Measures to Promote Green Cars: Evaluation at the Car Variant Level

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March 22, 2017 (University of Tokyo)

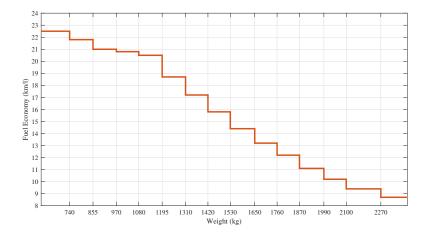
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Measures to Promote Green Cars

- One of the goals of auto-related policies: diffusion of green cars.
 - Tax incentives and subsidy for green cars
- Green cars in Japan
 - Next generation cars:
 - clean diesel cars, plug-in hybrid cars, electric cars, fuel cell vehicles, natural gas cars
 - Fuel efficient cars:
 - (1) Gasoline & hybrid cars complying with 2015 fuel economy standards
 - (2) Gasoline & hybrid cars complying with 2015 fuel economy stds+10%
 - (3) Gasoline & hybrid cars complying with 2015 fuel economy stds+20%

2015 fuel economy standards



Automobile related taxes in Japan

- Taxes on acquisition
 - Acquisition tax and consumption tax
- Taxes during ownership
 - Tonnage tax and automobile tax (mini-vehicle tax, -660cc)

On acquisition			
Acquisition tax Consumption tax			
5% of purchase price	5%		

During ownership			
Tonnage tax	Automobile tax	Mini-vehicle tax	
	- 1000cc: 29500 JPY/year		
(Before April 30, 2012)	1001 - 1500cc: 34500 JPY/year		
- 5000 JPY/0.5t per year	1501 - 2000cc: 39500 JPY/year		
(After May 1, 2012)	2501 - 3000cc: 45000 JPY/year		
- Vehicles complying with 2015 fe stds:	3001 - 3500cc: 51000 JPY/year	7200 JPY/year	
2500 JPY/0.5t per year	3501 - 4000cc: 66500 JPY/year		
- Other vehicles: 4100 JPY/0.5t per year	4001 - 4500cc: 76500 JPY/year		
	4501 - 6000cc: 88000 JPY/year		
	Over 6000cc: 111000 JPY/year		

Tax reduction & subsidy

• Tax incentives:

Target	Acquisition tax	Tonnage tax	Automobile tax
Next generation cars	Exempt	Exempt at 1st vehicle inspection (3 years);	50% reduction (1 year)
		50% reduction at 2nd inspection (2 years)	
Fuel efficient cars (+20%)	Exempt	Exempt at 1st vehicle inspection (3 years);	50% reduction (1 year)
		50% reduction at 2nd inspection (2 years)	
Fuel efficient cars (+10%)	75% reduction	75% reduction at 1st vehicle inspection	50% reduction (1 year)
		(3 years)	
Fuel efficient cars	50% reduction	50% reduction at 1st vehicle inspection	25% reduction (1 year)
		(3 years)	

• Subsidy: 2012.1 - 2012.9

Requirement	Standard-sized cars (≥ 660cc)	Mini-vehicle (< 660cc)
Compliant with 2015 fuel economy standards or	100,000 JPY	70,000 JPY
2010 fuel economy standards + 25%		

• 1USD ≈ 100 JPY.

- Discrete choice models a la Berry, Levinsohn and Pakes(1995, ECTA) have been applied in the studies of automobile markets, a market of product differentiation.
 - Berry, Levinsohn and Pakes (1999, AER), Golberg and Verboven(2001, ReStud), Petrin(2002, JPE), Bento et al.(2009, AER), Schiraldi (2011, RAND), Beresteanu and Li (2011, IER), Klier and Linn(2012, RAND), Kitano(2014), Wakamori(2015), Miravete et al.(2015) ...
- Demand estimation:

$$\underbrace{\ln s_{j} - \ln s_{0}}_{\text{quantity (sales)}} = \alpha \underbrace{p_{j}(tax_{j}, subsidy_{j})}_{\text{price}} + \sum_{k \in \mathcal{K}} \beta_{k} \underbrace{x_{jk}}_{\text{attributes}} + \underbrace{\xi_{j}}_{\text{error term}}, j \in \mathcal{J}$$

- The data consist of
 - (1) price and attributes
 - (2) sales

for each alternative in the choice set.

