Credible Spatial Preemption Revisited in Entry-Exit Game

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Abstract

Much literature investigates that how an incumbent firm deters an entrant firm by crowding in a differentiated product market. However, Judd (1985, RAND) shows that if the incumbent can exit its stores after observing entrant's location, then a spatial preemption is not credible because the incumbent exits its stores to reduce price competition. This paper analyzes an entry-exit game based on Hotelling's location-then-price competition model on a unit circle. I demonstrate that in some range of entry cost, spatial entry deterrence occurs in a subgame perfect Nash equilibrium if the incumbent can built five stores or more. In this case, the incumbent stays at least two stores so as to keep its market share. Since it intensifies price competition, it lowers an upper bound of entrant's equilibrium price in subgames and hence entrant's profit is strictly lower than a situation that only one incumbent store exists. Thus, entrant does not enter the market because its revenue never makes up for its entry cost. It implies that creating many stores itself can serve as a commitment device to crowd in the market.

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1 Introduction

It is well known that an incumbent firm hardly deters entry of an entrant firm. Much literature analyzes that how incumbent firms deter entry by spatial preemption (Schmalensee 1978; Salop 1979b; Eaton and Lipsey 1979). However, a seminal work Judd (1985) shows that if the incumbent can locate their stores only on two places and can exit its stores after observing a location of entrant, then it has an ex post incentive to withdraw its stores in order to reduce price competition. Thus the arguments of spatial entry deterrence do not hold unless the incumbent can commit not to withdraw its stores.

Few literature investigates how the incumbent commits to stay its stores. Hadfield (1991) shows that a franchise contract can serve as a commitment device. Ashiya (2000) analyzes that if multiple entrants exist, then the incumbent may allow the entry of a weak entrant in order to prevent strong one's entry. Ishibashi (2003) shows that if multiple entrants exist and their entry timing is endogenously determined, then spatial preemption is possible because each entrant wants to set its store only after observing other entrant's location. Wickelgren (2006) incorporates an uncertainty about entrant's production cost, and looks into properties of a mixed strategy equilibrium where the incumbent exits with some probability. Yet as far as I know, there is no literature which investigates whether credible spatial preemption is possible or not in an essentially same setting of Judd (1985).

This paper analyzes an entry-exit game when the incumbent firm can locate its stores more than two places. I investigate Hotelling's location-then-price competition model on a unit circle with linear shopping cost, which is explored by Vickrey (1999) and Salop (1979b). I demonstrate that in a certain range of fixed entry cost, spatial entry deterrence occurs in a subgame perfect Nash equilibrium if the incumbent can build five stores or more. It implies that the result in Judd (1985) depends on the number of locations where the incumbent can create its stores.

There are two main logics to constitute the result. First, if the incumbent creates five equidistant stores and stays at least two of them, entrant's profit is strictly lower than the case that only one incumbent store remains. This is because staying more than one incumbent's stores intensifies price competition, hence it lowers an upper bound of entrant's equilibrium price in subgames. Second, the incumbent stays at least two stores so as to keep its market share, if the distance between some of its stores and entrant's store is not too close.

The result means that the incumbent firm can commit to fill in the market by creating many stores. It can explain why incumbent firms in some industry produce differentiating similar products (or build many stores) as mentioned in Schmalensee (1978), though they can stop creating the products (exiting stores) with low costs.

In Section 2, I set up an entry-exit model based on Judd (1985). I analyze the model and prove the main result in Section 3. In Section 4, I conclude and illustrate further researches.

2 The Model

In this section, I set up a following sequential entry-exit game with price competition on a unit circle.

Suppose that a circular city model investigated by Vickrey (1999) and Salop (1979b). A continuum of consumer is uniformly located on a unit circle. Denote $d(i,j)$ be a usual distance of locations i and j on the circle.[1]

There are one incumbent and one entrant. They have a same fixed entry cost F per store and a same constant marginal cost of production, normalized to 0.

Consumers can buy the product at most one unit. Their monetary value of the good is $V > 0$. Suppose that consumer i pays a shopping cost $t \cdot d(i,j)$ if she buys the good in store j, where $t \in R_{++}$. Thus consumer i's utility when she buys the good from store j with price p_j is

$$V - p_j - t \cdot d(i,j).$$

If the consumer does not buy the good, then her utility is 0. Assume that $V \geq 3t$ so that every consumer buys the good in equilibrium and the incumbent firm prefers a monopoly. Suppose that

[1] Precisely, set $d(i,j) = |i-j|$ if $i > j$ and $i - j \leq \frac{1}{2}$, $d(i,j) = |(1-i)+j|$ if $i > j$ and $i - j > \frac{1}{2}$.

each consumer buys the good from the closest located store if her utility is indifferent.

