Notation

w = unit (wholesale) price that the entrant pays for access to the VIP's infrastructure.

u = incumbent VIP (V)'s upstream unit cost of production.

c_v = V's downstream unit cost of production.

c_e = entrant (E)'s downstream unit cost of production. ($c_e \in \{c_L, c_H\}$, where $c_L < c_H$).

F_u = V's fixed (upstream) cost of providing access.

F_d = V's fixed (downstream) cost of production.

F_e = E's fixed cost of production.

x_e = E's retail output.

x_v = V's retail output.

X = industry output. ($X = x_e + x_v$).

α = V's action, which affects E's cost. $\alpha \in \{0, \overline{\alpha}\}$.

$\underline{q}$ = probability $c_e = c_H$ when V undertakes the pro-competitive action ($\alpha = 0$).

$\overline{q}$ = probability $c_e = c_H$ when V undertakes the anticompetitive action ($\alpha = \overline{\alpha}$).

r = probability the regulator assesses V's action accurately. ($r \in [\frac{1}{2}, 1]$)
(Formally, r is the probability the regulator concludes $\alpha = \alpha_i$ when $\alpha = \alpha_i$, for $\alpha_i \in \{0, \overline{\alpha}\}$.)

$P(X) = a - bX$ = the industry (inverse) demand curve for the homogenous retail product.

$K(r)$ = regulator's cost of implementing detection probability r.

D_R = penalty V must pay when regulator determines $\alpha = \overline{\alpha}$. ($D_R \leq \overline{D}_R$)

f_R = the fraction of the regulatory penalty that is awarded to E.

D_C = penalty V must pay when convicted in court.

$\overline{d}$ = probability V incurs penalty D_C when $\alpha = \overline{\alpha}$.

$\underline{d}$ = probability V incurs penalty D_C when $\alpha = 0$. ($\underline{d} < \overline{d}$)

f_C = the fraction of the court penalty that is awarded to E.

$\underline{c} \equiv \underline{q}\, c_H + [1 - \underline{q}]\, c_L$; $\overline{c} \equiv \overline{q}\, c_H + [1 - \overline{q}]\, c_L$.

Assumptions

1. E requires access to V's infrastructure in order to produce.
   · Each unit of retail output requires one unit of access to produce.

2. E always enters the industry, and both firms serve some customers in equilibrium.

3. $\overline{q} > \underline{q}$, so V's anticompetitive activity increases the likelihood that E operates with the higher downstream unit cost of production.

4. $K(r)$ is increasing in r and $\lim_{r \to \frac{1}{2}} K_r(\cdot) = 0$.

5. The regulator always investigates V's behavior because there are no additional costs of monitoring once cost $K(\cdot)$ has been incurred.

6. $$[\, a - u + c_v - 2\,\underline{c}\,]^2 \;>\; 24\, b \left[\frac{\overline{D}_R}{2} + \underline{d}\; D_C + F_u \right] + \frac{4}{9} \left[\, \overline{q} - \underline{q} \,\right]^2 [\, c_H - c_L \,]^2 . \tag{1}$$

This assumption ensures: (i) industry demand (a) is sufficiently pronounced relative to V's upstream fixed cost of production and equilibrium expected penalties; and (ii) V's upstream unit cost (u) is sufficiently small that when w is set to ensure zero upstream profit for V, both firm produce strictly positive levels of retail output in equilibrium.

Timing

1. $\underline{d}$ and $\overline{d}$ are determined (exogenously for now).
   · $\overline{D}_R$ and D_C are also determined exogenously at the outset.

2. The regulator sets w, r, and D_R.

3. V chooses α and both firms observe E's downstream unit cost.

4. V chooses x_v and E chooses x_e, simultaneously and noncooperatively.

5. The regulatory investigation is undertaken.

6. It is determined whether V will face a judicial proceeding, and the outcome of any such proceeding is determined.

7. V pays any penalties that are levied.

Let $\underline{\pi}_v^u$ denote V's upstream profit when it undertakes the pro-competitive action. Suppose the regulator faces the participation constraint (PC) $\underline{\pi}_v^u \geq 0$. Also let $\underline{\pi}_v$ ($\overline{\pi}_v$) denote V's expected profit when it undertakes the pro-competitive (anticompetitive) action. Then the incentive compatibility constraint (IC) that the regulator must respect is $\underline{\pi}_v \geq \overline{\pi}_v$.