- A problem here is that
 - the sales data are available at the model-level, while
 - the data on prices and attributes are available at the variant-level.
- In constructing the database, the model-level sales data are associated with variant-level prices and attributes data.

Matching the price & attributes data with the sales data

• Polar weighting: use the price and attributes of base variants

- The base variant of a model is ordinary the cheapest one.
- Most previous studies apply this method.
- Equal weighting: use average price and attributes
 - A few previous studies apply this method.
 - Klier and Linn (2012, RAND)

 \Rightarrow Both of them are problematic in the analyses of automobile market because **the variant-level heterogeneity is substantial**.

- The effects of the policies can be different across variants within a model.
- Cf. Thomassen (2010): Variant-level sales data are available.

- Develop a method to estimate the demand for differentiated products by using the data of different levels of aggregation: the model-level sales data and the variant-level prices and attributes data.
 - Weights are endogenized: the weights depends on the differences in the prices and attributes among the variants of a model.
 - Variant-level demand can be obtained.
- Assess the impacts of the tax incentives and subsidy on the sales of the green cars.

Average fuel economy of newly sold cars

- Polar weighting: $AFE = \sum_{j} ms_{j} \left[\sum_{n \in \mathcal{B}_{j}} 1\{n = base\} fe_{j_{n}} \right]$
 - ms_j: market share of model j
 - fe_{j_n} : fuel economy of variant $n \in \mathcal{B}_j$
- Equal weighting: $AFE = \sum_{j} ms_{j} \left[\sum_{n \in \mathcal{B}_{j}} N_{j}^{-1} fe_{j_{n}} \right]$
 - N_j = #B_j: number of variants of model j
- Endogenous weighting: $AFE = \sum_{j} ms_{j} \left[\sum_{n \in \mathcal{B}_{j}} s_{n|j} f e_{j_{n}} \right]$
 - $s_{n|j}$: share of variant $n \in \mathcal{B}_j$ in the sales of model j
 - Function of variant n attributes and prices

Demand analyses under the various data availability conditions

- Petrin(2002, JPE): incorporating information relating consumer demographics to the characteristics of the products they purchase.
- Berry, Levinsohn and Pakes(2004, JPE): incorporating individual choice data.
- Bajari, Fox, and Ryan (2008, QME): estimation based on market share rank data (model-level shares are unavailable)
- Evaluation of environmental policies in the car markets
 - Goldberg (1998, JIndE), Bento et al. (2009, AER), Beresteanu and Li (2011, IER), Klier and Linn(2012, RAND), Miravete et al.(2015), ...

Variant-level heterogeneity matters: Summary statistics

Data:

- 2012.4 2014.3, monthly obs.
- # of variants: 1443, # of models (nameplates): 147.
 - Japanese firms: TOYOTA, HONDA, NISSAN, MAZDA, SUBARU, DAIHATSU MITSUBISHI, SUZUKI
- Catalog data are collected.
 - Detail data on variant-level car attributes are included.
 - The information on the model history is available.

Summary statistics

- The mean, std. dev., max-min, and (max-min)/min of the prices and attributes of the variants by models are computed.
- The values in the table show the average of them over models.

Variables	Mean	Std. Dev.	Max-Min	(Max - Min)/Min
Price (mil. JPY)	2.941	0.326	0.906	0.398
Car Size(m ³)	11.900	0.128	0.262	0.021
Wheelbase (m)	2.643	0.007	0.013	0.005
Engine Displacement (1000 cc)	1.949	0.085	0.194	0.098
Capacity (I)	53.235	0.842	1.735	0.042
HP (ps)/Weight (kg)	0.104	0.008	0.021	0.244
Fuel Economy (km/l)	16.216	1.293	3.211	0.229

Variant-level heterogeneity matters: Case of Subaru IMPREZA



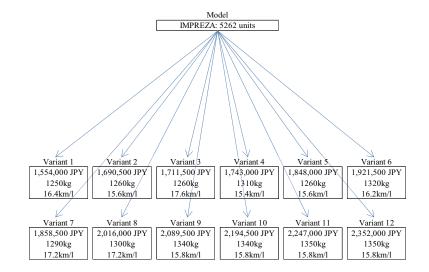
Source: Goo-net (http://www.goo-net.com/)

- There are several "models" under the name of IMPREZA.
- Each of the models has multiple variants.

