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# Central Bank Independence Promotes Budgetary Efficiency

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**Abstract**

This paper shows theoretically that inefficient public expenditure can be institutionally curtailed by an independent central bank. An advantage of our analysis is to employ a two-country model with cash-in-advance constraints. The model can deal with fiscal policy as well as monetary policy with considering international interdependence. Each government decides the levels of public goods provision and a lump-sum tax, and each central bank chooses the quantity of money supply, to maximize its own households' utility. When the central bank is not independent of the fiscal authority, that is, when fiscal policy is determined before monetary policy, the public good is oversupplied. When the central bank is independent (monetary policy is predetermined), however, the expenditure level is efficient. Because the government cannot decide the provision of public good in anticipation of seigniorage. Thus, an independent central bank can promote cuts of budgetary inefficiency.

Key words: Central Bank Independence, Public Goods, Cash-in-advance Model.

JEL classification: H41, E61, E62, E58

## I. Introduction

Recently, two topics relating to fiscal and monetary policies have been discussed in developed countries; budget deficits and central bank independence (CBI). The former has been caused by increasing inefficient public expenditure, and the latter prevents it from being monetized, which gives rise to inflation. There are the previous theoretical research on CBI, e.g. Rogoff (1985), Persson and Tabellini (1990), Cukierman (1992), Alesina and Gatti (1995), Walsh (1995), Waller and Walsh (1996) and so on. They investigated the relation between CBI and monetary policy. They showed mechanisms to induce inflationary policy, and concluded that CBI is important to prevent inflation.

We try to analyze the relation between CBI and fiscal policy as well as monetary policy in this paper. We will show that CBI is significant not only to prevent inflation but also to cut inefficient public expenditure. In previous studies, however, the relation between CBI and public expenditure were rarely considered theoretically. We need to discuss monetary and fiscal policies simultaneously.

Now we use a two-large-country model with cash-in-advance constraints in order to investigates the idea that inefficient public expenditure can be institutionally curtailed by an independent central bank.[1)] Households in both countries face the cash-in-advance constraints: they have to purchase goods with

[1] In Section III, we will define CBI in our model.

the producer’s currency. And the household's utility increases as not only a private good but a public good grows. Policymaker in each country decides the levels of a public good provision and a lump-sum tax, and the money supply, to maximize his own households’ utility. Hence, we can analyze fiscal and monetary policies simultaneously, and have the microeconomic foundation of the objective functions in the model.[2)]

We have other advantages in our model. First, we can examine welfare analysis of resource allocation. We will focus on the efficient provision of public goods supported by an independent central bank in this paper.

Second, we deal with policy in a *large* open economy. Previous theoretical studies on CBI mainly analyze monetary policy in a closed or small open economy. Developed countries are in fact large, and should be discussed in a *large* open economy model. In the middle of the 1980’s, policymakers in industrial countries argued as to whether or not they could cooperate on fiscal and monetary policies. In the 1990’s, the point at issue was important among European countries, especially. In Japan, the Bank of Japan law was revised to

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[2] We adopt a two-country model with cash-in-advance constraints; nevertheless we don't imply that we deny the loss function approach used in previous studies on CBI, and its conclusion. Their approach include a priori that the policymaker’s welfare is worse off by raising the inflation rate, were often used as the objective of the central bank. The reason for this is that the central bank stabilizes the price level, and prevents a household sustaining disutility due to inflation. That means that the central bank considers the household’s utility. In this sense, our model is relevant to these works.

enforce its independence regarding policymaking.

We will explain the following results using our model. When the central bank is not independent of the fiscal authority, that is, when fiscal policy is determined before monetary policy, the public good is oversupplied, because government can choose public good provision with respect to the issue of money. But when the central bank is independent, that is, when monetary policy is predetermined, the expenditure level is efficient. Because the government cannot control the public good in anticipation of seigniorage. Thus, an independent central bank promotes cuts of inefficient public expenditure.

This paper proceeds as follows. Section II demonstrates the model and analyzes the first best solution. Section III examines the results of policies when the fiscal and monetary authorities operate separately. Also we show that inefficient public expenditure can be cut down by an independent central bank. Section IV compares both regimes: one with central bank independence and the other without central bank independence. Finally, section V is the conclusion.

## II. The model

### II.1 A two-country model with cash-in-advance constraints

First, we show the model. This is a two-country model with cash-in-advance constraints, used by Lucas (1982), Helpman and Razin (1984), Canzoneri (1989), Martin (1994), and so on. The setting follows Canzoneri (1989) and Martin (1994).

The household in this model consumes a private good and a public good which the government provides. So this is appropriate for the analysis of fiscal and monetary policy in an international economy.

Suppose there are two countries, home country (country *h*) and foreign country (country *f*). They are symmetric and large: each one affects the other. We assume households are homogenous, live infinitely, and cannot migrate. The population in each country is assumed to be unity (constant). Both countries produce a single private good, whose (real) amounts of period $t$ are $y_t$ (> 0)and $y^*_t$ (> 0) units.[3)] To avoid unnecessary complications, we presumed $y_t$ and $y^*_t$ are exogenously given in each period.[4)] Hereafter, asterisks denote foreign country in all variables. These outputs are equally distributed in cash among households in both countries at the beginning of the next period.

The utility functions of representative households in both countries are given as follows:

$$u = \sum_{t=0}^{\infty} \beta^t (\log c_t + \log g_t), \qquad 0 < \beta < 1 \qquad (1)$$

$$u^* = \sum_{t=0}^{\infty} \beta^t (\log c_t^* + \log g_t^*), \qquad 0 < \beta < 1 \qquad (1')$$

where $c_t$ and $g_t$ are respectively (real) consumption of a private and a public good per capita. $\beta$ is a discount factor (the same in both countries). We assume that

[3] At period 0, the economy has the initial endowment $y_{-1}$ or $y^*_{-1}$.

[4] This assumption is the same as Canzoneri (1989) and Martin (1994). The assumption is also supported by the findings of Alesina and Summers (1993): There is no correlation between the degree of CBI and real growth rate. Incidentally they also find there is negative correlation between the degree of CBI and inflation rate.

the two countries' goods are perfect substitutes and have no trade costs. Hence the exchange rate between both currencies at period $t$, $e_t$, is satisfied as follows

$$p_t = e_t p^*_t, \qquad (2)$$

where $p_t$ and $p^*_t$ are home and foreign currency prices of the private good.