Expected consumers' surplus when V undertakes the pro-competitive action is:

$$S \equiv \underline{q}\, S(c_H) + \left[1 - \underline{q}\right] S(c_L) + [1 - r]\, D_R\, [1 - f_R] + [1 - f_C]\, \underline{d}\, D_C\,.$$

When she seeks to maximize expected consumers' surplus less monitoring costs, while inducing V to undertake the pro-competitive action, the regulator's problem, [RP], is to:

$$\underset{w \geq 0,\; r \in [\frac{1}{2}, 1]}{Maximize} \quad S - K(r) \quad \text{subject to} \quad \underline{\pi}_v^u \geq 0 \quad \text{and} \quad \underline{\pi}_v \geq \overline{\pi}_v.$$

The participation (IR) constraint and the incentive compatibility (IC) constraint each introduce a trade-off between w and r for the regulator.

<u>Trade-off along the IR Constraint</u>

An increase in the accuracy of the regulatory monitor (r) reduces the equilibrium expected regulatory penalty. Consequently, the regulator can reduce the access price (w) without reducing V's equilibrium expected upstream profit below 0.

<u>Trade-off along the IC Constraint</u>

An increase in the accuracy of the regulatory monitor (r) reduces the incremental gain that V anticipates from undertaking the anticompetitive action. Consequently, the regulator can reduce the access price (w) without inducing V to undertake the anticompetitive action. (The smaller is w, the more pronounced is V's incentive to undertake the anticompetitive action because V's opportunity cost of reducing E's retail output – reduced upstream profit due to reduced sales of access – declines as w declines).

Note 1. The ensuing analysis will consider settings where the participation constraint ($\underline{\pi}_v^u \geq 0$) and the incentive compatibility ($\underline{\pi}_v \geq \overline{\pi}_v$) constraint both bind at the solution to [RP].

Sufficient conditions are provided in the Appendix. The participation constraint will bind when V's upstream fixed cost of production (F_u) is sufficiently large. The incentive compatibility constraint will bind when a and $\left[\,\overline{q} - \underline{q}\,\right]\left[\,c_H - c_L\,\right]$ are large (so the impact of the anticompetitive action is pronounced) and $\left[\,\overline{d} - \underline{d}\,\right] D_C$ is not too large (so court oversight alone is insufficient to deter V from undertaking the anticompetitive action).

Note 2. We will also focus on settings in which it is prohibitively costly for the regulator to implement a perfect monitoring technology.

Sufficient conditions for $r \in (\frac{1}{2}, 1)$ at the solution to [RP] are provided in the Appendix.

Note 3. We will also focus on settings where the regulator optimally imposes the largest possible penalty on V when the regulatory monitor produces evidence that V has undertaken the anticompetitive action. Conclusion 1 provides a sufficient condition.

Conclusion 1. $D_R = \overline{D}_R$ *at the solution to [RP] if:*

$$[\,a - u + c_v - 2\,\underline{c}\,]^2 \;>\; \left[\frac{2\,a - 2\,u - \underline{c} - c_v}{3\,(1 - f_R\,)}\right]^2 + 24\,b\left[\frac{\overline{D}_R}{2} + \underline{d}\,D_C + F_u\right]. \tag{2}$$

Explanation of Conclusion 1.

Increasing D_R is costly because court monitoring is imperfect, and so V will incur a positive expected penalty even when it undertakes the pro-competitive action. Therefore, the regulator may not impose the maximum feasible regulatory penalty in order to avoid having to compensate V excessively for a very large equilibrium expected penalty.

However, the regulator will impose the maximum feasible regulatory penalty on V when:

1. $\overline{D}_R$ is relatively small, so even the maximum penalty will not increase V's expected equilibrium costs unduly.

2. $\underline{d}\,D_C$ is small, so imperfections in the court review process also do not serve to impose a large equilibrium expected penalty on V.

3. f_R is close to 0, so most of the regulatory penalty accrues to consumers (whose welfare the regulator values highly), rather than to E.