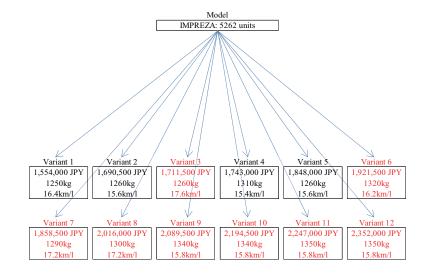
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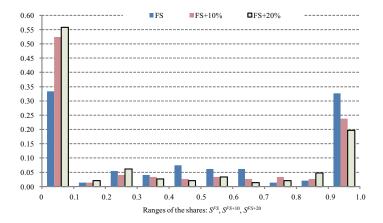
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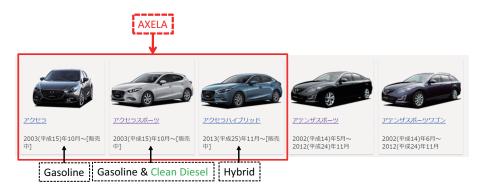
- In the case of Subaru IMPREZA,
 - the base variant (variant 1) is out of the policy target, though 8 variants are within the policy target, and
 - price differences among the variants are large: (max) 23,530 USD, (min) 15,540 USD
 - Cf. 1USD ≈ 100 JPY

Variant-level heterogeneity matters

S^{FS}_j = N^{FS}_j/N_j
 N_j: Number of variants of a model j
 N^{FS}_i: Number of variants meeting the 2015 FE stds within a model j



Variant-level heterogeneity matters: Case of Mazda AXELA



- Axela have three "models".
 - The variants of them include gasoline, hybrid and clean diesel cars.
 - Sales data by fuel types (engine types) are not available.
 - Variant-level analyses are necessary in order to assess the effects of the policies on the diffusion of the next generation cars.

Model: plan of presentation

• Simple case: common coefficients

- · Same levels of aggregation: standard logit
- Different levels of aggregation

⇒ Independence of Irrespective Alternatives (IIA)

- General case: random coefficients
 - Same levels of aggregation: Berry, Levinsohn and Pakes (1995, ECTA)
 - Different levels of aggregation

• Consumer *i*'s utility obtained from an alternative *j*:

- p_j : price of $j \in \mathcal{J}$ (incl. taxes and subsidy).
 - → *J* ∪ {0}: choice set,
 - $\{0\}$: outside option (not purchase). δ_0 is normalized to be zero.
- x_{jk} : attribute $k \in \mathcal{K}$ of j.
 - K: set of attributes
- ξ_j: unobserved attribute/demand shock specific to j
- ϵ_{ij} : idiosyncratic term (taste heterogeneity), i.i.d. type I extreme value
- (α, β) : parameters to be estimated.

Model: same levels of aggregation

• Choice probability of $j (\delta_0 = 0)$:

$$s_{ij} = s_j = \frac{e^{\delta_j}}{1 + \sum_{l \in \mathcal{J}} e^{\delta_l}}$$

• Demand function of *j*:

$$q_j = Ms_j$$

- *M*: market size (# of households).
- Estimation equation:

$$\ln s_j - \ln s_0 = \delta_j = \alpha p_j + \sum_{k \in \mathcal{K}} \beta_k x_{jk} + \xi_j$$

• Estimation can be done based on certain moment conditions on ξ_j .

