case 2, Equation (6) implies $y^* < q^M$. It follows that $P(y^*) > P(q^M)$ because P is strictly decreasing in quantity. It turns out that

$$-g_{-i}(m)P(y^*) < -g_{-i}(m)P(q^M)$$

Notice that the case $g_{-i}(m) \leq 0$ is ruled out since $y^* < q^M$. On the other hand, $y^*P(y^*) \leq q^M P(q^M)$ because $q^M = \underset{q}{\operatorname{argmax}}\, qP(q)$. Therefore, we have,

$$y^*P(y^*) - g_{-i}(m)P(y^*) < q^M P(q^M) - g_{-i}(m)P(q^M)$$

This contradicts the definition of y^*. That is $r(g_{-i}(m)) + g_{-i}(m) \geq q^M$ and we have $g_i(m) \leq r(g_{-i}(m))$. □

The above result shows that when the capacity size of the supplier is less than the market monopoly quantity, all retailers enjoy a higher price than the monopoly price, i.e., $P(K) \geq P(q^M)$, under any allocations. No retailers have incentive to charge less than $P(K)$ and no retailers can make greater profit by charging greater than $P(K)$, while the other retailers set price as $P(K)$. Therefore, the allocation mechanism does not affect the market price and the total retailers' profits as long as the capacity is significantly scarce. In other words, in this context, the supplier has no interests to choose a specific allocation mechanism as long as the mechanism is efficient.

Now let us consider the case where the capacity is relatively scarce that is $q^M < K \leq q^C$. In KS model, Cournot quantity q^c is the equilibrium capacity investment at zero cost. In our context of oligopolistic downstream market and scarce capacity, we have the following result:

Theorem 2. *Suppose $q^M < K \leq q^C$. Under any efficient allocation mechanism g, price equilibria are,*

1. *$p^* = P(K)$, if for all i $g_i(m) \leq r(g_{-i}(m))$;*
2. *$p^* > P(K)$, if there exists i such that $g_i(m) > r(g_{-i}(m))$.*

Proof. Since g is efficient, we have $\frac{K}{n} \leq g_l(m) \leq K$. It implies $\frac{n-1}{n}K \geq g_{-l}(m) \geq 0$. Since $K < q^C$, we have $\frac{n-1}{n}K \leq q^c_{-i}$. According to Equation (2), we obtain $r(q^c_{-i}) \leq r(\frac{n-1}{n}K) \leq r(g_{-l}(m)) \leq r(0) = q^M$. Therefore, we have either $g_l(m) \leq r(g_{-l}(m))$ or $g_l(m) > r(g_{-l}(m))$. In the former case, according to Lemma 1 and Lemma 2 case 1, we have $p^* = P(K)$. In the later case, according to Lemma 2 case 2, we have $p^* > P(K)$.

□

This theorem shows that when the capacity is relatively scarce the choice of the allocation mechanism has an effect on the pricing strategy in the downstream market. If allocation quantities are symmetric or do not exceed the best responses, the market price is stable and the equilibrium price reaches the price at the capacity size K. Otherwise, the market price becomes unstable and higher than the price at $P(K)$. In that case, the supplier prioritizes certain retailers in a significant way. This leads to higher and fluctuating retail prices, which is a unique phenomenon in the price competition. In the quantity competition, we have shown in [11] that the retail price is determined by the total allocation quantities. In other words, the difference of allocation among retailers does not affect the retail price. Hence, the supplier is more sensitive to choose allocation mechanism, when the downstream market is in the price competition.

Now we investigate the case when $q^C < K < \hat{q}$. In this case, the largest allocated retailer must be allocated greater than the Cournot quantity, since $q^C < K$ and the allocation mechanism is efficient. On the other hand, the allocation of the largest allocated retailer is always less than $\hat{q}$, since $K < \hat{q}$. Therefore, if the capacity is allocated to several retailers, the equilibrium price is in the mixed strategy as shown by the following theorem.

