# Bowley Duopoly under Vertical Relations

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*Abstract*. Compared to well-known oligopoly models of Cournot, etc, the so-called Bowley duopoly is less known, almost ignored in the literature. This neglect reflects the assumption that as a leader–leader model incorporating apparent excess rivalry it is presumably untenable, at least in theory. However, it is in fact observable in practice. Furthermore, the predicted excess competition is not only observable empirically but also accountable theoretically. We show how excess competition emerges when an upstream monopolist offers the downstream retailers a compensated game in which each acts as a leader. The outcome is not only stable but also benefits all involved actors, including consumers under vertically-related markets, such as those presided over by a monopolist producer. This result of emergent stability shows that the so-called Bowley duopoly should be considered alongside other oligopoly models.
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## 1. INTRODUCTION

According to Stackelberg (1934), a leader–follower (asymmetric) duopoly cannot maintain equilibrium indefinitely. Because the follower earns less profit than the leader, neither party will play the inferior role of the follower. Given that whichever party finds themselves in the role of follower will seek to overtake the other party to become leader, it is impossible to determine which one of the two will eventually win. Stackelberg apparently considers this to be a state of dynamics, saying, 'eventually the duopolist who initially gave in will make a new attempt to regain market dominance − so that in the end, the Bowley (symmetric) duopoly re-occurs' (Stackelberg, 2011, p.19).

But recent oligopoly theory (e.g. Kreps, 1990; Wolfstetter, 1999) confirms that a Bowley duopoly, a term coined by Stackelberg, lacks equilibrium inasmuch as both duopolists seek to lead the market. In quantity-setting simple duopolies featuring a homogeneous product, both duopolists will be thwarted in their attempts to lead the market because each will wrongly assume that the rival player will behave as a follower, not seek to usurp the leader position. This misunderstanding will produce a recurring competition that prevents equilibrium from emerging.

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The present paper examines whether or not such a self-generating theoretical result is in fact inescapable even under more realistic settings. For this purpose we consider two vertically related but decentralized markets with upstream and downstream stages of distribution under conditions of intra-brand competition. We will show that Bowley equilibrium can emerge in the downstream market despite the apparent superiority of the Stackelberg leader–follower competition, to its leader at least.

To empirically test this theory, two contemporary Japanese retail industries, such as frozen food and beer, are noted for engaging in endless price-cutting competition. In such industries, the so-called 'price destruction' may lead retail margins to become miniscule, or even negative. In such circumstances, producers have even reportedly offered rebates to offset retailers' losses (Itoh, 1995; Miwa, Nishimura and Ramseyer, 2002). Omoto (2006) in particular demonstrates that over the past two decades, processed-food manufacturer *Ajinomoto Co., Inc.* and beer producer *Asahi Breweries, Ltd.* have provided their retailers with rebates ranging from 3.5 to 5 and from 7 to 9 percent of gross sales proceeds per annum, respectively.

Getting back to theory, note that for the one firm to enjoy the role of a leader and impose the follower role on the other firm, the former needs to aggressively and irrevocably invest, e.g. by accumulating stock or constructing a production facility (Spence, 1977, 1979; Dixit, 1979, 1980).[1] Such logic led to studies on endogenous leader–follower relations in duopolies under uncertainty and on the order of decision-making by players (Spencer and Brander, 1992; Sadanand and Sadanand, 1996; van Damme and Hurkens, 1999).

The studies mentioned above, however, ignore the vertical structure of markets. So far as the goods sold in the retail markets are generally purchased in wholesale markets, upstream and downstream markets are vertically interdependent. Taking this into account, not only the recent quantity-setting intra-brand competition models by Whinston (2006), Ray and Tirole (2007) but also more traditional successive monopoly/oligopoly models (Greenhut and Ohta, 1976, 1979; Abiru et al., 1998) do investigate vertically related industries. But they usually treat retailers as symmetric duopolists or oligopolists.