I analyze a following four-stage entry-exit game based on Judd (1985). Suppose that firms can create their stores at any location on the circle.[2] I consider a large incumbent and a small entrant case: the incumbent can create stores as many as possible, though the entrant can manage at most one store.

Stage 1. The incumbent chooses the number of creating stores and their locations. Denote X be the set of selected locations and $|X| = m$. Though it can create its stores as many as possible, it pays a fixed entry cost $F > 0$ per store.

Stage 2. After observing incumbent's decision, the entrant chooses whether it enters the market by paying fixed entry cost $F > 0$ and sets its location. Denote $y \in [0, 1)$ be a selected location if it enters.

Stage 3. After observing entrant's decision, the incumbent chooses the number of exiting stores $n \in \{0, 1, \cdots, m\}$. Then it retrieves entry costs (or pays exit costs) $n \cdot \alpha F$ where $\alpha \in R$. $\alpha \geq 0$ represents a proportion of not-sunk cost, while $\alpha < 0$ means a proportion of exit cost. Assume that most of the fixed cost is sunk, or an exit cost exists.[3] Let $X' = (x_1, \cdots, x_{m-n}) \subset X$ be a set of remaining stores.

Stage 4. Firms simultaneously choose a price $p_s \in [0, V]$ for each own store s. Denote $\pi_f((p_{x_1}, \cdots, p_{x_{m-n}}), p_y)$ be a profit function of $f \in \{I, E\}$ excluding fixed costs.

The time line of the game is illustrated on Figure 1.

3 Analysis

I demonstrate that a spatial entry deterrence occurs in a subgame perfect Nash equilibrium by analyzing a case that the incumbent creates its stores on five equidistant locations, i.e., $X =$

[2] Judd (1985) assumes that firms can built their stores only on two locations.

[3] Precisely, I assume that $\alpha < 0.136$ so that Lemma 2 holds. Judd (1985) assumes that α is negative.

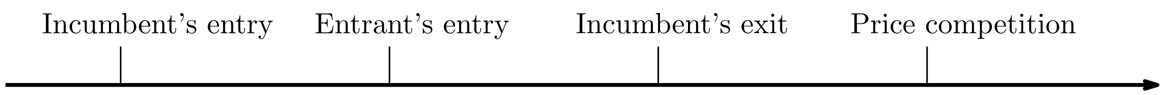

Figure 1: Timing of Entry-Exit Game.

$\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\}$.

3.1 Preliminaries

To prove the main result, I investigate equilibria in fourth stage subgames.

First, suppose that one of incumbent's store is on the same location as entrant's. Then a usual price competition occurs, and both stores' prices are zero in equilibrium regardless of incumbent's other locations. Hence $\pi_E = 0$ in this case.

Second, consider a case of one incumbent store on $x = 0$ and one entrant store on $y \in (0, \frac{1}{2}]$. I illustrate the demand of incumbent's store 0 in order to clarify the notion of *undercut*.

Suppose that $p_0 \in [p_y - ty, p_y + ty]$. Consumer $i \in (0, y]$ will purchase the product from store 0 rather than store y if

$$V - p_0 - t(i - 0) > V - p_y - t(y - i) \quad \Longleftrightarrow \quad i < \frac{1}{2t}(p_y - p_0 + ty).$$

Consumer $i \in (y, \frac{1}{2}]$ will never buy the product from store 0 in this case. Consumer $i \in (\frac{1}{2}, y + \frac{1}{2}]$ will purchase the product from store 0 rather than store y if

$$V - p_0 - t(1 - i) > V - p_y - t(i - y) \quad \Longleftrightarrow \quad 1 - i < \frac{1}{2t}(p_y - p_0 - ty + t).$$

Consumer $i \in (y + \frac{1}{2}, 1)$ will always buy the product from store 0 in this case. Thus if $p_0 \in [p_y - ty, p_y + ty]$, then the total demand of store 0 is $\frac{1}{2t}(2p_y - 2p_0 + t)$.

On the other hand, suppose that $p_0 < p_y - ty$. Then in addition to the demand $[0, y] \cup (\frac{1}{2}, 1)$, every consumer $i \in (y, \frac{1}{2}]$ will purchase the product from store 0 rather than store y, because

$$V - p_0 - t(i - 0) > V - p_y - t(i - y) \quad \Longleftrightarrow \quad p_0 < p_y - ty.$$

Thus if a store 0 sets its price $p_0 < p_y - ty$, the total demand is 1 and it sweeps all the demand of store y. This is called that store 0's price *undercuts* store y's price.