Model: different levels of aggregation

• Consumer *i*'s utility obtained from a variant $n \in \mathcal{B}_i$:

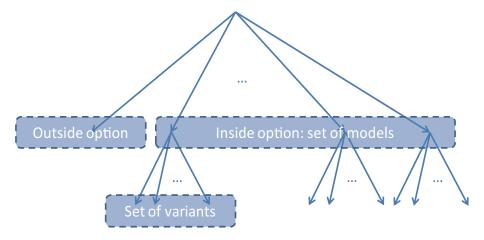
$$u_{ij_n} = \alpha p_{j_n} + \sum_{k \in \mathcal{K}_j} \beta_k x_{jk} + \sum_{k \in \mathcal{K} \setminus \mathcal{K}_j} \beta_k x_{j_n k} + \xi_j + \epsilon_{ij_n}$$
$$= \delta_j + \Delta \delta_{j_n} + \epsilon_{ij_n}$$
$$\delta_j = \sum_{k \in \mathcal{K}_j} \beta_k x_{jk} + \xi_j, \Delta \delta_{j_n} = \alpha p_{j_n} + \sum_{k \in \mathcal{K} \setminus \mathcal{K}_j} \beta_k x_{j_n k}$$

- \mathcal{B}_j : set of variants for the model j.
- $\mathcal{K}_j \subset \mathcal{K}$: set of attributes common to all variants

Key assumptions

- ξ_{j} , unobserved attribute/demand shock, is common to all variants within a model.
- ϵ_{ij_n} , an idiosyncratic term (taste heterogeneity), follows **GEV** that generates two stage nested logit structure.

Model: different levels of aggregation



Model: different levels of aggregation

- Choice probability for $n \in \mathcal{B}_j$: $s_{ij_n} = s_{j_n} = s_j \times s_{n|j}$
 - Probability of choosing model *j* (marginal choice probability):

$$s_j = \frac{e^{\delta_j + \lambda I_j}}{1 + \sum_{l \in \mathcal{J}} e^{\delta_l + \lambda I_l}}$$

Logit inclusive value: expected (mean) sub-utility obtained from j.

$$I_j = \ln\left[\sum_{n \in \mathcal{B}_j} e^{\Delta \delta_{j_n}/\lambda}\right]$$

 Probability of choosing variant *n* conditional on choosing model *j* (conditional choice probability):

$$s_{n|j} = \frac{e^{\Delta \delta_{j_n}/\lambda}}{\sum_{m \in \mathcal{B}_j} e^{\Delta \delta_{j_m}/\lambda}}$$

- *λ*: a parameter to be estimated, located between 0-1 interval if the model is consistent with utility maximization problem (McFadden, 1974).
 - $\lambda = 1$: the model reduces to a simple logit model.
 - λ → 0: the second stage of the model converges to the elimination-by-aspect model by Tversky.

• Demand functions for model j and variant $n \in \mathcal{B}_j$:

$$q_j = Ms_j$$
, and $q_{j_n} = Ms_{j_n}$

Estimation equation:

$$\ln(s_j) - \ln(s_0) = \delta_j + \lambda I_j = \sum_{k \in \mathcal{K}_j} \beta_k x_{jk} + \lambda \ln\left(\sum_{n \in \mathcal{B}_j} e^{(\alpha p_{jn} + \sum_{k \in \mathcal{K} \setminus \mathcal{K}_j} \beta_k x_{jnk})/\lambda}\right) + \xi_j$$
(1)

- Estimation can be done based on the moment condition on ξ_j as in the case of the same level of aggregation.
- Non-linear estimation.

Identification of λ

• λ is a key difference with the standard model.

- Can λ be identified from the data?
- Simple case: assume $\mathcal{K} = \mathcal{K}_j$ and $p_{j_n} = p_j$.
 - Eg. variant level heterogeneity due to color variation. (no color-specific dummy variables.)
- Eq. (1) can be rewritten as:

$$\ln(s_j) - \ln(s_0) = \alpha p_j + \sum_{k \in \mathcal{K}} \beta_k x_{jk} + \lambda \ln(N_j) + \xi_j$$

- $N_j = \# \mathcal{B}_j$, the number of variants
- λ is positive if there is taste heterogeneity in color variation.