Theorem 3. *Suppose $q^C < K < \hat{q}$. For any efficient allocation mechanism g, $p^* > P(K)$.*

Proof. Since $K < \hat{q}$, for any g, we have $g_l(m) \leq K < \hat{q}$. Suppose, for all i, we have $g_i(m) \leq r(g_{-i}(m))$. According to the definition of Cournot best response function, we obtain $g_i(m) \leq q_i^c$. It implies $\sum_{i=1}^{n} g_i(m) \leq q^C$ which is a contradiction of the supposition of an efficient allocation $\sum_{i=1}^{n} g_i(m) = K$. Hence, we have retailer i such that $g_i(m) > r(g_{-i}(m))$. According to Lemma 2 case 2, we have $p^* > P(K)$.

□

This theorem shows that when the capacity is greater than the Cournot quantity, the retail price is higher than $P(K)$ under any allocation mechanisms. Now we consider the final case when $K \geq \hat{q}$.

Theorem 4. *Suppose $K \geq \hat{q}$. For any efficient allocation mechanism,*

1. *$p^* = 0$, if $g_{-l}(m) \geq \hat{q}$.*
2. *$p^* > 0$, otherwise.*

Proof. Since $K \geq \hat{q}$ and g is efficient, g satisfies either $g_{-l}(m) \geq \hat{q}$ or $0 \leq g_{-l}(m) < \hat{q}$. The first case is the condition of Lemma 2 case 3. Hence, we have $p^* = 0$. In the later case, since $\frac{K}{n} > q^c$, we obtain $p^* > P(K)$ as same as Theorem 3.

□

Since the capacity range of this theorem is $K \geq \hat{q}$, the supplier has enough capacity to satisfy the market demand. Hence, allocation is not so important in this range.

Assuming that the allocation mechanisms are efficient, we have characterized the price equilibria in the pricing subgame according to capacity sizes of the supplier. Unlike the model having the quantity competition in the downstream market in [11], the equilibrium price can be greater than $P(K)$ in this model, when $K > q^M$. When the capacity is strictly limited, the selection of the allocation mechanism does not have a significant impact on the market behavior in the downstream market. On the other hand, when the capacity is relatively scarce, the mechanism selection affects the pricing strategy of the retailers and the price equilibria.

4 Total Retailers' Profits

Now we shift from the pricing subgame to the total retailers' profits. In general, the supplier is interested in the total retailers' profit. Therefore, it is a reasonable behavior for the supplier to select the allocation mechanism, which maximizes the total retailers' profits. However, the supplier since it is the primary member of the supply chain also has to consider the overall quantity of sold products. In other words, a low market price may decrease the retailers' profits.

In the previous section, we have shown that both the properties of the allocation mechanism and the capacity size of the supplier have an influence on the price equilibria. When the capacity of the supplier is in the range $q^M < K \leq q^C$, the selection of the allocation mechanism in the upstream market has an effect on the selection of the pricing strategy in the downstream market. Thus, we focus on this capacity range. Let $g^{\mathcal{P}}(m)$ be the mechanisms satisfying the condition of case 1 in Lemma 2 and $g^{\mathcal{M}}(m)$ for the condition of case 2. According to these conditions, mechanisms $g^{\mathcal{M}}(m)$ reflects the case where some retailers have been privileged. Notice that the symmetric allocation is included in $g^{\mathcal{P}}(m)$, since $K \leq q^C$. We show how heterogeneously distributed allocations make the total retailers' profit increase.

Theorem 5. *Suppose that $q^M < K \leq q^C$. For any efficient $g^{\mathcal{P}}(m)$ and $g^{\mathcal{M}}(m)$, $\sum_{i \in N} \pi_i(g_i^{\mathcal{M}}(m), g_{j \neq i}^{\mathcal{M}}(m)) > \sum_{i \in N} \pi_i(g_i^{\mathcal{P}}(m), g_{j \neq i}^{\mathcal{P}}(m))$.*

Proof. First we show the total retailers' profit under $g^{\mathcal{P}}(m)$. According to the case 1 of Lemma 2, the total retailers' profit is

$$\begin{aligned}\sum_{i \in N} \pi_i(g_i^{\mathcal{P}}(m), g_{j \neq i}^{\mathcal{P}}(m)) &= \sum_{i \in N} \left(P(K) g_i(m) - w g_i(m)\right), \\ &= P(K)K - wK. \end{aligned} \tag{7}$$