Denote $\overline{\pi}_f$ be an upper bound of firm f's profit. Kats (1995) shows that if there are only one incumbent and one entrant, then $\overline{\pi}_I, \overline{\pi}_E \leq 0.25t$ hold for any equilibrium in subgame.

3.2 Lemmas

In this subsection, I prove three lemmas which constitute the main result.

First, I show that if the incumbent stays more than one store, then entrant's profit is always less than $0.25t$. Denote y be entrant's store. Let l and r be the closest incumbent's stores such that $l < y < r$.

Lemma 1. Consider the fourth stage subgame.

(i) Suppose that $l - r = \frac{1}{5}$. Set $y \in (0, \frac{1}{10}]$, $l = 0$ and $r = \frac{1}{5}$ without loss of generality. Then for any equilibrium in subgame, the upper bound of entrant's profit is $\overline{\pi}_E \leq 0.02t$.

(ii) Suppose that $l - r = \frac{2}{5}$. Set $y \in (0, \frac{1}{5}]$, $l = 0$ and $r = \frac{2}{5}$ without loss of generality. Then for any equilibrium in subgame, the upper bound of entrant's profit is $\overline{\pi}_E \leq 0.16t$.

(iii) Suppose that $l - r = \frac{3}{5}$. Set $y \in (0, \frac{3}{10}]$, $l = 0$ and $r = \frac{3}{5}$ without loss of generality. Then for any equilibrium in subgame, the upper bound of entrant's profit is $\overline{\pi}_E \leq \frac{169}{900}t \simeq 0.188t$.

(iv) Suppose that $l - r = \frac{4}{5}$. Set $y \in (0, \frac{2}{5}]$, $l = 0$ and $r = \frac{4}{5}$ without loss of generality. Then for any equilibrium in subgame, the upper bound of entrant's profit is $\overline{\pi}_E \leq \frac{49t}{225} \simeq 0.218t$.

Proof. In appendix. ■

Intuitively, if the incumbent stays two stores or more, distance between incumbent stores and an entrant store is closer than one incumbent store case. Hence each firm is more easily to undercut other firm's price and sweep its market. Since it intensifies price competition, it lowers an upper bound of entrant's equilibrium price in subgames, and hence entrant's profit is strictly lower than a situation that only one incumbent store exists.

Second, I show that if the incumbent locates $\{0, \frac{4}{5}\}$ and the entrant locates $y \in [\frac{3}{10}, \frac{2}{5}]$, there exists a pure strategy equilibrium in subgame such that the incumbent profit is more than $0.25t$ and the entrant profit is less than $0.25t$.

Lemma 2. Consider the fourth stage subgame such that the incumbent locates $\{0, \frac{4}{5}\}$ and the entrant locates $y \in [\frac{3}{10}, \frac{2}{5}]$.

Then there exists a pure strategy equilibrium such that $p_0^* = (\frac{7}{15} + \frac{1}{6}y)t, p_{\frac{4}{5}}^* = (\frac{3}{5} - \frac{1}{6}y)t$ and $p_y^* = \frac{7}{15}t$. Entrant's equilibrium profit is $\pi_E^* = \frac{49}{225}t \simeq 0.218t$, and incumbent's equilibrium profit π_I is equal or more than $\geq \frac{64}{225}t \simeq 0.284t$ for any y.

Proof. In appendix. ■

The intuition is that, undercutting price is not profitable if a distance between each of incumbent's stores and entrant's store is not so close. Thus each firm does not undercut in equilibrium, and chooses a pure strategy. Then the incumbent can earn more than one incumbent store case by staying two stores, because it keeps larger market share in equilibrium.

Though other mixed strategy equilibria might exist in the subgames, it can be easily shown that the equilibrium is unique in pure strategies. Thus it is plausible to predict this equilibrium in the subgames.

Notice that if the incumbent sets five equidistant stores in stage 1 and it exits the closest three stores to entrant's store in stage 3, the above subgame occurs for any entrant's location. That is, Lemma 2 shows that if the incumbent creates five equidistant stores in stage 1, then the incumbent stays at least two stores so as to keep its market share.

Third, I show that incumbent's equilibrium profit is always less than $1.5t$ if the entrant enters in the market.