Households face cash-in-advance constraints. They need a home currency when they purchase the home good, and a foreign currency when they purchase the foreign good; they cannot purchase the foreign good with a home currency, or the home good with a foreign currency.[5] So they satisfy the following conditions at period $t$:

$$m_{ht} \geq p_t c_{ht}, \qquad m_{ft} \geq p_t^* c_{ft}, \qquad (3)$$

$$m_{ht}^* \geq p_t c_{ht}^*, \qquad m_{ft}^* \geq p_t^* c_{ft}^*, \qquad (3')$$

where $m_{ht}$ and $m_{ft}$ are respectively the home households' home and foreign currency (nominal) demand for private consumption at the beginning of period $t$ per capita, $c_{ht}$ and $c_{ft}$ are respectively the home households' home and foreign good (real) consumption per capita. Households can purchase the bonds issued by both governments in cash. We presume that the home bond is traded by only the home currency and the foreign bond is traded by the foreign currency. The bonds issued by both governments are assumed to be perfect substitutes. Since the bond markets are assumed to be perfect, the gross rates of interest are equal in both bonds by arbitrage (say $r_t$).

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[5] This is the seller's system as defined by Helpman and Razin (1984).

The cash flow of the home households for period $t$ is expressed as [6)]

$$m_{ht} + p_t b_{ht} + p_t \tau_t = p_{t-1} y_{t-1}/2 + p_t r_{t-1} b_{ht-1},$$

$$m_{ft} + p_t^* b_{ft} = p_{t-1}^* y_{t-1}^*/2 + p_t^* r_{t-1} b_{ft-1}.$$

So the budget constraint of the home households for period $t$ is expressed in money terms

$$m_{ht} + e_t m_{ft} + p_t b_{ht} + e_t p_t^* b_{ft} + p_t \tau_t$$

$$= p_{t-1} y_{t-1}/2 + e_t p_{t-1}^* y_{t-1}^*/2 + p_t r_{t-1} b_{ht-1} + e_t p_t^* r_{t-1} b_{ft-1}, \qquad (4)$$

where $b_{ht}$ and $b_{ft}$ are per capita (real) demand for home and foreign debt at the beginning of period $t$, $\tau_t$ is per capita (real) lump-sum tax. The households pay the tax in cash. We can also write the cash flow of the foreign households for period $t$ is expressed as

$$m_{ht}^* + p_t b_{ht}^* = p_{t-1} y_{t-1}/2 + p_t r_{t-1} b_{ht-1}^*,$$

$$m_{ft}^* + p_t^* b_{ft}^* + p_t^* \tau_t^* = p_{t-1}^* y_{t-1}^*/2 + p_t^* r_{t-1} b_{ft-1}^*.$$

Hence the budget constraint of the foreign households in a like manner;

$$m_{ht}^*/e_t + m_{ft}^* + p_t b_{ht}^*/e_t + p_t^* b_{ft}^* + p_t^* \tau_t^*$$

$$= p_{t-1} y_{t-1}/2e_t + p_{t-1}^* y_{t-1}^*/2 + p_t r_{t-1} b_{ht-1}^*/e_t + p_t^* r_{t-1} b_{ft-1}^*. \qquad (4')$$

Using (2) and (3), (4) and (4') are rewritten as

$$c_t + b_t + \tau_t = p_{t-1} y_{t-1}/2 p_t + p_{t-1}^* y_{t-1}^*/2 p_t^* + r_{t-1} b_{t-1}, \qquad (5)$$

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[6] Since the bonds issued by both governments are perfect substitutes and the home households satisfy (3), they adjust money demand for both currencies by the cash-in-advance constraints of private consumption. The same thing can be said of the foreign households.

$$c_t^* + b_t^* + \tau_t^* = p_{t-1} y_{t-1} / 2 p_t + p_{t-1}^* y_{t-1}^* / 2 p_t^* + r_{t-1} b_{t-1}^*, \tag{5'}$$

where $c_t \equiv c_{ht} + c_{ft}$, $b_t \equiv b_{ht} + b_{ft}$ $c_t^* \equiv c_{ht}^* + c_{ft}^*$, $b_t^* \equiv b_{ht}^* + b_{ft}^*$.

Second, both policymakers (fiscal and monetary authorities) collect lump-sum and seigniorage taxes and issue debt, and provide a public good. The public good in each country, however, is only supplied to the household in that country. We assume that the marginal rate of transformation between the public good and private good is unity in both countries for each period. Since we suppose policymakers purchase private good in their *own* country to provide public good, they face cash-in-advance constraints, too.

$$m_{ht}^g \geq p_t g_t, \qquad m_{ft}^g \geq p_t^* g_t^*.$$

Hence the budget constraint of the home and foreign policymaker at period $t$ are (analogous steps leading to the foreign constraint)

$$m_{ht}^g / p_t + r_{t-1} d_{t-1} = (M_t - M_{t-1}) / p_t + \tau_t + d_t$$

$$m_{ft}^g / p_t^* + r_{t-1} d_t^* = (M_t^* - M_{t-1}^*) / p_t^* + \tau_t^* + d_t^*$$

where $M_t$ is the total amount of (nominal) money supply in the beginning of period $t$ per capita. In our paper, we assume $\tau_t \geq 0$ in each country.[7)] $d_t$ denotes total amount of per capita (real) debt at the beginning of period $t$. Then the bond market clearing condition becomes

[7] Because, as shown later, if we allow lump-sum subsidy, the larger a seigniorage and lump-sum subsidy the government sets, the higher the utility of household becomes at the equilibrium. We set the assumption to avoid the situation that the government increases the levels of seigniorage and lump-sum subsidy to become infinite in this model.

$$b_{ht} + b^*_{ht} = d_t \,,$$

$$b_{ft} + b^*_{ft} = d^*_t \,.$$

For simplicity, we unify both conditions;

$$b_t + b^*_t = d_t + d^*_t \,. \tag{6}$$

In the bond market, both households and governments behave as price takers.

Similarly, the good market clearing condition in both countries becomes

$$c_{ht} + c^*_{ht} + g_t = y_t \,,$$

$$c_{ft} + c^*_{ft} + g^*_t = y^*_t \,.$$

It is convenient to combine both conditions;

$$c_t + g_t + c^*_t + g^*_t = y_t + y^*_t \,, \tag{7}$$

In the good market, both households and governments also behave as price takers.

The equilibrium conditions of the money market are

$$M_t = m_{ht} + m^*_{ht} + m^g_{ht} \,, \qquad M^*_t = m_{ft} + m^*_{ft} + m^g_{ft} \,.$$

Using the good market clearing conditions, the above conditions are rewritten as

$$M_t = p_t y_t \,, \qquad M^*_t = p^*_t y^*_t \,.$$

For given $y_t$ and $y*_t$, price levels are determined in both money markets when both policymakers choose the quantity of money.