Now we show the case of $g^{\mathcal{M}}(m)$. If $g_l(m) \neq K$, according to the case 2 of Lemma 2, the total retailers' profit is,

$$\sum_{i \in n} \pi_i(g_i^{\mathcal{M}}(m), g_{j \neq i}^{\mathcal{M}}(m)) = \sum_{i \in n} \left(\underline{\mathrm{p}}_i g_i^{\mathcal{M}}(m) - w g_i^{\mathcal{M}}(m)\right)$$

According to Theorem 2, we have $p^* > P(K)$ under $g^{\mathcal{M}}(m)$. Hence, we have

$$\begin{aligned}\sum_{i \in n} \pi_i(g_i^{\mathcal{M}}(m), g_{j \neq i}^{\mathcal{M}}(m)) &= \underline{\mathrm{p}}K - wK \\ &> P(K)K - wK. \end{aligned} \tag{8}$$

According to Equation (7) and (8), we obtain

$$\sum_{i \in n} \pi_i(g_i^{\mathcal{M}}(m), g_{j \neq i}^{\mathcal{M}}(m)) > \sum_{i \in N} \pi_i(g_i^{\mathcal{P}}(m), g_{j \neq i}^{\mathcal{P}}(m))$$

Now we consider the case $g_l(m) = K$, which is a special case of $g^{\mathcal{M}}(m)$. In this case, $g_l(m) > r(g_{-l}(m))$. For pricing, obviously, $P(q^M) > P(K)$. Since $K > q^M$, the profit of the monopolist is $\pi_l^M = q^M P(q^M) - wK$ from Equation (5). Since $\sum_{i \in n} \pi_i(g_i^{\mathcal{P}}(m)) = KP(K) - wK$ and $K > q^M$, we have $\pi_l^M > \sum_{i \in n} \pi_i(g_i^{\mathcal{P}}(m))$.

□

When the capacity of the supplier is relatively scarce, the very heterogeneous allocation $g^{\mathcal{M}}(m)$ makes the profits of the total retailers greater than ones under $g^{\mathcal{P}}(m)$. This is because the prioritized retailers enjoy benefits by charging higher retail price than the price under $g^{\mathcal{M}}(m)$. Meanwhile, under $g^{\mathcal{M}}(m)$, the products are less supplied under $g^{\mathcal{P}}(m)$, which is not beneficial for the end-users. If the supplier takes the profits of the end-users into account more than total retailers' profits, the supplier chooses allocation mechanism $g^{\mathcal{M}}(m)$ rather than $g^{\mathcal{P}}(m)$. In the supplier's point of view, $g^{\mathcal{P}}(m)$ is more preferable allocation than $g^{\mathcal{M}}(m)$ as long as allocation is efficient.

5 Equilibrium in Full Game

In this section, we consider how retailers set order quantities based on allocation mechanisms and determine selling prices according to allocated quantities in the full game. Since our interest is capacity allocation problems, we focus on the case when the demand exceeds the capacity size, $K \leq q^{Cw}$. We assume all retailers do not know the capacity size K, but they know that the capacity is less or equal to the total Cournot quantity at cost w. Prior to show how allocation mechanisms affect the market behaviors of retailers, we introduce mechanism design criteria on capacity allocation problems.

The main concerns of mechanism design are efficiency and stability. For a supplier, capacity utilization is a key performance index to evaluate its performance. If a mechanism makes the capacity utilization increase, it is a preferable mechanism for the supplier. A typical mechanism design criterion in capacity allocation is *individually responsive* (IR) which contributes to increase the capacity utilization.

Definition 2. *An allocation mechanism g is said to be IR if for any i,*

$$m_i' > m_i \text{ and } g_i(m) < K \text{ imply } g_i(m_i', m_{j\neq i}) > g_i(m_i, m_{j\neq i})$$

where m is a vector of retailers' orders, m_i is the i's component of m, $m_{j\neq i}$ is a vector of the other retailers' orders, and m_i' is a variation of m_i.