With these previous researches in mind, this paper considers two alternative distribution systems, each consisting of three participants: one producer and two distributors. In the upstream market, the producer and the two distributors act as a monopolistic seller and

[1] See Tirole (1988) and Vives (1999) for the investment effects on product positioning, product compatibility decision-making and incidence of switching costs: hasty decision-making may incur higher costs. Therefore, if uncertainty exists in demand, production costs and other factors, the advantages of 'bringing decision-making forward' tend to decrease as uncertainty increases. Brander and Spencer (1985), Karp and Perloff (1993) also show that similar relations arise in international rivalries when a government subsidises domestic firms to increase production or imposes tariffs on imports. Negishi and Okuguchi (1972) also consider the situation arising from introducing subjective cost functions, namely the possibility of a leader–leader duopoly equilibrium. Szidarovszky et al. (1991) generalise this result to an N-person game. However, these studies are predicated upon severe cost conditions for the equilibrium to be established.

competitive buyers, respectively. Upon purchasing, the two buyers in turn visit the final market downstream to sell their commodities, at which point they act as duopolistic sellers.

The upstream producer initially proposes that the distributors choose between two alternative modes of competition: Stackelberg (asymmetric) or Bowley (symmetric) duopoly. The retailers then adopt either a Stackelberg leader (or follower) or a Bowley leader (-leader) position (to be explained more fully below). The retailer's role as either a Stackelberg leader or a Bowley leader fundamentally *differs in terms of the rival retailer's reactions*, provided that the mode of duopoly is determined *in advance*.[2]

Next, we show a leader–leader equilibrium in which no struggle for dominance recurs. We show this by extending the analysis to a case in which the three players can make subsequent decisions. The retailers face a decision regarding which mode of competition to select between the choices presented by the producer, and make subsequent, not simultaneous decisions. We finally compare retail prices and social surpluses under alternative distribution systems available.

The remainder of the paper is organized as follows. Section 2 sets forth our basic assumptions to present the two alternative models of intra-brand competition in a downstream retail market: a Stackelberg versus Bowley duopoly. Section 3 investigates the upstream producer's feasible strategy of enticing the retailers to select the Bowley duopoly downstream as a first tactic, namely by offering them tentative compensation for any loss arising from their role in the duopoly. We show that this Bowley duopoly is superior to the alternative system in net profit, net of rebates. This result shows that the Bowley duopoly is indeed the rational mode of duopoly for the producer, as well as offers the optimal roles, i.e. leader–leader, to the two retailers. Section 4 investigates the effects of producer's choice of duopoly mode on society by comparing retail prices and social welfare under the two alternative distribution systems. Section 5 concludes the paper.

## 2. ALTERNATIVE DISTRIBUTION SYSTEMS WITH ONE PRODUCER AND TWO RETAILERS

Consider two vertically related but decentralized markets in upstream and downstream stages of distribution under conditions of intra-brand competition. Such a vertically related industry is typically characterized by the existence of many more firms downstream than upstream. The stylized facts of this structure may be incorporated in our simplifying the assumptions below:

[2] In this paper, a producer first selects one of the two possible alternative modes of duopolistic retail systems, and retailers in turn select their roles that are retrospectively characterized within the framework of the system. Therefore, the positioning of a Bowley leader and a Stackelberg leader distinctively differs from their rivals' perspectives, even though both are called leaders.

a) One upstream firm produces an identical product that the two retailers buy and then sell in the final market downstream.

b) The producer as a wholesaler upstream is a monopolist to the retailers, who in turn are quantity-setting duopolists in their final market downstream.

c) The producer offers a lump-sum rebate to a retailer as compensation for any loss incurred by playing the role of either a follower or a leader in their rivalry in the downstream market.

d) The downstream retailers can predict the upstream seller's incentives to provide the right amount of rebates; all three parties share this as a common knowledge.[3]

e) Final market demand $q^{\sigma}$ is a linear (inverse) function of retail price $p^{\sigma}$:

$$p^{\sigma} = a - q^{\sigma}\,, \quad \sigma = s \text{ or } b\,, \tag{1}^{\sigma}$$

where $a$ is a reservation price. Superscripts $s$ and $b$ denote Stackelberg and Bowley, respectively.

f) Unit costs of production $k$ are constant (so are marginal costs):

$$K(q^{\sigma})/q = K'(q^{\sigma}) = k\,, \qquad 0 < k < a\,, \tag{2}$$

where $K(q^{\sigma})$ is total cost as a function of $q^{\sigma}$, output produced.