Now, we define the growth rate of money:

$$h_t \equiv (M_t - M_{t-1})/M_t < 1, \qquad h^*_t \equiv (M^*_t - M^*_{t-1})/M^*_t < 1.$$

Then

$$(M_t - M_{t-1})/p_t = h_t y_t \,, \qquad (M^*_t - M^*_{t-1})/p^*_t = h^*_t y^*_t \,.$$

So the government budget constraints are rewritten as

$$g_t + r_{t-1}d_{t-1} = h_t y_t + \tau_t + d_t\,, \tag{8}$$

$$g_t^* + r_{t-1}d_{t-1}^* = h_t^* y_t^* + \tau_t^* + d_t^*\,, \tag{8'}$$

and the household budget constraints are rewritten as

$$c_t + b_t + \tau_t = (1-h_t)y_t/2 + (1-h_t^*)y_t^*/2 + r_{t-1}b_{t-1}, \tag{9}$$

$$c_t^* + b_t^* + \tau_t^* = (1-h_t)y_t/2 + (1-h_t^*)y_t^*/2 + r_{t-1}b_{t-1}^*. \tag{9'}$$

II.2 First best solution

We consider Pareto optimal allocation in the two-country economy as the benchmark case. In the same way as Canzoneri(1989), a world social planner maximizes the weighted sum of utilities of both households.

$$\max_{\{c_t, c_t^*, b_t, b_t^*, g_t, g_t^*, \tau_t, \tau_t^*, h_t, h_t^*, d_t, d_t^*\}} \quad \frac{1}{2}\sum_{t=0}^{\infty}\beta^t(\log c_t + \log g_t) + \frac{1}{2}\sum_{t=0}^{\infty}\beta^t(\log c_t^* + \log g_t^*)$$

s.t. (6), (7), (8), (8'), (9), (9'), $\tau_t \geq 0$, $\tau^*_t \geq 0$.

As we consider two symmetric countries, the weight of utility of each household is equated (the weight is 1/2). This optimal solution (first best solution) is[8)]

$$c_t = g_t = c_t^* = g_t^* = (y_t + y_t^*)/4\,.$$

The derivation of the above condition is given in Appendix A. The solution is efficient, because Samuelson (1954)'s rule is held in each country.[9)] In this model,

[8] In this solution, any levels of $\tau_t$, $\tau^*_t$, $h_t$, and $h^*_t$ satisfy the following conditions;

$$h_t y_t + \tau_t = \frac{y_t + y_t^*}{4}, \qquad h_t^* y_t^* + \tau_t^* = \frac{y_t + y_t^*}{4}.$$

[9] In this model, Samuelson rule is held, unless weights of each country is 1/2. Therefore the

the marginal rate of substitution between the public good and the private good for the household is $c_t / g_t$ (home country) or $c^*_t / g^*_t$ (foreign country) at period $t$, and the marginal rate of transformation between the public good and the private good is unity from the assumption. The above solution shows that Samuelson rule is held in each country. Moreover the solution implies the equilibrium when both policymakers take cooperative policies.

## III. Equilibria when the central bank is independent and not independent

### III.1 A definition of an independent central bank

Actually, fiscal and monetary authorities are separated in deciding policies whether they are interdependent or not. We consider policies when the fiscal authority (government) and the monetary authority (central bank) determine them separately. The fiscal authority determines fiscal policy: It can control the amount of a public good provision and a lump-sum tax. The monetary authority decides monetary policy: It can set the quantity of money supply and (non-monetized) debt. In this section, we consider what is central bank independence in our model.

When the central bank can decide a monetary policy without the interface of the government and the Congress, we call it an ‘independent central bank’. Hence, we define an independent central bank as a central bank that can choose

weights of utilities are not crucial.

the levels of $h_t$ and $d_t$ before the government decides fiscal policy in our model. In other words, an independent central bank can determine a monetary policy before the government chooses the levels of fiscal deficits (equal to lump-sum tax revenue minus expenditure). If a central bank determines a monetary policy after the government has already determined a fiscal policy, it can only choose a level of monetization to finance fiscal deficits decided by the government. Therefore, in this situation, a central bank is not independent of the government.

This definition is justified by previous research. Grillin, Masciandaro, and Tabellini (1991), Cukierman, Webb, and Neyapti (1992), and so on, which design the indexes of CBI, define the policymaking of the central bank without monetizing the fiscal deficit as one of factors regarding CBI. Also, Tabellini (1987) investigates a central bank which is freed from the obligation to monetize the fiscal deficit and as a result establishes a reputation of independence. So, we define an independent central bank as a central bank that decides a monetary policy before the government determines a fiscal policy.

We discuss two situations: 1) the central bank is not independent in deciding monetary policy, and 2) the central bank is independent. The former is the case where the government decides fiscal policy before the central bank: the government is the leader, and the central bank is the follower in deciding policy. Since the central bank must act under a given fiscal policy, the central bank is not independent. The latter is the case in which the central bank decides fiscal policy before the government: the central bank is the leader, and the government is the follower, so the central bank is independent.

Now, in order to keep our analysis simple, we assume that both a government and a central bank maximize the utility of the representative household in their own country. In other words, their objective functions are the same as the utility function of the representative household in each country. This assumption implies that there exist no conflict between a fiscal authority and a monetary authority with respect to preference. We will show that the outcome under an independent central bank is *different* from that under a dependent central bank even if a fiscal authority and a monetary authority have the *same* objective function. If the objective function of a central bank is different from that of a government, it is obvious that both outcomes may be different. We emphasize the difference of institutions rather than preference in our discussion.

Moreover, we consider two *equally large* countries in consideration of industrial countries. Hence we only examine simultaneous-move games between two countries in our model: Agents in each country maximize their objective functions given choices in the other country. Our analysis focuses on a Nash equilibrium. We are not interested in the leader-follower relationship between two countries.

### III.2 An equilibrium without an independent central bank

In this section, we analyze the case where a central bank is not independent in both countries. The process of decision making is as follows. In the first step, the government determines the amount of a public good provision and a lump-sum tax to maximize the household utility. Fiscal deficit, the difference between a public good provision and a lump-sum tax, is filled by issuing money or debt. In the second step, the central bank decides the quantity of money supply to maximize the household utility given his own fiscal policy and foreign policies. Finally, households choose their consumption and demand of debt to maximize their utility under the given policies. This structure is described as the extensive form game by Figure 1. Both countries make decisions simultaneously.