Under IR mechanisms, a retailer receives more allocations if it orders more. Consequently, retailers frequently place more orders than they actually need to acquire more allocation. On the other hand, the inflated orders prevent a right evaluation of the capacity investment. When demand is unstable, the supplier often fails to make decision on capacity planning due to lack of truthful order information. Hence, a mechanism inducing truthful order information from retailers is desirable mechanism design criteria. This criterion is formally represented as follows:

Definition 3. *An allocation mechanism g is said to be incentive compatible (IC) or truth-inducing if all retailers placing orders truthfully at their optimal sales quantities is a Nash equilibrium of g, formally for all i:*

$$\pi_i(g_i(m^*), g_{j\neq i}(m^*)) \geq \pi_i(g_i(m_i, m^*_{j\neq i}), g_{j\neq i}(m_i, m^*_{j\neq i})).$$

First we consider market behaviors under a representative allocation mechanism *proportional allocation*, which is the most commonly used allocation in industry. An allocation mechanism g is proportional allocation if

$$g_i(m) = min\left\{m_i, \frac{m_i}{\sum_{j=1}^{N} m_j}K\right\} \quad . \tag{9}$$

In other words, whenever capacity binds, allocated quantity to each retailer is the same fraction of its order under the proportional allocation. Obviously the proportional allocation is an IR allocation. As Cachon and Lariviere show in [5], the retailers inflate orders and there is no equilibrium under proportional allocation in the exclusive distribution model. We show a similar result in the selective distribution model with price competition.

Theorem 6. *Under proportional allocation g, there is no equilibrium in m, if $K \leq q^{Cw}$.*

Proof. Suppose there exists symmetric m^*. Since $K \leq q^{Cw}$, we have $g_i(m^*) \leq q^{cw}$ and $g_{-i}(m^*) \leq q^{cw}_{-i}$. According to Equation (2), we have $r(g_{-i}(m^*)) \geq r(q^{cw}_{-i})$. Since $g_i(m^*) \leq q^{cw}_i = r_w(q^{cw}_{-i}) < r(q^{cw}_{-i}) \leq r(g_{-i}(m^*))$, the profit of retailer i is

$$\pi_i(g_i(m^*), g_{j\neq i}(m^*)) = P(g_i(m^*) + g_{-i}(m^*))g_i(m^*) - wg_i(m^*),$$

according to Lemma 2 case 1. Let $m' = (m'_i, m^*_{j\neq i})$. Since g is IR, we have $g_i(m') > g_i(m^*)$ where $m'_i > m^*_i$.

If $\sum\limits_{i\in N} g_i(m^*) < K$, we have $g_l(m')$ such that $g_l(m') < q^{cw}_l$. According to Equation (9), we have $g_{-l}(m') \leq g_{-l}(m^*)$ that induces $r(g_{-l}(m')) \geq r(g_{-l}(m^*))$. Therefore, we have $g_l(m') < q^{cw}_l = r_w(q^{cw}_{-l}) < r(g_{-l}(m^*)) \leq r(g_{-l}(m'))$. According to Lemma 2 case 1, we have

$$\pi_l(g_l(m'), g_{j\neq l}(m')) = P(g_l(m') + g_{-l}(m'))g_l(m') - wg_l(m').$$

According to the concavity of the profit function and $g_i(m^*) < g_l(m') \leq r(g_{-l}(m'))$, we have $\pi_l(g_l(m'), g_{j\neq l}(m')) > \pi_i(g_i(m^*), g_{j\neq i}(m^*))$. Hence, $\sum\limits_{i\in N} g_i(m^*) < K$ is not the case of the existence of symmetric m^*.

If $\sum\limits_{i\in N} g_i(m^*) = K$, according to Equation (9), we have

$$g_l(m') - g_l(m^*) = -(g_{-l}(m') - g_{-l}(m^*)) \tag{10}$$

where $m'_l > m^*_l$. According to Equation (2), (10) and $g_i(m^*) < r(g_{-i}(m^*))$, we have m'_l such that $g_l(m') < r(g_{-l}(m'))$. The profit of retailer l is

$$\pi_l(g_l(m'), g_{j\neq l}(m')) = P(g_l(m') + g_{-l}(m'))g_l(m') - wg_l(m').$$

The difference between $\pi_l(g_l(m'), g_{j\neq l}(m'))$ and $\pi_i(g_i(m^*), g_{j\neq i}(m^*))$ is,

$$\begin{aligned}\pi_l(g_l(m'), g_{j\neq l}(m')) - \pi_i(g_i(m^*), g_{j\neq i}(m^*)) =& P(K)g_l(m') - wg_l(m') \\ & - (P(K)g_i(m^*) - wg_i(m^*)) \\ =& (P(K) - w)(g_l(m') - g_i(m^*)).\end{aligned}$$