Lemma 3. Consider the fourth stage subgames such that the entrant has a store in the market.

Then for any equilibrium in subgame, the upper bound of incumbent's profit is $\overline{\pi}_I \leq 1.5t$.

Proof. In appendix. ■

On the other hand, if the incumbent has one store or more and the entrant does not enter, then the incumbent can earn at least $V - 0.5t$ by setting its prices to $V - 0.5t$. Thus the incumbent prefers to deter entry if consumer's value of the good V is sufficiently large.

3.3 Main Result

Now I present the main result.

Proposition 1. If $0.218t \simeq \frac{49}{225}t < F < 0.25t$, then a spatial entry deterrence occurs in a subgame perfect Nash equilibrium.

Proof. By Lemma 2, in the third stage subgame the incumbent keeps at least two stores if it has five equidistant location stores, i.e., $X^* = \{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\}$. Then entrant's profit is always less than $0.218t$ by Lemma 1. Thus in the second stage subgame, the entrant does not enter if the incumbent builds its stores on five equidistant locations. In the first stage subgame, if the incumbent deters entrant's entry, it earns at least $V - 0.5t - 5F$ including fixed costs. On the other hand, if the incumbent does not deters the entry, it earns at most $1.5t - F$ including fixed costs by Lemma 3. Since $V \geq 3t$, in the first stage subgame the incumbent sets its stores so as to deter the entry. ■

The intuition of the proposition is as follows. As the distance between incumbent's store and entrant's store is close, the incentive to undercut the opponent's price increases (the price competition becomes harsh). However, if the incumbent keeps more stores, then it can get much market share in total. The effect of keeping the market share dominates the effect of reducing price competition in the case of Lemma 2. Hence the incumbent does not exit some stores so as to keep its market share. Moreover, the entrant does not have an incentive to undercut its price in this case, because the benefit of sweeping the market share is less than the loss of decreasing price for the existent consumers, if the increase of demand is not too big.[4]

[4]On the other hand, if the incumbent firm creates three or four equidistant stores and stays two or more stores, then there exists no pure strategy equilibrium. This is because the distance is relatively close and the entrant has an incentive to undercut incumbent's price and sweep its consumers.

Also, in any mixed strategy equilibrium in fourth subgame where the incumbent keeps two stores or more, the upper bound of entrant's profit is less than $0.218t$. This is because if the one firm sets too high price with positive probability, then the other firm has an incentive to undercut it with probability 1. Hence the upper bound price must not be too high for each firm.

Notice that Proposition 1 neither excludes the possibility that the incumbent deters the entry by building less than five stores, nor derives precise equilibrium strategies. This is because I derive only the upper bounds of equilibrium prices and profits, and these results are enough to prove that spatial preemption occurs in the above equilibrium.

4 Concluding Remarks

This paper investigates an entry-exit game based on Judd (1985). I demonstrate that in a certain range of fixed cost, spatial entry deterrence occurs in a subgame perfect Nash equilibrium. This is because creating many stores itself can serve as a commitment for an incumbent to keep more than one store. Therefore, this paper explains why incumbent firms in some industry create many stores to crowd in the market, although they can exit their stores with almost no costs.

To see the robustness of the result, suppose that both firms can exit in fourth stage subgame, which is assumed in Judd (1985). Then by the same logic of Proposition 1, the incumbent can credibly keep entrant's equilibrium profit less than $0.218t$. Thus the main result holds even if both firms can exit in fourth stage subgame.

To conclude the paper, I illustrate some possible future researches. First, a natural extension of the model is that how the credible spatial preemption result strengthens if the market is not a circler city or shopping cost is not linear. For example, linear city, multi-dimensional city (e.g., square-shape or disk) or convex shopping cost (e.g., quadratic) seems to be more reasonable to analyze in some industries. Second, the method of deriving the upper bound of prices and profits may be useful to characterize other Hotelling's location-then-price games. For example, the method may allow to partially characterize equilibria in entry game with a large incumbent and small multiple entrants.

5 Appendix

In the following, suppose that the entrant does not choose the same location with any incumbent's store. Let $F_I(p_{x_1}, \cdots, p_{x_{m-n}})$ be a mixed strategy of incumbent and $F_y(p_y)$ be a mixed strategy of entrant in a fourth stage subgame, where F is a cumulative probability distribution. Define $F_{x_k}(p_{x_k})$ be a k-th coordinate projection of $F_I(p_{x_1}, \cdots, p_{x_{m-n}})$. Denote S_s be a support of $F_s(p_s)$.[5] Since $p_s \in [0, V]$, each S_s is bounded. Let $\overline{p}_s = \max S_s$.