To investigate an equilibrium under this situation, we use the method of backward induction. So, in the first place, we solve the household's optimization problem. The home household's problem is as follows.

$$\max_{\{c_t, b_t\}} (1) \qquad \text{s.t. } (9) \qquad \text{given} \quad g_t, \tau_t, d_t, h_t, g^*_t, \tau^*_t, d^*_t, h^*_t.$$

The first-order conditions reduce to

$$c_t = \beta r_{t-1} c_{t-1}. \tag{10}$$

The derivation of the above condition is given in Appendix B. The foreign household's problem is similarly

$$\max_{\{c^*_t, b^*_t\}} (1') \qquad \text{s.t. } (9') \qquad \text{given} \quad g_t, \tau_t, d_t, h_t, g^*_t, \tau^*_t, d^*_t, h^*_t.$$

The first-order conditions reduce to

$$c^*_t = \beta r_{t-1} c^*_{t-1}. \tag{10'}$$

We can interpret (10) or (10') as the response function of the household.

The central bank chooses the amount of money or debt supply as given (10) or (10'), its own government's policy, and policies in the other country. Then the home central bank's problem is given by

$$\max_{\{h_t, d_t\}} (1) \qquad \text{s.t. } (8), (9), (10) \qquad \text{given} \quad g_t, \tau_t, g^*_t, \tau^*_t, d^*_t, h^*_t.$$

The first-order conditions reduce to

$$g_t = 2c_t \,, \tag{11}$$

$$g_t = \beta r_{t-1} g_{t-1} \,. \tag{12}$$

(12) is equivalent of (10). Then (12) will be omitted hereafter. Using (8), (9), and (11), the response function of the home central bank is written as

$$h_t y_t = -g_t - 2\tau_t + y_t + (1-h_t^*) y_t^* - 2(b_t - r_{t-1} b_{t-1})$$

$$d_t = 2g_t + \tau_t - y_t - (1-h_t^*) y_t^* + 2(b_t - r_{t-1} b_{t-1}) + r_{t-1} d_{t-1} \,. \tag{13}$$

where $b_t$ in (13) implicitly satisfies (10).

In the same way, the foreign central bank's problem is given by

$$\max_{\{h_t^*, d_t^*\}} \text{ (1')} \qquad \text{s.t. (8'),(9'), (10')} \qquad \text{given} \quad g_t,\ \tau_t,\ d_t,\ h_t,\ g^*_t,\ \tau^*_t.$$

The first-order conditions reduce to

$$g_t^* = 2c_t^* \,, \tag{11'}$$

$$g_t^* = \beta r_{t-1} g_{t-1}^* \,. \tag{12'}$$

(12') is equivalent of (10'). Then (12') will be omitted henceforth. Using (8), (9), and (11), the response function of the foreign central bank is written as

$$h_t^* y_t^* = -g_t^* - 2\tau_t^* + y_t^* + (1-h_t) y_t - 2(b_t^* - r_{t-1} b_{t-1}^*)$$

$$d_t^* = 2g_t^* + \tau_t^* - y_t^* - (1-h_t) y_t + 2(b_t^* - r_{t-1} b_{t-1}^*) + r_{t-1} d_{t-1}^* \,. \tag{13'}$$

where $b^*_t$ in (13') implicitly satisfies (10').

Finally, each government decides fiscal policy. The home government's objective is

$$\max_{\{g_t, \tau_t\}} \text{ (1)} \qquad \text{s.t. (8), (9), (10), (13), } \tau_t \geq 0 \qquad \text{given} \quad g^*_t,\ \tau^*_t,\ d^*_t,\ h^*_t.$$

This implies the lower $\tau_t$ is, the better it becomes. Hence, it sets $\tau_t = 0$, and

$$g_t^* = \frac{1}{2}\{y_t^* + (1-h_t)y_t - r_{t-1}d_{t-1}^* - 2(b_t^* - r_{t-1}b_{t-1}^*)\} \tag{14}$$

where $b_t$ in (14) implicitly satisfies (10), and $d_t$ in (14) implicitly satisfies (13).

The foreign government's problem reduces to

$$\max_{\{g_t^*, \tau_t^*\}} (1') \quad \text{s.t. } (8'), (9'), (10'), (13'), \tau^*_t \geq 0 \quad \text{given} \quad g_t, \tau_t, d_t, h_t.$$

Similarly, it sets $\tau^*_t = 0$, and

$$g_t^* = \frac{1}{2}\{y_t^* + (1-h_t)y_t - r_{t-1}d_{t-1}^* - 2(b_t^* - r_{t-1}b_{t-1}^*)\} \tag{14'}$$

where $b^*_t$ in (14') implicitly satisfies (10'), and $d^*_t$ in (14') implicitly satisfies (13').

Now, we discuss a Nash equilibrium under the above system in both countries. (11) and (11') are always held with any policy. These imply that this equilibrium is not efficient: these don't satisfy Samuelson rule. Why does the equilibrium become inefficient? In the above system, each government predetermines the provision of public good. We now consider the case that the home government raises $g_t$. In order to finance it, the home government can levy a lump-sum tax or delegate financing fiscal deficits to the home central bank. If $\tau_t$ increases by one unit for an increase of $g_t$, $c_t$ has to decrease by one unit in (9). While if $h_t y_t$ increases by one unit, $g_t$ increases by one unit in (8) and $c_t$ decreases a half unit in (9). Therefore the home government prefers a seigniorage tax to a lump-sum tax, and collects this seigniorage tax from the foreign household excessively. Because, in this case, $g_t$ increases by one unit and $c_t$ decreases half unit, that is, this relationship does not satisfy Samuelson rule.

Incidentally, from (11) and (11'), the government debts have no effect on both private and public goods consumption: Obviously the Ricardian equivalence is held. Then we assume $d_t = d^*_t = 0$ without loss of generality. So $b_t = b^*_t = 0$. Since they are symmetric, from (7), (11), and (11'), the quantity of consumption is

$$c_t = c_t^* = (y_t + y_t^*)/6,$$

$$g_t = g_t^* = (y_t + y_t^*)/3.$$

The policies of the central banks are, in this equilibrium,

$$h_t = (y_t + y_t^*)/3y_t, \qquad h_t^* = (y_t + y_t^*)/3y_t^*.$$

III.3 An equilibrium with an independent central bank

Next, we analyze the case where the central bank is independent in both countries. The process of decision making is as follows. In the first step, the central bank decides the quantity of money supply to maximize household utility. In the second step, the government determines the amount of a public good provision and a lump-sum tax to maximize household utility given its own monetary policies and foreign policies. Since the money supply is predetermined, the amount of a public good provision must be equal to a lump-sum tax and money or debt. Finally, households choose their consumption and demand of debt to maximize their utility are given policies. This structure is described as the extensive form game by Figure 2. Both countries make decisions simultaneously.