Using these assumptions, we now compare two alternative models of distribution systems in subsections 2.1 and 2.2. In particular, subsection 2.1 probes the fundamental features of a distribution system with a monopolistic wholesaler facing Stackelberg duopolistic retailers. Subsection 2.2, in turn, examines how a Bowley duopoly may be compared to its Stackelberg counterpart in the retail market.

### 2.1. Distribution system with a monopolistic producer and Stackelberg retailers [4]

First, consider the optimization problem for a retailer $j$ ($j \neq i; i = 1$ or $2$) as a Stackelberg follower, which can be formally stated by letting $\pi_j^{dsf}$ be follower $j$'s profit:

$$\max_{q_j^{sf}} \ \pi_j^{dsf} = (p^s - p^{ws})q_j^{sf} + A_j^{sf}$$

$$\text{s.t. } (1)^s \text{ and } q^s = q_i^{sl} + q_j^{sf} \text{ for any given } q_i^{sl} \text{ and } p^{ws} \qquad (j \neq i; i = 1 \text{ or } 2), \tag{3}$$

where $p^s$ is the retail price and $p^{ws}$ is the wholesale price, while $q_i^{sl}, q_j^{sf}$ are the quantities supplied by Stackelberg leader $i$ and follower $j$, respectively, $q^s$ is the total goods supplied, and $A_j^{sf}$ is the lump-sum rebate amount paid to follower $j$. Here superscripts $d$ and $w$ ($l$ and $f$) indicate downstream and wholesale (leader and follower),

[3] The right amount of rebate for either a Stackelberg follower or Bowley duopolists would offset their losses to make them as well-off as a Stackelberg leader.

[4] If we assume away the existence of wholesale markets, traditional Stackelberg and Bowley duopolies are extensive-form games of complete and perfect information in which one firm chooses its output first and the other firm, having observed the former's choice, chooses its output in turn (Gibbons, 1992).

respectively. While wholesale price $p^{ws}$ is given to the price-taking retailers, it is endogenously determined in the wholesale market.

Solving (3) for $q_j^{sf}$ yields optimal strategies for follower $j$ as shown in the following reaction function:

$$q_j^{sf} = \frac{a - p^{ws} - q_i^{sl}}{2}. \tag{4}$$

Anticipating the best-response strategies of Stackelberg follower $j$, the leader $i$ ($i = 1$ or 2) makes optimal decisions by solving the following optimization problem:

$$\max_{q_i^{sl}} \ \pi_i^{dsl} = (p^s - p^{ws})q_i^{sl} + A_i^{sl}$$

$$\text{s.t.}\ (1)^s \text{ and } q_j^{sf} = \left(a - p^{ws} - q_i^{sl}\right)/2 \text{ for any given } p^{ws}\ (j \neq i; i = 1 \text{ or } 2), \tag{5}$$

where $\pi_i^{dsl}$ is Stackelberg leader $i$'s profit, and $A_i^{sl}$ is a lump-sum rebate to the leader $i$.[5] Solving (5) for $q_i^{sl}$ yields

$$q_i^{sl} = \frac{a - p^{ws}}{2}. \qquad i = 1 \text{ or } 2 \tag{6}$$

Substituting (6) into (4) yields an optimal quantity choice for follower $j$:

$$q_j^{sf} = \frac{a - p^{ws}}{4}. \qquad j \neq i \tag{7}$$

Summing (6) and (7) in turn yields the derived demand for $q^s$ sold by the producer, given by the following inverse demand function:

$$p^{ws} = a - \frac{4}{3}q^s. \tag{8}$$

Now, consider the profit for an upstream producer $\pi^{u\sigma}(\sigma = s \text{ or } b)$ defined as net sales revenue from its two retailers minus relevant rebates paid, where superscript $u$ refers to upstream.[5] The profit maximization problem for the upstream producer under Stackelberg duopoly downstream therefore can be formally described as

$$\max_{q^s} \pi^{us} = (p^{ws} - k)q^s - A^s \ \text{s.t.}\ (8) \text{ and } A^s = A_i^{sl} + A_j^{sf} \qquad (j \neq i; i = 1 \text{ or } 2), \tag{9}$$

where $q^s$ is the quantity produced and $A^s$ is the total rebate offered by the producer (under duopoly $s$), respectively.[6] Solving (9) for $q^s$, we obtain

$$(1 - \alpha^s)\left(a - k - \frac{8}{3}q^s\right) = 0, \tag{10}$$

[5] One may wonder if offering rebates even in the case of Stackelberg competition serves as a rational strategy for the producer. In fact, it does insofar as the retailers are (assumed to be) free to choose their modes of competition. The producer is also free to choose a right rebate amount to offer any leader or follower.