5.1 Proof of Lemma 1

(i) It can be shown that no pure strategy equilibrium exists. However, a mixed strategy equilibrium exists by applying Theorem 5* in Dasgupta and Maskin (1986).[6]

To see it, take any pure strategy of incumbent $(p_{x_1}, \cdots, p_{x_{m-n}})$ such that p_k undercuts p_h (if more than one entrant's price undercuts p_h, let p_k be the lowest one among them). I show that it is not a best response of any entrant's mixed strategy $F_y(p_y)$. First, suppose that $\overline{p}_y$ undercuts p_k with probability 1. In this case, $\overline{p}_y$ also undercuts p_h with probability 1. Then the incumbent can increase its profit by setting p_h slightly higher than $\overline{p}_y$. Second, suppose that $\overline{p}_y$ does not undercut p_k with probability 1. In this case, the incumbent can increase its profit by setting p_h slightly higher than p_k. Therefore, incumbent's any pure strategy $(p_{x_1}, \cdots, p_{x_{m-n}})$ such that p_k undercuts p_h is never best response. Thus without loss of generality, I can restrict incumbent's action set such that undercutting does not occur between its stores, that is, $|p_k - p_h| \leq t|k - h|$ for any $k, h \in \{x_1, \cdots, x_{m-n}\}$. Then discontinuities of profit functions occur only on $|p_k - p_y| = t|k - y|$ for each $k \in \{x_1, \cdots, x_{m-n}\}$. Other conditions of Theorem 5* in Dasgupta and Maskin (1986) can be checked straightforwardly in the restricted action set.

Next, I show that $\overline{p}_0 \leq \overline{p}_y + yt$, $\overline{p}_{\frac{1}{5}} \leq \overline{p}_y + (\frac{1}{5} - y)t$, and $\overline{p}_y \leq \frac{1}{4}(\frac{1}{5}t + \overline{p}_0 + \overline{p}_{\frac{1}{5}})$ hold in any

[5] The definition of support is the smallest closed set whose complement is measure zero.

[6] Existences of an equilibrium in other fourth stage subgames can be shown by the same logic.

equilibrium.

Suppose that $\overline{p}_0 > \overline{p}_y + yt$. Notice that by the above argument, undercutting between incumbent's stores with positive measure never occurs in equilibrium. Then there exists a positive measure such that $\overline{p}_y$ undercuts a price of store 0 with probability 1. This is never optimal for the incumbent, because it can increase its payoff by removing the top of the support of store 0's mixed strategy and depositing it on slightly higher than $\overline{p}_y$ (or the highest price of other incumbent's store). Thus $\overline{p}_0 \leq \overline{p}_y + yt$ holds in equilibrium. By the same logic, $\overline{p}_{\frac{1}{5}} \leq \overline{p}_y + (\frac{1}{5} - y)t$ holds in equilibrium.

Notice that if p_y undercuts neither p_0 nor $p_{\frac{1}{5}}$, then the profit function of the entrant is $\pi_E(p_y, p_0, p_{\frac{1}{5}}) = \max\{\frac{1}{2t}(\frac{1}{5}t + p_0 + p_{\frac{1}{5}} - 2p_y), 0\}p_y$. Thus given that undercutting is not optimal for the entrant, the best response is $BR_E(p_0, p_{\frac{1}{5}}) = \frac{1}{4}(\frac{1}{5}t + p_0 + p_{\frac{1}{5}})$.