We discuss an equilibrium under this situation. The household's optimization problems that we solve first are the same as in section III.2. So we already gain the condition (10) and (10').

In the second stage, the government chooses the amount of public good provision and lump-sum tax given (10) or (10'), its own central bank's policy, and policies in the other country. Then the home government's problem is presented by

$$\max_{\{g_t,\tau_t\}} (1) \quad \text{s.t. } (8), (9), (10),\ \tau_t \geq 0 \qquad \text{given} \quad d_t, h_t, g^*_t, \tau^*_t, d^*_t, h^*_t.$$

From the first-order conditions, we obtain (12), and

$$g_t = c_t. \tag{15}$$

The derivation of the above condition is given in Appendix C. We can interpret (15) as the response function of the home government.

Similarly, the foreign government's problem given by

$$\max_{\{g_t^*,\tau_t^*\}} (1') \quad \text{s.t. } (8'), (9'), (10'),\ \tau^*_t \geq 0 \qquad \text{given} \quad g_t, \tau_t, d_t, h_t, d^*_t, h^*_t.$$

and, we obtain (12), and

$$g_t^* = c_t^*. \tag{15'}$$

We can interpret (15') as the response function of the foreign government.

Finally, each central bank decides monetary policy. The home central bank's objective is

$$\max_{\{h_t,d_t\}} (1) \quad \text{s.t. } (8), (9), (10), (15) \qquad \text{given} \quad g^*_t, \tau^*_t, d^*_t, h^*_t.$$

This implies that the larger $h_t$ or $d_t$ is, the better its utility becomes. So it sets

$$h_t y_t + 2d_t = 4g_t - y_t - (1-h_t^*)y_t^* + 2(b_t - r_{t-1}b_{t-1}) + 2r_{t-1}d_{t-1}, \tag{16}$$

or

$$3h_t y_t + 2d_t = -4\tau_t + y_t + (1-h_t^*)y_t^* - 2(b_t - r_{t-1}b_{t-1}) + 2r_{t-1}d_{t-1}.$$

The home central bank follows (16) and decides the amount of home money supply.

In a like manner, the foreign central bank's problem reduces to

$$\max_{\{h_t^*, d_t^*\}} \text{(1')} \qquad \text{s.t. (8'), (9'), (10'), (15')} \qquad \text{given} \quad g_t, \tau_t, d_t, h_t.$$

This implies that the larger $h^*_t$ or $d^*_t$ is, the better the utility of the foreign household becomes. So the foreign central bank sets

$$h_t^* y_t^* + 2d_t^* = 4g_t^* - y_t^* - (1-h_t)y_t + 2(b_t^* - r_{t-1}b_{t-1}^*) + 2r_{t-1}d_{t-1}^*, \qquad (16')$$

or

$$3h_t^* y_t^* + 2d_t^* = -4\tau_t^* + y_t^* + (1-h_t)y_t - 2(b_t^* - r_{t-1}b_{t-1}^*) + 2r_{t-1}d_{t-1}^*$$

The foreign central bank follows (16') and decides the amount of foreign money supply.

Now, we analyze a Nash equilibrium under the above system in both countries. According to (15) and (15'), these are consistent with the Pareto optimal allocation. In other words, Samuelson rules are held in both countries.

Moreover, to compare with the levels in section III.2, we suppose the governments set $\tau_t = \tau^*_t = 0$. From (10), (10'), (15), and (15'), the government debts have no effect on both private and public goods consumption: Obviously the Ricardian equivalence is held again. Then we assume $d_t = d^*_t = 0$ without loss of generality. So $b_t = b^*_t = 0$. Since they are symmetric, from (7), (15), and (15'), the amount of consumption is

$$c_t = g_t = c_t^* = g_t^* = (y_t + y_t^*)/4.$$

In this equilibrium, the central banks choose

$$h_t = (y_t + y_t^*)/4y_t, \qquad h_t^* = (y_t + y_t^*)/4y_t^*.$$

These imply the growth rates of money supply are lower than the rate in section III.2; this is inefficient. Why does this equilibrium become efficient? In the above system, each government determines the provision of public good after deciding on monetary policies. We now consider the case that the home government raises $g_t$. In order to finance, it can only levy a lump-sum tax. If $g_t$ increases by one unit, $\tau_t$ has to increase by one unit in (8). Then $c_t$ decreases by one unit in (9). Since the relationship between increase of $g_t$ and decrease of $c_t$ becomes one-to-one, Samuelson rule is satisfied. Thus, inefficient budgets are curtailed by an independent central bank.

These results also suggest that even if policymakers in both countries are *not* cooperative in their policies, the achieved equilibrium is Pareto optimal when the central bank is independent of the government in both countries.

## IV. The significance of central bank independence

What is the equilibrium if the central bank is not independent in either country? Now, consider in the situation where the home central bank is not independent and the foreign central bank is independent. Using the above results, the home policies are presented by (11), (14), and $\tau_t = 0$, and the foreign are (15') and (16'). Then assuming $\tau^*_t = 0$, $d_t = d^*_t = 0$, and $b_t = b^*_t = 0$, we obtain

$$c_t = c_t^* = g_t^* = (y_t + y_t^*)/5,$$

$$g_t = 2(y_t + y_t^*)/5.$$

The derivation of the above conditions is given in Appendix D. These suggest that, in this equilibrium, home households become better off and foreign households become worse off than when both central banks are independent. We gain the symmetric result when the foreign central bank is not independent and the home is independent. Table 1 shows the above results. It implies the game in this paper is the prisoners' dilemma. If either central bank is not independent, the equilibrium does not achieve Pareto optimal allocation. Moreover, the equilibrium is stable where both countries adopt the system in which the central bank is not independent.

If both countries adopt the system in which the central bank is independent, the equilibrium can achieve Pareto optimal allocation. Therefore CBI is significant in compelling the fiscal authority to provide public good efficiently.

## V. Concluding remarks

This paper discusses the relationship between fiscal and monetary policies, using a two-country cash-in-advance model. When the central bank is not independent of the government, that is, when fiscal policy is predetermined, the public good is oversupplied. Because the government can decide public good provision in anticipation of money supply. It forces the central bank to finance fiscal deficit. Furthermore, the central bank substantially monetizes the fiscal deficit, if it is forced. The fiscal deficit can be filled with a seigniorage tax which is the source of inefficiency, and a policymaker in one country has the incentive to levy with seigniorage tax upon citizens in the other country. In other words, each government decides fiscal policy without considering the negative externality of seigniorage to the other country. Hence the growth rate of money supply is excessively high and the public good is oversupplied.