4. F_u is small, so the regulator optimally sets a relatively low upstream profit margin $(w - c_u)$. Consequently, V has substantial incentive to raise E's costs (which increases V's

retail profit but reduces E's demand for the input), and so the maximum feasible penalty on V can be valuable in precluding V from undertaking the anticompetitive action.

5. c_v is large, so V is a relatively inefficient downstream producer. V will be relatively inclined to raise E's cost in this setting to help offset V's innate cost disadvantage (or less pronounced advantage).

Further Explanation of Conclusion 1.

$h \equiv [\, a - u + c_v - 2\,\underline{c}\,]^2 - 24\, b \left[\frac{\overline{D}_R}{2} + \underline{d}\; D_C + F_u \right] > \frac{4}{9} \left[\, \overline{q} - \underline{q} \,\right]^2 [\, c_H - c_L \,]^2$, by assumption. Conclusion 1 states that $D_R = \overline{D}_R$ if $h > \left[\frac{2\,a - 2\,u - \underline{c} - c_v}{3(\,1 - f_R\,)} \right]^2$. Therefore, given the maintained assumption, $D_R = \overline{D}_R$ if $\frac{4}{9} \left[\, \overline{q} - \underline{q} \,\right]^2 [\, c_H - c_L \,]^2 > \left[\frac{2\,a - 2\,u - \underline{c} - c_v}{3(\,1 - f_R\,)} \right]^2$. This inequality tends to hold when:

1. $\overline{q} - \underline{q}$ is large, so the regulatory monitor is relatively accurate;

2. $c_H - c_L$ is large, so the anticompetitive behavior can increase E's cost (and thereby harm consumers) substantially; and

3. $1 - f_R$ is large, so consumers receive a large fraction of regulatory penalties.

Conclusion 2. *$\frac{\partial r}{\partial D_R} < 0$ and $\frac{\partial w}{\partial D_R} > 0$ at the solution to [RP].*

Explanation of Conclusion 2.

A larger regulatory penalty allows the regulator to reduce her monitoring costs by reducing the accuracy of her monitoring technology while still dissuading V from undertaking the anticompetitive action. The reduced accuracy of the regulatory monitor implies that V anticipates a higher equilibrium regulatory penalty. Therefore, the regulator increases w to ensure that V continues to earn 0 expected profit.

We now address an issue of primary interest, namely, whether (imperfect) antitrust enforcement enhances welfare.

Conclusion 3. $\frac{dS^*}{dD_C} > 0$ *if* $f_C < f_R - \frac{k}{\overline{D}_R}$.

Explanation of Conclusion 3.

A higher court penalty increases the value of the regulator's objective function when: (i) f_C is small, so most of the expected court penalty is awarded to consumers, rather than to E; and (ii) f_R is large, so much of the expected regulatory penalty is awarded to E, rather than to consumers. Furthermore, the expected regulatory penalty is large because $\overline{D}_R$ is large and k is small (so the regulator undertakes a relatively large amount of regulatory monitoring).

Conclusion 4. $\frac{dS^*}{dD_C} < 0$ *if* $f_C > f_R - \frac{k}{\overline{D}_R}$ *and*

$$f_C \; > \; \frac{1}{3} + \left[f_C - f_R + \frac{k}{\overline{D}_R} \right] \left[\frac{\overline{d} - \underline{d}}{\underline{d} + \overline{d}} \right] . \tag{3}$$

Explanation of Conclusion 4.

An increased court penalty can be detrimental when: (i) much of the court penalty is awarded to E rather than to consumers (since f_C is large); (ii) a substantial fraction of the regulatory penalty accrues to consumers (since f_R is not too much larger than f_C, to ensure $f_R < f_C + \frac{k}{\overline{D}_R}$); (iii) the court monitor is relatively inaccurate (since $\frac{\overline{d} - \underline{d}}{\underline{d} + \overline{d}}$ is relatively small), so the increased court penalty provides relatively little incremental deterrence and V must be compensated for the increased equilibrium expected court penalty; and (iv) the regulatory instrument is potentially powerful because it can be employed to create substantial deterrence at relatively low cost (since $\overline{D}_R$ is large and k is small).