[6] The producer offers lump-sum rebate $A^\sigma$ to retailers under either duopoly $s$ or $b$, to compensate for their losses and motivate them to maintain the retail price suggested or desired by the producer (Wako and Ohta, 2005).

where $\alpha^s$ is defined as the rebate rate implicitly by $A^s = \alpha^s(p^{ws} - k)q^s$.

It is important to here note that Equation (10) yields a unique solution to $q^s$ regardless of whether $\alpha^s(< 1)$. Because the producer's net average revenue originates from the derived demand, which does not depend on $\alpha^s$, his/her optimal output quantity, determined when marginal revenue equals constant marginal cost, remains independent of the rebate amounts (Wako and Ohta, 2005).

Equations (10), (8) and $(1)^s$ are used to derive $q^{s*}, p^{ws*}$ and $p^{s*}$ in equilibrium as follows:

$$q^{s*} = \frac{3(a-k)}{8},\ p^{ws*} = \frac{a+k}{2},\ p^{s*} = \frac{5a+3k}{8}. \tag{11}$$

Equilibrium profits without rebates, $\pi_i^{dsl*}|_{\alpha^s=0}, \pi_j^{dsf*}|_{\alpha^s=0}$ and $\pi^{us*}|_{\alpha^s=0}$, are also derived from equations representing the related optimization problems introduced in (3), (5) and (9):

$$\pi_i^{dsl*}|_{\alpha^s=0} = \frac{1}{32}(a-k)^2,\ \pi_j^{dsf*}|_{\alpha^s=0} = \frac{1}{64}(a-k)^2,\ \pi^{us*}|_{\alpha^s=0} = \frac{3}{16}(a-k)^2 \tag{12}$$

With these outcomes in mind we now proceed to subsection 2.2.

### 2.2. Distribution system with a monopolistic producer and Bowley retailers

Each Bowley duopolist downstream under the considered vertical structure knows, unlike the classic Bowley leader, that the rival would never remain on his/her reaction function. Thus, our Bowley leader knows that he/she is unable to make the profits that a genuine classic Bowley duopolist aspires. Nevertheless, he/she also conjectures correctly that if he/she behaved as if he/she were in fact a genuine 'Bowley duopolist' and failed, his/her forgone profits will be completely indemnified by the upstream producer via rebating.

The producer conjectures that the retailers will behave as if they were Bowley leaders, who in turn conjecture that their rival will behave as a follower/price-taker. The retailers, in turn, comply with the producer's conjecture and behave as Bowley leaders despite knowing that their rival is not a price-taker. Correctly anticipating the resultant loss incurred or the profit foregone, the retailer in turn conjectures that the producer will compensate him/her for any such loss incurred due to the rival's rational reactions. The upshot: no Bowley leader downstream will make any profit, unlike the case for a Stackelberg leader. Nevertheless, behaving as Bowley leaders, i.e. not behaving as if they are the Stackelberg leaders, following a conjecture that the producer will rebate them *ex post*, is a successful strategy.

Should each retailer $j$ ($j$ =1, 2) have accepted the follower role, his/her profit maximization problem could have been expressed as follows by letting $\pi_j^{dbf}$ be follower $j$'s profit:

$$\max_{q_j^{bf}} \pi_j^{dbf} = (p^b - p^{wb})q_j^{bf}$$

$$\text{s.t. } (1)^b \text{ and } q^b = q_i^{bl} + q_j^{bf} \text{ for any given } q_i^{bl} \text{ and } p^{wb} \qquad (j = 1, 2; i \neq j), \qquad (13)$$

where $p^b$ is the retail price and $p^{wb}$ is the given wholesale price. In addition, $q_j^{bf}$ and $q_i^{bl}$ are the quantities supplied by the conjectured follower $j$ and Bowley leader $i$ ($i \neq j$), respectively, and $q^b$ is the total quantity supplied.