When the central bank is independent, monetary policy is predetermined, however, the expenditure level is efficient. Because the government cannot decide fiscal policy in anticipation of seigniorage. The central bank decides a monetary policy considering the response of the government. It sets the money supply rule. Moreover the government's only control is to levy a lump-sum tax in order to provide a public good. Also a lump-sum tax is not a distortionary tax. So the government appropriately collects the fiscal revenue. Therefore the public good is efficiently supplied.

We show that inefficient public expenditure can be cut down by an independent central bank. Notice that the role of the independent central bank is not to prevent budget deficits from monetizing, but to make the government observe Samuelson rule. So the provision of the public good is efficient. These findings is different from those of previous works.

As mentioned by previous studies, the main role of the central bank is the stabilization of the price level by controlling money supply or interest rates. To carry this out, it is necessary that the central bank be independent of the government or any political pressure. An independent central bank can prevent high inflation and fiscal deficits from monetizing.

We obtain a policy implication from our result. When the central bank is independent in each country, this equilibrium is Pareto efficient, even if each policymaker does not cooperate to decide its policies each other. We also say CBI is significant for efficiency when international policy coordination fails in world economy. In the middle of the 1980's, industrial countries cooperated to decide monetary policies in order to depreciate the value of dollar. This cooperation, however, did not fully succeed. After that, they moved onto CBI. Our result implies that it is important for budgetary efficiency that all policymakers establish independent central banks.

This paper shows CBI is important not only because the central bank averts high inflation and monetizing but also because inefficient public spending is curtailed. In other words, CBI becomes a commitment device for budget cuts. The source of inefficiency is not monetizing the fiscal deficit but excessive collection of seigniorage in our model. If policymakers create excessively high inflation rates, and collect more seigniorage, then policymakers excessively increase the quantity of a public good and the household decreases its consumption of a private good.[10)] Moreover the cause of excessive collection is that each government decides fiscal policy without considering the negative externality of seigniorage on the other country. Therefore, CBI is important because an independent central bank can play a role in preventing it.

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[10] In this model, money does not affect output that is exogenous, and the money illusion does not occur. If policymakers heighten the growth rate of money, the inflation rate increases at the same rate when output is constant. Money is used for exchange and the collection of tax.

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## Table 1
## Payoff Matrix in Both Countries

| | | Foreign | |
|---|---|---|---|
| | | independent | not independent |
| Home | independent | $(\frac{y_t + y_t^*}{4}, \frac{y_t + y_t^*}{4})$ ,<br>$(\frac{y_t + y_t^*}{4}, \frac{y_t + y_t^*}{4})$ | $(\frac{y_t + y_t^*}{5}, \frac{y_t + y_t^*}{5})$ ,<br>$(\frac{y_t + y_t^*}{5}, \frac{2(y_t + y_t^*)}{5})$ |
| | not independent | $(\frac{y_t + y_t^*}{5}, \frac{2(y_t + y_t^*)}{5})$<br>, $(\frac{y_t + y_t^*}{5}, \frac{y_t + y_t^*}{5})$ | $(\frac{y_t + y_t^*}{6}, \frac{y_t + y_t^*}{3})$ ,<br>$(\frac{y_t + y_t^*}{6}, \frac{y_t + y_t^*}{3})$ |

the upper row: $(c_t, g_t)$ ,and the lower row: $(c_t^*, g_t^*)$ in each cell.

**Figure 1**

The Timing of Decision Making When A Central Banks Is Not Independent

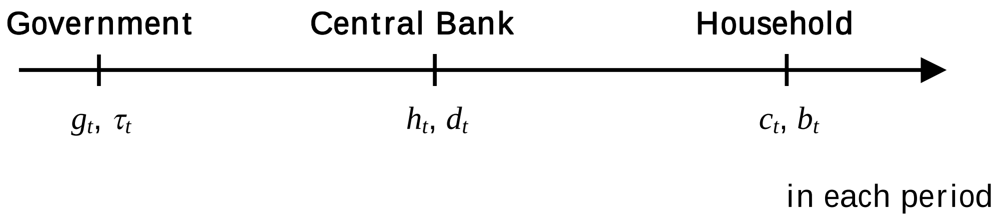
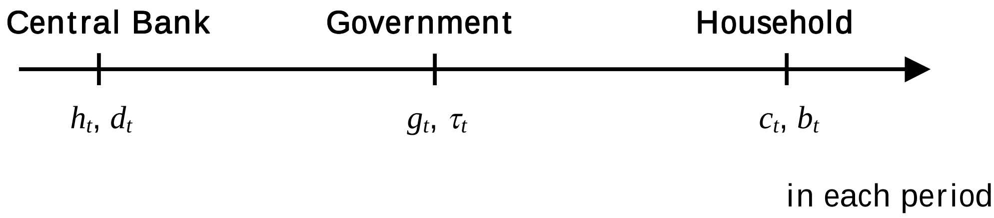



**Figure 2**

The Timing of Decision Making When A Central Banks Is Independent



## Appendix A: The derivation of the first best solution

The corresponding Lagrange function is expressed as

$$
\begin{aligned}
L = \textstyle\sum_{t=0}^{\infty} \beta^t [\{\frac{1}{2}(\log c_t + \log g_t) + \frac{1}{2}(\log c_t^* + \log g_t^*)\} + \xi_t (y_t + y_t^* - c_t - g_t - c_t^* - g_t^*) \\
+ \mu_t (h_t y_t + \tau_t + d_t - g_t - r_{t-1} d_{t-1}) + \mu_t^* (h_t^* y_t^* + \tau_t^* + d_t^* - g_t^* - r_{t-1} d_{t-1}^*) \\
+ \lambda_t \{(1-h_t) y_t / 2 + (1-h_t^*) y_t^* / 2 + r_{t-1} b_{t-1} - c_t - b_t - \tau_t\} \\
+ \lambda_t \{(1-h_t) y_t / 2 + (1-h_t^*) y_t^* / 2 + r_{t-1} b_{t-1}^* - c_t^* - b_t^* - \tau_t^*\}]
\end{aligned}
\quad ,
$$

where $\xi_t$, $\lambda_t$, $\lambda*_t$, $\mu_t$, and $\mu*_t$ are the Lagrange multipliers. Its first order conditions are