Suppose that $\overline{p}_y > \frac{1}{4}(\frac{1}{5}t + \overline{p}_0 + \overline{p}_{\frac{1}{5}})$. Then $\overline{p}_y$ is strictly higher than $BR_E(p_0, p_{\frac{1}{5}})$ for any $(p_0, p_{\frac{1}{5}})$ in the support, because $\overline{p}_y$ neither undercuts $\overline{p}_0$ nor $\overline{p}_{\frac{1}{5}}$, and $BR_E(p_0, p_{\frac{1}{5}})$ is strictly increasing in p_0 and $p_{\frac{1}{5}}$. Then the entrant can increase its profit by removing the top of the support of its mixed strategy and depositing it on a lower price. Formally, if $\overline{p}_0 < \overline{p}_y + yt$ and $\overline{p}_{\frac{1}{5}} < \overline{p}_y + (\frac{1}{5} - y)t$, then the entrant can increase its profit by removing the top of the support of its mixed strategy and depositing it on an adequate lower price, because there exists a positive measure of $F_y(p_y)$ that does not undercut any incumbent's prices.[7] Suppose that $\overline{p}_0 = \overline{p}_y + yt$. Then $F_y(p_y)$ has a mass at $\overline{p}_y$, otherwise the incumbent can increase its payoff by removing the top of the support of $F_0(p_0)$ and depositing it on $\overline{p}_y$. Since $\overline{p}_y$ never undercuts any incumbent's prices, the entrant can increase its profit by removing the mass $\overline{p}_y$ of its mixed strategy and depositing it on an adequate lower price. Proof of the case $\overline{p}_{\frac{1}{5}} = \overline{p}_y + (\frac{1}{5} - y)t$ is same. Thus $\overline{p}_y \leq \frac{1}{4}(\frac{1}{5}t + \overline{p}_0 + \overline{p}_{\frac{1}{5}})$ holds in equilibrium.

Since $\overline{p}_y \leq \frac{1}{4}(\frac{2}{5}t + 2\overline{p}_y)$ holds by the above three inequalities, I get $\overline{p}_y \leq \frac{1}{10}t$. Then the upper bound of entrant's equilibrium profit is $\pi_E(F_y(p_y), F_I(p_0, p_{\frac{1}{5}})) = \pi_E(\overline{p}_y, F_I(p_0, p_{\frac{1}{5}})) \leq \pi_E(\overline{p}_y, \overline{p}_0, \overline{p}_{\frac{1}{5}}) \leq \frac{1}{5} \cdot \frac{1}{10}t = 0.02t$. ■

[7] If $\frac{1}{4}(\frac{1}{5}t + \overline{p}_0 + \overline{p}_{\frac{1}{5}})$ is not undercut with probability 1, choosing it is fine. If so, then pick a slightly higher price than $\min\{\overline{p}_0, \overline{p}_{\frac{1}{5}}\}$.

(ii) By the same logic in (i), $\overline{p}_y \leq \frac{1}{4}(\frac{4}{5}t + 2\overline{p}_y)$ holds in any equilibrium. Thus $\overline{p}_y \leq \frac{2}{5}t$ holds. Then the equilibrium expected payoff of entrant is $\pi_E(F_y(p_y), F_I(p_0, p_{\frac{2}{5}})) = \pi_E(\overline{p}_y, F_I(p_0, p_{\frac{2}{5}})) \leq \pi_E(\overline{p}_y, \overline{p}_0, \overline{p}_{\frac{2}{5}}) \leq \frac{2}{5} \cdot \frac{2}{5}t = 0.16t$. ■

(iii) I need to check the following two cases; $X' = \{0, \frac{3}{5}\}$ and $X' = \{0, \frac{3}{5}, \frac{4}{5}\}$.

First, consider the case of $X' = \{0, \frac{3}{5}\}$. By the same logic in (i), $\overline{p}_y$ does not undercuts incumbent's upper bound prices with positive measure, and $\overline{p}_y \leq \frac{1}{4}(\frac{3}{5}t + \overline{p}_0 + \overline{p}_{\frac{3}{5}})$ holds in any equilibrium. Notice that the arguments in (i) also hold for the incumbent. Thus $\overline{p}_0 \leq \frac{1}{4}[(\frac{2}{5} + y)t + \overline{p}_y + 2\overline{p}_{\frac{3}{5}}]$ and $\overline{p}_{\frac{3}{5}} \leq \frac{1}{4}[(1-y)t + \overline{p}_y + 2\overline{p}_0]$ hold in any equilibrium. By adding these inequalities, $\overline{p}_0 + \overline{p}_{\frac{3}{5}} \leq \frac{7}{10}t + \overline{p}_y$ holds. By substituting this into $\overline{p}_y \leq \frac{1}{4}(\frac{3}{5}t + \overline{p}_0 + \overline{p}_{\frac{3}{5}})$, I get $\overline{p}_y \leq \frac{13}{30}t$ and $\overline{p}_0 + \overline{p}_{\frac{3}{5}} \leq \frac{17}{15}t$.