According to the assumption $w < P(q^{Cw})$, we have $w < P(K)$ that implies $\pi_l(g_l(m'), g_{j\neq l}(m')) > \pi_i(g_i(m^*), g_{j\neq i}(m^*))$. Hence, $\sum\limits_{i\in N} g_i(m^*) = K$ is not the case of the existence of symmetric m^*.

Suppose there exists asymmetric m^* such that $g_l(m^*) < r(g_{-l}(m^*))$. According to Lemma 1 and Lemma 2 case 1, the profit of retailer l is

$$\pi_l(g_l(m^*), g_{j\neq l}(m^*)) = P(g_l(m^*) + g_{-l}(m^*))g_l(m^*) - wg_l(m^*),$$

and the profit of retailer i such that $g_i(m^*) < g_l(m^*)$ is

$$\pi_i(g_i(m^*), g_{j\neq i}(m^*)) = P(g_i(m^*) + g_{-i}(m^*))g_i(m^*) - wg_i(m^*).$$

Similarly to the case of symmetric m^* and $\sum\limits_{i\in N} g_i(m^*) < K$, we have $\pi_l(g_l(m^*), g_{j\neq l}(m^*)) > \pi_i(g_i(m^*), g_{j\neq i}(m^*))$. Hence, asymmetric m^* such that $g_l(m^*) < r(g_{-l}(m^*))$ does not exist.

Suppose there exists asymmetric m^* such that $g_l(m^*) > r(g_{-l}(m^*))$. The profit of retailer l is

$$\pi_l(g_l(m^*), g_{j\neq l}(m^*)) = P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*),$$

Since m^* is an equilibrium and $g_l(m^*) \leq K \leq q^{Cw}$, we have

$$\pi_l(g_l(m^*), g_{j\neq l}(m^*)) > 0. \tag{11}$$

The profit of retailer i where $m_i^* < m_l^*$ is

$$\pi_i(g_i(m^*), g_{j\neq i}(m^*)) = \frac{P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*))}{g_l(m^*)} g_i(m^*) - wg_i(m^*),$$

according to Lemma 2 case 2. The difference of profits between retailer l and i is

$$\begin{aligned}
&\pi_l(g_l(m^*), g_{j\neq l}(m^*)) - \pi_i(g_i(m^*), g_{j\neq i}(m^*)) \\
&= P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) \left(\frac{g_l(m^*) - g_i(m^*)}{g_l(m^*)} \right) - w(g_l(m^*) - g_i(m^*)) \\
&= \frac{g_l(m^*) - g_i(m^*)}{g_l(m^*)} (P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*)).
\end{aligned}$$

According to Equation (11), we have $\pi_l(g_l(m^*), g_{j\neq l}(m^*)) > \pi_i(g_i(m^*), g_{j\neq i}(m^*))$. Hence, asymmetric m^* such that $g_l(m^*) > r(g_{-l}(m^*))$ does not exist.

□

When the demand exceeds the capacity size, retailers inflate orders to be allocated more than they need. The different behavior in order quantity from the KS result is observed under proportional allocation in our model. While the supply limit determination for each retailer is independent in the KS model, each retailer is not able to choose allocation quantity independently under proportional allocation in our model, but it is determined by its order quantity, competitors order quantity and allocation mechanisms. Hence, by increasing order quantity the retailer is able to decrease the allocation quantity for competitors.