Solving (13) for $q_j^{bf}$ yields the following optimal strategies for retailer $j$ ($j$ = 1, 2) as a conjectural follower:

$$q_j^{bf} = \frac{a - p^{wb} - q_i^{bl}}{2}, \qquad j = 1, 2;\ i \neq j. \qquad (14)$$

However, each retailer $i$ ($i$ = 1, 2) intends to act as a leader.[7] Therefore, we denote Bowley leader $i$'s profit by $\pi_i^{dbl}$. In a case in which the behaviour of rival $j$ ($j \neq i$) is governed by its reaction function, Bowley leader $i$ can make optimal choices by solving

$$\max_{q_i^{bl}} \pi_i^{dbl} = (p^b - p^{wb})q_i^{bl} + A_i^b$$

$$\text{s.t. } (1)^b, q_j^{bl} = \left(a - p^{wb} - q_i^{bl}\right)/2 \ \ and\ q^b = \sum_{i=1}^{2} q_i^{bl} \text{ for any given } p^{wb}\ (i = 1, 2; j \neq i), \qquad (15)$$

where $A_i^b$ is the rebate allotted to Bowley leader $i$ and $q_j^{bl}$ is the conjectural quantity choice by rival $j$ ($j \neq i$) via (14). Solving (15) for $q_i^{bl}$ yields

$$q_i^{bl} = \frac{a - p^{wb}}{2}, \qquad i = 1, 2 \qquad (16)$$

Summing (16) over $i$ = 1, 2 yields (in inverse form) the derived demand for $q^b$ wholesaled by the producer:

$$p^{wb} = a - q^b. \qquad (17)$$

The profit maximization problem for an upstream monopolistic producer intending to establish a Bowley duopoly downstream can be formally described as

$$\max_{q^b} \pi^{ub} = (p^{wb} - k)q^b - A^b \text{ s.t. } (17) \text{ and } A^b = \sum_{i=1}^{2} A_i^b, \qquad (18)$$

where $A^b = \alpha^b(p^{wb} - k)q^b$, $\alpha^b (\geq 0)$ is the rebate rate and $q^b$ is the quantity supplied

[7] In a traditional Bowley duopoly, the equilibrium actions of the two firms are inconsistent with the conjectures each firm is supposed to hold. We propose an alternative hypothesis: each retailer believes that the other's behaviour is not governed by his reaction function and neither one of the reaction functions is obeyed. Instead, each retailer believes that acting as a leader leads to loss and related compensation from the producer, while the producer offers to compensate their loss in fact. Under these conditions, equilibrium actions of the three parties are consistent with their initial conjectures. Thus, while a traditional Bowley duopoly equilibrium is untenable, our Bowley leader–leader duopoly equilibrium is stable and optimal.

by the producer. Solving (18) for $q^b$ yields

$$(1-\alpha^b)(a-k-2q^b)=0\,. \tag{19}$$

Note that (19) yields a unique solution to $q^b$, like (10), to yield $q^s$, regardless of $\alpha^b(<1)$.

Obtained from Equations (19), (17) and( 1 )$^b$, $q^{b*}$, $p^{wb*}$ and $p^{b*}$ in the equilibrium are:

$$q^{b*}=\frac{a-k}{2},\ p^{wb*}=\frac{a+k}{2},\ p^{b*}=\frac{a+k}{2}. \tag{20}$$

Note that equilibrium output $q^{b*}$ under the distribution system formed by a monopolistic wholesale market and a Bowley duopolistic retail market equals the output $(a-k)/2$ under a simple monopoly. This reflects the fact that Bowley leaders downstream fail to obtain any retail margins *ex post* due to excess competition. Equilibrium profits without rebates from the producer, $\pi_i^{dbl*}|_{\alpha^b=0}$ and $\pi^{ub*}|_{\alpha^b=0}$, are also derived from equations (15) and (18) as related optimization problems:

$$\pi_i^{dbl*}|_{\alpha^b=0}=0,\ \pi^{ub*}|_{\alpha^b=0}=\frac{1}{4}(a-k)^2\,. \tag{21}$$

Thus with no rebates, as (21) clearly shows, Bowley duopoly brings no profits at all to retailers despite the maximum possible profit to the producer. This outcome being untenable, calls for further analysis below to show how rebating works.