Then the upper bound of entrant's equilibrium profit is $\pi_E(F_y(p_y), F_I(p_0, p_{\frac{3}{5}})) = \pi_E(\overline{p}_y, F_I(p_0, p_{\frac{3}{5}})) \leq \pi_E(\overline{p}_y, \overline{p}_0, \overline{p}_{\frac{3}{5}}) = \max\{\frac{1}{2t}(\frac{3}{5}t + \overline{p}_0 + \overline{p}_{\frac{3}{5}} - 2\overline{p}_y), 0\}\overline{p}_y = \frac{1}{2t}(\frac{3}{5}t + \overline{p}_0 + \overline{p}_{\frac{3}{5}} - 2\overline{p}_y)\overline{p}_y \leq \frac{169}{900}t \simeq 0.188t$.

Second, consider the case of $X' = \{0, \frac{3}{5}, \frac{4}{5}\}$. By the same logic in (i), $\overline{p}_y$ does not undercuts incumbent's upper bound prices with positive measure, and $\overline{p}_y \leq \frac{1}{4}(\frac{3}{5}t + \overline{p}_0 + \overline{p}_{\frac{3}{5}})$ holds in any equilibrium. Notice that the arguments in (i) also hold for the incumbent. Thus $\overline{p}_0 \leq \frac{1}{4}[(\frac{1}{5} + y)t + \overline{p}_y + 2\overline{p}_{\frac{4}{5}}]$, $\overline{p}_{\frac{3}{5}} \leq \frac{1}{4}[(\frac{4}{5} - y)t + \overline{p}_y + 2\overline{p}_{\frac{4}{5}}]$ and $\overline{p}_{\frac{4}{5}} \leq \frac{1}{4}(\frac{2}{5}t + 2\overline{p}_0 + 2\overline{p}_{\frac{3}{5}})$ hold in any equilibrium. By adding these inequalities, $\overline{p}_0 + \overline{p}_{\frac{3}{5}} \leq \frac{7}{10}t + \overline{p}_y$ holds. The rest of proof is just same in the above case. ■

(iv) By the same logic in (i), $\overline{p}_y$ does not undercuts incumbent's upper bound prices with positive measure, and $\overline{p}_y \leq \frac{1}{4}(\frac{4}{5}t + \overline{p}_0 + \overline{p}_{\frac{4}{5}})$ holds in any equilibrium. Notice that the arguments in (i) also hold for the incumbent. Thus $\overline{p}_0 \leq \frac{1}{4}[(\frac{1}{5} + y)t + \overline{p}_y + 2\overline{p}_{\frac{4}{5}}]$ and $\overline{p}_{\frac{4}{5}} \leq \frac{1}{4}[(1-y)t + \overline{p}_y + 2\overline{p}_0]$ hold in any equilibrium. By adding these inequalities, $\overline{p}_0 + \overline{p}_{\frac{4}{5}} \leq \frac{3}{5}t + \overline{p}_y$ holds. By substituting this into $\overline{p}_y \leq \frac{1}{4}(\frac{4}{5}t + \overline{p}_0 + \overline{p}_{\frac{4}{5}})$, I get $\overline{p}_y \leq \frac{7}{15}t$ and $\overline{p}_0 + \overline{p}_{\frac{4}{5}} \leq \frac{16}{15}t$.

Then the upper bound of entrant's equilibrium profit is $\pi_E(F_y(p_y), F_I(p_0, p_{\frac{4}{5}})) = \pi_E(\overline{p}_y, F_I(p_0, p_{\frac{4}{5}})) \leq \pi_E(\overline{p}_y, \overline{p}_0, \overline{p}_{\frac{4}{5}}) = \max\{\frac{1}{2t}(\frac{4}{5}t + \overline{p}_0 + \overline{p}_{\frac{4}{5}} - 2\overline{p}_y), 0\}\overline{p}_y = \frac{1}{2t}(\frac{4}{5}t + \overline{p}_0 + \overline{p}_{\frac{4}{5}} - 2\overline{p}_y)\overline{p}_y \leq \frac{49}{225}t \simeq 0.218t$. ■

5.2 Proof of Lemma 2

First, given $p_0^*, p_{\frac{4}{5}}^*$ and check the entrant's incentive. Suppose that the entrant does not undercut any incumbent's store price. The demand of the entrant without undercutting is $D_E = \max\{\frac{1}{2t}(\frac{4}{5}t + p_0^* + p_{\frac{4}{5}}^* - 2p_y), 0\} = \max\{\frac{1}{2t}(\frac{28}{15}t - 2p_y), 0\}$. By maximizing $\pi_E = D_E p_y$, it can be shown that $p_y^* = \frac{7}{15}t$ and $\pi_E^* = \frac{49}{225}t$ for any y. On the other hand, suppose that the entrant undercuts some incumbent's price. In order to undercut p_0^*, the entrant has to set $p_y < (\frac{7}{15} + \frac{1}{6}y)t - yt = (\frac{7}{15} - \frac{5}{6}y)t$. Also, in order to undercut $p_{\frac{4}{5}}^*$, the entrant has to set $p_y < (\frac{3}{5} - \frac{1}{6}y)t - (\frac{4}{5} - y)t = (-\frac{1}{5} + \frac{5}{6}y)t$. Since $y \in [\frac{3}{10}, \frac{2}{5}]$, the upper bound of entrant's profit is

$$\pi_E \leq p_y < (\frac{7}{15} - \frac{5}{6}y)t \leq \frac{13}{60}t < \frac{49}{225}t.$$