To induce truthful orders from the retailers is important for the supplier to determine its production capacity and its selling price from a long-term perspective. In other words, by choosing truth-inducing mechanisms, the supplier acquires truthful demand information of the downstream market which leads the supplier's secure decision-makings on capacity planning and sales planning. As a representative mechanism of a truth-inducing mechanism, we show the equilibrium in order and price under uniform allocation presented in [29]. Under uniform allocation, the retailers are indexed in

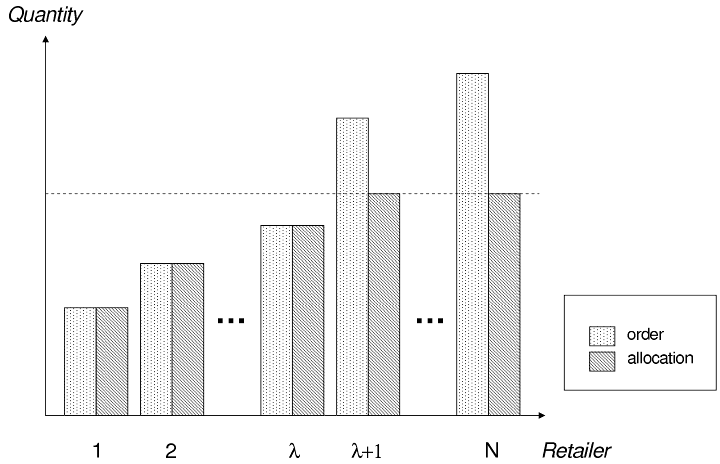

Figure 2: Uniform Allocation Mechanism

ascending order of their order quantity, i.e., $m_1 \leq m_2 \leq \ldots \leq m_n$. Let $\lambda = \max\left\{ i : K - nm_1 - \sum_{j=2}^{i}(n-l)(m_j - m_{j-1}) > 0 \right\}$ and uniform allocation is,

$$g_i(m) = \begin{cases} K/n, & \text{if } nm_1 > K, \\ m_i, & \text{if } i \leq \lambda, \\ m_\lambda + \left(K - (n-\lambda+1)m_\lambda - \sum_{j=1}^{\lambda-1} m_j \right) /(n-\lambda), & \text{otherwise.} \end{cases}$$

Under uniform allocation mechanism, the retailers with orders less than a threshold m_λ receive the same quantities as respective orders, and the rest of retailers receive m_λ and the rest of capacity divided by the number of retailers ordered greater than m_λ. The threshold of m_λ is led by the following procedure. If $m_1 \times n$ is greater than the capacity, all retailers receive $\frac{K}{n}$, otherwise, there is a threshold m_λ where λ is greater than or equal to 1. In case of $m_1 \times n \leq K$, the supplier counts up the number of retailers, while $\sum_{j=1}^{i} m_j + m_j \times (n-i) \leq K$ (the sum of i-th smallest orders and the quantity of the i-th order times the number of the rest of retailers is less than the capacity size of the supplier). If the sum of orders is greater than or equal to the capacity, the allocation quantity is equal to the capacity, which is the area below the horizontal dashed line in Figure 2.

Under uniform allocation, we have the following order quantities equilibria and the price equilibria.

Theorem 7. *Under uniform allocation, $m_i^* = q^{cw}$ is an equilibrium inducing a price equilibrium $p^* = P(K)$.*

Proof. If $m_i^* = q^{cw}$ for all i, we have $g_i(m^*) = K/n \leq q^{cw}$. Since $g_i(m^*) \leq r_w(g_{-i}(m^*)) < r(g_{-i}(m^*))$, the profit of retailer i is

$$\pi_i = P(g_i(m^*) + g_{-i}(m^*))g_i(m^*) - wg_i(m^*). \tag{12}$$

If $m_i' > m_i^*$ and $m_{j\neq i}^* = q^{cw}$, we have $g_i(m') = g_{j\neq i}(m') = K/n$, which is the same allocation quantity to the case at m^*. Hence, by increasing order m_i', retailer i cannot increase its profit. If $m_i' < m_i^*$, we have $g_i(m') \leq \frac{K}{n} \leq g_{j\neq i}(m') \leq q^{cw}$. We check whether this case fits to the condition of case 2 of Lemma 2. According to Equation (2), we have $r(g_{-i}(m')) \leq r(g_{-j}(m'))$. Since $g_{j\neq i}(m') \leq q^{cw} = r_w(q_{-i}^{cw}) < r(q_{-i}^{cw}) \leq r(g_{-i}(m')) \leq r(g_{-j}(m'))$, this case does not satisfies the the condition of case 2 of Lemma 2. Hence, we only consider the case of the pure strategy. The profit of retailer i is same as Equation (12). Recall that Equation (12) is concave in $g_i(m)$ and maximized at $r_w(g_{-i}(m))$. Since $g_i(m') \leq r_w(q_{-i}^{cw})$, by decreasing m_i', π_i is not increased. Therefore, we have an equilibrium order quantity $m_i^* = K/n$ and an equilibrium price $p_i^* = P(K)$ □

Since uniform allocation is a truth inducing mechanism, all retailers place truthful order quantities. The allocations at the truth orders induce the equilibrium price $P(\sum g_i(m^*))$ in the price competition of the downstream market.