• Unobserved attribute is uncorrelated with the observable attributes within a model.

$$E(\xi_j|X) = 0$$

Alternative assumption:

$$E(\xi_j|Std.Dev.(X)) = 0$$

• Unobserved attribute can be correlated with the average attributes.

Introducing random coefficients

• Problem of common coefficient model:

 Independence of Irrespective Alternatives (IIA): relative shares of two alternatives (say j and j') are independent of prices and attributes of the other alternatives (say j'').

$$\frac{s_j}{s_{j'}} = \frac{e^{\delta_j + \lambda I_j}}{e^{\delta_{j'} + \lambda I_{j'}}}$$

Independence of Irrespective Nests (IIN)

• Relaxing the IIA restriction: introducing random coefficients.

 Berry, Levinsohn and Pakes (1995, ECTA) for the estimation based on same level of aggregation

Utility function

$$u_{ij} = \alpha_i p_j + \sum_{k \in \mathcal{K}} \beta_{ik} x_{jk} + \xi_j + \epsilon_{ij}$$
$$= \delta_j + \mu_{ij} + \epsilon_{ij}$$

•
$$\alpha_i p_j = (-\alpha/y_i) p_j \approx \alpha \ln(y_i - p_j)$$

- y_i: consumer i's income.
- $\beta_{ik} = \beta_k + \sigma_k v_{ik}$: random coefficient
 - $v_{ik} \sim N(0, 1)$: taste heterogeneity for attribute k
- Common term: $\delta_j = \sum_{k \in \mathcal{K}} \beta_k x_{jk} + \xi_j$
- Heterogeneous term: $\mu_{ij} = \alpha_i p_j + \sum_{k \in \mathcal{K}} x_{jk} v_{ik} \sigma_k$

• Choice probability:

$$s_{ij} = \frac{e^{\delta_j + \mu_{ij}}}{1 + \sum_{l \in \mathcal{J}} e^{\delta_l + \mu_{il}}}$$

• Share function:

$$s_j = \int_y \int_\nu s_{ij} dF_\nu(\nu) dF_y(y)$$

Demand function:

$$q_j = Ms_j$$

- Estimation can be done by incorporating a contraction mapping procedure proposed by BLP.
 - Given the values of the parameters in μ_{ij} , namely $\theta_2 = (\alpha, \sigma)$, δ_j can be obtained by computing the following series:

$$\delta_j^h = \delta_j^{h+1} + (\ln S_j - \ln s_j)$$

- $S_j = Q_j/M$: observed share.
- s_j is computed by simulation:

$$s_j \approx \frac{1}{ns} \sum_{i=1}^{ns} \frac{e^{\delta_j + \alpha_i p_j + \sum_{k \in \mathcal{K}} x_{jk} v_{ik} \sigma_k}}{1 + \sum_{l \in \mathcal{J}} e^{\delta_l + \alpha_i p_l + \sum_{k \in \mathcal{K}} x_{lk} v_{ik} \sigma_k}}$$

• Note that δ_i is the function of θ_2 . Then, the following can be obtained:

$$\xi_j = \delta_j(\theta_2) - \sum_{k \in \mathcal{K}} \beta_k x_{jk}$$

 As in the case of logit, parameters in the model can be obtained based on certain moment conditions on ξ_j. • Utility function:

$$u_{ij_n} = \alpha_i p_{j_n} + \sum_{k \in \mathcal{K}_j} \beta_{ik} x_{jk} + \sum_{k \in \mathcal{K} \setminus \mathcal{K}_j} \beta_{ik} x_{j_n k} + \xi_j + \epsilon_{ij_n}$$
$$= (\delta_j + \Delta \delta_{j_n}) + (\mu_{ij} + \Delta \mu_{ij_n}) + \epsilon_{ij_n}$$