Conclusion 5. $\frac{dS^*}{d\overline{d}} > 0$ *at the solution to [RP].*

Explanation of Conclusion 5. As $\overline{d}$ increases, the court becomes more likely to detect V's anticompetitive action accurately. This reduces V's incentive to undertake the anticompetitive action, relaxing the binding IC constraint.

Conclusion 6. $\frac{dS^*}{dk} < 0$ *at the solution to [RP].*

Explanation of Conclusion 6. When the IR and IC constraints both bind, the values of r^* and w^* do not vary with k. However, the increased cost of securing the same level of accuracy reduces welfare.

Conclusion 7. *If* $f_C < f_R - \frac{k}{\bar{D}_R}$, *then* $\frac{dS^*}{d\underline{d}} > 0$ *at the solution to [RP].*

Explanation of Conclusion 7. As $\underline{d}$ increases, V is more likely to incur the court penalty in equilibrium. To offset this higher expected court penalty, the regulator increases the accuracy of her monitor in order to reduce the equilibrium expected regulatory penalty. (See Conclusion 10 below.) The increase in expected court penalty and reduction in expected regulatory penalty increases expected consumer welfare if the fraction of the court penalty that accrues to consumers $(1-f_C)$ is sufficiently large relative to the fraction of the regulatory penalty that accrues to consumers $(1 - f_R)$.

Note: This finding is surprising. It suggests that consumers may be better off as the court system becomes a less reliable predictor of V's action.

Conclusion 8. *Suppose* $f_C > \max\{\, f_R - \frac{k}{\bar{D}_R},\ \frac{1}{2} f_R + \frac{1}{6}\,\}$ *and a is sufficiently large. Then* $\frac{dS^*}{d\underline{d}} < 0$ *at the solution to [RP].*

Explanation of Conclusion 8.

As $\underline{d}$ increases, V becomes more likely to incur the court penalty in equilibrium. The increased expected court penalty reduces expected consumer welfare if a sufficiently large fraction of the court penalty accrues to E rather than to consumers.

Conclusion 9. $\frac{\partial r}{\partial D_C} < 0$ *and* $\frac{\partial w}{\partial D_C} > 0$ *at the solution to [RP].*

Explanation of Conclusion 9.

A higher court penalty reduces V's incentive to undertake the anticompetitive action, so the regulator can reduce her monitoring costs by reducing the accuracy of the regulatory monitor (r) without violating the incentive compatibility constraint. When the court penalty increases, V faces a higher expected court penalty even when it undertakes the pro-competitive action. The associated reduction in r also increases the expected (regulatory) penalty that V incurs when it undertakes the pro-competitive action. Therefore, w is increased to ensure V continues to earn its reservation level of expected profit (i.e., to ensure the participation constraint is not violated).

Conclusion 10. $\frac{\partial r}{\partial \underline{d}} > 0$ *and* $\frac{\partial w}{\partial \underline{d}} > 0$ *at the solution to [RP].*

Explanation of Conclusion 10.

When $\underline{d}$ increases, the court monitor becomes less reliable in the sense that the court becomes more likely to impose the penalty on V even when it undertakes the pro-competitive action. This reduces V's equilibrium expected profit. The regulator increases w to increase V's equilibrium expected profit to ensure V's participation constraint is not violated.

The less accurate court monitor also reduces V's incentive to undertake the pro-competitive action. To ensure the incentive compatibility constraint is not violated, the regulator increases the accuracy of her monitoring technology.

Conclusion 11. $\frac{\partial r}{\partial \overline{d}} < 0$ *and* $\frac{\partial w}{\partial \overline{d}} > 0$ *at the solution to [RP].*

Explanation of Conclusion 11.

When $\overline{d}$ increases, the court monitor becomes more reliable in the sense that the court becomes more likely to impose the penalty on V if it undertakes the anticompetitive competitive action. Consequently, the regulator can reduce the (costly) accuracy of her monitoring technology without violating the incentive compatibility constraint.

The reduced accuracy of the regulatory monitor implies that V is more likely to incur the regulatory penalty in equilibrium. Consequently, the regulator must increase w to ensure V continues to earn 0 expected profit.

Conclusion 12. $\frac{\partial r}{\partial F_u} < 0$ *and* $\frac{\partial w}{\partial F_u} > 0$ *at the solution to [RP].*

Explanation of Conclusion 12.