6 Conclusion

This paper describes how allocation mechanisms affect the market behaviors in the connected downstream market where several retailers are in the price competition. We have shown that the heterogeneous allocations in the upstream market affect the pricing strategy of the retailers is dependent on the capacity size of the supplier in the upstream market. The interesting case is when the capacity is relatively scarce. The heterogeneous allocations lead the higher equilibrium price with fluctuation in the downstream market compared to the equilibrium price under homogeneous allocations. At the same time, even though the higher equilibrium price makes the lower market penetration, the heterogeneous allocations make the total retailers' profits greater. Hence, if there exists an influential retailer in the supply chain network, the supplier prioritizes the retailer in allocation, even though it may yield the undesirable market behaviors for the supplier that are the lower market penetration and the unstable retail price. These market behaviors are unique in this model, compared to other models such as the exclusive distribution model in [5] and the selective distribution model with quantity competition in [11].

Unlike the Kreps and Scheinkman model, each retailer is not able to choose its own allocation, supply limit, which is determined by its order quantity, the competitors' order quantity and the allocation mechanism. Hence, the market behaviors are more complex than the ones in Kreps and Scheinkman model. We show the equilibrium order quantity does not always equal to Cournot quantity, since the allocation mechanisms play important roles to determine the equilibrium order quantity. Under proportional allocation, a representative allocation in IR mechanisms, retailers place greater orders to be allocated even more than Cournot quantity. Contrary, under uniform allocation, a representative allocation in Truth-inducing mechanisms, the equilibrium order quantity is symmetric similar to Vives [**?**]. Especially, when the capacity is greater or equal to the total Cournot quantity, the equilibrium order quantity and the equilibrium price are equal to the results of Kreps and Scheinkman model.

This research sheds light that mechanism design considering supply chain is a fruitful research direction. By considering connected markets, the market behaviors are explained differently and closer to ones in real business.

References

[1] G. Anandalingam, R. Day and S. Raghavan, The Landscape of Electronic Market Design , *Management Science*, Vol. 51, No. 3, pp. 316-327, March, 2005.

[2] D. Besanko and U. Doraszelski, Capacity Dynamics and Endogenous Asymmetries in Firm Size, *The RAND Journal of Economics*, Vol. 35, No. 1, pp. 23-49, Spring, 2004.

[3] N. Boccard and X. Wauthy, Bertrand competition and Cournot outcomes: further results, *Economics Letters*, Vol. 68, No. 3, pp. 279-285, 2000.

[4] N. Boccard and X. Wauthy, Bertrand competition and Cournot outcomes: a correction, *Economics Letters*, Vol. 84, No. 2, pp. 163-166, 2004.

[5] G. P. Cachon and M. A. Lariviere, Capacity choice and allocation: strategic behavior and supply chain performance, *Management Science*, Vol. 45, No. 8, 1091-1108, 1999.

[6] V. Conitzer and T. Sandholm, Automated mechanism design: Complexity results stemming from the single-agent setting, *Proceedings of the 5th International Conference on Electronic Commerce (ICEC-03)*, 17-24, 2003.

[7] M. De Francesco, On a property of mixed strategy equilibria of the pricing game, *Economics Bulletin* , Vol. 4, No. 30, 1-8, 2003.

[8] R. J. Deneckere and D. Kovenock, Bertrand-Edgeworth duopoly with unit cost asymmetry, *Journal of Economic Theory*, Vol. 8, pp. 1-25, 1996.

[9] M. Erkoc and S. D. Wu, Managing high-tech capacity expansion via capacity reservation, *Production and Operations Management*, 14(2), 232-251, 2005.

[10] C. Frank Jr. and R Quandt, On the Existence of Cournot Equilibrium, *International Economic Review*, Vol. 4, No. 1, 92-96, 1963.