$$\frac{\partial L}{\partial c_t} = \beta^t \{\frac{1}{2c_t} - \xi_t - \lambda_t\} = 0 , \qquad \frac{\partial L}{\partial c_t^*} = \beta^t \{\frac{1}{2c_t^*} - \xi_t - \lambda_t^*\} = 0 ,$$

$$\frac{\partial L}{\partial b_t} = \beta^{t+1} \lambda_{t+1} r_t - \beta^t \lambda_t = 0 , \qquad \frac{\partial L}{\partial b_t^*} = \beta^{t+1} \lambda_{t+1}^* r_t - \beta^t \lambda_t^* = 0 ,$$

$$\frac{\partial L}{\partial g_t} = \beta^t \{\frac{1}{2g_t} - \xi_t - \mu_t\} = 0 , \qquad \frac{\partial L}{\partial g_t^*} = \beta^t \{\frac{1}{2g_t^*} - \xi_t - \mu_t^*\} = 0 ,$$

$$\frac{\partial L}{\partial \tau_t} = \beta^t \{\mu_t - \lambda_t\} = 0 , \qquad \frac{\partial L}{\partial \tau_t^*} = \beta^t \{\mu_t^* - \lambda_t^*\} = 0 ,$$

$$\frac{\partial L}{\partial h_t} = \beta^t \{\mu_t y_t - \frac{\lambda_t y_t}{2} - \frac{\lambda_t^* y_t^*}{2}\} = 0 , \qquad \frac{\partial L}{\partial h_t^*} = \beta^t \{\mu_t^* y_t^* - \frac{\lambda_t y_t}{2} - \frac{\lambda_t^* y_t^*}{2}\} = 0 ,$$

$$\frac{\partial L}{\partial d_t} = -\beta^{t+1} \mu_{t+1} r_t + \beta^t \mu_t = 0 , \qquad \frac{\partial L}{\partial d_t^*} = -\beta^{t+1} \mu_{t+1}^* r_t + \beta^t \mu_t^* = 0 .$$

Hence $\mu_t$ = $\mu*_t$ = $\lambda_t$ = $\lambda*_t$. So $c_t = g_t = c_t^* = g_t^*$. From (7), $c_t = g_t = c_t^* = g_t^*$ $= (y_t + y_t^*)/4$.

## Appendix B: The derivation of (10) ~ (14)

The corresponding Lagrange function of the home household is given as

$$L = \sum_{t=0}^{\infty} \beta^t [\{\log c_t + \log g_t\} \\ + \lambda_t \{(1-h_t) y_t/2 + (1-h_t^*) y_t^*/2 + r_{t-1} b_{t-1} - c_t - b_t - \tau_t\}],$$

where $\lambda_t$ is the Lagrange multiplier. Its first order conditions are

$$\frac{\partial L}{\partial c_t} = \beta^t \{\frac{1}{c_t} - \lambda_t\} = 0, \qquad \frac{\partial L}{\partial b_t} = \beta^{t+1} \lambda_{t+1} r_t - \beta^t \lambda_t = 0.$$

Then these satisfy (10). From(10),

$$c_t = \beta r_{t-1} c_{t-1} = \beta^{t+1} r_{t-1} r_{t-2} \cdots r_0 r_{-1} c_{-1}.$$

So $c_t$ can be expressed as the function of $\beta$, $r_{t-1}$, $r_{t-2}$, …, $r_{-1}$, and $c_{-1}$. Hereafter, $c_t$ and $b_t$ in (9) are assumed to satisfy (10).

The corresponding Lagrange function of the home central bank is given as

$$L = \sum_{t=0}^{\infty} \beta^t [\{\log c_t + \log g_t\} \\ + \mu_t (h_t y_t + \tau_t + d_t - g_t - r_{t-1} d_{t-1}) \\ + \lambda_t \{(1-h_t) y_t/2 + (1-h_t^*) y_t^*/2 + r_{t-1} b_{t-1} - c_t - b_t - \tau_t\}],$$

where $\lambda_t$, and $\mu_t$ are the Lagrange multipliers. Its first order conditions are

$$\frac{\partial L}{\partial h_t} = \beta^t \{-\frac{y_t}{2c_t} - \frac{y_t}{g_t}\} = 0, \qquad \frac{\partial L}{\partial d_t} = -\beta^{t+1} r_t / g_{t+1} + \beta^t / g_t = 0,$$

Then these satisfy (11) and (12).

Substituting (8) into (9),

$$c_t + b_t + \tau_t = y_t/2 + (1-h_t^*) y_t^*/2 + r_{t-1} b_{t-1} - (g_t + r_{t-1} d_{t-1} - \tau_t + d_t)/2$$

And substituting (8) into the above equation,

$$c_t + b_t + \tau_t = y_t/2 + (1-h_t^*) y_t^*/2 + r_{t-1} b_{t-1} - (g_t + r_{t-1} d_{t-1} - \tau_t)/2 \\ - \{2g_t + \tau_t - y_t - (1-h_t^*) y_t^* + r_{t-1} d_{t-1} + 2(b_t - r_{t-1b} b_{t-1})\}/2.$$

Moreover substituting (11) into the above equation

$$g_t = \{-\tau_t + y_t + (1-h_t^*)y_t^* - 2(b_t - r_{t-1}b_{t-1}) - r_{t-1}d_{t-1}\}/2$$

Therefore the optimization problem of the home government is rewritten as

$$\max_{\{g_t,\tau_t\}} \quad u = \sum_{t=0}^{\infty} \beta^t (2\log g_t - \log 2)$$

s.t. $g_t = \{-\tau_t + y_t + (1-h_t^*)y_t^* - 2(b_t - r_{t-1}b_{t-1}) - r_{t-1}d_{t-1}\}/2$ , $\tau_t \geq 0$

given $g^*_t$, $\tau^*_t$, $d^*_t$, $h^*_t$.

The corresponding problem of the home government is rewritten as

$$u = \sum_{t=0}^{\infty} \beta^t [2\log\{-\tau_t + y_t + (1-h_t^*)y_t^* - 2(b_t - r_{t-1}b_{t-1}) - r_{t-1}d_{t-1}\} - 3\log 2].$$

Its first order condition is

$$\frac{\partial u}{\partial \tau_t} = 2\beta^t \frac{-1}{g_t} < 0.$$

Then $\tau_t = 0$. Therefore $g_t$ is given as (14). Analogous steps lead to the foreign.