## 3. PRODUCER'S CONTRIVANCE FOR BOWLEY DUOPOLY

We are finally in a position to show that if the producer upstream guarantees the retailers downstream the same profit regardless of competition mode they may choose, then he/she can contrive Bowley competition downstream. Analytical results of Section 2 reveal that with no rebates the retailer can maximize profit only as a Stackelberg leader. Hence each player would opt for being a Stackelberg leader rather than a Bowley leader with no follower possibly existing.[8] Thus, the two retailers have a clear conflict over their roles, leading to the difficult question of who should take the lead in the alternative modes of duopoly. This issue arises because the producer has an incentive to contrive a Bowley rather than a Stackelberg duopoly downstream, while the retailers find a Stackelberg leader's profits more attractive.

Can the producer guarantee retailers a Stackelberg leader's profits while maintaining a Bowley duopoly without provoking economic warfare and ruinous competition as a result? To resolve this question, we consider a tentative allotment of rebates for two retailers in the two possible alternative (but not necessarily actual) duopolies. Note that they have no strategic incentive to decline compensation for their losses (assumptions c)

[8] Retailers raise their opportunity costs by accepting any role but that of a Stackelberg leader. Thus, struggles over a leadership position may recur unless a more advantageous external opportunity arises.

and d)).

In addition, note that only in a Stackelberg duopoly will retail profits exhibit asymmetry with respect to the retailer's role in the duopoly. We therefore specify the relations between the total amount of rebates $A^{\sigma}(\sigma = s \text{ or } b)$ and allotted rebates $A_{\varphi}^{\sigma\tau}(\tau = f \text{ or } l; \varphi = i \text{ or } j)$ in two possible alternative duopolies, which are established to be equally profitable through optimal rebating: rebates allotted to each retailer ($i$ = 1, 2) in Bowley duopolies will be half the amount of total rebates paid, whereas in a Stackelberg duopoly, the entire rebate amount is offered to follower $j$ ($j \neq i$). No rebate is paid to leader $i$ ($i$ = 1 or 2).

$$A_i^{sl} = 0,\ A_j^{sf} = A^s;\ A_i^b = \frac{A^b}{2} \tag{22}$$

According to the specifications above, profits for the Stackelberg leader, Stackelberg follower and producer, namely, $\pi_i^{dsl*}, \pi_j^{dsf*}$ and $\pi^{us*}$, respectively, are modified as

$$\pi_i^{dsl*} = \frac{1}{32}(a-k)^2,\ \pi_j^{dsf*} = \frac{(1+12\alpha^s)(a-k)^2}{64},\ \pi^{us*} = \frac{3(1-\alpha^s)(a-k)^2}{16} \qquad \alpha^s \geq 0 \quad (12)'$$

Similarly, profits for Bowley leaders $\pi_i^{dbl*}$ and profit for producer $\pi^{ub*}$ can also be rewritten as

$$\pi_i^{dbl*} = \frac{\alpha^b(a-k)^2}{8},\ \pi^{ub*} = \frac{(1-\alpha^b)(a-k)^2}{4} \qquad \alpha^b \geq 0 \quad (21)'$$

Next, suppose that producer assures a Stackelberg leader's profits to retailers under Stackelberg and Bowley duopolies by optimally adjusting rebate rates $\alpha^{\sigma}(\sigma = s \text{ or } b)$. Then, *pro forma* equilibrium profits for the Stackelberg follower and its producer, $\pi_j^{dsf*}$and $\pi^{us*}$, adjusted by an optimal rebate rate $\alpha^s = 1/12$ are:

$$\pi_j^{dsf*}\big|_{\alpha^s=1/12} = \pi_i^{dsl*}\big|_{\alpha^s=0},\ \pi^{us*}\big|_{\alpha^s=1/12} = \frac{11}{64}(a-k)^2 \tag{23}$$