Therefore, given incumbent's prices the entrant chooses $p_y^* = \frac{7}{15}t$ in equilibrium.

Similarly, given $p_y^* = \frac{7}{15}t$, the demand of store 0 without undercutting is $D_0 = \max\{\frac{1}{2t}(\frac{1}{5}t + ty + p_y^* + p_{\frac{4}{5}} - 2p_0), 0\}$ and the demand of store $\frac{4}{5}$ without undercutting is $D_{\frac{4}{5}} = \max\{\frac{1}{2t}(t - ty + p_y^* + p_0 - 2p_{\frac{4}{5}}), 0\}$. Since the incumbent's profit is $\pi_I = D_0 p_0 + D_{\frac{4}{5}} p_{\frac{4}{5}}$, it can be shown that the incumbent does not have an incentive to undercut entrant's price in equilibrium, and optimal price tuple is $p_0^* = (\frac{7}{15} + \frac{1}{6}y)t, p_{\frac{4}{5}}^* = (\frac{3}{5} - \frac{1}{6}y)t$. The equilibrium profit satisfies $\pi_I^* \geq \frac{64}{225}t \simeq 0.284t$ for all $y \in [\frac{3}{10}, \frac{2}{5}]$. ■

5.3 Proof of Lemma 3

Denote y be entrant's store. Let l and r be the closest incumbent's stores such that $l < y < r$. Set $l = 0$ without loss of generality.

By the same logic in Lemma 1, $\overline{p}_y$ does not undercuts incumbent's upper bound prices with positive measure, and $\overline{p}_0 \leq \overline{p}_y + yt$, $\overline{p}_r \leq \overline{p}_y + (r - y)t$, and $\overline{p}_y \leq \frac{1}{4}(rt + \overline{p}_0 + \overline{p}_r)$ hold in any equilibrium. By combining these inequalities, $\overline{p}_y \leq rt$ holds. Since $r \in (0, 1)$, $\overline{p}_y \leq t$ holds for any locations and equilibrium in subgames.

Take any incumbent's store. If its price is more than $1.5t$, then the entrant undercuts it with probability 1. It implies the upper bound of incumbent's equilibrium profit is less than $1.5t$. ■

REFERENCES

Ashiya, M. (2000): "Weak Entrants Are Welcome," *International Journal of Industrial Organization*, 18, 975–984.

Dasgupta, P. and E. Maskin (1986): "The Existence of Equilibrium in Discontinuous Economic Games, I and II" *Review of Economic Studies*, 53, 1–41.

Eaton, C. and R. Lipsey (1979: "The Theory of Market Pre-emption: The Persistence of Excess Capacity and Monopoly in Growing Spatial Markets" *Economica*, 46, 149–158.

Hadfield, G. (1991): "Credible Spatial Preemption through Franchising," *RAND Journal of Economics*, 22(4), 531–543.

Ishibashi, I. (2003): "A Note on Credible Spatial Entry Deterrence," *International Journal of Industrial Organization*, 21, 283–289.

Judd, K. (1985): "Credible Spatial Preemption," *RAND Journal of Economics*, 16(2), 153–166.

Kats, A. (1995): "More on Hotelling's Stability in Competition," *International Journal of Industrial Organization*, 13, 89–93.

Salop, S. (1979a): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10, 141–156.

Salop, S. (1979b): "Strategic Entry Deterrence," *American Economic Review, Papers and Proceedings*, 69(2), 335–338.

Schmalensee, R. (1978): "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry," *Bell Journal of Economics*, 9, 305–327.

Vickrey, W. (1999): "Spatial competition, Monopolistic Competition, and Optimum Product Diversity," *International Journal of Industrial Organization*, 17, 953–963.

Wickelgren, A. L. (2006): "The Effect of Exit on Entry Deterrence Strategies," *Games and Economic Behavior*, 54, 226–240.