- Common term: $\delta_j = \sum_{k \in \mathcal{K}_j} \beta_k x_{jk} + \xi_j$, $\Delta \delta_{j_n} = \sum_{k \in \mathcal{K} \setminus \mathcal{K}_j} \beta_k x_{j_n k}$
- Heterogeneous term: $\mu_{ij} = \sum_{k \in \mathcal{K}_j} x_{jk} v_{ik} \sigma_{ik}$, $\Delta \mu_{ij_n} = \alpha_i p_{j_n} + \sum_{k \in \mathcal{K} \setminus \mathcal{K}_j} x_{jk} v_{ik} \sigma_{ik}$

- Choice probability of
 - model j:

$$s_{ij} = \frac{e^{\delta_j + \mu_{ij} + \lambda I_{ij}}}{1 + \sum_{l \in \mathcal{J}} e^{\delta_l + \mu_{il} + \lambda I_{il}}}$$

► logit inclusive value:
$$I_{ij} = \ln \left[\sum_{n \in \mathcal{B}_j} e^{(\Delta \delta_{j_n} + \Delta \mu_{ij_n})/\lambda} \right]$$

• variant $n \in \mathcal{B}_j$ conditional on choosing model *j*:

$$s_{in|j} = \frac{e^{(\Delta \delta_{j_n} + \Delta \mu_{ij_n})/\lambda}}{\sum_{m \in \mathcal{B}_j} e^{(\Delta \delta_{j_m} + \Delta \mu_{ij_m})/\lambda}}$$

Model-level share function:

$$s_j = \int_y \int_v \frac{e^{\delta_j + \mu_{ij} + \lambda I_{ij}}}{1 + \sum_{l \in \mathcal{J}} e^{\delta_l + \mu_{il} + \lambda I_{il}}} dF(v) dF(y)$$

Introducing random coefficients: different levels of aggregation

- Estimation can be done by adopting BLP's contraction mapping:
 - Define

$$\bar{\delta}_j \equiv \sum_{k \in \bar{\mathcal{K}}} x_{jk} \beta_k + \xi_j \text{ and } \bar{I}_{ij} \equiv \frac{\sum_{k \in \mathcal{K}_j \setminus \bar{\mathcal{K}}} x_{jk} \beta_k + \mu_{ij}}{\lambda} + I_{ij}$$

- The individual choice probability can be rewritten as:

$$s_{ij} = \frac{e^{\bar{\delta}_j + \lambda \bar{I}_{ij}}}{1 + \sum_{l \in \mathcal{J}} e^{\bar{\delta}_l + \lambda \bar{I}_{il}}}$$

δ̄ can be calculated by using BLP contraction mapping given the parameters in λI
_{ij}, namely θ₂ = ((β_k)_{k∈K\K̄}, (σ_k)_{k∈K}, λ).

	(i) NLS		(ii) G	MM
Variables	Coef.	S. E.	Coef.	S. E.
Price	-0.225	0.037	-0.763	0.203
Fuel Cost	-0.256	0.016	-0.280	0.019
Car Space	0.430	0.071	0.635	0.093
Car Size	-0.048	0.040	-0.183	0.062
Engine Displacement	0.141	0.109	1.044	0.345
Horse Power/Weight	7.641	2.049	7.177	2.117
Diameter	-0.035	0.162	-0.154	0.164
4WD	0.913	0.131	1.225	0.159
FR	0.375	0.117	0.725	0.192
Cruise Control	-0.308	0.113	-0.170	0.131
Power Seat	0.023	0.142	0.720	0.257
Stability Control System	0.644	0.074	0.623	0.088
Const	-8.600	0.666	-6.900	0.875
λ	0.554	0.128	0.308	0.187

• λ lies between 0 and 1.

Price coefficient get larger in absolute value after instrumenting.

Estimation results: random coefficients

	Mean(<i>β</i>)		Std. D	$ev.(\sigma)$
Variables	Coef.	S. E.	Coef.	S. E.
Price	-	-	-1.782	0.766
Fuel Cost	-0.249	0.030	-	-
Car Space	0.406	0.200	0.012	2.074
Car Size	-0.047	0.054	-	-
Engine Displacement	0.186	0.216	-	-
Horse Power/Weight	6.282	2.238	-	-
Diameter	0.078	0.196	-	-
4WD	0.853	0.105	-	-
FR	0.429	0.121	-	-
Cruise Control	-0.283	0.244	-	-
Power Seat	0.043	0.216	-	-
Stability Control System	0.658	0.115	-	-
Const	-8.757	2.912	0.066	6.364
λ	0.597	0.213	-	-

• λ lies between 0 and 1.