Similarly, respective *pro forma* equilibrium profits for Bowley leaders and their producer, $\pi_i^{dbl*}$and $\pi^{ub*}$, adjusted by an optimal rate $\alpha^b = 1/4$ are:

$$\pi_i^{dbl*}\big|_{\alpha^b=1/4} = \pi_i^{dsl*}\big|_{\alpha^s=0},\ \pi^{ub*}\big|_{\alpha^b=1/4} = \frac{3}{16}(a-k)^2 \tag{24}$$

Comparing producers' *pro forma* profits under the two alternative systems leads to:

$$\pi^{ub*}\big|_{\alpha^b=1/4} = \frac{3}{16}(a-k)^2 > \pi^{us*}\big|_{\alpha^s=1/12} = \frac{11}{64}(a-k)^2 \tag{25}$$

We have yet to demonstrate how a unique vertical industry incorporating a rational mode of downstream duopoly may emerge. If the retailers eventually accept possible loss compensation from the producer via rebates in each suggested retail system [assumptions c) and d)], then the analysis above reveals that every alternative role under any given system yields the same profit.

Under these circumstances, first, the retailers have no incentive to change their *pro forma* roles in every alternative retail system suggested by the producer, since they will not be better off by doing so; second, the retailers are indifferent between the two possible alternative modes of competition. This implies that they have little or no incentive to reject any particular retail system that the producer may offer/propose. Therefore, the producer can indeed entice the retailers to behave as he/she desires.

The producer's profits from the Stackelberg and Bowley retail systems, by contrast, are $11/64(a-k)^2$ and $3/16(a-k)^2$, respectively, in (25). The latter being higher, he/she will clearly prefer the Bowley retail system to Stackelberg retail system. Therefore, unique and rational choices of the three players thus yield a vertically related industry with a downstream Bowley duopoly, thereby avoiding recurring warfare or ruinous competition.

## 4. EFFECTS OF PRODUCER'S CHOICE ON THE SOCIETY

We now compare retail prices (social surpluses) $p^s$ and $p^b$ ($SS^s$ and $SS^b$) under alternative Stackelberg and Bowley downstream duopolies, respectively.

$$SS^s = \frac{39}{128}(a-k)^2 \tag{26}$$

$$SS^b = \frac{3}{8}(a-k)^2 \tag{27}$$

where $SS^s = \pi_i^{dsl*} + \pi_j^{dsf*} + \pi^{us*} + CS^s$ and $SS^b = 2\pi_i^{dbl*} + \pi^{ub*} + CS^b$, while $CS^s and\ CS^b$ stand for consumer surpluses under the possible alternative distribution systems embodying Stackelberg and Bowley duopolies downstream, respectively.

Comparing (26) and (27) while utilizing (11) and (20) yields the following relations:

$$p^s > p^b \tag{28}$$

$$SS^s < SS^b \tag{29}$$

Thus, retail prices (social surplus) under the distribution system incorporating a Bowley duopolistic market downstream are lower (greater) than those under the distribution system incorporating a Stackelberg duopolistic market. Therefore, the distribution system with a Bowley retail market benefits the upstream seller, downstream buyers and thus society as a whole.

## 5. CONCLUSION

Apparent aggressive competition causing retail prices to fall below wholesale prices is a frequently observed phenomenon in the Japanese frozen food and beer industries and may seem irrational, if not illegal, at first glance. But such pricing neither constrains competition nor steals customers from rival firms. In this sense, such competition may be within the bounds of fair trade. However, for this type of competition to be fair, it must sufficiently increase potential demand to increase the entire channel's profits. The reason is as follows: if these conditions are met, the producer remains profitable even after compensating retailers with rebates funded by its 'increased net profits, net of rebating expenditures'.

The final market equilibrium reaches its optimum indeed under a distribution system formed by a monopolistic wholesale market and a Bowley duopoly retail market.[9] Within the confines of our present model, the producer can guarantee retailers a Stackelberg leader's profits while retailers maintain aggressive competition. Moreover, the producer attains maximum profit under the identical distribution system with a Bowley retail market downstream. Furthermore, since the retail price falls while the social surplus is greater than under the Stackelberg counterpart, a vertically related Bowley retail market creates in effect a trilateral win-win-win game in which producers, distributors and society all benefit.

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[9] This outcome does not even require aforementioned increase in 'demand' at all. What actually increases is the 'quantity of demand' along a given demand (curve) as price decreases under our distribution system herein.

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