When F_u increases, the regulator increases w to ensure that V continues to receive 0 expected profit. The increase in w makes sales of the input to E more profitable for V. Consequently, V perceives a reduced gain from reducing E's equilibrium output. Therefore, the regulator can reduce the (costly) accuracy of her monitoring technology without inducing V to undertake the anticompetitive action.

Conclusion 13. $\frac{\partial r}{\partial u} < 0$ *and* $\frac{\partial w}{\partial u} > 0$ *at the solution to [RP].*

Explanation of Conclusion 13.

When u increases, the regulator increases w to ensure that V continues to receive 0 expected profit. The higher w reduces E's equilibrium output, and so w must increase by more than u increases to ensure that V continues to earn 0 expected profit. The increase in $w - u$ makes sales of the input to E more profitable for V. Consequently, V perceives a reduced gain from reducing E's equilibrium output. Therefore, the regulator can reduce the (costly) accuracy of her monitoring technology without inducing V to undertake the anticompetitive action.

Conclusion 14. $\frac{\partial r}{\partial a} > 0$ *and* $\frac{\partial w}{\partial a} < 0$ *at the solution to [RP].*

Explanation of Conclusion 14.

When a increases, E produces more output, and so purchases more of the input from V. Consequently, the regulator can reduce w without reducing V's expected upstream profit below 0. The reduction in w and the increase in the scale of the retail market increases V's incentive to increase E's unit cost of production. To ensure that V continues to undertake the pro-competitive action, the regulator increases the accuracy of her monitoring technology.

Conclusion 15. $\frac{\partial r}{\partial z} = 0$ *and* $\frac{\partial w}{\partial z} = 0$ *at the solution to [RP] for* $z \in \{k, F_d, F_e\}$*.*

Explanation of Conclusion 15.

Changes in F_e and F_d do not affect either V's participation constraint (which reflects only V's upstream earnings) or V's incentive compatibility constraint (since, by assumption, both firms always serve retail customers in equilibrium). Therefore, changes in F_e and F_d do not affect the optimal regulatory policy.

k also does not enter either the IR or the IC constraint directly. Consequently, changes in k do not affect the trade-off between w and r the regulator faces either as it pertains to the IR constraint or as it pertains to the IC constraint. Consequently, the optimal resolution of these trade-offs does not vary as k changes, so there is a single (w, r) pair that continues to solve these equations simultaneously as k changes. The level of expected consumer welfare (S) that arises at the identified (w, r) pair declines as k increases, but the best way to achieve the (now lower) S does not change.

Conclusion 16. $\frac{\partial r}{\partial \underline{q}} < 0$ *and* $\frac{\partial w}{\partial \underline{q}} > 0$ *at the solution to [RP].*

Explanation of Conclusion 16. When $\underline{q}$ increases, E's cost becomes more likely to be high in equilibrium. Consequently, E's expected equilibrium output declines, which reduces V's equilibrium expected upstream profit. The regulator increases w to ensure V secures nonnegative profit on its upstream operation. The increase in V's upstream profit margin reduces V's incentive to increase E's unit cost of production (and thereby reduces E's expected retail output and thus its demand for the input). Consequently, the regulator can reduce the (costly) accuracy of the regulatory monitor without inducing V to undertake the anticompetitive action.

Conclusion 17. $\frac{\partial r}{\partial \overline{q}} > 0$ *and* $\frac{\partial w}{\partial \overline{q}} < 0$ *at the solution to [RP].*

Explanation of Conclusion 17. When $\overline{q}$ increases, V's anticompetitive action becomes more effective at raising E's cost. To induce V to refrain from the anticompetitive action, the regulator increases the accuracy of her monitoring technology. The increased accuracy reduces the expected regulatory penalty that V anticipates in equilibrium. Consequently, the regulator can reduce w without reducing V's equilibrium expected profit below zero.