In this equilibrium, if $d_t = 0$ and $b_t = 0$, from (8) $g_t = h_t y_t$, and from (9) $g_t = (y_t + y^*_t)/2 - h^*_t y^*_t/2$. Similarly, if $d^*_t = 0$ and $b^*_t = 0$, from (8') $g^*_t = h^*_t y^*_t$, and from (9') $g^*_t = (y_t + y^*_t)/2 - h_t y_t/2$. Moreover,

$$g_t = (y_t + y_t^*)/2 - g_t^*/2 = (y_t + y_t^*)/2 - (y_t + y_t^*)/4 + g_t/4.$$

So $g_t = (y_t + y_t^*)/3$ and $c_t = (y_t + y_t^*)/6$. From $\tau_t = 0$, $h_t = (y_t + y_t^*)/3y_t$. Similarly,

$$g_t^* = (y_t + y_t^*)/2 - g_t/2 = (y_t + y_t^*)/2 - (y_t + y_t^*)/4 + g_t^*/4.$$

So $g_t^* = (y_t + y_t^*)/3$ and $c_t^* = (y_t + y_t^*)/6$. From $\tau^*_t = 0$, $h_t^* = (y_t + y_t^*)/3y_t^*$. These quantities, $c_t$, $g_t$, $c^*_t$, and $g^*_t$, satisfy (7).

## Appendix C: The derivation of (15) and (16)

The first order condition of the home household is given as (10). Hereafter we presume $c_t$ and $b_t$ in (9) satisfy (10).

The corresponding Lagrange function of the home government is given as

$$L = \sum_{t=0}^{\infty} \beta^t [\log\{(1-h_t)y_t/2 + (1-h_t^*)y_t^*/2 + r_{t-1}b_{t-1} - b_t - \tau_t\} \\ + \log(h_t y_t + \tau_t + d_t - r_{t-1}d_{t-1})],$$

Its first order condition is

$$\frac{\partial L}{\partial \tau_t} = \beta^t \{-\frac{1}{c_t} + \frac{1}{g_t}\} = 0$$

Then these satisfy (15). Also it sets $\tau_t = g_t - h_t y_t - d_t + r_{t-1}d_{t-1}$

Substituting (8) and (15) into (9),

$$2c_t + b_t + r_{t-1}d_{t-1} - h_t y_t - d_t = (1-h_t)y_t/2 + (1-h_t^*)y_t^*/2 + r_{t-1}b_{t-1},$$

Moreover

$$c_t = h_t y_t/4 + (d_t - r_{t-1}d_{t-1})/2 + y_t/4 + (1-h_t^*)y_t^*/4 - (b_t - r_{t-1}b_{t-1})/2.$$

Therefore the optimization problem of the home central bank is rewritten as

$$\max_{\{h_t, d_t\}} \quad u = \sum_{t=0}^{\infty} \beta^t (2\log c_t)$$

s.t. $c_t = h_t y_t/4 + (d_t - r_{t-1}d_{t-1})/2 + y_t/4 + (1-h_t^*)y_t^*/4 - (b_t - r_{t-1}b_{t-1})/2$,

given $g^*_t$, $\tau^*_t$, $d^*_t$, $h^*_t$.

The corresponding problem of the home central bank is rewritten as

$$u = \sum_{t=0}^{\infty} \beta^t [2\log\{h_t y_t/4 + (d_t - r_{t-1}d_{t-1})/2 + y_t/4 + (1-h_t^*)y_t^*/4 - (b_t - r_{t-1}b_{t-1})/2\}].$$

Its first order conditions are

$$\frac{\partial u}{\partial h_t} = 2\beta^t \frac{1}{c_t}\frac{1}{4y_t} > 0, \qquad \frac{\partial u}{\partial d_t} = 2\beta^t \frac{1}{c_t}\frac{1}{2} > 0.$$

This implies that the larger $h_t$ or $d_t$ is, the better its utility becomes. So it sets (16). Analogous steps lead to the foreign.

In this equilibrium, if $\tau_t = 0$, $d_t = 0$, and $b_t = 0$, from (8) $g_t = h_t y_t$, and from (9) $g_t = (y_t + y^*_t)/3 - h^*_t y^*_t/3$. Similarly, if $\tau^*_t = 0$, $d^*_t = 0$, and $b^*_t = 0$, from (8') $g^*_t = h^*_t y^*_t$, and from (9') $g^*_t = (y_t + y^*_t)/3 - h_t y_t/3$. Moreover,

$$g_t = (y_t + y_t^*)/3 - g_t^*/3 = (y_t + y_t^*)/3 - (y_t + y_t^*)/9 + g_t/9.$$

So $g_t = (y_t + y_t^*)/4$ and $c_t = (y_t + y_t^*)/4$. From $\tau_t = 0$, $h_t = (y_t + y_t^*)/4y_t$. Similarly,

$$g_t^* = (y_t + y_t^*)/3 - g_t/3 = (y_t + y_t^*)/3 - (y_t + y_t^*)/9 + g_t^*/9.$$

So $g_t^* = (y_t + y_t^*)/4$ and $c_t^* = (y_t + y_t^*)/4$. From $\tau^*_t = 0$, $h^*_t = (y_t + y_t^*)/4y_t^*$. These quantities, $c_t$, $g_t$, $c^*_t$, and $g^*_t$, satisfy (7).

## Appendix D: The derivation of an equilibrium when the home central bank is not independent and the foreign central bank is independent

We now presume $d_t = d^*_t = 0$ and $b_t = b^*_t = 0$. Using the above results, the home policies are presented by (11), (14), and $\tau_t = 0$. Namely, $g_t = 2c_t$, $\tau_t = 0$, and $g_t = h_t y_t$. From (9), $g_t = (y_t + y^*_t)/2 - h^*_t y^*_t/2$. The foreign policymaker are (15') and (16'). The foreign is assumed to set $\tau^*_t = 0$. Namely, $g^*_t = c^*_t$, $\tau^*_t = 0$, and $g^*_t = h^*_t y^*_t$. From (9'), $g^*_t = (y_t + y^*_t)/3 - h_t y_t/3$. Then we obtain

$$g_t = (y_t + y_t^*)/2 - g_t^*/2 = (y_t + y_t^*)/2 - (y_t + y_t^*)/6 + g_t/6.$$

So $g_t = 2(y_t + y_t^*)/5$ and $c_t = (y_t + y_t^*)/5$. Similarly,

$$g_t^* = (y_t + y_t^*)/3 - g_t/3 = (y_t + y_t^*)/3 - (y_t + y_t^*)/6 + g_t^*/6.$$

Then $g_t^* = c_t^* = (y_t + y_t^*)/5$. These quantities, $c_t$, $g_t$, $c^*_t$, and $g^*_t$, satisfy (7).