• The standard errors of σ estimates are huge.

(i) Green Cars (1000 unit)					
Year	With Policies	Without Policies	Difference	Rate of Change (%)	
2012	2415	2310	105	4.56	
2013	2646	2545	102	4.00	
(ii) Clean Diesel and Plug-in Hybrid (1000 unit)					
Year	With Policies	Without Policies	Difference	Rate of Change (%)	
2012	23	21	2	8.96	
2013	45	42	3	7.65	

• Policy evaluation:

- The sales of next generation cars (clean diesel and PHV) in the presence of the policies are simulated.
- Compute the sales under the prices in the absence of the subsidy and tax incentives.
- Results:
 - Green car sales were increased by about 4.3%.
 - Next generation car sales were increased by about 8%.

Summary and remaining tasks

- Summary
 - Variant-level heterogeneity in attributes matters.
 - Variant-level analyses are necessary when assessing the attribute-based policy interventions.
 - However, the sales data are available at the model-level, while the price and attributes data are available at the variant-level.
 - This paper develops the method to estimate the variant-level demand based on the data on the different levels of aggregation.

Key assumption: ξ is common to the variants of the model.

Based on the estimates of the model, this paper shows that the policies increased the sales of green cars by 4 – 4.56%, and that of the next generation cars by 7.65 – 8.96% from 2012 to 2013.

Remaining tasks

- More estimation.
- Comparison with the case of the other weighting assumptions:
 - polar weighting & equal weighting.
- Supply side

• Profit function of firm f producing the set of models \mathcal{J}_f :

$$\pi_f = \sum_{j \in \mathcal{J}_f} \sum_{n \in \mathcal{B}_j} \left(p_{j_n} - c_{j_n} \right) q_{j_n}$$

FOCs for the profit maximization problem under Bertrand competition:

$$s_{j_n} + \sum_{l \in \mathcal{J}_f} \sum_{m \in \mathcal{B}_l} (p_{l_m} - mc_{l_m}) \frac{\partial s_{l_m}}{\partial p_{j_n}} = 0, \forall j \in \mathcal{J}_f, \forall f$$

• By rearranging the FOCs, we have

$$\mathbf{mc} = \mathbf{p} - \Delta^{-1}\mathbf{s}$$

where,

•
$$\mathbf{s} = (\mathbf{s}_j)_{j \in \mathcal{J}}, \mathbf{p} = (\mathbf{p}_j)_{j \in \mathcal{J}}, \mathbf{mc} = (\mathbf{mc}_j)_{j \in \mathcal{J}},$$

- $\mathbf{s}_j = (s_{j_n})_{n \in \mathcal{B}_j}, \mathbf{p}_j = (p_{j_n})_{n \in \mathcal{B}_j}, \mathbf{mc}_j = (mc_{j_n})_{n \in \mathcal{B}_j}, \text{ and}$
- Δ is $\sum_{j \in \mathcal{J}} N_j \times \sum_{j \in \mathcal{J}} N_j$ matrix specified as

$$\Delta = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1J} \\ \Delta_{21} & \Delta_{22} & \cdots & \Delta_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{J1} & \Delta_{J2} & \cdots & \Delta_{JJ} \end{pmatrix}.$$

 $\Delta_{jr} = \Delta_{jr}^* \circ \mathbf{H}_{jr}$, where Δ_{jr}^* is $N_j \times N_r$ substitution matrix whose m - n element is $\frac{\partial s_{r_n}}{\partial p_{j_m}}$, and H_{jr} is all ones matrix if j and r are supplied by the same firm and all zeros matrix otherwise. (\circ indicates element wise multiplication operator, or Hadamard product.)

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