Definitions.

$$Z_1 \equiv [a - u + c_v - 2\underline{c}]^2 - 25b\left[\frac{D_R}{2} + \underline{d}\, D_C + F_u\right].$$

$$Z_2 \equiv 9b\left[-\frac{D_R}{2} + (\overline{d} - 2\,\underline{d})\, D_C - F_u\right] + 2\left[\overline{q} - \underline{q}\right]\left[c_H - c_L\right]\left[3\,c_v - 2\,\underline{c} - \frac{c_L + c_H}{2}\right].$$

$$Z_3 \equiv 9b\left[D_R + (\overline{d} - 2\,\underline{d})\, D_C - F_u\right] + 2\left[\overline{q} - \underline{q}\right]\left[c_H - c_L\right]\left[3\,c_v - 2\,\underline{c} - \frac{c_L + c_H}{2}\right].$$

Conclusion 18. *At the solution to [RP]: (i)* $\frac{\partial r}{\partial c_v} < 0$ *if* $Z_1 > 0$*; (ii)* $\frac{\partial w}{\partial c_v} > 0$ *if* $Z_2 > 0$*; and (iii)* $\frac{\partial w}{\partial c_v} < 0$ *if* $Z_3 < 0$*.*

Explanation of Conclusion 18. The ambiguous impact of an increase in c_v may reflect two countervailing effects. First, when c_v increases, V's downstream profit margin declines, and so V anticipates a smaller increase in profit from increasing E's unit cost of production (since the resulting shift of output to V provides a smaller per-unit profit). The reduced incentive to undertake the anticompetitive action allows the regulator to reduce the accuracy of her monitor (r) without violating the IC constraint. The reduced accuracy implies an increased expected regulatory penalty on V, and so an increase in w is needed to ensure V continues to earn zero expected profit.

Second, when c_v increases, E produces more output in equilibrium, which increases V's upstream profit. Therefore, the regulator can reduce w without reducing V's expected profit below zero. The reduction in w reduces V's upstream profit margin, which increases V's incentive to raise E's cost (because the opportunity cost of doing so – foregone upstream profit – has declined). The regulator is thus inclined to increase r to reduce V's incentive to undertake the anticompetitive action.

Definitions

$$Z_4 \equiv \left[\overline{d} - \underline{d} \right] D_C - \frac{2}{9\,b} \left[4\,\underline{q} - \frac{1}{2} \right] \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]^2 .$$

$$Z_5 \equiv \left[\overline{q} - \underline{q} \right] \left\{ \frac{\left[\overline{d} - \underline{d} \right] D_C}{\frac{2}{9\,b} \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]} + \frac{c_H - c_L}{2} \right\}$$
$$- \frac{4\,\underline{q} \left[\frac{D_R}{2} + \underline{d}\, D_C + F_u \right] \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]}{\left[\overline{d} - \underline{d} \right] D_C + \left[3\,c_v - \frac{c_L + c_H}{2} - 2\,\underline{c} \right] \frac{2}{9\,b} \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]} .$$

$$Z_6 \equiv \left[\overline{q} - \underline{q} \right] \left\{ \frac{D_R + \left[\overline{d} - \underline{d} \right] D_C}{\frac{2}{9\,b} \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]} + \frac{c_H - c_L}{2} \right\}$$
$$- \frac{4\,\underline{q} \left[\underline{d}\, D_C + F_u \right] \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]}{D_R + \left[\overline{d} - \underline{d} \right] D_C + \left[3\,c_v - \frac{c_L + c_H}{2} - 2\,\underline{c} \right] \frac{2}{9\,b} \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]} .$$

Conclusion 19. *At the solution to [RP]: (i)* $\frac{\partial r}{\partial c_H} > 0$ *if* $Z_1 > 0$ *and* $Z_4 > 0$*; (ii)* $\frac{\partial w}{\partial c_H} < 0$ *if* $Z_5 > 0$*; and (iii)* $\frac{\partial w}{\partial c_H} > 0$ *if* $Z_6 < 0$.

Explanation of Conclusion 19. The ambiguous impact of c_H likely reflects the following countervailing forces. An increase in c_H increases V's incentive to undertake the anticompetitive action because the action now increases the likelihood of a relatively large increase in E's unit cost of production (since $c_H - c_L$ has increased). In contrast, an increase in c_H reduces V's incentive to undertake the anticompetitive action because E's equilibrium expected output is lower due to its higher expected production cost, and so V's expected gain from increasing E's unit cost declines.