[11] M. Furuhata and D. Zhang, Capacity allocation with competitive retailers, *Proceedings of the 8st International Conference on Electronic Commerce (ICEC-06)*, 31-37, 2006.

[12] J. Gallien, Dynamic mechanism design for online commerce , *Technical report* Sloan School of Management, MIT Press, Cambridge, 2002.

[13] O. Hart and J. Tirole, Vertical Integration and Market Foreclosure; Comments and Discussion, *Brookings Papers on Economic Activity*, pp. 205-286, Brookings Institution Press, 1990.

[14] M. T. Hajiaghayi, R. Kleinber and T Sandholm, Automated Online Mechanism Design and Prophet Inequalities, *Proceedings of the Twenty-Second National Conference on Artificial Intelligence (AAAI-07)*, pp. 58-65, Vancouver, British Columbia, Canada, 2007

[15] R. Johari and J. Tsitsiklis, Efficiency loss in a network resource allocation game, *Mathematics of Operations Research*, 29(3), 407-435, 2004.

[16] J. Kalagnanam and D. C. Parkes, Auctions, Bidding and Exchange Design, in Handbook of Qualitative Supply Chain Analysis: Modeling in the E-Business Era, Kluwer Academic Publishers. D. Simichi-Levi, S. D. Wu and Z. Max Shen (Eds), pp.143-212, 2004.

[17] J. F. Kurose and R. Simha, A Microeconomic Approach to Optimal Resource Allocation in Distributed Computer Systems, IEEE Transaction on Computers, pp.705-717, 38, 5, 1989.

[18] D. M. Kreps and J. A. Scheinkman, Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes, *Bell Journal of Economics*, Vol. 14, 326-337, 1983.

[19] D. Lehmann, L. I. O'Callaghan and Y. Shoham, Truth Revelation in Approximately Efficient Combinatorial Auctions, *Journal of the ACM*, Vol. 49, Issue 5, September 2002, pp. 577-602.

[20] R. Levitan and M. Shubik, Price Duopoly and Capacity Constraints, *International Economic Review*, Vol. 13, No. 1, pp. 111-122, 1972.

[21] P. Madden, Elastic demand, sunk costs and the Kreps-Scheinkman extension of the Cournot model, *Journal of Economic Theory*, Vol. 12, pp. 199-212, 1998.

[22] S. Mallik and P. Harker, Coordinating supply chains with competition: capacity allocation in semiconductor manufacturing, *European Journal of Operational Research*, 159(2), 300-347, December, 2004.

[23] N. Nisan and A. Ronen, Algorithmic Mechanism Design (extended abstract), *Proceedings of the thirty-first annual ACM symposium on Theory of computing*, Atlanta, USA, pp.129-140, 1999.

[24] R. Porter, Y. Shoham and M. Tennenholtz, Fair Imposition, *Journal of Economic Theory*, Vol. 118, No. 2, pp. 209-228, October, 2004.

[25] D. C. Parkes and L. H. Ungar, Iterative Combinatorial Auctions: Theory and Practice, *Proceedings of Seventeenth National Conference on Artificial Intelligence (AAAI-00)*, Austin, USA, pp. 74-81, 2000.

[26] S. S. Reynolds and B. J. Wilson, Bertrand-Edgeworth Competition, Demand Uncertainty, and Asymmetric Outcomes, *Journal of Economic Theory*, Vol. 92, pp.122-141, 2000.

[27] T. Sandholm, eMediator: a next generation electronic commerce server, *Proceedings of the fourth international conference on Autonomous agents*, Barcerona, Spain, pp.341-348, 2000.

[28] O. Shehory and S. Kraus, Methods for task allocation via agent coalition formation, *Artificial Intelligence*, Vol. 101, No. 1-2, pp.165-200, 1998.

[29] Y. Sprumont, The Division Problem with Single-Peaked Preferences: A Characterization of the Uniform Allocation Rule, *Econometrica*, Vol. 59, No. 2, 509-519, 1991.

[30] X. Vives, Rationing Rules and Bertrand-Edgewroth Equilibria in Large Markets, *Economics Letters*, Vol. 21, pp. 113-116, 1986.