Conclusion 20. *If* $\left[\overline{d} - \underline{d} \right] D_C > \frac{1}{9\,b} \left[\overline{q} - \underline{q} \right] \left[c_H - c_L \right]^2$*, then* $\frac{\partial r}{\partial c_L} < 0$ *and* $\frac{\partial w}{\partial c_L} > 0$ *at the solution to [RP].*

Explanation of Conclusion 20. The anticompetitive action becomes less profitable for V as c_L increases, because the action promotes a smaller increase in E's unit cost of production. Consequently, V becomes less inclined to undertake the anticompetitive action, and so the regulator can reduce the (costly) accuracy of her monitor without violating the IC constraint. When r declines, V becomes more likely to incur the regulatory penalty in equilibrium. The regulator increases w to ensure V continues to earn zero expected profit.

Appendix

Definitions.

$$w\,(1)\big|_{IR} \;=\; \frac{1}{4}\,[\,a + 3\,u + c_v - 2\,\underline{c}\,] \\ \qquad - \;\frac{1}{4}\left(\,[\,a - \;u + c_v - 2\,\underline{c}\,]^2 - 24\,b\,[\,\underline{d}\;D_C + F_u\,]\,\right)^{\frac{1}{2}}. \tag{4}$$

$$w\left(\frac{1}{2}\right)\bigg|_{IC} \;=\; -\,\frac{\left[\,\overline{d} - \underline{d}\,\right] D_C}{\frac{4}{9\,b}\left[\,\overline{q} - \underline{q}\,\right][\,c_H - c_L\,]} + \frac{1}{4}\,[\,2\,a + 2\,u + c_L + c_H - 4\,c_v\,]\,. \tag{5}$$

Conclusion 21. *If* $D_R = \overline{D}_R$*, then* $r \in \left(\frac{1}{2}, 1\right)$ *at the solution to [RP] if:*

$$\frac{\left[\,\overline{d} - \underline{d}\,\right] D_C}{\frac{1}{9\,b}\left[\,\overline{q} - \underline{q}\,\right][\,c_H - c_L\,]} - \left([\,a - u + c_v - 2\,\underline{c}\,]^2 - 24\,b\left[\,\frac{D_R}{2} + \underline{d}\;D_C + F_u\,\right]\right)^{\frac{1}{2}} \\ <\; a - \;u + c_L + c_H - 5\,c_v + 2\,\underline{c} \;<\; \frac{D_R + \left[\,\overline{d} - \underline{d}\,\right] D_C}{\frac{1}{9\,b}\left[\,\overline{q} - \underline{q}\,\right][\,c_H - c_L\,]} \\ \qquad - \left(\,[\,a - u + c_v - 2\,\underline{c}\,]^2 - 24\,b\,[\,\underline{d}\;D_C + F_u\,]\,\right)^{\frac{1}{2}}\,; \tag{6}$$

$$\frac{\frac{1}{9\,b}\left[\,2\,a - w\left(\frac{1}{2}\right)\big|_{IC} - u - \underline{c} - c_v\,\right]}{1 - f_R} \;>\; \frac{2}{9\,b}\left[\,\overline{q} - \underline{q}\,\right][\,c_H - c_L\,]\;;\;\; \textit{and} \tag{7}$$

$$\frac{\frac{1}{9\,b}\,[\,2\,a - w\,(1)|_{IR} - u - \underline{c} - c_v\,]}{k + D_R\,[\,1 - f_R\,]} \;<\; \frac{a + 3\,u + c_v - 2\,\underline{c} - 4\;\,w\,(1)|_{IR}}{3\,b\,D_R}\,. \tag{8}$$

Conclusion 22. *Suppose* $r \in \left(\frac{1}{2}, 1\right)$ *at the solution to [RP]. Then* $D_R = \overline{D}_R$ *and the participation and the incentive compatibility constraints both bind if (2) holds and:*

$$7\,a - 7\,u - 5\,c_v - 2\,\underline{c} \;>\; 8\left[\,\overline{q} - \underline{q}\,\right][\,c_H - c_L\,]\left[\,\frac{k}{\overline{D}_R} + 1 - f_R\,\right]. \tag{9}$$

Conclusion 23. *Suppose (2), (6), (7), (8), and (9) hold. Then* $D_R = \overline{D}_R$, $r \in (\frac{1}{2}, 1)$*, and both the IR and the IC constraints bind at the solution to [RP].*